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V.I. Abrosimov and J. Randrup

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Further studies of the macroscopic nuclear surface response*

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Abstract:

The macroscopic response of a semi-infinite Fermi fluid is reconsidered. By retaining the first *two* terms in the expansion of the normal surface stress in powers of the external frequency, we obtain an improved dispersion relation which yields explicit expressions for the friction coefficient and the inertial mass for the damped surface motion. The resulting response function has a finite integral and yields a good absolute reproduction of the response observed via inelastic proton scattering.

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1 Introduction

Recently, the macroscopic response of the nuclear surface was studied within the framework of Landau's kinetic equation.[1] In that work, the idealized limit of vanishing quasiparticle interaction was considered and a surface mode with a purely imaginary frequency was found. In the present work, we incorporate the quasiparticle interaction and obtain a dispersion relation that admits a truly complex eigenfrequency. This mode can be described as a damped harmonic oscillator and the associated parameters are determined. In this connection, our previous attempt to do this for the purely damped surface mode [1] is discussed and corrected. Finally, it is demonstrated how the ensuing response function gives a good quantitative reproduction of the gross behavior of the experimental data on inelastic proton scattering.

2 Dispersion relation

We consider the free surface oscillations of a semi-infinite Fermi fluid described by Landau's semiclassical theory. The surface is located near the plane $z = 0$ and undergoes harmonic oscillations characterized by the wave vector $\mathbf{k}_\perp$. The expression for the normal stress at the surface can be written as

$$P_{zz}(z=0,\mathbf{k}_\perp) \;=\; \frac{3}{4}\rho_0 p_F U_{k_\perp}\mathcal{P} = \frac{3}{4}\rho_0 p_F U_{k_\perp}\sum_{n\geq 0}\mathcal{P}_n c^n \;. \tag{1}$$

Here ρ_0 is the standard density of nuclear matter, $p_F = mv_F$ is the corresponding Fermi momentum, and $U_{k_\perp} = -i\omega Z_{k_\perp}$ is the (maximum) surface velocity. Furthermore, the quantity $\mathcal{P}$ has been expanded in powers of the (supposedly small) dimensionless frequency $c = \omega/v_F k_\perp$. In ref. [1] only the first (zeroth-order) term was considered, corresponding to the limit $F_0 \to 0$. In the present work, we admit a finite value of F_0 by incorporating the next (first-order) term in c. In the Appendix, the first three terms in the power expansion (1) are derived, with the following result,

$$\begin{aligned}
\mathcal{P}_0 &= -1\;, \\
\mathcal{P}_1 &= i\,\frac{3\pi^2}{32}\frac{F_0}{1+F_0}\;, \\
\mathcal{P}_2 &= \frac{8}{5}\frac{F_0}{1+F_0}\left[1+\frac{\pi^2}{12}\frac{F_0}{1+F_0}\right]\;.
\end{aligned} \tag{2}$$

Since the normal stress must be balanced by the surface tension, $P_{zz}(z=0,\mathbf{k}_\perp) = \sigma k_\perp^2 Z_k$, the following dispersion relation is obtained,

$$i\frac{3}{4}\rho_0 p_F\omega + \frac{1}{2}\left(\frac{3\pi}{8}\right)^2\frac{F_0}{1+F_0}\frac{m\rho_0}{k_\perp}\omega^2 \;=\; \sigma k_\perp^2\;, \tag{3}$$

when the first two orders in the frequency are retained. This result agrees with the more general relation derived by Ivanov.[2] Moreover, when $F_0 \to 0$ the second term on the left-hand side vanishes and the result reduces to the simpler relation obtained in ref. [1].

3 Equation of motion

The surface mode described by the above dispersion relation can be described by a standard damped harmonic oscillator,

$$B_{k_\perp}\ddot{Z}_{k_\perp}(t) \; + \; \gamma_{k_\perp}\dot{Z}_{k_\perp}(t) \; + \; C_{k_\perp}Z_{k_\perp}(t) \; = \; 0 \; . \tag{4}$$

Since the stiffness coefficient is given by $C_{k_\perp} = \sigma k_\perp^2$, it follows that the friction coefficient is

$$\gamma_{k_\perp} \; = \; \frac{3}{4}\rho_0 p_F = \; \gamma_{\rm wf} \; , \tag{5}$$

where $\gamma_{\rm wf}$ is the result of the standard wall formula [3], and the inertial-mass parameter is

$$B_{k_\perp} \; = \; \frac{1}{2}\,(\frac{3\pi}{8})^2\,\frac{F_0}{1+F_0}\,\frac{m\rho_0}{k_\perp} \; . \tag{6}$$

Although the same dispersion relation was already derived by Ivanov [2], he did not make the above inferences about the dynamical coefficients. While the result for the friction coefficient is expected on general grounds [3], the present treatment provides a different derivation. Moreover, the expression (6) for the inertia is novel. It shows that $B_{k_\perp}$ is proportional to the irrotational inertial-mass parameter, which is given by $B_{\rm irr} = m\rho_0/k_\perp$.[4] The factor of proportionality is $\approx 0.7F_0/(1+F_0)$, which vanishes for $F_0 = 0$ and approaches 0.7 for $F_0 \to \infty$.

It might cause some concern that various recent calculations yield a negative value for the parameter F_0.[5] However, our result (6) pertains to the simplified scenario when the momentum-dependent part of the quasi-particle interaction vanishes, $F_1 = 0$. When a finite value of F_1 is admitted [2], the expression for the inertia is modified to

$$B_{k_\perp} = \frac{1}{2}\left[(\frac{3\pi}{8})^2\frac{F_0}{1+F_0} - \frac{F_1}{1+F_1/3}\right]\frac{m\rho_0}{k_\perp} \; . \tag{7}$$

The inertia then remains positive when the typical values $F_0 \approx -0.4$ and $F_1 \approx -0.9$ are employed. (These values are representative of those calculated in various models and they also follow directly from the relations $m^\star/m = 1 + F_1/3$ and $K = 6\epsilon_F(1+F_0)m/m^\star$, when the values $m^\star \approx 0.7m$ and $K \approx 200$ MeV are used.[5])

In our previous treatment [1], we considered the limit $F_0 \to 0$, corresponding to a vanishing quasiparticle interaction. Accordingly, the dispersion relation was obtained as

$$\omega \; = \; -i\frac{\sigma k_\perp^2}{\frac{3}{4}\rho_0 p_F} \; . \tag{8}$$

The eigenfrequency is then purely imaginary and the surface mode can be described by a first-order equation of the form

$$\gamma_{k_\perp} \dot{Z}_{k_\perp}(t) \; + \; C_{k_\perp} Z_{k_\perp}(t) \; = \; 0 \; . \tag{9}$$

The dispersion relation (7) and the fact that $C_{k_\perp} = \sigma k_\perp^2$ then imply that $\gamma_{k_\perp} = \gamma_{\rm wf}$, as in the more general case described above.[1]

In general, the equation of motion (4) yields a complex eigenfrequency,

$$\begin{aligned} \omega &= \omega_1 + i\omega_2 \\ &= \frac{1}{2B}[-i\gamma \pm (-\gamma^2 + 4BC)^{1/2}] \end{aligned} \tag{10}$$

However, when the inertia B is sufficiently small the real part vanishes, $\omega_1 = 0$, and the surface exhibits a purely exponential relaxation, $Z_{k_\perp}(t) \sim \exp(-i\omega_2 t)$. An expansion of the above expression (9) through second order in B yields

$$\omega \approx -i\frac{C}{\gamma}(1 - \frac{BC}{2\gamma^2}) \; . \tag{11}$$

Here the first term, $\omega = i\omega_2 = -iC/\gamma$, is the frequency pertaining to the limiting case when the inertia vanishes, cf. eq. (7). When the inertia is finite (but small), the frequency is decreased by the relative amount $BC/2\gamma^2$.

In a recent RPA study of the surface response in Fermi liquids [6], Bertsch and Esbensen calculated the values of the friction and inertial-mass coefficients for the vibrating nuclear surface. For the friction coefficient γ they obtain a numerical result which is very close to the standard wall-formula value, $\gamma_{\rm wf} = m\rho\bar{v} \approx 1.05$ MeV $\cdot$ 10^{-22}s/fm^4. Moreover, for small values of the wave number $k_\perp$, they find that the inertia approaches a small but finite value given by $B(k_\perp \to 0) = 0.26$ MeV $\cdot$ $(10^{-22}\text{s})^2/\text{fm}^4$. The smallness of the inertial mass supports the description of nuclear shape distortions in terms of a first-order equation of motion, as was first advocated in ref. [3]. Indeed, using these values, we find that the above finite-mass correction to the frequency is 0.06 fm$^2 k_\perp^2$, which amounts to $\approx$ 10% for $k_\perp = k_F$. This estimate indicates that that the overdamped equation (8), with $\gamma_{k_\perp} = \gamma_{\rm wf}$, provides an excellent dynamical description of the nuclear surface motion.

[1] In ref. [1] the ansatz for the equation of motion was of the general second-order form (4), rather that the first-order form (8) pertaining to purely damped motion. Unfortunately, in connection with that analysis, it was erroneously assumed that the stiffness coefficient was given by *one half* of $\sigma k_\perp^2$. This would indeed have been the correct value if a surface wave of trigonometric form had been considered, $Z \sim \cos(\mathbf{k}_\perp \cdot \mathbf{r}_\perp)$, but in that case the friction coefficient would also be given by one half of the wall-formula value, since the average of $\cos^2$ is one half. However, when considering a surface wave of exponential form, $Z \sim \exp(i\mathbf{k}_\perp \cdot \mathbf{r}_\perp)$, as we do throughout for convenience, the appropriate coefficients are twice as large, since the norm of the exponential is unity. As a consequence of that confusion, it was mistakingly concluded that the purely damped surface mode is *critically* damped. In fact, when the proper value of the stiffness coefficient is used in conjunction with the general equation of motion (4), it follows that the inertial-mass coefficient is zero and, consequently, the above first-order equation (8) holds, with $\gamma_{k_\perp} = \gamma_{\rm wf}$.

4 Response function

It was shown in ref. [1] that if the surface oscillator (6) is driven by a harmonic external force of the form $f_0\cos(\omega_{\rm ext}t)$, then the response function is given by

$$R(\omega_{\rm ext}, k_\perp) \;=\; \gamma\,\frac{f_0^2}{B^2}\,\frac{\omega_{\rm ext}}{(\omega_{\rm ext}^2-\omega_0^2)^2+4\lambda^2\omega_{\rm ext}^2}\,. \tag{12}$$

Therefore, in the limit of vanishing quasiparticle interaction, $F_0 \to 0$, where $B = 0$, the response function becomes

$$R(\omega_{\rm ext}, k_\perp) \;=\; \gamma_{\rm wf}\,f_0^2\,\frac{\omega_{\rm ext}}{\gamma_{\rm wf}^2\omega_{\rm ext}^2+C_{k\perp}^2}\,. \tag{13}$$

Consequently, since $R \sim 1/\omega_{\rm ext}$ for $\omega_{\rm ext} \to \infty$, the integrated response diverges, $\int d\omega_{\rm ext}R(\omega_{\rm ext}, k_\perp) \to \infty$. Contrary to this rather unsatisfactory result, the integrated response is finite when the quasiparticle interaction is taken into account, since the inertia is then finite, though small, so that $R \sim 1/\omega_{\rm ext}^3$ for $\omega_{\rm ext} \to \infty$.

For values of $\omega_{\rm ext}$ relevant for the inelastic proton scattering experiments [7], the above limiting response function (12) is still a quite accurate approximation, because of the smallnes of the inertia, which implies that $\lambda \gg \omega_0$. This is illustrated in fig. 1. For the values of $\omega_{\rm ext}$ substantially above the frequency corresponding to the maximum of the response function (11), we may employ the asymptotic approximation

$$R(\omega_{\rm ext}, k_\perp) \approx \frac{f_0^2}{\gamma\omega_{\rm ext}} \tag{14}$$

This approximation is included in fig. 1 and it is used for the calculation of differential cross sections. Furthermore, the smooth part of the observed differential cross section is rather well reproduced with this response function, without invoking any renormalization factor. The comparison between theory and experiment is shown in fig. 2.

5 Concluding Remarks

Because the effective quasiparticle interaction depends on the local nucleon density, the Landau parameters F_0 and F_1 vary strongly in the nuclear surface region. As a consequence, the present results, to the extent that they depend on these quantities, can not be expected to be quantitatively accurate. The result for the friction coefficient (that it equals identically that of the wall formula) does not depend on the quasiparticle interaction and is therefore expected to be retain its validity in a more refined treatment. This expectation is verified by the considerably more realistic RPA calculation by Bertsch and Esbensen.[6]

On the other hand, the expression derived for the inertial mass is directly proportional to F_0 and should not be regarded as reliable. One obvious problem is the fact that F_0 may conceivably be negative, in which case the interpretation would

be dubious. Another problem is the proportionality to the irrotational-flow inertia, which diverges as $k_\perp$ becomes small; the more realistic RPA calculations [6] yield a finite (and small) limiting value.

The dependence of the response function on the parameters characterizing the quasiparticle interaction is rather weak (the response function changes by only 5% when F_0 varies in the interval $0.5 - 3$). However, a non-zero value of F_0 is essential for obtaining a non-divergent result for the total response. The macroscopic surface response reproduces rather well the smooth part of the nuclear response function observed via inelastic proton scattering at small momentum transfer.

In summary then, it appears that the damping of the nuclear surface motion is well described by the wall-formula friction coefficient and the associated inertial mass is sufficiently small to be immaterial for the dynamics of the surface, although a reliable analytical derivation of this quantity has not yet been made.

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Appendix

In this appendix we show the coefficient in eq. (2) are derived. The staring point is the Fourier representation of the normal stress at the surface (see eqs. (2.15, 2.17, 2.18) of ref. [1]).[2] The normal stress at the surface can be written

$$\begin{aligned} P_{zz}(z=0,\mathbf{k}_\perp) &= \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} P_{zz}(\mathbf{k}) = \frac{3}{4}\rho_0 p_F U_{k_\perp} \mathcal{P} \ , \qquad (101) \\ \mathcal{P} &= -i\frac{8}{c}\int_{-\infty}^{\infty}\frac{dx}{2\pi}\left\{\frac{F_0}{4\Delta(s)}\left[\frac{s^2}{c^2}(w+1)+(2-3\frac{s^2}{c^2})s^2 w\right]^2\right. \\ &\quad +s^2(s^2 w-\frac{1}{3})+\frac{s^4}{c^2}\left[3w-5(s^2 w-\frac{1}{3})\right] \\ &\quad \left.+\frac{1}{8}\frac{s^4}{c^4}\left[3(w+1)-30s^2 w+35s^2(s^2 w-\frac{1}{3})\right]\right\} \ . \end{aligned}$$

Here $c = \omega/v_F k_\perp$ is the dimensionless frequency and $s = c/\sqrt{x^2+1}$ with $x = k_z/k_\perp$. Moreover,

$$w(s) = \frac{s}{2}\ln\left|\frac{s+1}{s-1}\right| - 1 - i\frac{\pi}{2}s\theta(|s|-1) \ , \qquad (102)$$

where $\Delta(s) = 1 - F_0 w(s)$ and $\theta(x)$ denotes the truncation function.

In the further derivation, it will be assumed that the dimensionless frequency is small, $|c| \ll 1$. By exploting this assumption we can expand the quantity (A-3) in

[2] Unfortunately, there are some misprints in (2.15) and (2.16) in ref. [1]: In (2.15), an overall factor of $-\frac{1}{k_\perp}$ is missing from the last expression; furthermore, the bracket following F_0 should be squared. In (2.16), the last term should have the opposite sign; also note that the θ-function denotes the *truncation* function, *i.e.* $\theta(x<0)=1$ and $\theta(x>0)=0$, rather than the step(-up) function.

powers of c,

$$
\begin{aligned}
w(s) &= -1 - i\frac{\pi}{2}s + s^2 + \frac{s^4}{3} + \dots \\
&= -1 - i\frac{\pi}{2}\frac{c}{\sqrt{x^2+1}} + \frac{c^2}{x^2+1} + \frac{c^4}{3(x^2+1)^2} + \dots .
\end{aligned}
\tag{103}
$$

By inserting this latter result into the expression (A-2) for $\mathcal{P}$, we find the quoted result (2) for the normal stress at the surface,

$$
\begin{aligned}
\mathcal{P} &= -i\frac{8}{c}\int_{-\infty}^{\infty}\frac{dx}{2\pi}\left\{\frac{F_0}{4}\left[1 - F_0(-1 - i\frac{\pi}{2}\frac{c}{\sqrt{x^2+1}} + \frac{c^2}{x^2+1} + \dots)\right]^{-1}\right. \\
&\times\left[\frac{1}{x^2+1}(-i\frac{\pi}{2}\frac{c}{\sqrt{x^2+1}} + \frac{c^2}{x^2+1} + \dots)\right. \\
&\left. +(2 - \frac{3}{x^2+1})\frac{c^2}{x^2+1}(-1 - i\frac{\pi}{2}\frac{c}{\sqrt{x^2+1}} + \frac{c^2}{x^2+1} + \dots)\right]^2 \\
&+\frac{c^2}{x^2+1}\left[\frac{c^2}{x^2+1}(-1 - i\frac{\pi}{2}\frac{c}{\sqrt{x^2+1}} + \frac{c^2}{x^2+1} + \dots) - \frac{1}{3}\right] \\
&+\frac{c^2}{(x^2+1)^2}\left[3(-1 - i\frac{\pi}{2}\frac{c}{\sqrt{x^2+1}} + \frac{c^2}{x^2+1} + \dots)\right. \\
&\left. -5[\frac{c^2}{x^2+1}(-1 - i\frac{\pi}{2}\frac{c}{\sqrt{x^2+1}} + \frac{c^2}{x^2+1} + \dots) - \frac{1}{3}]\right] \\
&+\frac{1}{8}\frac{1}{(x^2+1)^2}\left[3(-i\frac{\pi}{2}\frac{c}{\sqrt{x^2+1}} + \frac{c^2}{x^2+1} + \dots)\right. \\
&-30\frac{c^2}{x^2+1}(-1 - i\frac{\pi}{2}\frac{c}{\sqrt{x^2+1}} + \frac{c^2}{x^2+1} + \dots) \\
&\left.\left. + 35\frac{c^2}{x^2+1}[\frac{c^2}{x^2+1}(-1 - i\frac{\pi}{2}\frac{c}{\sqrt{x^2+1}} + \frac{c^2}{x^2+1} + \dots) - \frac{1}{3}]\right]\right\} \\
&= -\frac{3}{4}\int_{-\infty}^{\infty} dx(x^2+1)^{-\frac{5}{2}} \\
&+\left\{i\frac{4}{3\pi}\left[\int_{-\infty}^{\infty} dx(x^2+1)^{-1} + 4\int_{-\infty}^{\infty} dx(x^2+1)^{-2}\right.\right. \\
&\left.\left. -8\int_{-\infty}^{\infty} dx(x^2+1)^{-3}\right] + i\frac{\pi}{4}\frac{F_0}{1+F_0}\int_{-\infty}^{\infty} dx(x^2+1)^{-3}\right\} c \\
&+\left\{6\int_{-\infty}^{\infty} dx(x^2+1)^{-\frac{5}{2}} - \frac{15}{2}\int_{-\infty}^{\infty} dx(x^2+1)^{-\frac{7}{2}}\right. \\
&+2\frac{F_0}{1+F_0}\left[-\int_{-\infty}^{\infty} dx(x^2+1)^{-\frac{5}{2}} + 2\int_{-\infty}^{\infty} dx(x^2+1)^{-\frac{7}{2}}\right] \\
&\left.+\frac{\pi^2}{8}(\frac{F_0}{1+F_0})^2\int_{-\infty}^{\infty} dx(x^2+1)^{-\frac{7}{2}}\right\} c^2 + \dots
\end{aligned}
\tag{104}
$$

$$= \quad -1 + i\frac{3\pi^2}{32}\frac{F_0}{1+F_0}\,c + \frac{8}{5}\frac{F_0}{1+F_0}\left[1 + \frac{\pi^2}{12}\frac{F_0}{1+F_0}\right]c^2 + \dots\,.$$

In deriving the above result, we assumed that $|c| \ll 1$. After obtaining the dispersion relation (3), we can verify that this assumption is justified. Indeed, by using (10) we find

$$|c| = \frac{|\omega|}{v_F k_F} \approx \left[\frac{C}{\gamma}(1 - \frac{BC}{\gamma^2})\right]\frac{1}{v_F k_F} < \frac{C}{\gamma}\frac{1}{v_F k_F} = \frac{\sigma k_\perp}{v_F \gamma} \approx 0.1 k_\perp \ll 1\,. \qquad (105)$$

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Figure captions

Figure 1: Response function.
Surface response function (11) for two different values of the momentum transfer $\hbar k_\perp$, using a strength of $f_0 = 1$ MeV/fm^3. Also shown are the response function corresponding to the limit of vanishing quasiparticle interaction, $F_0 \to 0$, (dot-dashed curve) and the asymptotic approximation (13) (dashed curve).

Figure 2: Differential cross sections.
Observed [7] and calculated differential cross sections for inelastic scattering of 800 MeV protons off ^{116}Sn to an angle of 5°. Also shown is the differential cross section calculated with the RPA response function (dashed curve).[6]

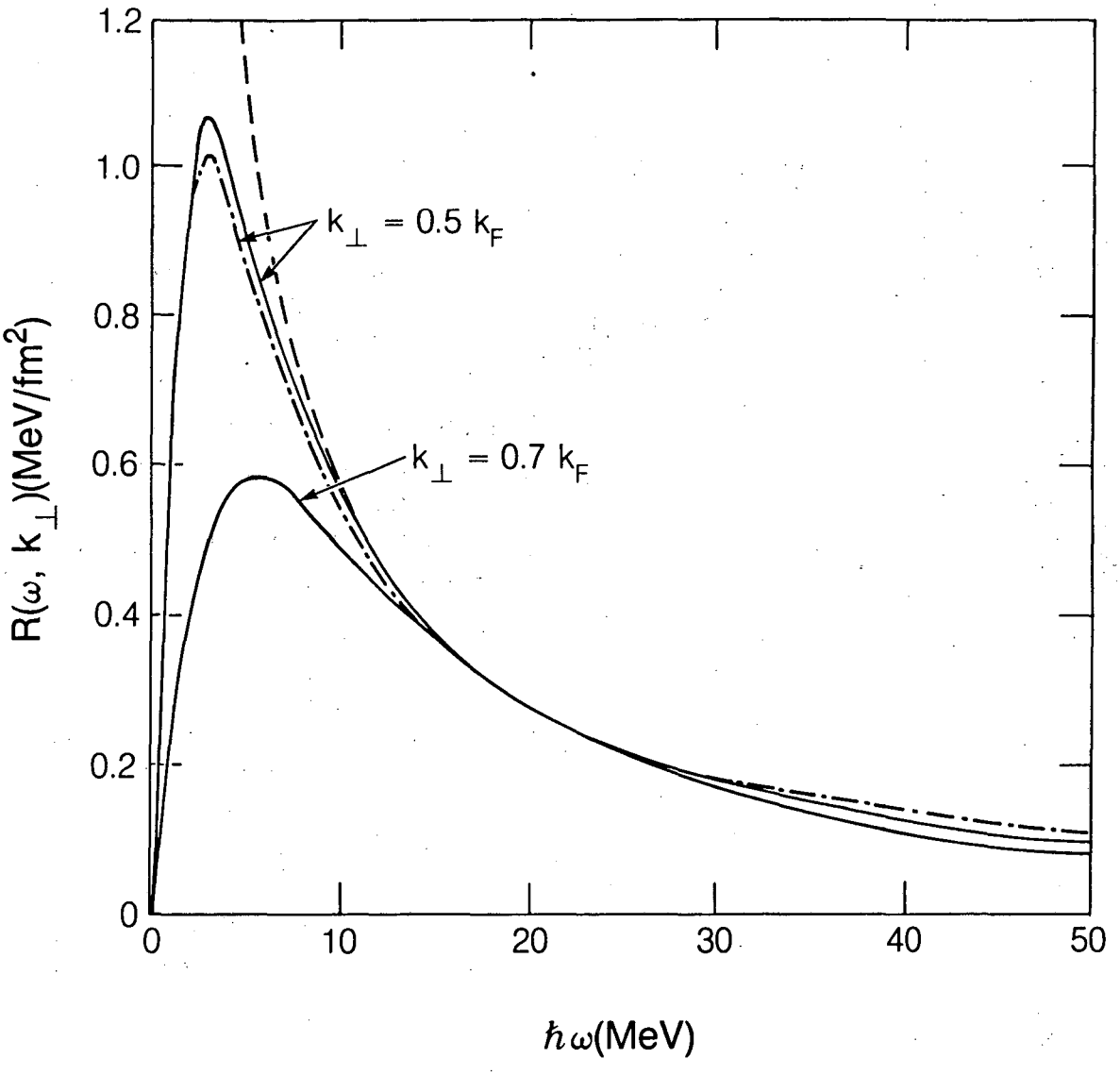

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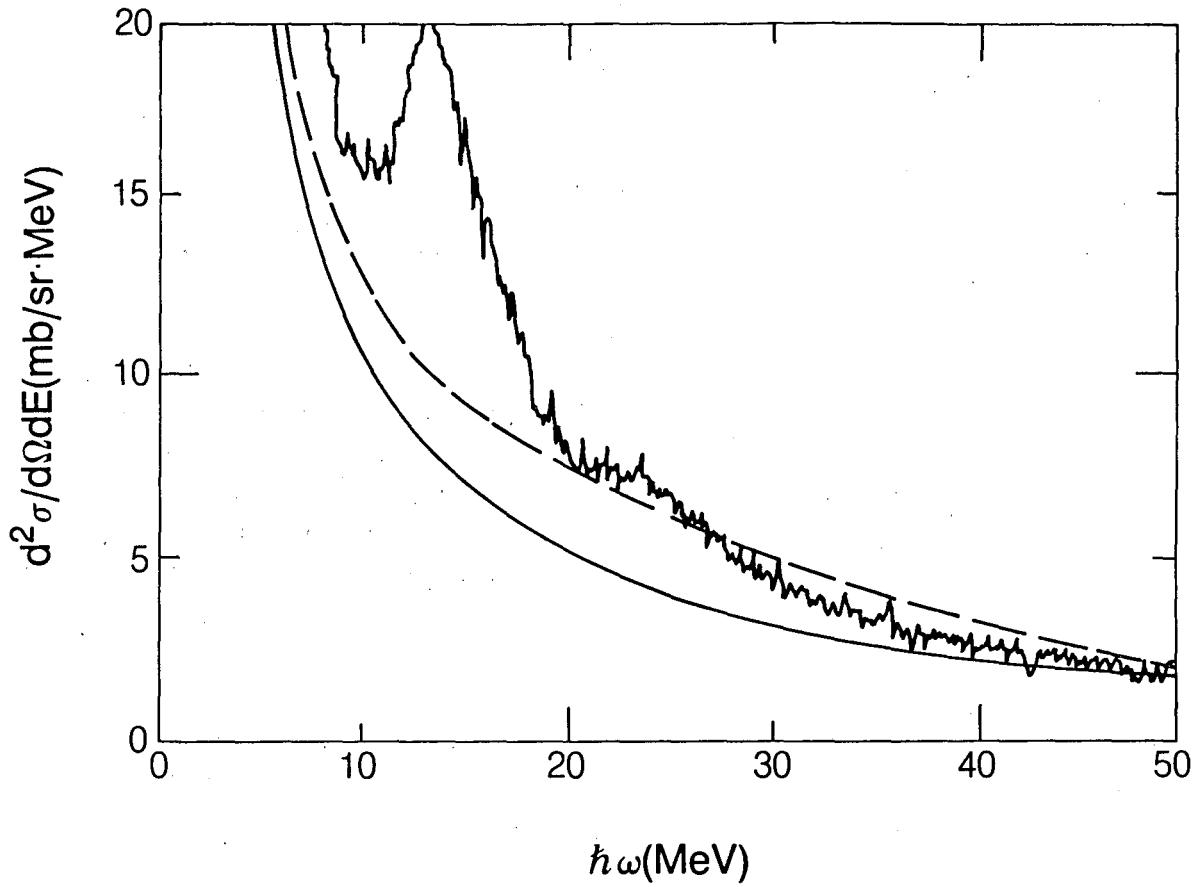
Figure 1: Response function.

Surface response function (11) for two different values of the momentum transfer $\hbar k_\perp$, using a strength of $f_0 = 1$ MeV/fm^3. Also shown are the response function corresponding to the limit of vanishing quasiparticle interaction, $F_0 \to 0$, (dot-dashed curve) and the asymptotic approximation (13) (dashed curve).



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Observed [7] and calculated differential cross sections for inelastic scattering of 800 MeV protons off ^{116}Sn to an angle of 5°. Also shown is the differential cross section calculated with the RPA response function (dashed curve).[6]

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