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The Capillary Hysteresis Model HYSTR–User's Guide

A. Niemi and G.S. Bodvarsson

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1.0 INTRODUCTION

The potential disposal of nuclear waste in the unsaturated zone at Yucca Mountain, Nevada, has generated increased interest in the study of fluid flow through unsaturated media. In the near future, large-scale field tests will be conducted at the Yucca Mountain site, and work is now being done to design and analyze these tests (Montazer and Wilson, 1984).

As part of these efforts a capillary hysteresis model has been developed. A computer program to calculate the hysteretic relationship between capillary pressure ϕ and liquid saturation (S_l) has been written that is designed to be easily incorporated into any numerical unsaturated flow simulator that computes capillary pressure as a function of liquid saturation. This report gives a detailed description of the model along with information on how it can be interfaced with a transport code.

Although the model was developed specifically for calculations related to nuclear waste disposal, it should be applicable to any capillary hysteresis problem for which the secondary and higher order scanning curves can be approximated from the first order scanning curves.

2.0 BACKGROUND

Due to the complex nature of the liquid-phase configuration in an unsaturated porous medium, the functional relationship between capillary pressure (ϕ) and liquid saturation (S_l) (or capillary pressure and water content (θ)) is not unique, but is characterized by a multiple-valued, hysteretic relationship (Narasimhan, 1975). The saturation associated with a given capillary pressure is less during wetting than during drainage from fully saturated condition. An example of a hysteretic $\phi = \phi(\theta)$ relationship is shown in Figure 1.

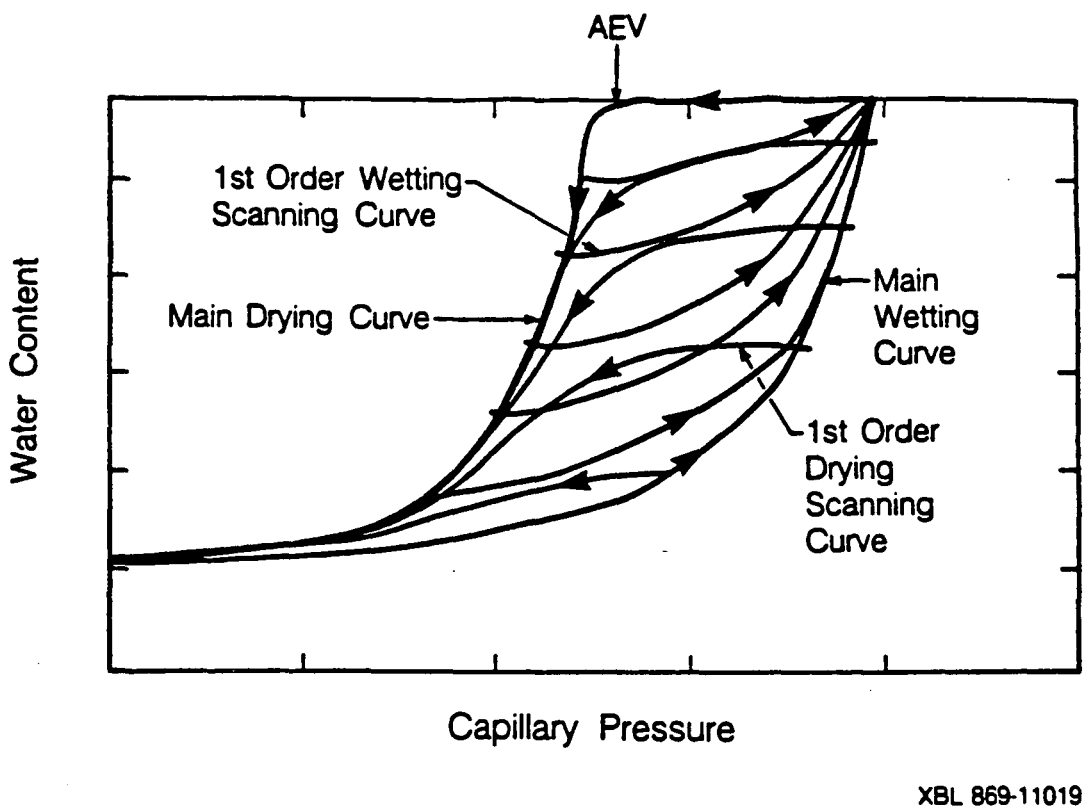

Figure 1. Example of a hysteretic $\phi\ \phi(\theta)$ relationship (modified from Banerjee and Watson, 1984).

If, for example, the system begins to dry out from a fully saturated condition, the capillary pressure must first increase (in Figure 1 capillary pressure increases from right to left) to a certain

air-entry value (AEV) before drying begins. After this, drying follows the main drying curve. If saturation then begins to increase, wetting will take place along the first order wetting scanning curves (see Figure 1). Subsequent drying would follow a second-order drying scanning curve. The second-order scanning curves are not shown in Figure 1.

The presence of hysteresis in the $\phi = \phi(\theta)$ (or in the $\phi = \phi(S_l)$) relationship as well as in the relative permeability-capillary pressure ($k_{rl} = k_{rl}(\phi)$) relation is well established for many materials. There is, however, disagreement as to whether there is significant hysteresis in the $k_{rl} = k_{rl}(\theta)$ (or in the $k_{rl} = k_{rl}(S_l)$) relationship.

Pickens and Gillham (1980), reviewed related measurements published in the literature and concluded that the majority of published data indicates that $k_{rl} = k_{rl}(\theta)$ can be assumed to be nonhysteretic for all practical purposes. The maximum difference in $k_{rl} = k_{rl}(\theta)$ values due to different wetting/drying histories, reported by Poulovassilis and Tzimas (1975), amounts to a factor of 2. This appears not to be very significant given the typically large variation in permeability with saturation and inaccuracies in field data. When modeling hysteresis in unsaturated flow, nonhysteretic $k_{rl} = k_{rl}(\theta)$ has been used by Banerjee and Watson (1984), Perrens and Watson (1977), Scott et al. (1983), and Curtis and Watson (1984), among others.

The different approaches that have been used to numerically describe the hysteretic $\phi = \phi(\theta)$ relationship can be categorized into three main groups: (1) methods employing empirical closed-form expressions, (2) methods based on domain theory, and (3) interpolation methods. Scott et al. (1983), Kool and Parker (1987), Pickens and Gillham (1980), Hoa et al. (1977), Killough (1976) and Dane and Wierenga (1975) all have used closed-formed empirical expressions to calculate the scanning curves. Empirical models are often developed for a specific soil and they do not claim general validity. In some cases their predictions are tested with several different soil types (Scott et al., 1983; Kool and Parker, 1987) but their derivation is not based on physical presentation of hysteresis. The so-called domain models have been developed and used by Mualem (1984), Banerjee and Watson (1984), Mualem (1974), Mualem and Dagan (1975), Mualem (1976), Topp (1971), Lees and Watson (1975), Poulovassilis and Childs (1971) and Poulovassilis and Tzimas (1975) among others. The domain models are based on theoretical presentation of capillary hysteresis in which the porous medium is viewed as a collection of pore

domains with characteristic wetting and drying radii (Kool and Parker, 1987).

A third approach involves interpolation between tabulated data points to provide values inside the hysteresis loop. Whisler and Watson (1969), Watson and Perrens (1973), Curtis (1977), Perrens and Watson (1977) and Curtis and Watson (1984) tabulate the main boundary curves and primary scanning curves and then use the simplifying assumption that higher order scanning curves can be obtained through interpolation between primary scanning curves. A similar assumption is used also by Klute and Heermann (1974) in connection with analytically-expressed primary scanning curves.

In comparison with the other methods, the most sophisticated dependent domain models are probably the most accurate in predicting scanning curves for different media (Kool and Parker, 1984). The accuracy of such models as that of Mualem and Dagan (1975) and Banerjee and Watson (1987) is, however, achieved at the expense of using more data for calibration and more complicated models. The so-called modified dependent domain model by Mualem (1984) only needs the main wetting and drying curves for calibration. This model, however, only gives expressions for the first and second order scanning curves.

The interpolation model has the advantage that any kind of measured data can easily be incorporated into it and the model has a simple formulation. Because the model approximates the second and higher order scanning curves from the first order scanning curves, it can produce erroneous results if frequent reversals from wetting to drying and vice versa take place. This so-called "pumping phenomenon" has been demonstrated by Klute and Heermann (1974) and Jaynes (1984) and it is not limited into interpolation models, but can occur with other models as well for which the closing of the hysteresis loops is not required (for example models of Scott et al. (1983) and Dane and Wierenga (1975)).

3.0 COMPUTER PROGRAM HYSTR

3.1. The Model

The capillary hysteresis model HYSTR was developed for the modeling of the unsaturated zone at Yucca Mountain. The interpolative approach was determined to be most appropriate, especially in light of its general applicability to any kind of experimental data that might later be obtained. It is assumed that the secondary and higher order scanning curves can be obtained through interpolation from the first-order scanning curves. Possible inaccuracies in second and higher order scanning curves resulting from this assumption are discussed in Chapter 3.4.

Hysteresis is only considered in the capillary pressure - liquid saturation relationship. The relative permeability - liquid saturation relationship is assumed non-hysteretic based on the literature cited in Section 2.

3.2. Structure of the Program

Given the saturation value and the previous wetting/drying history of an element, the program HYSTR computes the corresponding capillary pressure through interpolation between tabulated curves. For each hysteretic material the main wetting and drying curves as well as the first-order scanning curves are input in tabulated form. This data is held in storage throughout the computations to be used for the interpolation.

HYSTR consists of three main parts and can be incorporated into a numerical simulator through three call statements. Subroutine INHYST reads in the data, subroutine PCHYST performs all computations by using several lower level subroutines, and subroutine SAVEHS saves information from one time step to the next. A flow chart of the subroutine PCHYST is shown in Figure 2.

The information concerning the previous wetting/drying history that is needed to uniquely define the current location in the hysteresis loop includes:

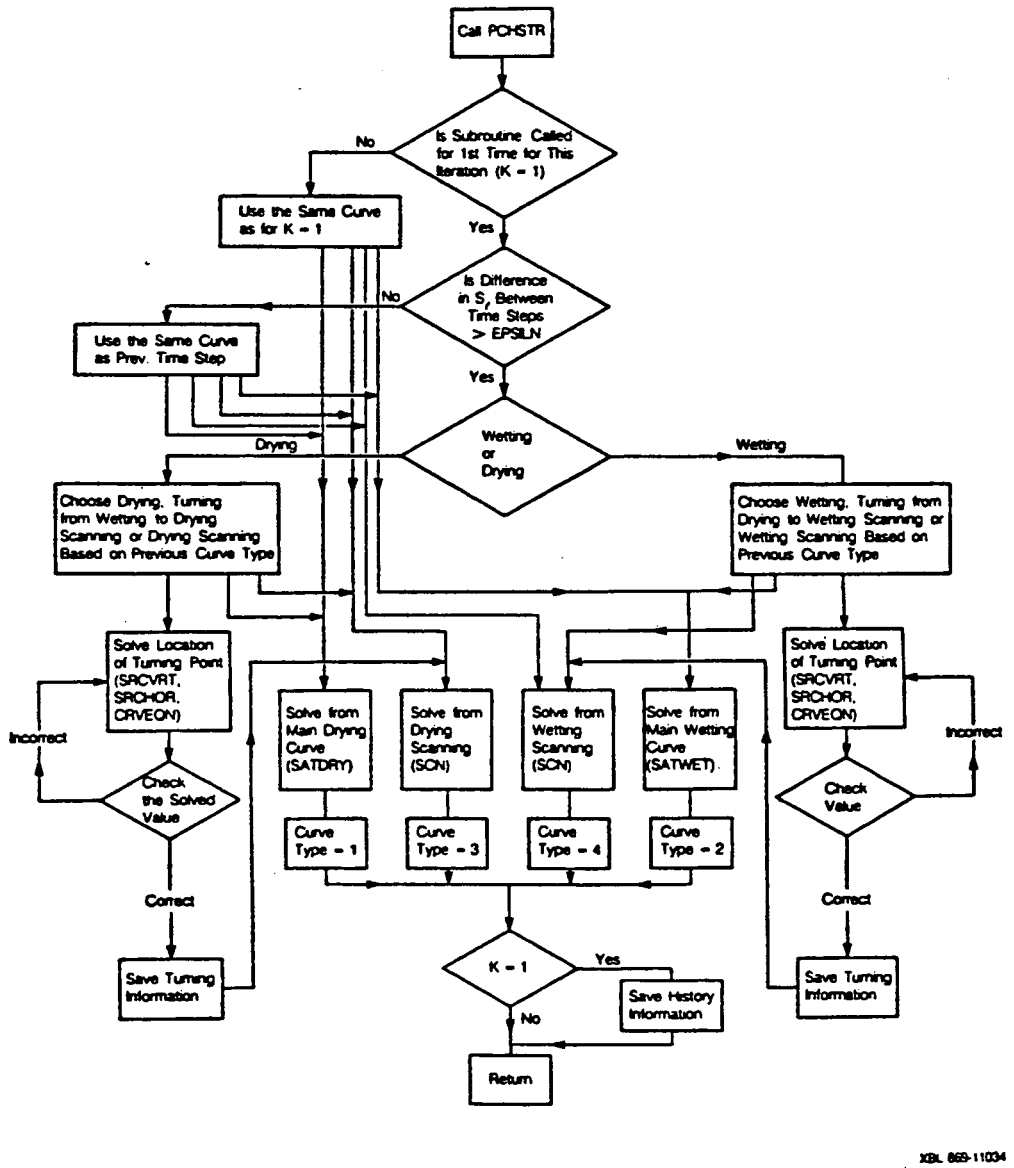

Figure 2. Flow chart of subroutine PCHYST.

(1) Liquid saturation at the end of the previous time step.

(2) Capillary pressure at the end of the previous time step. This information is needed in case of a reversal from wetting to drying or vice versa, when the location of the turning point is needed.

(3) Type of curve used at the end of the previous time step. The program specifies four different curve types: main wetting curve, main drying curve, wetting scanning curve and drying scanning curve.

This history information must be given as initial conditions for each element and saved from one time step to the next. Together this information and the saturation at the present time step determine the type of curve to be followed. Different situations that can occur are shown in Figure 3.

Drying and wetting along the main curves are treated by simply interpolating between values tabulated along these curves. This is done independently from the scanning curve calculations, for which continuous lines are interpolated between the tabulated scanning curves. In all cases the equation-solving and interpolation is done linearly. Because logarithmic capillary pressure scales are often encountered, the code has an option to accommodate these type of data.

In case of a reversal from the curve used in the previous time step (cases 2, 4, 5 and 8 in Figure 3), the location of the new scanning curve must be determined. This is done by identifying the two scanning curves between which the turning point falls. The turning point is always assumed to be located at the solution point from the previous time step. Then, the vertical distance of the turning point from the adjacent tabulated curves of the proper type (drying vs wetting) is calculated. Subsequent scanning is assumed to take place along a continuous line with a constant vertical position relative to the adjacent tabulated curves.

To avoid nonphysical reversals from drying to wetting and vice versa due to small saturation changes, a reversal criterion (EPSLN in Figure 2) must be selected. This reversal criterion defines the smallest difference in liquid saturation with which a reversal from the old curve can take place. In the simulations that have been carried out so far, reversal criteria ranging from 10^{-4} to 10^{-7} have been used. This factor is an input parameter and can be specified by the user.

HYSTR has been designed to compute capillary pressure several times per iteration; once for the calculation of the capillary pressures at the state point and then several times using

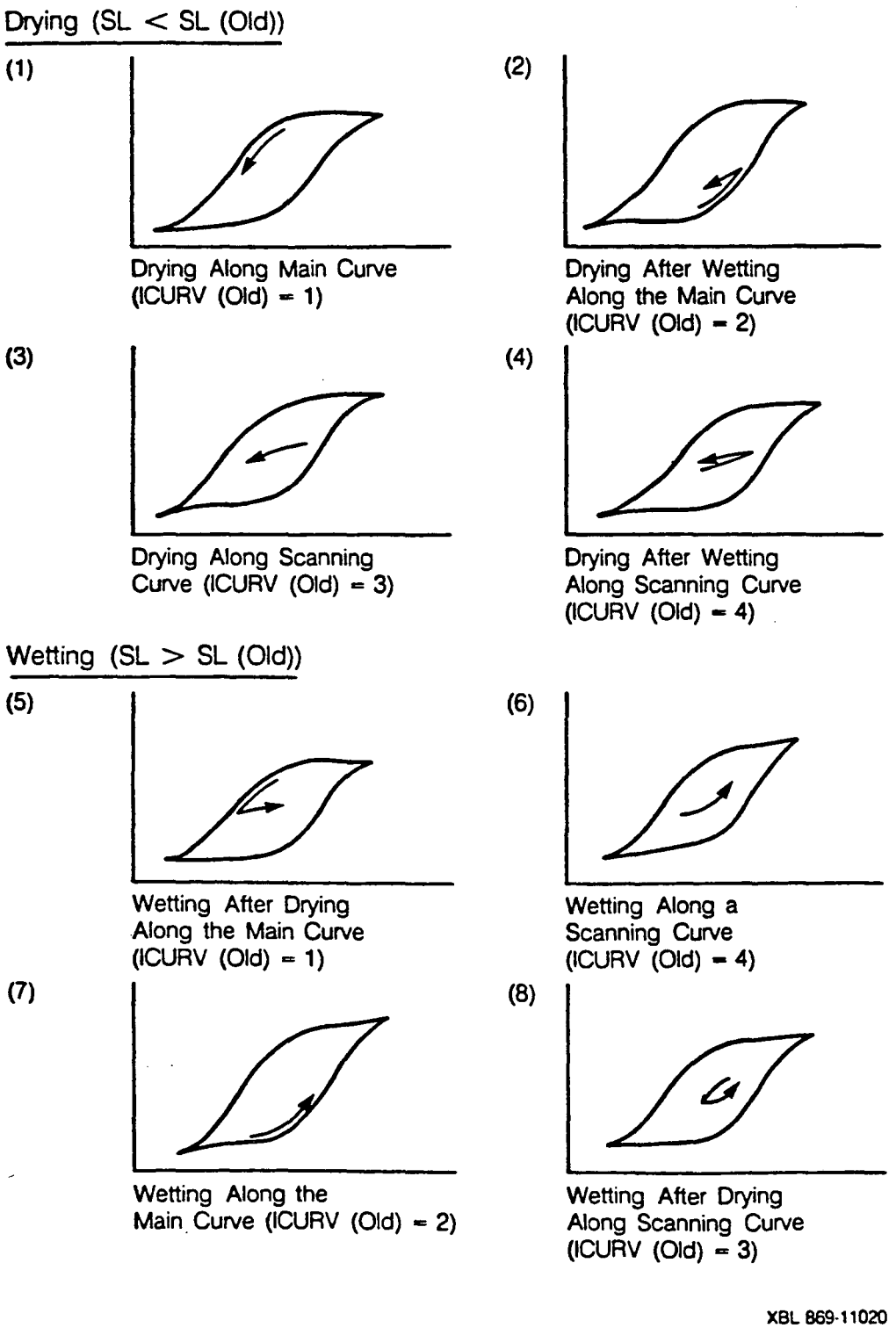

Figure 3. The eight possible situations during a setting/drying process.

incremented values of the primary variables to obtain the derivatives. The curve to be followed is determined once per iteration, during the state-point calculation, and is used during the derivative calculations as well. This is done to assure the continuity of the derivatives, which is important for a rapid convergence of the iterations. Only the results obtained with the state-point calculation are saved after each iteration to be used as "history" information for the next time step if convergence is achieved. The use of this multiple HYSTR calls per iteration is optional and the program can also be used with numerical schemes that calculate capillary pressures only once during each iteration.

Some parts of the program (interpolation along the main boundary curves, vertical search algorithm and part of the data input section are modified from the works of Narasimhan and Witherspoon (1978) and Wang et al. (1981).

3.3. List of the Subroutines

HYSTR consists of nine subroutines; three are called independently (INHYST, PCHYST and SAVEHS) by the main program, and the remaining six are called by subroutine PCHYST. A brief description of each subroutine is given below.

INHYST is called by the main program to read in the necessary initial conditions for each element and the tabulated hysteresis loops for all the different materials.

PCHYST is the main subroutine called for solving the capillary pressure for a given element with known liquid saturation (see Figure 2). In this subroutine first a decision is made as to which type of curve should be used given the previous wetting history of the element in question. If the wetting/drying is taking place along the same path as during the previous time step, PCHYST calls the appropriate subroutine to perform the calculation. If a reversal from the previous curve is necessary, the location of the turning point is solved in PCHYST by calling several lower level routines.

SATDRY and SATWET are called by PCHYST to solve $\phi = \phi\,(S_l)$ through interpolation between the tabulated nodes on the main drying and main wetting curves, respectively.

SRCVRT is called by PCHYST to determine between which two tabulated scanning curves the turning point falls in the vertical direction. This search only gives a first estimate for the

correct interval; when turning from wetting to drying the actual interval can be below the first guess, and when turning from drying to wetting it can be above.

SRCHOR is called by PCHYST to determine between which two tabulated pressure values the turning point falls in the horizontal direction.

CRVEQN is called by PCHYST to solve for the coefficients in the equation $S_l = A + B * \phi$ for the tabulated curves below and above the turning point in the pressure interval solved in SRCHOR.

SCN is called by PCHYST to find the capillary pressure for a given liquid saturation from an interpolated scanning curve.

SAVEHS is called by the main program after convergence has been achieved (at the end of each time step). In SAVEHS the capillary pressure, liquid saturation and curve type from the final iteration are assigned to be used as "history" information during the next time step. Several variables defining the location of the interpolated scanning curve are also saved.

3.4. Limitations of the Model

(1) HYSTR is based on the assumption that second and higher order scanning curves can be approximated from the tabulated first order scanning curves. With frequent reversals from wetting to drying and vice versa, this can produce and erroneous "pumping effect" as shown in Figure 4 adopted from Jaynes (1984). From Figure 4 we can see that the wetting periods are too short for the scanning path to reach a looplike shape. It has been suggested (Banerjee and Watson, 1984) that this kind of method should not be used when reversals are too closely spaced in time. Perrens and Watson (1977) also refer to this limitation under rapidly recurring intermittency, but conclude that the approach generally describes most redistribution events with sufficient accuracy. We recommend that, when modelling continuously alternating wetting/drying pulses, use of some other hysteresis model specifically designed to avoid "pumping," should be considered. Niemi and Bodvarsson (1988) give examples of the performance of the HYSTR model as compared to the performance of two other hysteresis models (the empirical closed-form model by Killough (1976) and the modified dependent domain

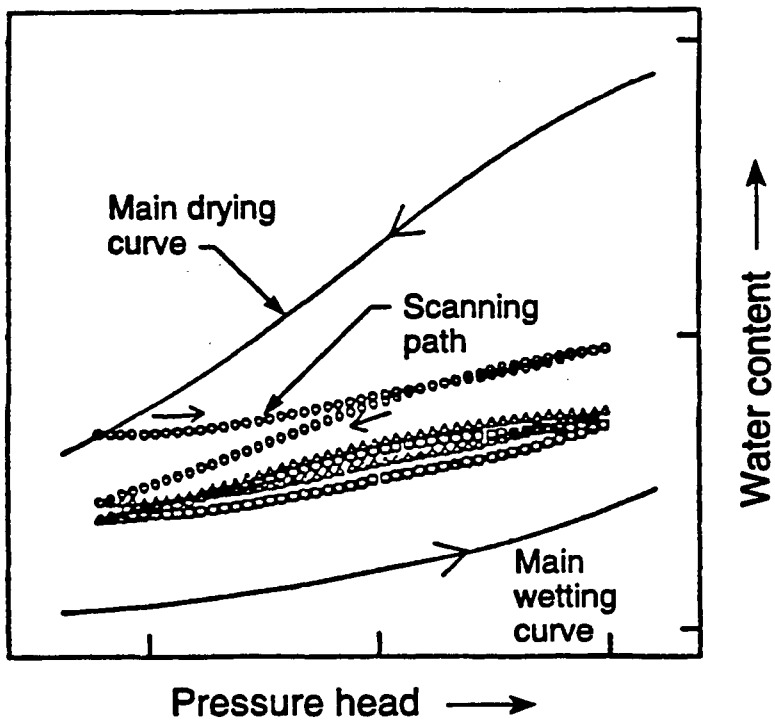

Figure 4. Example of the "pumping effect" (modified from Jaynes, 1984).

model by Mualem (1984)) both of which are based on the assumption that hysteresis loops have to close at the original point of reversal.

(2) Throughout the figures in the report the main hysteresis envelope is shown to close at maximum saturation (S_{max} of the main drying curve is equal to S_{max} of the main wetting curve). This is the assumption employed in most of the hysteresis models. Recently, however, Kool and Parker (1987) have introduced a model that takes into account the air entrapment during the wetting stage, which causes the maximum saturation of the main wetting curve to be less that the maximum saturation of the main drying curve. If this kind of phenomenon is observed, it can be easily included into the HYSTR model by an appropriate tabulation of the curves given as input data for the program.

4.0 INCORPORATION OF HYSTR INTO A NUMERICAL SIMULATOR

The HYSTR set of subroutines is ready to be incorporated into any numerical program that simulates unsaturated flow. The incorporation is done by adding three different call statements to the numerical code. A flow chart of the process is shown in Figure 5.

(1) The first statement to be added is:

```
CALL INHYST(NELE)                                    (1)
```

where

NELE = variable denoting the total number of elements.

This subroutine reads the input data and performs initiatizations.

(2) The next statement, which allows calculation of hysteretic capillary pressure as a function of liquid saturation, is:

```
CALL PCHYST(SL,P,NM,NE,K)                            (2)
```

where

SL = degree of liquid saturation (given to the subroutine)

P = capillary pressure (to be calculated in the subroutine)

NM = material number of the element in question (material properties are read in INHYST for NM = 1,2...)

NE = element number

K = option identifier for the cases, in which PCHYST is called several times during one iteration

(a) K = 1, if PCHYST is being called for the first time during the iteration in question and the selection for the appropriate curve needs to be made. Setting K = 1 also causes the history information from that particular calculation to be saved as the actual result from that iteration.

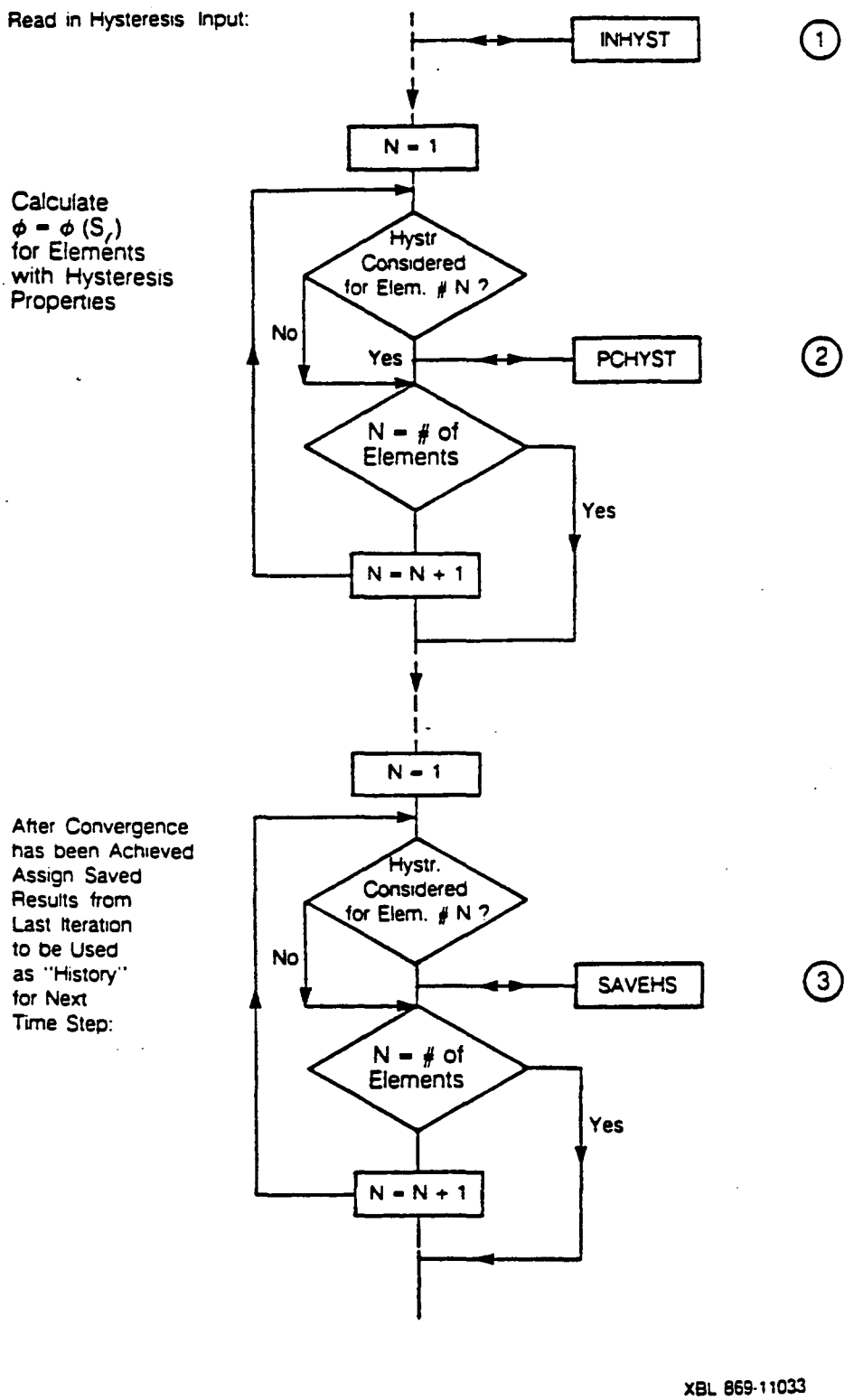

Figure 5. Flow chart of the incorporation of HYSTR into a numerical simulator.

(b) $K \neq 1$, if the same curve as for the $K = 1$ case should be used. The history results are not saved.

For the numerical schemes where PCHYST is only called once for each iteration, K is always 1. For schemes where derivatives are calculated and PCHYST is called several times per iteration, $K = 1$ for the state-point calculation (the first PCHYST call per iteration) and $K \neq 1$ for the derivative calculations.

Subroutine PCHYST is called for each element separately. The call statement should be incorporated into the main program at the point where capillary pressure normally is calculated. Calling should only be done for elements that are of "hysteretic" material. In the flow chart (Figure 5) this call statement is shown as a part of a loop over all the elements to emphasize the fact that calling is done element-wise. Usually this loop, however, already exists in the main program.

(3) The third call statement to be added is

CALL SAVEHS(NE) (3)

where

NE = element number.

This subroutine is called after convergence for the time step has been achieved. This routine is also designed to be called element-wise and only for the elements for which hysteresis is considered (see Figure 5). With little effort the user can change subroutine SAVEHS so that it only has to be called once; in this case the element loop should be inside SAVEHS. However, one must then determine inside SAVEHS, whether an element is of "hysteretic" material which requires more information to be transferred to the subroutine.

After the call statements have been added to the numerical simulator, the dimensions of the HYSTR common blocks and the I/0 commands should be changed to be consistent with the main program (see Appendix A, a listing of HYSTR, for dimension requirements).

5.0 DATA INPUT

5.1. Input Cards

All the data is read in from one file (specified as unit 7 in the program) and read statements are in free format.

DATA SET HYSTR

The data set HYSTR is used to read in general hysteresis data.

Card HYSTR.1

LOGR	= 1, if the capillary pressures in the hysteresis loop are input on a logarithmic scale (as in Figure 6) = 0, if capillary pressures are input in linear scale.
NTOT	total number of materials for which hysteresis loops will be read in
EPSLN	reversal criterion (determines the minimum difference in liquid saturation between two consecutive time steps for which a reversal from the previous curve is possible)

The following cards (HYSTR.n.1-n.4) define the tabulated hysteresis curves; they should be read in NTOT times, once for each material with hysteretic properties. The order in which the materials are read in determines the material number, NM, that is introduced into argument list of the CALL PCHYST-statement (see Section 4).

Card HYSTR.n.1

L1	number of tabulated sets of values on the main boundary curves
L2	number of tabulated drying scanning curves
L3	number of tabulated wetting scanning curves

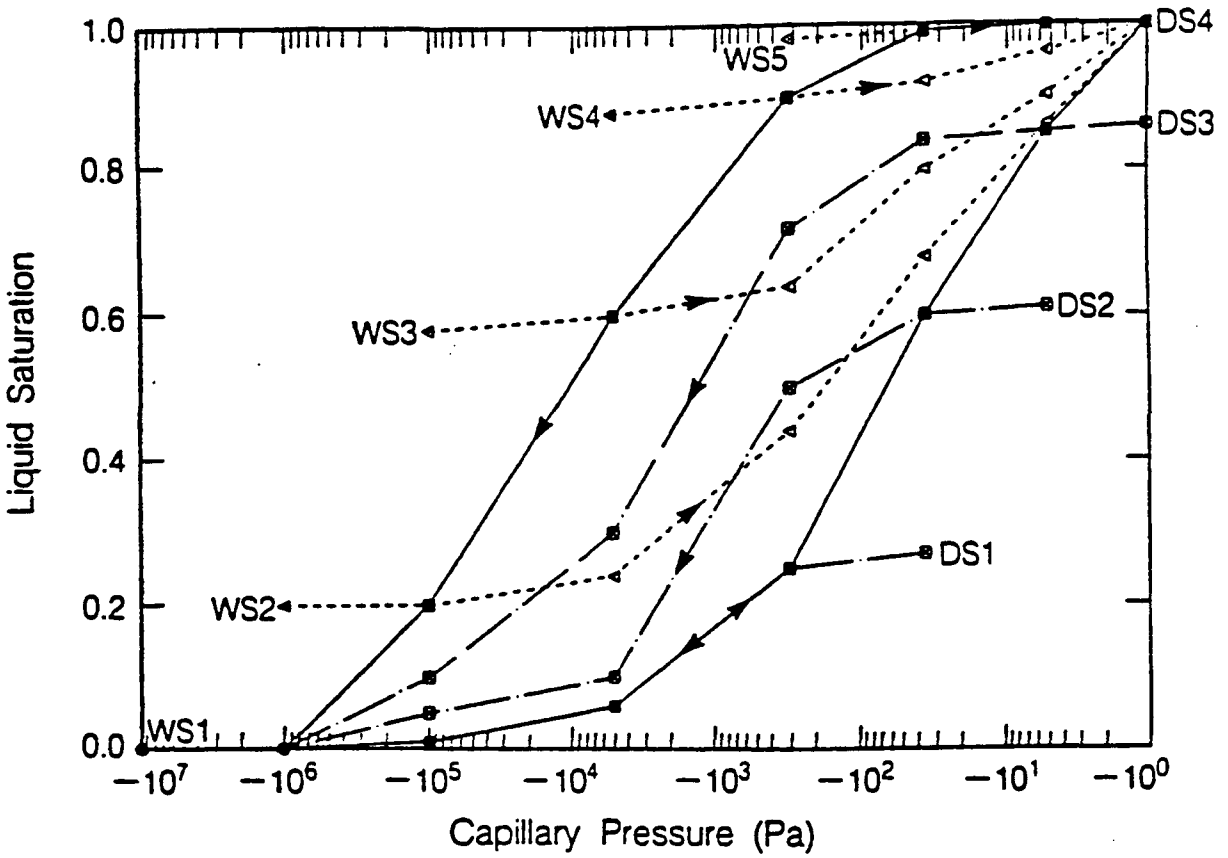

Figure 6. Example of a tabulated hysteresis data. WS1 . . . WS5 are tabulated wetting scanning curves. DS1 . . . DS4 are tabulated drying scanning curves.

Card HYSTR.n.2

This card defines the main boundary curves. Input L1 sets of values running from left to right (across a hysteresis loop such as the one shown in Figure 6)

PSIVAR(j,n)	capillary pressure value (Pa)
SDRY(j,n)	liquid saturation value on the main drying curve corresponding to PSIVAR(j,n)
SWET(j,n)	liquid saturation value on the main wetting curve corresponding to PSIVAR(j,n)

Card HYSTR.n.3

This card defines the primary drying scanning curves. Tabulate for L2 curves starting from the lowest curve (see Figure 6). Values on each curve must be tabulated from right to left.

Card HYSTR.n.3.1

KK1	number of tabulated point nodes on the drying scanning curve

Card HYSTR.n.3.2

Read in KK1 sets of the following values:

DDP(i,j,n)	capillary pressure value (Pa)
DDS(i,j,n)	liquid saturation on the tabulated drying scanning curve corresponding to DDP(i,j,n)

Card HYSTR.n.4

This card defines the primary wetting scanning curves. Tabulate for L3 curves starting from the lowest curve (see Figure 6). Values on each curve must be tabulated from left to right.

Card HYSTR.n.4.1

KK2	number of tabulated points on the wetting scanning curve

Card HYSTR.n.4.2

Read in KK2 sets of the following values

WP(i,j,n)	capillary pressure value (Pa)
WS(i,j,n)	liquid saturation on the tabulated wetting scanning curve corresponding to WP(i,j,n)

DATA SET INC

The following cards (INC.1 and INC.2) are used to read in the element-wise initial conditions. These cards are read in for each element, including those with non-hysteretic properties. For non-hysteretic elements, however, these values are not used and thus any values can be input.

Card INC.1

POLD(NE)	initial capillary pressure of the element, (Pa); solved from the chosen curve (see ICURV) for the given initial liquid saturation
SOLD(NE)	initial liquid saturation of the element
ICURV(NE)	type of the initial curve = 1, start from the main drying curve = 2, start from the main wetting curve = 3, start from a drying scanning curve = 4, start from a wetting scanning curve

Card INC.2

This card specifies the location of the starting point (defined by POLD(NE) and SOLD(NE)) relative to the adjacent tabulated scanning curves of corresponding type. These values are only needed if the starting point is on a scanning curve; if ICURV(NE) is 1 or 2, these values should be input as zeros (blank cards).

NN10(NE)	number of the tabulated curve of proper type above the starting point (numbering starts from the lowest one)
NN20(NE)	number of the tabulated curve of proper type below the starting point
K1 (NE)	number of tabulated values on curve number NN10(NE)
K2 (NE)	number of tabulated values on curve number NN20(NE)
RATIO(NE)	= X1/X2 where X1 = difference in S_l between the starting point and curve number NN10(NE), at pressure POLD(NE) X2 = difference in S_l between curve number NN10 and curve number NN20 at pressure POLD(NE)

An example of an input file is shown in Appendix B.

5.2. Tabulation of the Curves

When tabulating the hysteresis loop, the user should be aware of the following factors:

(1) For both "families" of scanning curves, the pressure values should be given so that tabulated values form straight lines parallel to the S_l-axis (for example, for wetting scanning curves, the pressure value for the second point on the lowest curve should be the same as the pressure of the first point on the second lowest curve).

(2) Imaginary points (those falling outside the actual hysteresis loop) must be introduced at the beginning of each scanning curve (LHS for wetting scanning curves and RHS for drying scanning curves). When interpolating between an imaginary section of a scanning curve (section falling outside the actual loop) and a real scanning curve, interpolation uses the imaginary section also, which should be taken into consideration when the locations of the imaginary nodes are selected.

(3) The number of tabulated points on the wetting scanning curves decreases by one when moving up from one curve to the next; on drying curves the number increases by one.

(4) Scanning curves should be tabulated so that all areas are covered inside the hysteresis loop (see the extreme wetting scanning curves WS1 and WS5 and the extreme drying scanning curves DS1 and DS4 in Figure 6). As in Figure 6, it is a good practice to tabulate the main wetting curve as the first wetting scanning curve and the main drying curve as the last drying scanning curve.

(5) Tabulation along the main boundary curves is independent of the scanning curve tabulation. However, it is wise to choose the points so that resulting interpolation along the main boundary curves gives approximately the same solution as interpolation done between the scanning curves in the same location. This assures that when reversal from the main boundary curve takes place, an appropriate scanning curve interval - fulfilling the $\phi = \phi(S_l)$ as solved from the main curve - can be found.

(6) The program does all the interpolations linearly. If the capillary pressure range is logarithmic as in the Figure 6, this should be specified in the data deck (LOGR = 1). The program will then take logarithms of all the input pressure values, do the

calculations using these logarithmic values and then convert the results back into the exponential form. If LOGR = 1 is used, give $\phi = 10^0$ Pa as input for fully saturated capillary pressure instead of $\phi = 0$ Pa (see Figure 6 and Appendix B).

(7) Even when the slope of a scanning curve is zero, the neighboring nodes on the curve should not have exactly the same S_l. The program divides by the difference of these values.

(8) The program assumes that all the wetting scanning curves converge toward one point ($S_l = 1$) and all the drying scanning curves toward another point ($S_l = 0$). If the user wants the drying scanning curves to end somewhere on the main drying curve, for example, this can be done by tabulating the lower part of the curve so that it follows the main drying curve.

As an example, the hysteresis loop in Figure 6 has been tabulated. It is shown in the input file in Appendix B.

6.0 EXAMPLE PROBLEM

To demonstrate the use of HYSTR, a simple sample problem has been developed. Three elements - two with hysteretic capillary pressure-liquid saturation relationships and one with no hysteresis - are examined. The element properties are shown in Table 1. The corresponding input file is shown in Appendix B.

Table 1. Element properties in the sample problem.

Element	Initial liquid saturation	Initial curve type	Capillary pressure relationship
E1	0.40	1	Hysteresis loop as shown in Figure 6
E2	0.40	0	No hysteresis
E3	0.40	4	Hysteresis loop as shown in Figure 6, with capillary pressure values multiplied by 10^{+2}

Rather than incorporating HYSTR into a numerical simulator and solving an actual problem, a simple main program was written to stepwise increase and decrease the liquid saturation of the elements and to call HYSTR to calculate the corresponding capillary pressures. The main routine is shown in Appendix C. Using this main program together with HYSTR and the input file in Appendix B, the output file in Appendix D was produced. The hysteresis path for element E1 as obtained from the output is graphically represented in Figure 7.

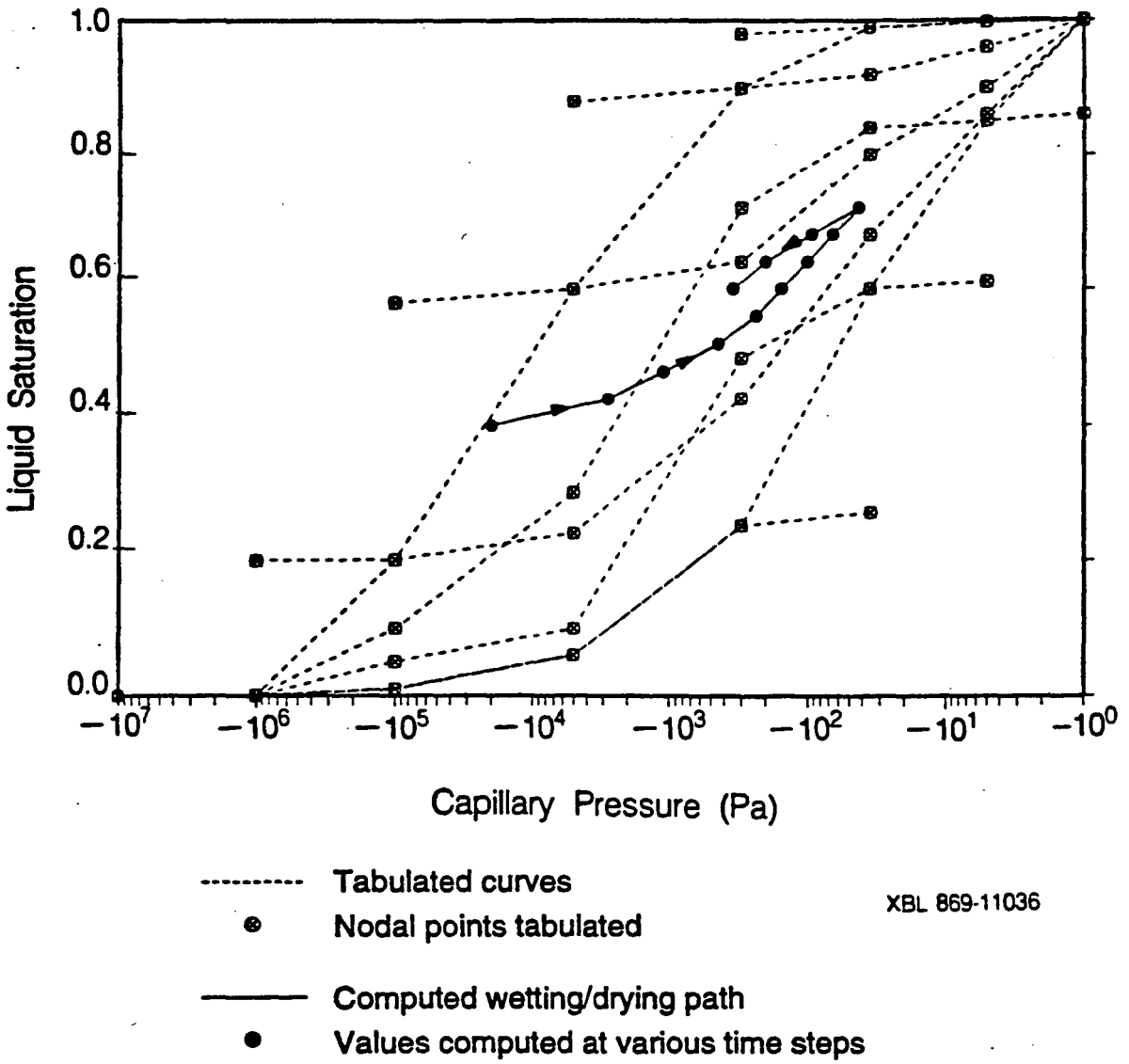

Figure 7. Wetting/drying path of element E1 in the sample problem.

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```
c
c-----------------------------------------------------------------------
c
c    This is HYSTR , a set of subroutines to calculate capillary pressure for
c    a given liquid saturation under hysteretic conditions
c
c    Program was written by Auli Niemi at Lawrence Berkeley Laboratory
c
c    For information on the program see the users guide
c    Niemi,A and Bodvarsson,G.S. Capillary Hysteresis Model HYSTR - Users Manual
c    Lawrence Berkeley Laboratory ,LBL        ,1986.
c
c-----------------------------------------------------------------------
c    Subroutines satdry,satwet,idea of vertical search in srcvrt and part of
c    data input are modifided from Wang,Bodvarsson,Wan and Tsang (1981)
c-----------------------------------------------------------------------
c    When program is incorporated into a numerical simulator
c
c   1) incorporation is done through 3 call statements (see users manual)
c   2) dimensions in the common blocks should be checked and possibly changed
c      ( see tabulation below)
c   3) I/O commands should be checked for possible disagreement with other
c      I/O commands in the actual numerical simulator (see file opening
c      statements (commented out)in subroutine inhyst and the read
c      and write statements)
c
c_______________________________________________________________________
c
c     LIST OF ALL THE COMMON BLOCKS WITH THEIR PRESENT DIMENSIONS
c
c     dimensions for the tabulated values used in hysteresis calculations
c     common/prssat/psivar(20,27),sdry(20,27),swet(20,27)
c     common/dscan/ddp(40,15,27),dds(40,15,27)
c     common/wscan/wp(40,15,27),ws(40,15,27)
c     common/tab/ltabps(27),ltabd(27),ltabw(27),
c    1           nunsat,aunsat(27),nsat(27)
c     common/tab2/kd(15,27),kw(15,27)
c     common/icon/icurv(500),pcin(500)
c     common/savehr/pold(500),sold(500)
c     common/lg/logr,epsln
c     common/tmphys/ptmpo(500),stmpo(500),ktmpo(500)
c     common/calc/k1(500),k2(500),ratio(500),nn10(500),nn20(500)
c     common/savecr/nn1sa(500),nn2sa(500),ratisa(500),icntr(500),
c    1 k1sa(500),k2sa(500)
c
c     dimension a(40,15,27)
c     dimension b(40,15,27)
c
c     CHANGES IN THE DIMENSIONS SHOULD BE DONE ACCORDING TO THE FOLLOWING
c     ------------------------
c     TABULATION  IN WHICH :
c
c     ne  = number of elements
c     nmt = number of materials
c     nns = max. number of tabulated nodes on any one scanning curve
c     ncs = max. of the following two values:
c           number of tabulated drying scanning curves or number of tabulated
c           wetting scanning  curves
c     nb = number of tabulated nodes on main boundary curves
c
```

```
c
c     common/prssat/psivarc(nb,nmt),sdry(nb,nmt),swet(nb,nmt)
c     common/dscan/ddp(nns,ncs,nmt),dds(nns,ncs,nmt)
c     common/wscan/wp(nns,ncs,nmt),ws(nns,ncs,nmt)
c     common/tab/ltabps(nmt),ltabd(nmt),ltabw(nmt),
c    1 nunsat,aunsat(nmt),nsat(nmt)
c     common/tab2/kd(ncs,nmt),kw(ncs,nmt)
c     common/icon/icurv(ne),pcin(ne)
c     common/savehr/pold(ne),sold(ne)
c     common/tmphys/ptmpo(ne),stmpo(ne),ktmpo(ne)
c     common/calc/k1(ne),k2(ne),ratio(ne),nn10(ne),nn20(ne)
c     common/savecr/nn1sa(ne),nn2sa(ne),ratisa(ne),icntr(ne),
c    1 k1sa(ne),k2sa(ne),kcyc0
c
c     dimension a(nns,ncs,nmt)
c     dimension b(nns,ncs,nmt)
c
c_____________________________________________________________________________
c
c
      subroutine inhyst(nele)
c     -----------------------
c
c--open the files
c
c     open(7,file='hsin',status='old')
c     open(8,file='hsout',status='new')
c
c--subroutine to read in the initial conditions and tabulated curves
c
c     dimensions for the tabulated values used in hysteresis calculations
      common/prssat/psivar(20,27),sdry(20,27),swet(20,27)
      common/dscan/ddp(40,15,27),dds(40,15,27)
      common/wscan/wp(40,15,27),ws(40,15,27)
      common/tab/ltabps(27),ltabd(27),ltabw(27),
     1           nunsat,aunsat(27),nsat(27)
      common/tab2/kd(15,27),kw(15,27)
      common/icon/icurv(500),pcin(500)
      common/savehr/pold(500),sold(500)
      common/lg/logr,epsin
      common/savecr/nn1sa(500),nn2sa(500),ratisa(500),icntr(500),
     1 k1sa(500),k2sa(500)
      common/calc/k1(500),k2(500),ratio(500),nn10(500),nn20(500)
c--- read in the hysteresis loop
c
        logr=0
        read(7,*)logr,ntot,epsin
        write(8,1554)ntot,epsin
        if(logr.eq.1)write(8,1564)
c
        n=1
  801   read(7,*) l1,l2,l3
  565   write(8,122)n,l1,l2,l3
c
        ltabps(n) = l1
        ltabd(n) = l2
        ltabw(n) = l3
c
        write(8,1560)
        write(8,1561)
```

```
      do 811 j=1,l1
      read(7,*) psivar(j,n),sdry(j,n),swet(j,n)
      if(logr.eq.1)psivar(j,n)=-alog10(-psivar(j,n))
      write(8,1559) psivar(j,n),sdry(j,n),swet(j,n)
 811  continue
c
      write(8,1562)
      do 821 j=1,l2
      read(7,*) kk1
      write(8,1555)kk1
      kd(j,n)=kk1
      do 821 i=1,kk1
      read(7,*) ddp(i,j,n),dds(i,j,n)
      if(logr.eq.1)ddp(i,j,n)=-alog10(-ddp(i,j,n))
      write(8,1558) ddp(i,j,n),dds(i,j,n)
 821  continue
c
      write(8,1563)
      do 831 j=1,l3
      read(7,*) kk2
      write(8,1555)kk2
      kw(j,n)=kk2
      do 831 i=1,kk2
      read(7,*) wp(i,j,n),ws(i,j,n)
      if(logr.eq.1)wp(i,j,n)=-alog10(-wp(i,j,n))
      write(8,1558)wp(i,j,n),ws(i,j,n)
 831  continue
      n=n+1
      if(n.gt.ntot)goto 802
      goto 801
 802  continue
c
c-- read in the initial conditions for hysteresis calculation---------------
c
      write(8,1901)
      write(8,1902)
c
      do 1010 i=1,nele
      read(7,*) pold(i),sold(i),icurv(i)
      read(7,*)nn10(i),nn20(i),k1(i),k2(i),ratio(i)
      ratisa(i)=ratio(i)
      nn1sa(i)=nn10(i)
      nn2sa(i)=nn20(i)
      k1sa(i)=k1(i)
      k2sa(i)=k2(i)
      icntr(i)=0
      write(8,1922)i,pold(i),sold(i),icurv(i),nn10(i),nn20(i),k1(i),
     1k2(i),ratio(i)
      if(logr.eq.1)pold(i)=-alog10(-pold(i))
 1010 continue
c
c
 121   format(3i5)
 122   format(/,10x,' material no =',i5,/,
     110x,' no of tabul. sets of values on main curves =',i5,/,
     210x,' no of tabulated drying scanning curves =',i5,/,
     310x,' no of tabulated wetting scanning curves =',i5,/)
 1554 format(/,10x,' HYSTERESIS TABULATION FOR ',I5,' MATERIALS',/,
     110x,' reversal criterion = ',e10.3,/)
 1555 format(/,10x,' number of tabulated nodes',i5,/)
```

```
 1556 format(i5)
 1557 format(2i5,e10.3)
 1558 format(10x,2e10.3)
 1559 format(10x,3e10.3)
 1560 format(10x,' tabulation of the main curves ',/,
     110x,' ------------------------',/)
 1561 format(10x,' psivar= ','   sdry=  ','   swet=   ',/)
 1562 format(//,10x,'drying curves',/,
     110x,'--------------',/,
     110x,'   ddp=   ','    dds=   ')
 1563 format(//,10x,'wetting curves',/,
     110x,'--------------',/,
     110x,'   wp=   ','    ws=   ')
 1564 format(/,10x,' CAPILLARY PRESSURES ARE READ IN EXPONENTIAL FORM')
 1901 format(/,10x,' INITIAL CONDITIONS FOR HYSTERESIS CALCULATION ',/)
 1902 format(/,10X,'  ne  pold(ne)  sold(ne)  icurv  nn1  nn2   k1   k2
     1  ratio',/)
 1921 format(2e10.3,i5)
 1922 format(10x,i5,2e10.3,5i5,2x,e10.3)
c
      return
      end

c
c
      subroutine pchyst (s,p,nm,ne,k)
c     ____________________________________
c
c--- subroutine to calculate capillary pressure for given saturation and
c    save the calculated values from each iteration
c
c
      common/prssat/psivar(20,27),sdry(20,27),swet(20,27)
      common/dscan/ddp(40,15,27),dds(40,15,27)
      common/wscan/wp(40,15,27),ws(40,15,27)
      common/tab/ltabps(27),ltabd(27),ltabw(27),
     1           nunsat,aunsat(27),nsat(27)
      common/tab2/kd(15,27),kw(15,27)
      common/savehr/pold(500),sold(500)
      common/calc/k1(500),k2(500),ratio(500),nn10(500),nn20(500)
      common/cyc/kcyc,iter,iterc,timin,sumtim,gf
      common/konit/kon,delt,igood
      common/icon/icurv(500),pcin(500)
      common/tmphys/ptmpo(500),stmpo(500),ktmpo(500)
      common/savecr/nn1sa(500),nn2sa(500),ratisa(500),icntr(500),
     1 k1sa(500),k2sa(500)
      common/lg/logr,epsln,npr
      common/svz/noite,mop(24)

c
c first choose the type of the curve
c
c turning information calculated only once for each element in one iteration
c for incremental calculations (k=2,3,4) use the same curves than for k=1 to
c assure that derivatives will be calculated correctly
        if(k.ne.1) goto 19
        if(sold(ne).eq.1..and.s.lt.1.)goto 7
        if(sold(ne).eq.0..and.s.gt.0.)goto 6
        if(abs(sold(ne)-s).lt.epsln) goto 16
        if(sold(ne).lt.s) goto 15
```

```
c
c  cases s < sold (drying,turning from wetting to drying  scanning, drying
c  scanning,turning from wetting scanning to drying scanning)
        if(icurv(ne).ne.3)goto 14
        nn10(ne)=nn1sa(ne)
        nn20(ne)=nn2sa(ne)
        ratio(ne)=ratisa(ne)
        k1(ne)=k1sa(ne)
        k2(ne)=k2sa(ne)
 14     goto(7,70,55,70) icurv(ne)
c
c  cases s > sold (turning from drying to wetting scanning,wetting,turning
c  from drying scanning to wetting scanning, wetting scanning)
 15     continue
        if(icurv(ne).ne.4)goto 155
        nn10(ne)=nn1sa(ne)
        nn20(ne)=nn2sa(ne)
        ratio(ne)=ratisa(ne)
        k1(ne)=k1sa(ne)
        k2(ne)=k2sa(ne)
 155    goto (80,6,80,50) icurv(ne)
c
c cases where reversal in saturation so small that the same curve is used
  16     if(icntr(ne).ne.1)goto 166
         nn10(ne)=nn1sa(ne)
         nn20(ne)=nn2sa(ne)
         ratio(ne)=ratisa(ne)
         k1(ne)=k1sa(ne)
         k2(ne)=k2sa(ne)
 166     goto(7,6,55,50)icurv(ne)
c
c for k=2,3,4, use the same curve than for k=1
  19    goto(7,6,55,50)ktmpo(ne)
c
c  solution from the drying curve
  7    call satdry(nm,p,s)
       kcurv=1
       goto 201
c
c  solution from the wetting curve
   6  call satwet(nm,p,s)
       kcurv=2
       goto 201
c
c
c wetting scanning
 50    ich=2
       call scn(ws,wp,s,p,nm,ich,ne)
       kcurv=4
       goto 201
c
c drying scanning
 55    ich=1
       call scn(dds,ddp,s,p,nm,ich,ne)
       kcurv=3
       goto 201
c
c turn from drying or drying scanning to wetting scanning
  80   if(k.ne.1)goto 50
       idch=6
```

```
      ktab=ltabw(nm)
      call srcvrt(ktab,ws,nm,idch,ne)
  81  nn1=nn10(ne)
      nn2=nn20(ne)
      k1(ne)=kw(nn1,nm)
      k2(ne)=kw(nn2,nm)
      call srchor(pold(ne),wp,i,nm,ne,idch)
      ich=2
      call crveqn(aup,bup,adn,bdn,i,nm,ws,wp,ich,ne)
      sdn=adn+bdn*pold(ne)
      sup=aup+bup*pold(ne)
      if(sold(ne).ge.sdn.and.sold(ne).le.sup)goto 83
      nn10(ne)=nn1-1
      nn20(ne)=nn2-1
      goto 81
 83   ratio(ne)=(sup-sold(ne))/(sup-sdn)
      goto 50
c
c
c turn from wetting or wetting scanning to drying scanning
  70  if(k.ne.1) goto 55
      idch=5
      ktab=ltabd(nm)
      call srcvrt(ktab,dds,nm,idch,ne)
  71  nn1=nn10(ne)
      nn2=nn20(ne)
      k1(ne)=kd(nn1,nm)
      k2(ne)=kd(nn2,nm)
      call srchor(pold(ne),ddp,i,nm,ne,idch)
      ich=1
      call crveqn(aup,bup,adn,bdn,i,nm,dds,ddp,ich,ne)
      sdn=adn+bdn*pold(ne)
      sup=aup+bup*pold(ne)
      if(sold(ne).ge.sdn.and.sold(ne).le.sup)goto 73
      nn10(ne)=nn1+1
      nn20(ne)=nn2+1
      goto 71
 73   ratio(ne)=(sup-sold(ne))/(sup-sdn)
c      write(8,*)ratio(ne),sup,sold(ne),sdn
      goto 55
c
c come here to save the calculated values
c save values from k=1. to be used as 'old' values at next time ste
c
 201  if(k.ne.1)goto 200
      ptmpo(ne)=p
      stmpo(ne)=s
      ktmpo(ne)=kcurv
      if(ktmpo(ne).ne.icurv(ne))icntr(ne)=1
 200  if(logr.eq.1) p=-10.**(-p)
      return
      end
c

        subroutine satdry(nm,p,s)
c        ------------------------
c
c       subroutine to calculate pressure on drying curve
c       (saturation-pressure table)
c
```

```
      common/prssat/psivar(20,27),sdry(20,27),swet(20,27)
      common/tab/ltabps(27),ltabd(27),ltabw(27),
     1          nunsat,aunsat(27),nsat(27)
c
      ll = ltabps(nm)
      n1=0
      n2=-1
c
      do 110 j=1,ll
      if(s.eq.sdry(n1+j,nm)) goto 121
      if(s .gt. sdry(n1+j,nm)) go to 110
      if(j .eq. 1) stop 1003
c     write(8,15)j,sdry(n1+j,nm),sdry(n2+j,nm),psivar(n1+j,nm),
c    1 psivar(n2+j,nm)
c
      p=psivar(n2+j,nm)+(psivar(n1+j,nm)-psivar(n2+j,nm))*
     1 (s-sdry(n2+j,nm))/(sdry(n1+j,nm)-sdry(n2+j,nm))
      goto 122
  121 p=psivar(n1+j,nm)
  122 goto 120
  110 continue
      stop 1004
c
  120 return
      end
c
c
      subroutine satwet(nm,p,s)
c      ------------------------
c
c     subroutine to calculate pressure on wetting curve
c     (saturation pressure table)
      common/prssat/psivar(20,27),sdry(20,27),swet(20,27)
      common/tab/ltabps(27),ltabd(27),ltabw(27),
     1          nunsat,aunsat(27),nsat(27)
c
      ll = ltabps(nm)
      n1=0
      n2=-1
c
      do 150 j=1,ll
      if(s.eq.swet(n1+j,nm)) goto 121
      if(s .gt. swet(n1+j,nm)) go to 150
      if(j .eq. 1) stop 1006
c      write(8,15)j,swet(n1+j,nm),swet(n2+j,nm),psivar(n1+j,nm),
c    1 psivar(n2+j,nm)
c
      p=psivar(n2+j,nm)+(psivar(n1+j,nm)-psivar(n2+j,nm))*
     1 (s-swet(n2+j,nm))/(swet(n1+j,nm)-swet(n2+j,nm))
     goto 122
  121 p=psivar(n1+j,nm)
  122 go to 160
  150 continue
      stop 1007
c
  160 return
      end
c
c
      subroutine scn(a,b,s,p,nm,ich,ne)
```

```
c
        common/calc/k1(500),k2(500),ratio(500),nn10(500),nn20(500)
        common/savehr/pold(500),sold(500)
        dimension a(40,15,27)
        dimension b(40,15,27)
c
        if(s.eq.sold(ne))p=pold(ne)
        if(s.eq.sold(ne))return
c
        nn1=nn10(ne)
        nn2=nn20(ne)
        m=1
        if(ich.eq.1)m=2
        k3=k1(ne)-1
  123   do 121 i=m,k3
        if(ich.eq.2)goto 133
c
c drying scanning
        sen1=a(i,nn1,nm)-ratio(ne)*(a(i,nn1,nm)-a(i-1,nn2,nm))
        sen2=a(i+1,nn1,nm)-ratio(ne)*(a(i+1,nn1,nm)-a(i,nn2,nm))
        if(b(i,nn1,nm).eq.b(i+1,nn1,nm)) goto 126
        slope=(sen1-sen2)/(b(i,nn1,nm)-b(i+1,nn1,nm))
        aa=sen1-slope*b(i,nn1,nm)
        if(slope.eq.0)goto 125
        pp=(s-aa)/slope
        if(pp.le.b(i,nn1,nm).and.pp.ge.b(i+1,nn1,nm)) goto 122
        goto 121
c
  133   sen1=a(i+1,nn1,nm)-ratio(ne)*(a(i+1,nn1,nm)-a(i+2,nn2,nm))
        sen2=a(i,nn1,nm)-ratio(ne)*(a(i,nn1,nm)-a(i+1,nn2,nm))
        if(b(i+1,nn1,nm).eq.b(i,nn1,nm))goto 126
        slope=(sen1-sen2)/(b(i+1,nn1,nm)-b(i,nn1,nm))
        aa=sen1-slope*b(i+1,nn1,nm)
        if(slope.eq.0)goto 125
        pp=(s-aa)/slope
        if(pp.le.b(i+1,nn1,nm).and.pp.ge.b(i,nn1,nm)) goto 122
  121   continue
        stop 1177
  122   p=pp
        return
  125   write(8,223)
  223   format(' interpolated scanning curve slope equal to zero,',/,
     1 ' does not provide unique solution for capillary pressure ')
        stop
  126   write(8,224)
  224   format(' erroneous scanning curve tabulation ')
        stop
        end
c
c
        subroutine srcvrt(ktab,a,nm,idch,ne)
        dimension a(40,15,27)
        common/calc/k1(500),k2(500),ratio(500),nn10(500),nn20(500)
        common/savehr/pold(500),sold(500)
c
c compare location of the turning point always to the nodes laying on
c main wetting and drying curves ( second tabulated node for each scanning
c curve
        n1=0
        n2=-1
```

```
      do 120 j=1,ktab
      if(sold(ne).gt.a(2,n1+j,nm)) goto 120
      if(j.eq.1)goto 122
      goto 121
  120 continue
      if(idch.ne.6)stop 7777
      j=ktab
      goto 121
  122 if(idch.ne.5)stop 6666
      j=2
c
c
 121  nn10(ne)=n1+j
      nn20(ne)=n2+j
      return
      end
c
c
      subroutine crveqn(aup,bup,adn,bdn,i,nm,a,b,ich,ne)
c
      common/calc/k1(500),k2(500),ratio(500),nn10(500),nn20(500)
      dimension a(40,15,27)
      dimension b(40,15,27)
      nn1=nn10(ne)
      nn2=nn20(ne)
c
      if(b(i+1,nn1,nm).eq.b(i,nn1,nm))goto 211
      bup=(a(i+1,nn1,nm)-a(i,nn1,nm))/(b(i+1,nn1,nm)-b(i,nn1,nm))
      aup=a(i,nn1,nm)-bup*b(i,nn1,nm)
      if(ich.eq.2) goto 209
      if(b(i,nn2,nm).eq.b(i-1,nn2,nm)) goto 211
      bdn=(a(i,nn2,nm)-a(i-1,nn2,nm))/(b(i,nn2,nm)-b(i-1,nn2,nm))
      adn=a(i,nn2,nm)-bdn*b(i,nn2,nm)
      goto 208
c
 209  if(b(i+2,nn2,nm).eq.b(i+1,nn2,nm))goto 211
      bdn=(a(i+2,nn2,nm)-a(i+1,nn2,nm))/(b(i+2,nn2,nm)-b(i+1,nn2,nm))
      adn=a(i+1,nn2,nm)-bdn*b(i+1,nn2,nm)
c
 208  return
 211  write(8,443)
 443  format('  erroneous scanning curve tabulation ,'/,
     1 'execution terminated ')
      stop
      end
c
      subroutine srchor(p,b,i,nm,ne,idch)
c
      common/icon/icurv(500),pcin(500)
      common/calc/k1(500),k2(500),ratio(500),nn10(500),nn20(500)
      dimension b(40,15,27)
c
      nn1=nn10(ne)
      m=2
      if(idch.eq.5)m=3
c
      do 210 i=m,k1(ne)
      if(idch.eq.5)goto 207
      if(p.lt.b(i,nn1,nm)) goto 211
      goto 210
```

```
 207    if(p.gt.b(i,nn1,nm)) goto 211
 210    continue
        stop 4444
 211    i=i-1
        return
        end
c
c
        subroutine savehs(ne)
c       ____________________
c
c---subroutine to save the results from the timestep after convergence '
c   has bee achievd
c
      common/savehr/pold(500),sold(500)
      common/icon/icurv(500),pcin(500)
      common/tmphys/ptmpo(500),stmpo(500),ktmpo(500)
      common/nn/nel,ncon,nogn
      common/rpcap/irp(27),rp(7,27),icp(27),cp(7,27),irpd,rpd(7),
     xicpd,cpd(7)
      common/e2/matx(1)
      common/savecr/nn1sa(500),nn2sa(500),ratisa(500),icntr(500),
     1 k1sa(500),k2sa(500)
      common/calc/k1(500),k2(500),ratio(500),nn10(500),nn20(500)
      common/lg/logr,epsln
c
c   this subroutine is called only when convergence has been achieved
c
        pold(ne)=ptmpo(ne)
        sold(ne)=stmpo(ne)
        icurv(ne)=ktmpo(ne)
        nn1sa(ne)=nn10(ne)
        nn2sa(ne)=nn20(ne)
        ratisa(ne)=ratio(ne)
        icntr(ne)=0
        k1sa(ne)=k1(ne)
        k2sa(ne)=k2(ne)
c
        return
        end
```

APPENDIX B

```
1    2     1.e-4          LOGR,NTOT,EPSLN
7    4    5               L1,L2,L3 FOR MATERIAL NO 1
     -0.100E+07 0.000E+00 0.000E+00 tabulation of boundary curves
     -0.100E+06 0.200E+00 0.100E-01
     -0.500E+04 0.600E+00 0.600E-01
     -0.300E+03 0.900E+00 0.200E+00
     -0.350E+02 0.990E+00 0.600E+00
     -0.500E+01 0.997E+00 0.850E+00
     -0.100E+01 0.100E+01 0.100E+01
5                                    lowest drying scanning curve (DS1)
     -0.350E+02 0.270E+00
     -0.300E+03 0.250E+00
     -0.500E+04 0.600E-01
     -0.100E+06 0.100E-01
     -0.100E+07 0.000E+00
6                                     (DS2)
     -0.500E+01 0.610E+00
     -0.350E+02 0.600E+00
     -0.300E+03 0.500E+00
     -0.500E+04 0.100E+00
     -0.100E+06 0.500E-01
     -0.100E+07 0.000E+00
7                                     (DS3)
     -0.100E+01 0.860E+00
     -0.500E+01 0.850E+00
     -0.350E+02 0.840E+00
     -0.300E+03 0.720E+00
     -0.500E+04 0.300E+00
     -0.100E+06 0.100E+00
     -0.100E+07 0.000E+00
8                                     uppermost drying scanning curve (DS4)
     -0.500E+00 0.100E+01  --actual value of this imag.node doesnt matter-
     -0.100E+01 0.999E+00
     -0.500E+01 0.997E+00
     -0.350E+02 0.990E+00
     -0.300E+03 0.900E+00
     -0.500E+04 0.600E+00
     -0.100E+06 0.200E+00
     -0.100E+07 0.000E+00
8                                  lowest tabulated wetting curve (WS1)
     -0.100E+08 0.000E+00  --actual value of this imag.node doesnt matter--
     -0.100E+07 0.100E-02
     -0.100E+06 0.100E-01
     -0.500E+04 0.800E-01
     -0.300E+03 0.250E+00
     -0.350E+02 0.600E+00
     -0.500E+01 0.850E+00
     -0.100E+01 0.100E+01
7                                    (WS2)
     -0.100E+07 0.199E+00
     -0.100E+06 0.200E+00
     -0.500E+04 0.240E+00
     -0.300E+03 0.440E+00
     -0.350E+02 0.680E+00
     -0.500E+01 0.860E+00
     -0.100E+01 0.100E+01
6                                    (WS3)
     -0.100E+06 0.580E+00
     -0.500E+04 0.600E+00
     -0.300E+03 0.640E+00
```

```
      -0.350E+02 0.800E+00
      -0.500E+01 0.900E+00
      -0.100E+01 0.100E+01
5                                   (WS4)
      -0.500E+04 0.880E+00
      -0.300E+03 0.900E+00
      -0.350E+02 0.920E+00
      -0.500E+01 0.960E+00
      -0.100E+01 0.100E+01
4                                   (WS5)
      -0.300E+03 0.989E+00
      -0.350E+02 0.990E+00
      -0.500E+01 0.997E+00
      -0.100E+01 0.100E+01
7     4    5                        L1,L2,L3 FOR MATERIAL N;O 2
      -0.100E+09 0.000E+00 0.000E+00 tabulation of boundary curves
      -0.100E+08 0.200E+00 0.100E-01
      -0.500E+06 0.600E+00 0.600E-01
      -0.300E+05 0.900E+00 0.200E+00
      -0.350E+04 0.990E+00 0.600E+00
      -0.500E+03 0.997E+00 0.850E+00
      -0.100E+03 0.100E+01 0.100E+01
5                                    lowest drying scanning curve (DS1)
      -0.350E+04 0.270E+00
      -0.300E+05 0.250E+00
      -0.500E+06 0.600E-01
      -0.100E+08 0.100E-01
      -0.100E+09 0.000E+00
6                                    (DS2)
      -0.500E+03 0.810E+00
      -0.350E+04 0.600E+00
      -0.300E+05 0.500E+00
      -0.500E+06 0.100E+00
      -0.100E+08 0.500E-01
      -0.100E+09 0.000E+00
7                                    (DS3)
      -0.100E+03 0.860E+00
      -0.500E+03 0.850E+00
      -0.350E+04 0.840E+00
      -0.300E+05 0.720E+00
      -0.500E+06 0.300E+00
      -0.100E+08 0.100E+00
      -0.100E+09 0.000E+00
8                                    uppermost drying scanning curve (DS4)
      -0.500E+02 0.100E+01  --actual value of this imag.node doesnt matter-
      -0.100E+03 0.999E+00
      -0.500E+03 0.997E+00
      -0.350E+04 0.990E+00
      -0.300E+05 0.900E+00
      -0.500E+06 0.600E+00
      -0.100E+08 0.200E+00
      -0.100E+09 0.000E+00
8                                 lowest tabulated wetting curve (WS1)
      -0.100E+08 0.000E+00  --actual value of this imag.node doesnt matter--
      -0.100E+09 0.100E-02
      -0.100E+08 0.100E-01
      -0.500E+06 0.600E-01
      -0.300E+05 0.250E+00
      -0.350E+04 0.600E+00
      -0.500E+03 0.850E+00
```

```
          -0.100E+03 0.100E+01
    7                                        (WS2)
          -0.100E+09 0.199E+00
          -0.100E+08 0.200E+00
          -0.500E+06 0.240E+00
          -0.300E+05 0.440E+00
          -0.350E+04 0.680E+00
          -0.500E+03 0.860E+00
          -0.100E+03 0.100E+01
    6                                        (WS3)
          -0.100E+08 0.580E+00
          -0.500E+06 0.600E+00
          -0.300E+05 0.640E+00
          -0.350E+04 0.800E+00
          -0.500E+03 0.900E+00
          -0.100E+03 0.100E+01
    5                                        (WS4)
          -0.500E+06 0.880E+00
          -0.300E+05 0.900E+00
          -0.350E+04 0.920E+00
          -0.500E+03 0.960E+00
          -0.100E+03 0.100E+01
    4                                        (WS5)
          -0.300E+05 0.989E+00
          -0.350E+04 0.990E+00
          -0.500E+03 0.997E+00
          -0.100E+03 0.100E+01
    -2.e+4        .40    1          pold(1),sold(1),icurv(1)
0     0   0   0   0                 dummy values
-   -2.e+0        .40    0          no hystr. for this element
0     0   0   0   0                 dummy values
    -9.e+5        .40    4          pold(3),sold(3),icurv(3)
3     2   6   7     0.5454545       nn10,nn20,k1,k2,ratio
```

APPENDIX C

```
c sample main program
c
      dimension s(3),p(3)
      k=1
      a=0.04
      do 10 i=1,3
 10   s(i)=.4
c
c-----------------------------------------------------------------------
      call inhyst(3)
c-----------------------------------------------------------------------
c
      write(8,12)
c
 20   continue
      k=k+1
      do 30 i=1,3
      s(i)=s(i)+a
      sl=s(i)
      if(i.eq.1)nm=1
      if(i.eq.2)goto 40
      if(i.eq.3)nm=2
c-----------------------------------------------------------------------
      call pchyst(sl,pc,nm,i,1)
c-----------------------------------------------------------------------
      p(i)=pc
      goto 30
 40   p(2)=0.
 30   write(8,11)i,s(i),p(i)
c
      do 60 i=1,3
c-----------------------------------------------------------------------
      if(i.ne.2)call savehs(i)
c-----------------------------------------------------------------------
 60   continue
c
      if(k.gt.8)a=-0.04
      if(k.eq.12)goto 50
      goto 20
c
 11  format(' ne=',i5,' s(ne)=',e10.3,' p(ne)= ',e10.3)
 12  format(/,' WETTING/DRYING RESULTS FOR THE 3 ELEMENTS',/)
c
 50   stop
      end
```

APPENDIX D

```
HYSTERESIS TABULATION FOR     2 MATERIALS
reversal criterion =  0.100E-03

CAPILLARY PRESSURES ARE READ IN EXPONENTIAL FORM

material no =    1
no of tabul. sets of values on main curves =    7
no of tabulated drying scanning curves =    4
no of tabulated wetting scanning curves =    5

tabulation of the main curves
-----------------------

psivar=    sdry=     swet=

-0.600E+01 0.000E+00 0.000E+00
-0.500E+01 0.200E+00 0.100E-01
-0.370E+01 0.600E+00 0.600E-01
-0.248E+01 0.900E+00 0.200E+00
-0.154E+01 0.990E+00 0.600E+00
-0.699E+00 0.997E+00 0.850E+00
 0.000E+00 0.100E+01 0.100E+01


drying curves
---------------
   ddp=       dds=

 number of tabulated nodes    5

-0.154E+01 0.270E+00
-0.248E+01 0.250E+00
-0.370E+01 0.600E-01
-0.500E+01 0.100E-01
-0.600E+01 0.000E+00

 number of tabulated nodes    6

-0.699E+00 0.610E+00
-0.154E+01 0.600E+00
-0.248E+01 0.500E+00
-0.370E+01 0.100E+00
-0.500E+01 0.500E-01
-0.600E+01 0.000E+00

 number of tabulated nodes    7

 0.000E+00 0.860E+00
-0.699E+00 0.850E+00
-0.154E+01 0.840E+00
-0.248E+01 0.720E+00
-0.370E+01 0.300E+00
-0.500E+01 0.100E+00
-0.600E+01 0.000E+00

 number of tabulated nodes    8
```

```
 0.301E+00 0.100E+01
 0.000E+00 0.999E+00
-0.699E+00 0.997E+00
-0.154E+01 0.990E+00
-0.248E+01 0.900E+00
-0.370E+01 0.600E+00
-0.500E+01 0.200E+00
-0.600E+01 0.000E+00

wetting curves
---------------
   wp=        ws=

 number of tabulated nodes    8

-0.700E+01 0.000E+00
-0.600E+01 0.100E-02
-0.500E+01 0.100E-01
-0.370E+01 0.600E-01
-0.248E+01 0.250E+00
-0.154E+01 0.600E+00
-0.699E+00 0.850E+00
 0.000E+00 0.100E+01

 number of tabulated nodes    7

-0.600E+01 0.199E+00
-0.500E+01 0.200E+00
-0.370E+01 0.240E+00
-0.248E+01 0.440E+00
-0.154E+01 0.680E+00
-0.699E+00 0.860E+00
 0.000E+00 0.100E+01

 number of tabulated nodes    6

-0.500E+01 0.580E+00
-0.370E+01 0.600E+00
-0.248E+01 0.640E+00
-0.154E+01 0.800E+00
-0.699E+00 0.900E+00
 0.000E+00 0.100E+01

 number of tabulated nodes    5

-0.370E+01 0.880E+00
-0.248E+01 0.900E+00
-0.154E+01 0.920E+00
-0.699E+00 0.960E+00
 0.000E+00 0.100E+01

 number of tabulated nodes    4

-0.248E+01 0.989E+00
-0.154E+01 0.990E+00
-0.699E+00 0.997E+00
 0.000E+00 0.100E+01

 material no =    2
```

```
 no of tabul. sets of values on main curves =    7
 no of tabulated drying scanning curves =    4
 no of tabulated wetting scanning curves =    5

 tabulation of the main curves
 -----------------------

 psivar=    sdry=     swet=

-0.800E+01 0.000E+00 0.000E+00
-0.700E+01 0.200E+00 0.100E-01
-0.570E+01 0.600E+00 0.600E-01
-0.448E+01 0.900E+00 0.200E+00
-0.354E+01 0.990E+00 0.600E+00
-0.270E+01 0.997E+00 0.850E+00
-0.200E+01 0.100E+01 0.100E+01


drying curves
---------------
   ddp=       dds=

 number of tabulated nodes    5

-0.354E+01 0.270E+00
-0.448E+01 0.250E+00
-0.570E+01 0.600E-01
-0.700E+01 0.100E-01
-0.800E+01 0.000E+00

 number of tabulated nodes    6

-0.270E+01 0.610E+00
-0.354E+01 0.600E+00
-0.448E+01 0.500E+00
-0.570E+01 0.100E+00
-0.700E+01 0.500E-01
-0.800E+01 0.000E+00

 number of tabulated nodes    7

-0.200E+01 0.860E+00
-0.270E+01 0.850E+00
-0.354E+01 0.840E+00
-0.448E+01 0.720E+00
-0.570E+01 0.300E+00
-0.700E+01 0.100E+00
-0.800E+01 0.000E+00

 number of tabulated nodes    8

-0.170E+01 0.100E+01
-0.200E+01 0.999E+00
-0.270E+01 0.997E+00
-0.354E+01 0.990E+00
-0.448E+01 0.900E+00
-0.570E+01 0.600E+00
-0.700E+01 0.200E+00
-0.800E+01 0.000E+00
```

```
wetting curves
---------------
   wp=        ws=

 number of tabulated nodes    8

-0.700E+01 0.000E+00
-0.800E+01 0.100E-02
-0.700E+01 0.100E-01
-0.570E+01 0.600E-01
-0.448E+01 0.250E+00
-0.354E+01 0.600E+00
-0.270E+01 0.850E+00
-0.200E+01 0.100E+01

 number of tabulated nodes    7

-0.800E+01 0.199E+00
-0.700E+01 0.200E+00
-0.570E+01 0.240E+00
-0.448E+01 0.440E+00
-0.354E+01 0.680E+00
-0.270E+01 0.860E+00
-0.200E+01 0.100E+01

 number of tabulated nodes    6

-0.700E+01 0.580E+00
-0.570E+01 0.600E+00
-0.448E+01 0.640E+00
-0.354E+01 0.800E+00
-0.270E+01 0.900E+00
-0.200E+01 0.100E+01

 number of tabulated nodes    5

-0.570E+01 0.880E+00
-0.448E+01 0.900E+00
-0.354E+01 0.920E+00
-0.270E+01 0.960E+00
-0.200E+01 0.100E+01

 number of tabulated nodes    4

-0.448E+01 0.989E+00
-0.354E+01 0.990E+00
-0.270E+01 0.997E+00
-0.200E+01 0.100E+01

 INITIAL CONDITIONS FOR HYSTERESIS CALCULATION

  ne  pold(ne)  sold(ne)  icurv  nn1  nn2   k1   k2   ratio

    1-0.200E+05 0.400E+00    1    0    0    0    0   0.000E+00
    2-0.200E+01 0.400E+00    0    0    0    0    0   0.000E+00
    3-0.900E+06 0.400E+00    4    3    2    6    7   0.545E+00

WETTING/DRYING RESULTS FOR THE 3 ELEMENTS
```

```
ne=    1 s(ne)= 0.440E+00 p(ne)= -0.276E+04
ne=    2 s(ne)= 0.440E+00 p(ne)=  0.000E+00
ne=    3 s(ne)= 0.440E+00 p(ne)= -0.224E+06
ne=    1 s(ne)= 0.480E+00 p(ne)= -0.110E+04
ne=    2 s(ne)= 0.480E+00 p(ne)=  0.000E+00
ne=    3 s(ne)= 0.480E+00 p(ne)= -0.924E+05
ne=    1 s(ne)= 0.520E+00 p(ne)= -0.440E+03
ne=    2 s(ne)= 0.520E+00 p(ne)=  0.000E+00
ne=    3 s(ne)= 0.520E+00 p(ne)= -0.382E+05
ne=    1 s(ne)= 0.560E+00 p(ne)= -0.234E+03
ne=    2 s(ne)= 0.560E+00 p(ne)=  0.000E+00
ne=    3 s(ne)= 0.560E+00 p(ne)= -0.221E+05
ne=    1 s(ne)= 0.600E+00 p(ne)= -0.153E+03
ne=    2 s(ne)= 0.600E+00 p(ne)=  0.000E+00
ne=    3 s(ne)= 0.600E+00 p(ne)= -0.145E+05
ne=    1 s(ne)= 0.640E+00 p(ne)= -0.996E+02
ne=    2 s(ne)= 0.640E+00 p(ne)=  0.000E+00
ne=    3 s(ne)= 0.640E+00 p(ne)= -0.949E+04
ne=    1 s(ne)= 0.680E+00 p(ne)= -0.650E+02
ne=    2 s(ne)= 0.680E+00 p(ne)=  0.000E+00
ne=    3 s(ne)= 0.680E+00 p(ne)= -0.622E+04
ne=    1 s(ne)= 0.720E+00 p(ne)= -0.424E+02
ne=    2 s(ne)= 0.720E+00 p(ne)=  0.000E+00
ne=    3 s(ne)= 0.720E+00 p(ne)= -0.408E+04
ne=    1 s(ne)= 0.680E+00 p(ne)= -0.921E+02
ne=    2 s(ne)= 0.680E+00 p(ne)=  0.000E+00
ne=    3 s(ne)= 0.680E+00 p(ne)= -0.887E+04
ne=    1 s(ne)= 0.640E+00 p(ne)= -0.200E+03
ne=    2 s(ne)= 0.640E+00 p(ne)=  0.000E+00
ne=    3 s(ne)= 0.640E+00 p(ne)= -0.193E+05
ne=    1 s(ne)= 0.600E+00 p(ne)= -0.342E+03
ne=    2 s(ne)= 0.600E+00 p(ne)=  0.000E+00
ne=    3 s(ne)= 0.600E+00 p(ne)= -0.338E+05
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