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QUANTUM MECHANICAL TREATMENT OF PARTICLE TRANSFER BETWEEN HEAVY IONS NEAR THE COULOMB BARRIER IN THE PRESENCE OF COULOMB EXCITATION*

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Abstract

The theory of particle transfer between heavy ions when Coulomb excitation of the ions can take place has been formulated quantum mechanically. It is pointed out that the large ℓ and r spaces required for a quantum mechanical treatment of Coulomb excitation alone are not required for transfer reactions even when the former process plays an important role. This makes the quantum description of the transfer process calculationally possible.

1. Introduction

In the collision, at energies near the Coulomb barrier, between an ion such as oxygen, and a deformed nucleus such as Samarium, the probability that the deformed nucleus is left in an excited state is very high. Indeed it is near unit probability that the nucleus is in the 2^+ state[1]). In the treatment of particle transfer between the colliding ions, it is therefore essential that inelastic processes be included in the description. Now there is no difficulty in formulating the quantum mechanical description of Coulomb excitation. Indeed the Coulomb and nuclear forces were treated on an equal basis in a very accurate description of 50 MeV alpha scattering on rare earth nuclei which yielded the first information on higher multipoles in the nuclear shape[2]). There however the Coulomb field was much weaker than the nuclear. In the typical Coulomb excitation experiment, the reverse is true, and then while the theory is the same as before, the numerical problems become critical. The Coulomb field being the strong one; and the quadrupole Coulomb field falling off with distance so slowly as $1/r^3$, the differential equations describing the collision have to be integrated to great distance, R, (hundreds of Fermi's) and corresponding large angular momenta ($\ell m \simeq kR$ i.e. hundreds). For this reason Coulomb excitation can and has been treated semi-classically, so long as the energy is sufficiently low that nuclear interaction does not take place, and the enormous experimental literature on Coulomb excitation has been based on such a treatment[1]).

With the burgeoning interest in heavy ion reactions, a semi-classical theory of particle transfer has been developed in analogy to the Coulomb excitation theory[3]). One might think at first, if Coulomb excitation is

important in these transfer reactions, that the same difficulties as to large interaction regions and high partial waves will be present here also. This is in fact not the case and can be understood as follows. At Coulomb, or sub-Coulomb energies, the ions are prevented from interpenetrating by the Coulomb barrier. (Quantum mechanically we know that there is a small but finite probability for this. In such cases massive rearrangement will result and the particles are lost to the particular simple transfer of one or several nucleons that we envision.) Particles can be exchanged between the ions in slightly more distant collisions however because of the finite probability of their being found beyond the nuclear surface, where their wave functions are described by exponentially decaying tails. However because of the exponential decay of the tails, the probability of transfer falls off rapidly with distance. So there is some region bounded on the lower side by the sum of the radii $R_1 + R_2$ of the two ions and some larger but not so much larger radius R_3 where the product of the two exponential tails, the particle bound in one nucleus, and then transferred to the other, produces negligible probability that the transfer takes place. This outer radius for particle transfer is certainly only several fermi's larger than $R_1 + R_2$. On the other hand, because of the slowly decreasing Coulomb field, Coulomb excitation between states in the target nucleus as mentioned earlier can take place even at several hundred fermi beyond $R_1 + R_2$. However any such collision with impact parameter R which is much larger than $R_1 + R_2$ <u>cannot</u> contribute to the transfer of particles between the ions. Therefore only such partial waves are relevant for the transfer of particles which lie within $\ell \leqslant R_3/k$. (The radial wave function corresponding to angular momentum ℓ increases from zero and has its first maximum near $R = \ell/k$ where k is the wave

number of relative motion.) Higher partial waves, while they may excite the target will not give rise to particle transfer!

The above argument establishes that particle transfer reactions between heavy ions, even when Coulomb excitation of an excited state has a high probability, is governed by a much more modest number of partial waves than the Coulomb cross section, namely those corresponding to the nuclear region, $\ell \leqslant R_3/k$. However, considering the final partition of the system after particle transfer has taken place, Coulomb excitation of the final nucleus among the above limited number of partial waves is still possible and implies, because of the slow fall off of the Coulomb field, that the equations describing the reaction will have to be integrated to distances considerably in access of the nuclear region, though much smaller than the distance required to describe Coulomb excitation alone.

For the above reason it is feasible to do a fully quantum mechanical treatment of particle transfer reactions between heavy ions at Coulomb energies when Coulomb excitation is present.

In this paper we formulate the solution to the above problem using the source term method[4]). The zero-range approximation, which is a plausible approximation for transfer from light nuclei, is untenable for transfer from ions of a few mass units or more, and so the finite-range of the stripping interaction is taken into account[5,6]). However the means by which this is done limits its validity up to energies which are not much greater than the Coulomb barrier where the transfer of particles occur from the exponential tail regions of their wave functions.

In the following, when we speak of the transfer of a particle, it may be a nucleon or several particles such as an alpha-like group.

2. Formulation of the Coupled Equations

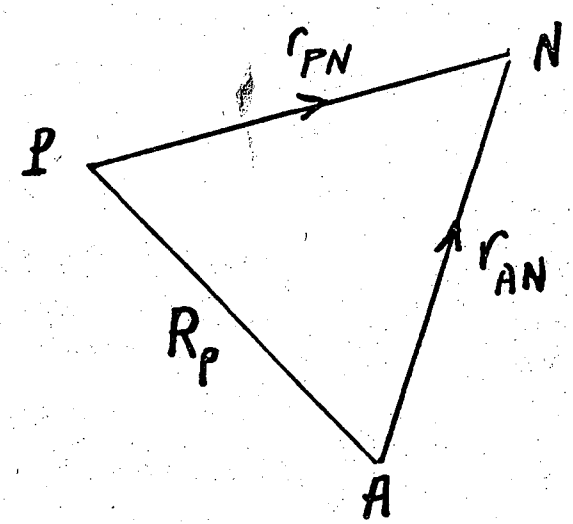
We use the following suggestive notation to denote the particles of the reaction

$$D + A \rightarrow P + B$$

$$D = P + N \quad , \quad B = A + N \quad , \tag{1}$$

in which for the usual (d,p) reaction D would stand for the deuteron, P for proton, and N for neutron. Here we mean to use this notation only to help the reader to keep in mind which are the "fundamental" particles of the reaction. Thus both P and N may be complex particles. If they are, then the coordinates used below refer to their centers of mass. Their internal structure does not enter into the basic equations described below save as constants in well isolated places which we point out when they arise. Their values can be computed from the detailed structure of the nuclei involved and as such does not fall within the specific province of this paper.

The coordinate system is shown in fig. 1 which shows the most massive "elementary" particle as A, (in our example in the introduction, samarium) the transferred object N which with A constitutes the other massive ion B, and P the outgoing light ion, (some derivative of oxygen in our earlier example after N has been removed).



In the source term method[4]), the problem breaks up into two parts, one corresponding to the initial partition D + A, and one corresponding to the final partition P + B.

In the initial partition we introduce the channel functions

$$\phi_{d\pi I} = [\mathcal{Y}_{j_d} \Phi_A]_I \quad , \tag{2}$$

where Φ_A is the nuclear wave function for A and $\mathcal{Y}_{j_d}$ is a spin-orbit function for the projectile constructed from its internal wave function Φ_D, and the spherical harmonic describing the relative angular momentum between D and A

$$\mathcal{Y}_{j_d} = [Y_{\ell_d}(\hat{R}_d)\ \Phi_D]_{j_d} \quad . \tag{3}$$

Finally d stands for the collection of quantum numbers

$$d \equiv \ell_d,\ j_d,\ D,\ A \quad .$$

Also I is the total angular momentum and π the parity

$$\pi = (-)^{\ell_d}\ \pi_A\ \pi_D \quad .$$

The solution to the scattering of D by A is governed by the Hamiltonian

$$H = H_A + H_D + T + V(A,D) \quad , \tag{4}$$

where the notation is obvious, T being the relative kinetic energy. One seeks a solution to the Schroedinger equation by expanding its solution as

$$\Psi^M_{d\pi I} = R_d^{-1} \sum_d u_d^{\pi I}(R_d) \quad \phi^M_{d\pi I} \quad , \tag{5}$$

and substituting into $(H-E)\Psi = 0$ one obtains in the usual way a set of coupled equations for each channel d:

$$[T_d - E_d] \; u_d^{\pi I}(R_d) + \sum_{d'} V^{\pi I}_{dd'}(R_d) \; u^{\pi I}_{d'}(R_d) = 0 \tag{6}$$

with

$$E_d = E - E_D - E_A \tag{7}$$

$$V_{dd'}(R_d) = \langle \phi_{d\pi I} | V(A,D) | \phi_{d'\pi I} \rangle \tag{8}$$

Further details on definitions and boundary conditions can be found in ref. 4

In an analogous way, for the final partition, P + B we introduce the channel functions

$$\phi_{p\pi I} = [\mathcal{Y}_{j_p} \; \phi_B]_I \tag{9}$$

$$\mathcal{Y}_{j_p} = [Y_{\ell_p} \; (\hat{R}_p) \; \phi_P]_{j_p} \tag{10}$$

$$p \equiv \ell_p, j_p, B, P \quad .$$

An analogous system of equations to (6) would result, save according to the rationale given in ref. 4, there should appear a source term corresponding to the production of the particles P by means of the transfer of N onto the various states of A that are excited as a result of the collision of D and A.

The equation therefore reads

$$[T_p - E_p]\, w_p(R_p) + \sum_{p'} V_{pp'}(R_p)\; w_{p'}(R_p) = \sum_d \rho_{d,p}(R_p) \quad , \qquad (11)$$

where $\rho_{d,p}$ corresponds to the source of P at R_p due to the transfer in the channel D of N onto A forming the channel p. Also $V_{pp'}$ on the channel matrix elements of V(P,B). Since we are assuming that the experiments are carried out at such an energy that there is little interpenetration of the particles, the interactions V(P,B) and V(D,A) are Coulombic. They could be expressed directly in terms of measured moments, or alternately in terms of the nuclear shape[2]).

These equations are to be solved with boundary conditions as given in Ref. 4. This yields the S-matrix elements from which the amplitude for the reaction can be constructed in analogy with that reference.

3. Evaluation of the Source Term

As discussed in our earlier work the structure of the source term is[4])

$$\rho_{dp}^{\pi I}(R_p) = R_p \left\langle \phi_{p\pi I} | V(r_{PN}) | \phi_{d\pi I} \frac{u_d^{\pi I}(R_d)}{R_d} \right\rangle \quad , \tag{12}$$

where all coordinates are integrated, save R_p. With this source, and the neglect of inelastic excitation, (i.e. off-diagonal elements of V(D,A) and V(P,B)) the above equations lead to the usual DWBA theory for particle transfer[5]).

Now if we restrict ourselves to situations where only a few nucleons of D are transferred, i.e.

$$N \ll D \quad , \tag{13}$$

then we may make the approximation

$$\underset{\sim}{R}_d \simeq \underset{\sim}{R}_p \quad , \tag{14}$$

so that the source term now appears as

$$\rho_{dp}^{\pi I}(R_p) = \langle \phi_{p\pi I} | V(r_{PN}) | \phi_{d\pi I} \rangle \, u_d(R_p) \quad . \tag{15}$$

The task now is to evaluate the matrix element appearing in this equation. To begin, we introduce the parentage expansions

$$\Phi_D = \sum_{\ell_1 j_1 P'} \beta(P', j_1) \, [\Phi_{P'} \psi_{\ell_1 j_1}(r_{PN})]_D \tag{16}$$

$$\Phi_B = \sum_{\ell_2 j_2 A'} \beta(A', j_2) \, [\Phi_A \, \psi_{\ell_2 j_2}(r_{AN})]_B \quad . \tag{17}$$

Here j_1, which may have several values, corresponds to the angular momentum carried by the center-of-mass of N with respect to P, and j_2, the angular momentum of N with respect to A. In brief N is transferred from orbit j_1 in D to orbit j_2 in B. The particular values that the parentage amplitudes take is of course a question of the detailed structure of the nuclei involved.

Writing now the full meaning of the matrix element in (15)

$$\langle \phi_p | V(r_{PN}) | \phi_d \rangle = \sum_{\ell_1 j_1} \sum_{\ell_2 j_2} \beta(P,j_1)\, \beta(A,j_2) \Big\langle \{[Y_{\ell_p}(\hat{R}_p),\ \Phi_P]_{j_p}\ ,$$

$$[\Phi_A,\ \psi_{\ell_2 j_2}(\underset{\sim}{r}_{AN})]_B\}\ I \Big| V(r_{PN}) \Big| \{[Y_{\ell_d}(\hat{R}_p),$$

$$(\Phi\ ,\ \psi_{\ell_1 j_1}(\underset{\sim}{r}_{PN}))_D]_{j_d},\ \Phi_A\}\ I \Big\rangle \quad . \tag{18}$$

Note that on account of the orthogonality of the nuclear functions $(\Phi_P, \Phi_{P'}) = \delta_{PP'}$ and $(\Phi_A, \Phi_{A'}) = \delta_{AA'}$ sums on P' and A' do not occur.

To proceed further, we introduce explicitly the assumption that the reaction occurs at an energy near or below the Coulomb barrier, so that the transfer of particles involves only the exponential tails of their wave functions. We may write therefore

$$\psi_{\ell_1 j_1}(r_{PN}) = N_1\, i^{\ell_1}\, h^{(1)}_{\ell_1}(iK_1\, r_{PN})\, \mathcal{Y}_{j_1} \quad \text{(outside)} \tag{19}$$

$$\psi_{\ell_2 j_2}(r_{AN}) = N_2\, i^{\ell_2}\, h^{(1)}_{\ell_2}(iK_2\, r_{AN})\, \mathcal{Y}_{j_2} \quad \text{(outside)} \quad , \tag{20}$$

where $h^{(1)}$ is a spherical Hankel function and

$$\mathcal{Y}_{j_1} = [Y_{\ell_1}(\hat{r}_{PN})\ \Phi_N]_{j_1} \tag{21}$$

$$\mathcal{Y}_{j_2} = [Y_{\ell_2}(\hat{r}_{AN})\ \Phi_N]_{j_2} \quad . \tag{22}$$

Note that an additional assumption has been made at this point, namely that the same internal function describing the transferred particle appears in (21) and (22). If N were a nucleon there would be no other choice. If N is composite the function which describes its internal motion may be different in D than in A. However in the matrix element (18) the intrinsic spin of this particle cannot change. So at most an overlap appears, say Ω_N, which depends as do the β's in (16) and (17) on the detailed structure of D and A.

Having made the assumption leading to (19) and (20) the addition theorem of Buttle and Goldfarb[5]) can now be used to eliminate $\underset{\sim}{r}_{AN}$ in favor of $\underset{\sim}{r}_{PN}$ and $\underset{\sim}{R}_p$. The integration on $\underset{\sim}{r}_{PN}$ and $\hat{R}_p$ can then be done leaving the matrix element as a function of R_p alone, as anticipated.

The addition theorem appears, in terms of the vectors shown in fig. 1 as

$$h_{\ell_2}(iK_2\ r_{AN})\ Y^m_{\ell_2}(\hat{r}_{AN}) = \sqrt{4\pi} \sum_{\Lambda\lambda} i_2^{\ell+\Lambda-\lambda} \left(\frac{\hat{\lambda}\hat{\Lambda}}{\hat{\ell}_2}\right)^{1/2} C^{\Lambda\lambda\ell_2}_{000}$$

$$(-)^{\Lambda}\ h^{(1)}_{\Lambda}(iK_2\ R_p)\ j_{\lambda}\ (iK_2\ r_{PN})\ [Y_{\Lambda}(\hat{R}_p)\ Y_{\lambda}\ (\hat{r}_{PN})]^m_{\ell_2} \quad . \tag{23}$$

We abbreviate

$$A_{\Lambda\lambda\ell_2} \equiv \sqrt{4\pi}\; i^{\ell_2+\Lambda-\lambda} \left(\frac{\lambda\Lambda}{\hat{\ell}_2}\right)^{1/2} C^{\Lambda\lambda\ell_2}_{000} \quad . \tag{24}$$

Then

$$\langle \phi_p | V(r_{PN}) | \phi_d \rangle = \sum_{\ell_1 j_1} \sum_{\ell_2 j_2} \beta(P,j_1)\, \beta(A,j_2)\, N_1 N_2\, i^{\ell_1-\ell_2} \sum_{\Lambda\lambda} (-)^{\Lambda} A^{*}_{\Lambda\lambda\ell_2}\, I_{\lambda\ell_1}$$

$$\Big\langle \{[Y_{\ell_p}(\hat{R}_p)\, \Phi_P]_{j_p}\, ,\ \Big[\Phi_A,\ \Big[\Big(Y_\Lambda(\hat{R}_p)\, Y_\lambda(\hat{r}_{PN})\Big)_{\ell_2} \Phi_N\Big]_{j_2}\Big]_B\} \; I$$

$$\Big| \{\Big[Y_{\ell_d}(\hat{R}_p),\ (\Phi_P,\ [Y_{\ell_1}(\hat{r}_{PN})\, \Phi_N]_{j_1})_D\Big]_{j_d}\, ,\ \Phi_A\} \; I \Big\rangle\; h^{(1)*}_{\Lambda}(iK_2\, R_p) \quad , \tag{25}$$

where

$$I_{\lambda\ell_1} = \int_0^\infty j_\lambda^{*}(iK_2\, r_{PN})\; V(r_{PN})\; h^{(1)}_{\ell_1}(iK_1\, r_{PN})\; r^2_{PN}\, dr_{PN} \quad . \tag{26}$$

The angle integral, which appears very complicated, can be done most easily by doing various recouplings. We now drop explicit reference to the angle coordinates, of which there are only $\hat{R}_p$ and $\hat{r}_{PN}$ since their association with an angular momentum symbol in what follows can be identified through (26). We write the bracket in (26) as

$$B \equiv \langle (\ell_p P)_{j_p}\, ,\ (A,\ [(\Lambda\lambda)\ell_2,\ N]_{j_2})_B;\ I \Big| \{\ell_d,\ [P,\ (\ell_1 N)_{j_1}]_D\}\ j_d,\ A;\ I \rangle \tag{27}$$

The bra we transform as

$$\langle (\ell_p P) j_p, (A, [(\Lambda\lambda)\ell_2, N] j_2) B; I |$$

$$= \sum_j U(\Lambda\lambda\, j_2\, N;\, \ell_2 j) \langle (\ell_p P) j_p, \left(A, [\Lambda, (\lambda N) j] j_2\right) B; I |$$

$$= \sum_j U_j \sum_j (-)^{A+j_2-B} U(j_p j_2\, I\, A;\, j' B) \langle \{(\ell_p P) j_p, [\Lambda, (\lambda N) j] j_2\}\, j', A; I |$$

$$= \sum_{j,j'} U_j U_{j'} (-)^{A+j_2-B} \sum_{\ell x} \begin{Bmatrix} \ell_p & P & j_p \\ \Lambda & j & j_2 \\ \ell & x & j' \end{Bmatrix} (\hat{\ell}\, \hat{x}\, \hat{j}_p\, \hat{j}_2)^{1/2}$$

$$\langle \{(\ell_p \Lambda)\ell, [P(\lambda N) j]\, x\}\, j', A; I | \quad . \tag{28}$$

In the above the U functions are related to the 6-j coefficients as defined in Edmonds[7]). At this point by means of the three transformations the bra has been arranged to have the same ordering in most of its dependences as the ket of (27) so that we arrive immediately at

$$B = U(\Lambda\ell_1\, j_2 N;\, \ell_2 j_1)\, U(j_p j_2\, I\, A;\, j_d B)\, (-)^{A+j_2-B}$$

$$\begin{Bmatrix} \ell_p & P & j_p \\ \Lambda & j_1 & j_2 \\ \ell_d & D & j_d \end{Bmatrix} (\hat{\ell}_d\, \hat{D}\, \hat{j}_p\, \hat{j}_2)^{1/2}\, \delta_{\lambda\ell_1} \langle (\ell_p \Lambda)\ell_d | \ell_d \rangle \quad . \tag{29}$$

The evaluation of the last bracket is easily accomplished through the addition theorem of the two spherical harmonics on the left, since the argument of all three, ℓ_p Λ and ℓ_d is $\hat{R}_p$

$$\langle (\ell_p \Lambda)\ell_d | \ell_d \rangle = (-)^{\ell_d} \left(\frac{\hat{\ell}_p \hat{\Lambda}}{4\pi}\right)^{1/2} \begin{pmatrix} \ell_p & \Lambda & \ell_d \\ 0 & 0 & 0 \end{pmatrix} \quad . \tag{30}$$

The final result for the source term is

$$\rho_{dp}^{\pi I}(R_p) = \Omega_N \sum_{\ell_1 j_1} \sum_{\ell_2 j_2} N_1 N_2 \, \beta(P,j_1) \, \beta(A,j_2) \sum_{\Lambda} \hat{\Lambda} \, i^{\Lambda}$$

$$(-)^{\ell_1+\ell_d+A+j_2-B} \, (\hat{\ell}_1 \hat{\ell}_p)^{1/2} \begin{pmatrix} \ell_1 & \Lambda & \ell_2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_p & \Lambda & \ell_d \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{Bmatrix} \ell_p & P & j_p \\ \Lambda & j_1 & j_2 \\ \ell_d & D & j_d \end{Bmatrix} \quad U(\Lambda \ell_1 j_2 N; \ell_2 j_1) \, (\hat{\ell}_d \hat{D} \hat{J}_p \hat{J}_2)^{1/2}$$

$$U(j_p j_2 \, I \, A; \, j_d B) \, I_{\ell_1} \, h_{\Lambda}^{(1)*}(iK_2 \, R_p) \, u_d^{\pi I}(R_p) \quad . \tag{31}$$

The overlap Ω_N between the internal motion of the cluster which is transferred appears as a multiplicative factor which merely effects the overall magnitude of the cross sections but not their relative values between transitions.

4. Summary and Discussion of the Approximations

a) The transfer process is weak: This assumption is implicit in the fact that we write a source term only in the equations describing the final partition (11) and not in (6), and corresponds to treating the reaction in first order, although the Coulomb excitation is treated to all orders. This assumption can be relaxed by introducing the inverse source term into eq. (6) so that the transferred particle N can go back and forth between the two ions.

b) It is assumed that the mass of the transferred particle N is small compared to the masses of the other ions. It would be numerically very awkward to evaluate the source term without this assumption which allows us to use (14).

c) It is assumed if several nucleons are transferred, that their relative state of motion amongst themselves does not change; nor does the intrinsic spin of the transferred object change. This assumption is implicit in the absence of any reference to the internal motion of N in the parentage expansions (16) and (17) where only reference is made to the angular momenta they possess in their host nuclei. The assumption manifests itself again in (21) and (22) and in the final result for the source term (31) where a simple overlap Ω_N appears (which would be unity for nucleon transfer). This assumption could be relaxed by summing over various Ω_N corresponding to various intrinsic spin and internal motions of the transferred particle. There would remain the implicit assumption that these states of N are not disturbed in the course of the transfer.

d) The reaction takes place near or below the Coulomb barrier: This assumption appears in the replacement of the wave functions by their Hankel function tails (19) and (20) and further implies that no charge is transferred. However it is probably not a serious approximation to use these same functions for a small charge transfer.

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