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COMPARISON OF NEUTRON RESONANCE SPACINGS WITH MICROSCOPIC THEORY FOR NUCLEI WITH STATIC DEFORMATION*

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Abstract

A level density formula including low-energy rotational levels for nuclei with axially symmetric deformation is tested with neutron resonance data for lanthanide and actinide nuclides. The calculations with the microscopic theory including nuclear pairing utilize deformed single particle levels of Nilsson et al. The experimental data for the actinide nuclei and part of the lanthanide nuclei are consistent with a theory which includes collective rotational levels. The derivation and applicability of a level density formula which includes collective rotational levels are discussed.

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1. Introduction

The experimental information on the nuclear level densities of statically deformed nuclei comes principally from neutron capture resonance measurements. The experimental nuclear level spacings determined from the resonance experiments have been compiled by several groups[1-3]). This information is summarized in Tables 1-4. A number of attempts have been made to analyze the neutron resonance data with a Bethe type formula containing phenomenological modifications to account for nuclear pairing and shells[4]). In addition, Ericson[5]) has analyzed several lanthanide and actinide level densities in terms of a unified model which included both particle excitations and collective excitations. However, in this analysis[5]) the particle excitations were calculated on the assumption that the single particle levels are equally spaced.

The purpose of the present paper is to make an up-to-date comparison of the experimental level spacings of statically deformed nuclei with the corresponding spacings calculated with a microscopic theory of interacting Fermions[6]) and realistic sets of single particle levels. The microscopic theory in the present calculations gives the number of particle or intrinsic states and each of these states serves as the parent of a rotational band as in the Ericson calculation.

2. Calculational Procedure

The theoretical calculations of the level density were performed for nuclei with axial symmetry which implies that the particle states can be characterized by the quantum number Ω which represents the total projection of the particle angular momentum on the nuclear symmetry axis. The projection Ω for a given particle state results from combining the projections of all the unpaired single particle excitations. The density of particle states of a particular Ω is approximated by a normal distribution,

$$\omega_{intr}(E,\Omega) = [\omega_{intr}(E)/(2\pi\sigma_{\parallel}^{2}(E))^{1/2}] \exp[-\Omega^2/2\sigma_{\parallel}^{2}(E)] \qquad (1)$$

where $\omega_{intr}(E)$ is the total density of particle or intrinsic states of both parities. The state density $\omega_{intr}(E)$ and spin cutoff factor $\sigma_{\parallel}^{2}(E)$ are calculated with a microscopic theory including nuclear pairing by a method outlined previously[4,7]. In the present context, $\sigma_{\parallel}^{2}(E)$ is defined by,

$$\sigma_{\parallel}^{2}(E) = \mathcal{J}_{\parallel}T/\hbar^2 = \frac{1}{2}\left\{\sum_i \Omega_{pi}^{2}\,\mathrm{sech}^2\left(\tfrac{1}{2}\beta E_{pi}\right) + \sum_i \Omega_{ni}^{2}\,\mathrm{sech}^2\left(\tfrac{1}{2}\beta E_{ni}\right)\right\} \qquad (2)$$

where β, E_{pi} and E_{ni} are the inverse of the temperature T, the proton quasiparticle energy and the neutron quasiparticle energy, respectively. The quantity $\sigma_{\parallel}^{2}(E)$ is analogous to the spin cutoff factor $\sigma^2(E)$ for a spherical nucleus, except for an axially deformed nucleus, $\sigma_{\parallel}^{2}(E)$ is related to the total projection of the particle angular momentum on the nuclear symmetry axis. The quantity $\mathcal{J}_{\parallel}$ is the moment-of-inertia about an axis parallel to the symmetry axis.

In the present calculation we assume that each particle state characterized by $|\Omega|$ has a rotational band built upon it. For this assumption to be valid the particle degrees of freedom must be completely decoupled from the collective

rotational degrees of freedom. If the contribution of states with $K \neq \Omega$ is small at the neutron binding energy, then one may assume $K = \Omega$, where K is the projection of the total angular momentum I on the nuclear symmetry axis. This approximation neglects all the vibrational excitations. The level density for a specific I is then obtained[5,8]) by summing over all particle states with $|K| \leqslant I$,

$$\rho(E,I) = \sum_{K \leqslant I} \omega_{intr} [E - E_{rot}(K,I), K] \tag{3}$$

where $E_{rot}(K,I)$ is the rotational energy expended for a rotational level I built on a particular K value. A level is characterized by a positive value of Ω (or K) and only values equal to or greater than zero are included in the sum. The fact that the particle states are degenerate with respect to the sign of Ω (or K) is a result of rotational invariance for axially symmetric nuclei.

If one assumes that the rotational energy $E_{rot}(K,I)$ is small compared to the total excitation energy, then eq. (3) may be rewritten,

$$\rho(E,I) = \sum_{K \leqslant I} \omega_{intr}(E,K) \exp\left[-\frac{I(I+1) - K^2}{2\sigma_\perp^2(E)}\right] \tag{4}$$

where $\sigma_\perp^2(E) = \mathcal{J}_\perp T/\hbar^2$ and $\mathcal{J}_\perp$ is the moment-of-inertia about an axis perpendicular to the nuclear symmetry axis. With the assumption that $K = \Omega$, the quantity $\omega_{intr}(E,\Omega)$ given by eq. (1) is substituted into eq. (4) to give,

$$\rho(E,I) = [1/(8\pi)^{\frac{1}{2}}\sigma_\parallel(E)]\, \omega_{intr}(E) \sum_{K=-I}^{K=+I} \exp\left\{-\frac{K^2}{2\sigma_\parallel^2(E)} - \frac{I(I+1)-K^2}{2\sigma_\perp^2(E)}\right\} \tag{5}$$

where the sum extends from -I to +I and an additional factor of $\frac{1}{2}$ is introduced. This gives the proper weight to the K = 0 bands, since for a given I, only one-half of the K = 0 bands contribute[8]. For small values of I where the exponential term in eq. (5) is near unity, the sum in eq. (5) is approximately equal to 2I+1 and eq. (5) reduces to,

$$\rho(E,I) \approx [(2I + 1)/(8\pi)^{1/2} \sigma_{\parallel}(E)] \omega_{intr}(E) \quad . \tag{6}$$

This equation is to be compared to eq. (3) of the previous paper[4]) where in the same approximation the spin dependent level density for a spherical nucleus is given by,

$$\rho(E,I) \approx [(2I + 1)/(8\pi)^{1/2} \sigma^3(E)] \omega(E) \quad . \tag{7}$$

The latter equation for spherical nuclei contains an extra factor of $\sigma^2(E)$ in the denominator. This factor is discussed in the following paragraphs.

Equation (7) is derived for spherical nuclei on the basis that no rotational states contribute to the state density and, hence, $\omega(E)$ in eq. (7) is the total state density. In contrast to eq. (7), eq. (6) is derived on the basis that particle and rotational states contribute to the state density. The intrinsic state density $\omega_{intr}(E)$ is only that part of the total state density coming from the particle states. When rotational states are included for deformed nuclei, the total state density is given approximately by,

$$\omega(E) \approx \omega_{intr}(E) \sigma_{\perp}^{2}(E) \quad , \tag{8}$$

where $\sigma_{\perp}^{2}(E)$ ranges from 35 to 65 for nuclei with A values ranging from 150 to 250, rigid-body moments of inertia and excitation energies equal to the neutron binding energy.

Equation (8) is derived from the rotational partition function in the approximation that the particle and rotational energies are independent. The total state density, including particle and rotational states, for a deformed nucleus with axially symmetry is to good approximation given by,

$$\omega(E) = \frac{1}{2} \sum_{I=0}^{\infty} \sum_{X=1}^{2I+1} \omega_{intr} [E - E_{rot}(X,I)] \quad . \tag{9}$$

The factor of $\frac{1}{2}$ is a symmetry number for axially symmetric nuclei due to invariance with respect to rotation of 180° about an axis perpendicular to the symmetry axis. Due to this property, only one-half of the particle states serve as parent levels for the bands of rotational levels. The individual levels in the rotational band built on each particle level are labelled by X. The sum in eq. (9) runs over all angular momentum values I=0 to I=∞. In addition, for each level X with angular momentum I, there are 2I+1 eigenstates corresponding to the same magnitude of the rotational energy, i.e., each rotational level has a degeneracy of 2I+1. If the rotational energy $E_{rot}(I)$ given by

$$E_{rot}(I) = \frac{\hbar^2}{2\mathcal{J}_\perp} I(I+1) \tag{10}$$

is small compared to the total energy E, then eq. (9) may be rewritten as

$$\omega(E) \approx \frac{1}{2} \omega_{intr}(E) \sum_{I=0}^{\infty} \sum_{X=1}^{2I+1} \exp\left[-\frac{E_{rot}(X,I)}{T}\right] \quad . \tag{11}$$

With the (2I+1) degeneracy factor and the rotational energy of eq. (10), one may rewrite eq. (11) as follows

$$\omega(E) \approx \frac{1}{2} \omega_{intr}(E) \sum_{I=0}^{\infty} (2I+1) \exp\left[-\frac{\hbar^2 I(I+1)}{2\mathcal{J}_\perp T}\right] \quad . \tag{12}$$

If $\hbar^2/2\mathcal{J}_\perp T$ is small compared to unity, the sum in eq. (12) can be replaced by an integral. On integration of this expression, one obtains eq. (8). Equation (5) may now be rewritten in terms of the total state density,

$$\rho(E,I) = [1/(8\pi)^{\frac{1}{2}}\, \sigma_\parallel(E)\, \sigma_\perp^{\,2}(E)]\, \omega(E) \sum_{K=-I}^{K=+I} \exp\left\{-\frac{K^2}{2\sigma_\parallel^{\,2}(E)} - \frac{I(I+1)-K^2}{2\sigma_\perp^{\,2}(E)}\right\} \quad . \tag{13}$$

This equation is analogous to eq. (3) of the previous paper and in the limit of small I gives,

$$\rho(E,I) = [(2I+1)/(8\pi)^{\frac{1}{2}}\, \sigma_\parallel(E)\, \sigma_\perp^{\,2}(E)]\, \omega(E) \quad . \tag{14}$$

Equation (14) is analogous to the spherical nucleus result given by eq. (7). Since eq. (13) contains the total state density, it satisfies the condition that $\int_0^\infty (2I+1)\, \rho(E,I) = \omega(E)$, just as does eq. (3) of the previous paper[4]. However, $\omega(E)$ in eq. (13) is equal to $\sigma_\perp^{\,2}\, \omega_{intr}(E)$.

In performing calculations of the level density for a deformed nucleus of particular angular momentum I and both parities, we use eq. (5). The intrinsic or particle state density $\omega_{intr}(E)$ is calculated with the microscopic theory with values of the nuclear pairing energies given in the previous paper[4]. The microscopic theory is used also to compute $\sigma_\parallel^{\,2}(E)$ by way of eq. (2). Values of $\sigma_\perp^{\,2}(E)$ are calculated with the rigid-body moment-of-inertia. Although this assumption leads to an upper limit for $\sigma_\perp^{\,2}(E)$, values of $\rho(E,I)$ are not very sensitive to $\sigma_\perp(E)$ for small I. The energies of the single particle levels were calculated for quadrupole and hexadecapole deformations with a program and parameters of Nilsson et al.[9]. In addition, some calculations were performed

with the single particle levels of Nix *et al.*[10]) and the single particle levels calculated with the revised parameters of Tsang *et al.*[11]). For the odd particle system, the statistical functions were calculated for the adjacent even-even nucleus and then the energy scale was shifted by an energy equivalent to that required to produce one quasiparticle.

3. Results and Discussion

The results of the theoretical calculations are summarized in Tables 1-4. In each of these tables both the experimental data and the results of the theoretical calculations are presented in terms of level spacings. The level spacing D in eV for a single parity is defined in relation to eq. (5) by,

$$D(eV) = 2\times10^6/[\rho(E, I_o - \tfrac{1}{2}) + \rho(E, I_o + \tfrac{1}{2})] \qquad (15)$$

where each level density is defined in terms of the number of levels per MeV of a particular angular momentum and both parities and I_o refers to the target spin. The factor of 2 in eq. (15) is a result of the assumption that the number of levels of each parity are equal[4]. All of the values of $D_{theo.}$ for the deformed nuclei are calculated with eqs. (5) and (15), whereas for the spherical nuclei the formalism in the previous paper is used[4].

The ratios of $D_{theo.}/D_{exp.}$ for deformed nuclei in the mass regions $150 < A < 185$ and $A > 228$ are plotted as solid symbols in fig. 1. The theoretical spacings for these nuclei as well as for ^{188}Os and ^{196}Pt were calculated with single particle levels generated with the deformations and parameters of Nilsson et al.[9]. The ratios of $D_{theo.}/D_{exp.}$ for spherical nuclei in the mass region $186 \leqslant A \leqslant 208$ are plotted as open symbols in fig. 1. Again the theoretical spacings for the spherical nuclei were calculated with single particle levels generated with the parameters of Nilsson et al.[9], however, no enhancement due to the collective rotations are included in $D_{theo.}$.

The question of the sensitivity of $D_{theo.}$ to a particular set of single particle levels was investigated by performing calculations with single particle levels calculated by Nix et al.[10]) and with single particle levels calculated with the deformations and parameters of Tsang et al.[11]). The results are compared in Table 5 and are quite similar.

In the present formulation, any enhancement in the level density due to vibrations has been neglected. For the case of an axially symmetric nucleus, the enhancement in the level density due to vibrations is calculable with the same type of formalism as that discussed for rotations and is given approximately by[8]) $[1 - \exp(-\hbar\omega/T)]^{-g}$. The exponential factor is for a vibrational mode with g-fold degeneracy. The predicted enhancement in the level density depends on the ratio of $\hbar\omega/T$ and g. For heavy nuclei, $\hbar\omega/T$ is of the order of 1 to 2. Hence, the enhancements in the level density for heavy nuclei due to vibrational excitations is expected to be an order of magnitude less than that due to rotations.

In adding the collective rotational levels to the intrinsic or particle level density, one is concerned about the redundance in the two types of degrees of freedom. One knows that at low excitation energy, each intrinsic level has built upon it a rotational band. At these low energies, the nucleus has a well-defined deformation and one can clearly separate the particle and collective motion. However, as the temperature and excitation energy increase, it is probably no longer a good approximation to assume that the particle and collective degrees of freedom are independent of each other. In the limit where the temperature is very large and the particle and collective degrees of freedom are thoroughly mixed, then $\omega_{intr}(E)$ contains all of the available states and there is no enhancement in the level density due to collective rotations.

The question remains as to whether the level density will be enhanced for deformed lanthanide and actinide nuclei at excitation energies corresponding to the neutron binding energy where the temperature is about 0.5 MeV. It has been estimated[8]) that at a nuclear temperature of the order of $40A^{-1/3}\ \delta$ MeV (where δ is the deformation) the fluctuations in the nuclear orientation becomes so large that it is not meaningful to speak of a separation of the rotational and intrinsic motion. For the actinide nuclei this estimate of 1.6 MeV for this nuclear temperature is well in excess of the actual temperature of 0.5 MeV at the neutron binding energy. The good agreement between the experimental spacings and the theoretical spacings (eqs. (5) and (15)) for the actinide nuclei supports the general validity of the formalism described in this paper where collective rotations are added to the level density for axially deformed nuclei. In the limit where no enhancement in the level density due to collective rotations is included, [this calculation assumes $\omega(E) = \omega_{intr}(E)$ and makes use of eq. (13)], the experimental spacings of the actinide nuclei are not even reproduced when the pairing gap parameters are reduced to zero.

The results for the nuclei in the mass region $150 < A < 185$ are somewhat ambiguous. Although the lighter lanthanide nuclei appear to require the enhancement in the level density associated with the rotational levels, the heavier lanthanides require only a partial enhancement in the level density. One puzzling feature is the decrease in the ratio $D_{theo.}/D_{exp.}$ with A in the A region, $150 < A < 185$. Although the magnitude of this ratio can be shifted up or down by varying the pairing gap parameters, the downward trend remains. As an example of the influence of pairing, an increase in the gap parameters

of 15% increases the ratios of $D_{theo.}/D_{exp.}$ by approximately a factor of 3. The trend in the ratio $D_{theo.}/D_{exp.}$ observed for the lanthanide nuclei may possibly reflect a deficiency in the various sets of theoretical single particle levels used in the calculation of the particle state density. Similiar calculations need to be done with other sets of theoretical single particle levels.

The transitional nuclei ^{184}W, ^{188}Os and ^{196}Pt represent interesting cases where the theoretical spacings are much too small when calculated on the basis of a deformed nucleus formalism (eqs. (5) and (15)). However, the agreement with experiment is good for ^{188}Os and ^{196}Pt if these nuclei are assumed to be spherical. As can be seen in fig. 1, the ratios of $D_{theo.}/D_{exp.}$ are near unity for these nuclei when spherical single particle levels are used. This result may be due to the fact that these nuclei are very soft to changes in shape. By the time the neutron and proton numbers are increased sufficiently to reach ^{196}Pt, the nuclear potential energy surface has become very shallow with slight minima at oblate and prolate deformations[9,11]). However, the depths of these minima are only about 1/4 MeV. The criterion for the temperature at which the enhancement in the level density due to rotations vanishes for these soft nuclei must be substantially different from the criterion discussed earlier for rigidly deformed nuclei where the minimum in the potential energy is of the order of the neutron binding energy. For these soft transitional nuclei, an excitation energy corresponding to the neutron binding energy may give a nuclear temperature sufficiently high, such that there is not a well defined deformation and, hence, no observable enhancement in the level density due to collective rotations.

In summary, the neutron resonance spacings give information on the A dependence of the nuclear level density at an almost fixed excitation energy. Although the rather low excitation energy is advantageous for testing theoretical models for the effects of nuclear shells and structure, it is disadvantageous in that a number of factors can substantially alter the theoretical level density. We have presented evidence from comparisons of experimental and theoretical neutron resonance spacings in support of a level density formula which includes low-energy rotational levels. The evidence is somewhat convincing for the actinide nuclei where the microscopic state density calculations were performed with three different sets of single particle levels. The evidence for the rotational enhancement for the lanthanide nuclei is less definitive where calculations have been performed only for Nilsson single particle levels. Theoretical calculations with single particle levels from other potentials are needed.

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Footnotes and References

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Table 1. Experimental and theoretical level spacings of even Z-even N deformed nuclei.

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV	$\sigma_{\parallel}$
^{151}Sm	$\frac{5}{2}-$	^{152}Sm	2-,3-	8.27	1.3	1.3	1.3	1.1	6.2
^{155}Gd	$\frac{3}{2}-$	^{156}Gd	1-,2-	8.53	2.1	2.0	1.9	1.1	6.5
^{157}Gd	$\frac{3}{2}-$	^{158}Gd	1-,2-	7.93	12	6.1	6.0	2.1	6.3
^{161}Dy	$\frac{5}{2}+$	^{162}Dy	2+,3+	8.20	2.1	2.6	2.9	0.7	6.7
^{163}Dy	$\frac{5}{2}-$	^{164}Dy	2-,3-	7.66	11	9.6	9.6	1.2	6.6
^{167}Er	$\frac{7}{2}+$	^{168}Er	3+,4+	7.77	3	4	4.1	0.4	6.7
^{171}Yb	$\frac{1}{2}-$	^{172}Yb	0-,1-	8.02		7.2	6.5	0.7	6.9
^{173}Yb	$\frac{5}{2}-$	^{174}Yb	2-,3-	7.47		7.8	8.4	0.8	6.8
^{177}Hf	$\frac{7}{2}-$	^{178}Hf	3-,4-	7.63	2.9	3.2	2.4	0.4	7.0
^{179}Hf	$\frac{9}{2}+$	^{180}Hf	4+,5+	7.39	4.0	5.8	4.4	0.7	
^{183}W	$\frac{1}{2}-$	^{184}W	0-,1-	7.41	15	16	12	0.7	8.0
^{187}Os	$\frac{1}{2}-$	^{188}Os	0-,1-	7.99		14	9.1	0.09	9.3
^{195}Pt	$\frac{1}{2}-$	^{196}Pt	0-,1-	7.92	18	19	12	0.1	8.5
^{229}Th	$\frac{5}{2}+$	^{230}Th	2+,3+	6.79		0.58	0.60	0.56	5.6

(continued)

Table 1. (continued)

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]	$\sigma_{\parallel}$
^{233}U	$\frac{5}{2}+$	^{234}U	2+,3+	6.84		0.99	0.61	0.34	6.0
^{235}U	$\frac{7}{2}-$	^{236}U	3-,4-	6.55		0.67	0.53	0.43	6.2
^{239}Pu	$\frac{1}{2}+$	^{240}Pu	0+,1+	6.53		2.3	2.4	2.3	6.6
^{241}Pu	$\frac{5}{2}+$	^{242}Pu	2+,3+	6.30		1.2		1.7	6.7
^{245}Cm	$\frac{7}{2}+$	^{246}Cm	3+,4+	6.45			1.5	1.2	7.2

[a] Data compiled by Lynn[1]).

[b] Data compiled by Baba[2]).

[c] Data compiled by Vonach et al.[3]).

[d] Calculated with deformed single particle levels of Nilsson et al.[9]). The theoretical spacing is for two spin states.

Table 2. Experimental and theoretical level spacings of even Z-odd N deformed nuclei.

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]
^{150}Sm	0+	^{151}Sm	$\frac{1}{2}$ +	5.59		24	68	135
^{152}Sm	0+	^{153}Sm	$\frac{1}{2}$ +	5.87		60	52	75
^{154}Sm	0+	^{155}Sm	$\frac{1}{2}$ +	5.81			115	90
^{152}Gd	0+	^{153}Gd	$\frac{1}{2}$ +	6.49			11.5	25
^{154}Gd	0+	^{155}Gd	$\frac{1}{2}$ +	6.45			10.2	28
^{156}Gd	0+	^{157}Gd	$\frac{1}{2}$ +	6.37		75	47	30
^{158}Gd	0+	^{159}Gd	$\frac{1}{2}$ +	5.94			85	60
^{160}Gd	0+	^{161}Gd	$\frac{1}{2}$ +	5.63			170	100
^{156}Dy	0+	^{157}Dy	$\frac{1}{2}$ +	6.98			3.4	6
^{158}Dy	0+	^{159}Dy	$\frac{1}{2}$ +	6.83			30	9
^{160}Dy	0+	^{161}Dy	$\frac{1}{2}$ +	6.45			11	13
^{162}Dy	0+	^{163}Dy	$\frac{1}{2}$ +	6.27	130	42	72	15
^{164}Dy	0+	^{165}Dy	$\frac{1}{2}$ +	5.71			200	50
^{162}Er	0+	^{163}Er	$\frac{1}{2}$ +	6.91		7.1	6.9	3

(continued)

Table 2. (continued)

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]
^{164}Er	0+	^{165}Er	$\frac{1}{2}$ +	6.66		17	22	5
^{166}Er	0+	^{167}Er	$\frac{1}{2}$ +	6.44		47	38	7
^{168}Er	0+	^{169}Er	$\frac{1}{2}$ +	6.00		100	95	15
^{170}Er	0+	^{171}Er	$\frac{1}{2}$ +	5.68			155	27
^{168}Yb	0+	^{169}Yb	$\frac{1}{2}$ +	6.87		20		3
^{170}Yb	0+	^{171}Yb	$\frac{1}{2}$ +	6.62		22	37	5
^{172}Yb	0+	^{173}Yb	$\frac{1}{2}$ +	6.37		75	63	9
^{174}Yb	0+	^{175}Yb	$\frac{1}{2}$ +	5.82		250	164	25
^{176}Yb	0+	^{177}Yb	$\frac{1}{2}$ +	5.56		250	185	40
^{174}Hf	0+	^{175}Hf	$\frac{1}{2}$ +	6.85			16	4
^{176}Hf	0+	^{177}Hf	$\frac{1}{2}$ +	6.38		41	32	8
^{178}Hf	0+	^{179}Hf	$\frac{1}{2}$ +	6.10		55	64	16
^{180}Hf	0+	^{181}Hf	$\frac{1}{2}$ +	6.70		140	125	30
^{230}Th	0+	^{231}Th	$\frac{1}{2}$ +	5.12		11	7.7	22

(continued)

Table 2. (continued)

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]
^{232}Th	0+	^{233}Th	$\frac{1}{2}$ +	4.79	17.5	12.4	16.7	40
^{232}U	0+	^{233}U	$\frac{1}{2}$ +	5.74	7.6	14.2	4.2	5
^{234}U	0+	^{235}U	$\frac{1}{2}$ +	5.31	13	18	12.3	12
^{236}U	0+	^{237}U	$\frac{1}{2}$ +	5.12	17	27	15.4	18
^{238}U	0+	^{239}U	$\frac{1}{2}$ +	4.80	17.7	18.1	20.8	30
^{238}Pu	0+	^{239}Pu	$\frac{1}{2}$ +	5.66	13	16	11.7	8
^{240}Pu	0+	^{241}Pu	$\frac{1}{2}$ +	5.24	13	14	12.5	17
^{242}Pu	0+	^{243}Pu	$\frac{1}{2}$ +	5.04	15			26
^{244}Cm	0+	^{245}Cm	$\frac{1}{2}$ +	5.52	13	20	14	18
^{246}Cm	0+	^{247}Cm	$\frac{1}{2}$ +	5.16			38	40
^{248}Cm	0+	^{249}Cm	$\frac{1}{2}$ +	4.71			35	130

[a]Data compiled by Lynn[1]). [b]Data compiled by Baba[2]). [c]Data compiled by Vonach *et al.*[3]).

[d]Calculated with deformed single particle levels of Nilsson *et al.*[9]). The theoretical spacing is for one spin state.

Table 3. Experimental and theoretical level spacings of odd Z-even N deformed nuclei.

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]
^{176}Lu	7-	^{177}Lu	$\frac{13}{2}-, \frac{15}{2}-$	7.07	2.3	2.4	2.3	0.14
^{242}Am	5-	^{243}Am	$\frac{9}{2}-, \frac{11}{2}-$	6.38			0.6	0.3

[a] Data compiled by Lynn[1]).

[b] Data compiled by Baba[2]).

[c] Data compiled by Vonach *et al.*[3]).

[d] Calculated with deformed single particle levels of Nilsson *et al.*[9]).

Table 4. Experimental and theoretical level spacings of odd Z-odd N deformed nuclei.

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]
^{151}Eu	$\frac{5}{2}+$	^{152}Eu	2+,3+	6.31	0.75	0.72	0.7	1.3
^{153}Eu	$\frac{5}{2}+$	^{154}Eu	2+,3+	6.44	1.4	1.3	1.1	1.1
^{159}Tb	$\frac{3}{2}+$	^{160}Tb	1+,2+	6.38	3.9	4.3	4.2	1.0
^{165}Ho	$\frac{7}{2}-$	^{166}Ho	3-,4-	6.24	6.1	5.7	5.5	0.3
^{169}Tm	$\frac{1}{2}+$	^{170}Tm	0+,1+	6.59	6.0	6.6	7.3	0.4
^{175}Lu	$\frac{7}{2}+$	^{176}Lu	3+,4+	6.29	3.7	3.6	3.0	0.3
^{181}Ta	$\frac{7}{2}+$	^{182}Ta	3+,4+	6.06	4.4	4.3	4.3	0.1
^{231}Pa	$\frac{3}{2}-$	^{232}Pa	1-,2-	5.56	1.0	0.44		0.5
^{233}Pa	$\frac{3}{2}-$	^{234}Pa	1-,2-	5.20	0.8	1.0		0.9
^{237}Np	$\frac{5}{2}+$	^{238}Np	2+,3+	5.48		0.72	0.67	0.5
^{241}Am	$\frac{5}{2}-$	^{242}Am	2-,3-	5.53	0.77	0.58		0.7
^{243}Am	$\frac{5}{2}-$	^{244}Am	2-,3-	5.36	1.4	1.5		1.0

[a]Data compiled by Lynn[1]. [b]Data compiled by Baba[2]. [c]Data compiled by Vonach *et al.*[3].

[d]Calculated with deformed single particle levels of Nilsson *et al.*[9].

Table 5. Comparison of experimental and theoretical level spacings for three sets of single particle levels.

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	D_{exp} in eV (b)	D_{Theo} in eV (c)	D_{Theo} in eV (d)	D_{Theo} in eV (e)
^{229}Th	$\frac{5}{2}+$	^{230}Th	2+,3+	6.79	0.58	0.60	0.56	0.55	
^{235}U	$\frac{7}{2}-$	^{236}U	3-,4-	6.55	0.67	0.53	0.43	1.0	
^{241}Pu	$\frac{5}{2}+$	^{242}Pu	2+,3+	6.30	1.2		1.7	1.8	2.4
^{245}Cm	$\frac{7}{2}+$	^{246}Cm	3+,4+	6.45		1.5	1.2	1.0	

[a] Data compiled by Baba[2]).

[b] Data compiled by Vonach et al.[3]).

[c] Calculated with deformed single particle levels of Nilsson et al.[9]).

[d] Calculated with deformed single particle levels of Tsang et al.[11]).

[e] Calculated with deformed single particle levels of Nix et al.[10]).

Figure Caption

Fig. 1. The ratios of the theoretical spacings to the experimental spacings are plotted as a function of mass number. The points represented by solid symbols are for calculations of $D_{theo.}$ based on single particle levels calculated for deformed nuclei with the deformations and parameters of Nilsson et al.[9]). The actual calculations of $D_{theo.}$ are performed with eqs. (5) and (15). The open symbols are the same except for spherical nuclei where $D_{theo.}$ is calculated with the equations for spherical nuclei in the previous paper[4]).

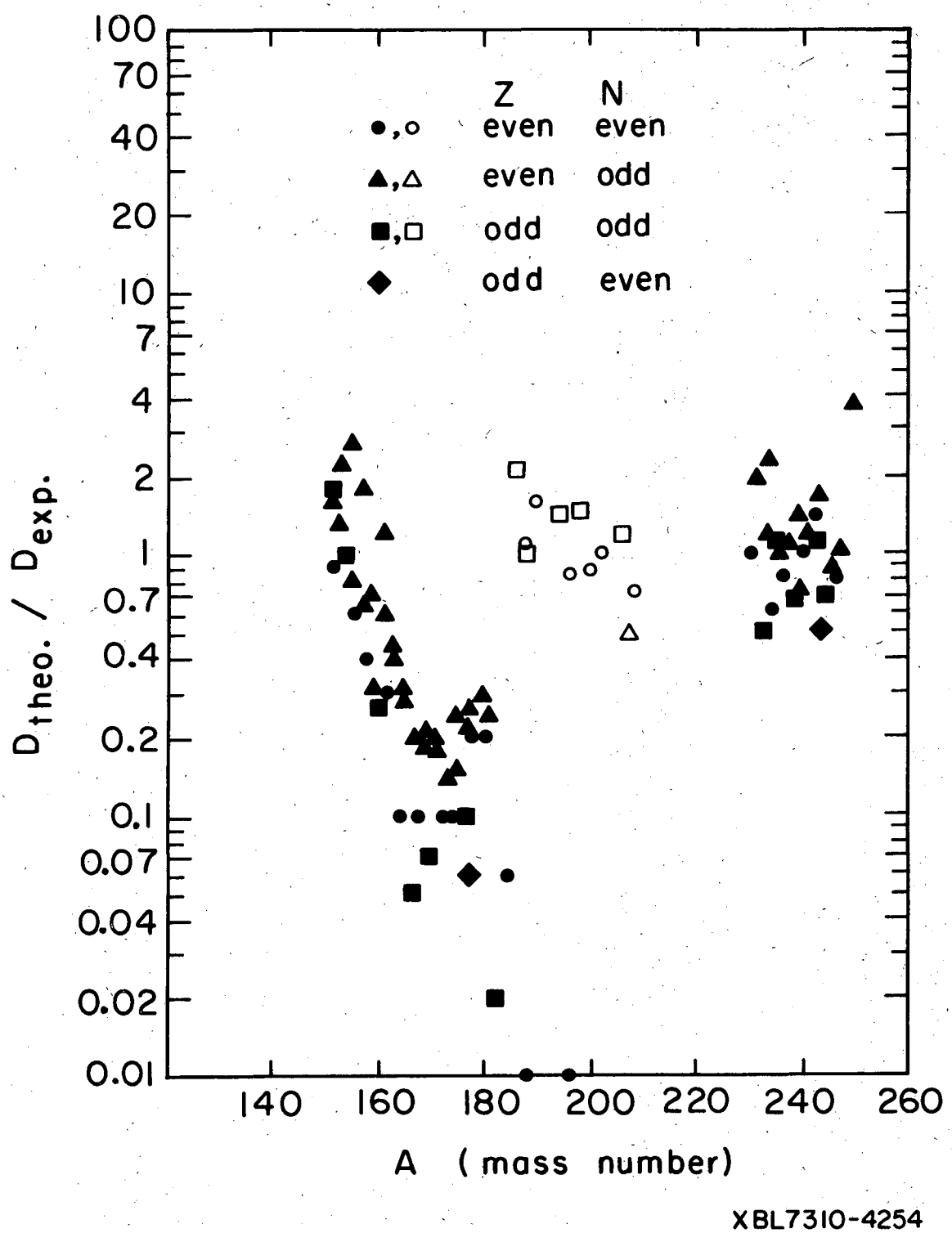
Z N
●,○ even even
▲,△ even odd
■,□ odd odd
◆ odd even
100
70
40
20
10
7
4
2
1
0.7
0.4
0.2
0.1
0.07
0.04
0.02
0.01
D theo. / D exp.
140 160 180 200 220 240 260
A (mass number)
XBL7310-4254

Fig. 1

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