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February 1974

Prepared for the U. S. Atomic Energy Commission
under Contract W-7405-ENG-48

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KEYWORD ABSTRACT

NUCLEAR STRUCTURE: $^{58,60,62}Ni$, $^{64,66,68}Zn$, $^{70,72,74,76}Ge$, $^{74,76,78}Se$, $^{80,82,84}Kr$, $^{84,86,88}Sr$, ^{92}Zr, $^{94,96,98}Mo$, $^{100,102,104}Ru$, $^{104,106,108,110}Pd$, $^{106,108,110,112,114,116}Cd$, ^{116}Sn, $^{122,124,126}Te$, $^{126,128,130,132}Xe$, $^{132,134,138}Ba$, $^{140,142}Ce$, ^{144}Nd, $^{148,150}Sm$, ^{152}Gd; calculated E2/M1 mixing ratio δ.

E2/M1 MULTIPOLE MIXING RATIOS OF GAMMA TRANSITIONS IN EVEN-EVEN SPHERICAL NUCLEI*

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ABSTRACT

A systematic survey is presented of E2/M1 mixing ratios of gamma transitions between the low-lying even-parity levels of even-even nuclei in the mass range $58 \leqslant A \leqslant 152$. Particular attention is given to the variations in the phase of the mixing ratios, which are deduced from the literature in a systematic manner. It is shown that the systematics of both magnitudes and phases of the mixing ratios are explained quite well for a number of nuclei by a model proposed by Greiner, in which the magnitude of the mixing ratio is parameterized in terms of the deviation of the g-factor of the first 2^+ state from the value Z/A. It is further shown that a semi-microscopic description, in terms of small admixtures of two-particle components to the phonon basis states, yields reasonable agreement with the observed phase variations and absolute magnitudes, even when only very few two-particle states are considered.

I. INTRODUCTION

The interpretation of the magnitudes and relative phases of the electromagnetic transition matrix elements between the low-lying excited states of even-even nuclei in the mass range $40 < A < 150$ may be attempted in the basis states of three nonequivalent models: The excited states may be described as vibrations about a spherical equilibrium shape, as rotations of a "soft" deformed core, or as excitations of two particles (quasiparticles) from the ground state. These various models lead to quite different predictions for the static and dynamic electromagnetic multipole moments, and it is to be expected that detailed study of the systematic behavior of these moments can result in an indication of the extent to which the low-lying levels can be understood in terms of collective or single-particle effects.

In a previous communication by the author,[1] a study was presented of E2/M1 mixing ratios in deformed even-Z, even-N nuclei ($150 < A < 190$). The results of that study indicated that a phenomenological interpretation of the mixing ratios between states of the β- or γ-vibrational bands and the ground-state band was possible if the appropriate mixing of intrinsic states were included. In the present work, a similar study of previously measured E2/M1 mixing ratios of even-even nuclei in the mass range $60 \lesssim A \lesssim 150$ is presented; the compiled mixing ratios are compared with other experimentally determined static and dynamic electromagnetic multipole moments and with the structure of the spectrum of excited states in order to determine the applicability of the appropriate model. Particular attention is given to the phases of the mixing ratios, which are deduced in a systematic manner from the literature; these phases yield additional insight into the structure of the excited states.

The properties of the levels of nuclei of the f-p shell ($40 < A < 60$) cannot, in general, be dealt with in phenomenological terms, and must be treated more microscopically; the same is true of most neutron- or proton-closed-shell nuclei in other mass regions. These will be discussed in a subsequent publication. In the present communication, we deal with those nuclei for which the lower excited states can be interpreted primarily in collective terms.

A number of similar compilations have been undertaken in the past.[2] However, in view of the success of high-resolution detectors and electronics in eliminating ambiguities and conflicts in the angular distribution and angular correlation literature in recent years, it seems worthwhile to offer a more current compilation.

II. COMPILATION OF VALUES

The E2/M1 mixing ratios have been obtained from a survey of the angular distribution and correlation literature. In extracting the mixing ratios from the quoted angular correlation coefficients, the phase convention of Krane and Steffen[3] has been employed, in which emission matrix elements are always used for the multipole operators. The amplitude mixing ratio is given in this convention by the ratio of the reduced emission matrix elements of the multipole operators as

$$\delta = \frac{\langle J_f \| \vec{j}_N \vec{A}(E2) \| J_i \rangle}{\langle J_f \| \vec{j}_N \vec{A}(M1) \| J_i \rangle}, \tag{1}$$

where $\vec{A}$ represents the appropriate electromagnetic vector field and $\vec{j}_N$ is the nuclear current. The angular correlation coefficients for the case in which the initial transition in a cascade $J_1 \rightarrow J_2 \rightarrow J_3$ is of mixed E2/M1 character are written as

$$A_{kk} = \frac{F_k(11\ J_1J_2) - 2\delta F_k(12\ J_1J_2) + \delta^2 F_k(22\ J_1J_2)}{1 + \delta^2} F_k(L_2L_2J_3J_2). \tag{2}$$

If the mixed transition is the final transition in a cascade (or if the transition is observed following the decay of a nuclear state oriented by, for example, nuclear reactions or cryogenic methods), the interference term is written with a + sign. The phase of the mixing ratio so defined may be compared with the frequently employed Biedenharn-Rose (BR)[4] and Rose-Brink (RB)[5] conventions for a γ_1-γ_2 cascade as follows:

$$\delta(\gamma_1)_{BR} = -\delta(\gamma_1)_{KS}, \quad \delta(\gamma_1)_{RB} = -\delta(\gamma_1)_{KS} ,$$

$$\delta(\gamma_2)_{BR} = \delta(\gamma_2)_{KS}, \quad \delta(\gamma_2)_{RB} = -\delta(\gamma_2)_{KS} . \tag{3}$$

The mixing ratios may be compared with theoretical values through the expression

$$\frac{\delta}{E_\gamma} = 0.835 \frac{\langle J_f \| \mathcal{M}(E2) \| J_i \rangle}{\langle J_f \| \mathcal{M}(M1) \| J_i \rangle} , \tag{4}$$

where the reduced matrix elements of the multipole operators are those used, for example, by Bohr and Mottelson,[6] and are given in units of electron-barns (eb) for E2 and nuclear magnetons (nm) for M1. The γ-ray energy E_γ is measured in MeV.

For purposes of theoretical comparisons, it is useful to define the mixing ratio Δ:

$$\Delta = \frac{\langle J_f \| \mathcal{M}(E2) \| J_i \rangle}{\langle J_f \| \mathcal{M}(M1) \| J_i \rangle} , \tag{5}$$

where Δ is given in units of eb/nm.

The results of the literature survey for transitions from the second excited 2^+ states (referred to as 2' states) to the first excited 2^+ state are given in Table I. In a number of cases, two close-lying 2^+ levels exist having similar properties, and both are included in Table I. Also shown in Table I are the reduced E2 transition probabilities B(E2) for the decay of the first excited 2^+ states, as well as the ratios of the reduced E2 transition probabilities describing the decay modes of the 2' states.

A cursory inspection of Table I illustrates a number of systematic features of the 2' $\rightarrow$ 2 mixing ratios. In general, the magnitudes are large,

in agreement with the expected forbiddance of M1 transitions between collective states. The phases seem to show little systematic variation and, indeed, one seems to find nearly equal frequencies for the occurrence of positive or negative phases. This is illustrated by the histograms shown in Fig. 1, from which it can be seen that for nuclei at least four valence particles (or holes) away from a closed shell, the mixing ratios have their largest values and also have roughly equal numbers of cases with positive as with negative phases. As one approaches a closed shell, one phase clearly begins to dominate. As shown below, in the lowest-order approximations, the two-particle contribution to the M1 matrix element depends on the single particle g-factor; the largest contributions are expected to arise from proton states, which always have positive g-factors.

The dependence of the magnitudes of the mixing ratios on shell effects is illustrated in Fig. 2, which shows the data of Table I as a function of A. A dramatic decrease in the magnitude of Δ as one approaches a shell closure is apparent. One can also infer again the dominance of proton over neutron contributions; the minima of the data are smaller (i.e., less collective and more two-particle) for closed-neutron configurations (in which the proton configurations dominate) than for closed-proton configurations.

In Table II are listed the E2/M1 mixing ratios from higher-lying 2^+ levels, which are referred to as 2" levels. Also shown in Table II are the relative transition probabilities for transitions depopulating the 2" levels. The relative reduced transition probabilities B(E2) are given for the de-excitation to the ground state and first excited 2^+. For de-excitations to the 2' levels, the reduced transition probabilities are generally not derivable, owing to the lack of information on the multipolarity of the 2" $\rightarrow$ 2 transition; hence only the ratios of the total transition intensities T are given.

Most of the measured mixing ratios are for 2" → 2 transitions, with a few indicated cases of 2" → 2' transitions. In general, these mixing ratios are smaller than those of the 2' → 2 type, indicating somewhat less collective behavior. Comparison of the systematic behavior of both the magnitudes and phases can provide an indication of the similarity of states in neighboring nuclei. For example, it is apparent that the 2" levels in the cadmium isotopes have a similar structure, as evidenced by the agreement in magnitudes and phases of the 2" → 2 mixing ratios. In the A ≈ 100 region, one finds the lowest 2" → 2 mixing ratios in ^{102}Ru, ^{106}Pd and ^{108}Pd to be quite similar; it is also possible to compare the higher-lying 2" levels of ^{102}Ru and ^{106}Pd in a similar manner.

Table III shows the collected results for transitions depopulating the 3^+ states; these transitions are of the type 3 → 2, 3 → 2', and 3 → 4. In addition, the ratios of the total transition intensities T and reduced transition probabilities B(E2) are shown for de-excitations of the 3^+ level to the 2 and 2' levels. In Table IV are listed the mixing ratios of transitions from the second 4^+ level (referred to as the 4' level) to the first 4^+ level, as well as the relative transition probabilities for de-excitations of the 4' level to the 4 and 2 levels. Similar data for transitions from the spin 5 and 6' (second excited 6^+) levels are shown in Table V.

The references for the results given in Tables I-V are listed in the Appendix. Often the tabulated value is an average of several results found in the literature; in such cases, the reference gives a representative result, and generally references to additional work may be found therein.

In the following section these tabulated mixing ratios and transition probabilities are compared with the predictions based on the interpretation of the low-lying even-parity levels in terms of various nuclear models.

III. COMPARISON WITH NUCLEAR MODELS

A. I = 2' Levels

1. Phonon (Vibrational) Model

In the harmonic vibrational model, the low-lying even-parity excited states are treated as arising from quadrupole vibrations of the nuclear surface.[7] The energy spectrum of the excited states expected in this model is shown in Fig. 3; in practice, the degeneracy of the N-phonon levels is split by various residual interactions. Also shown in Fig. 3, for comparison, is the energy spectrum of ^{120}Te, with the energy spacing normalized such that the excitation energy of the first 2^+ state is equal to the phonon energy. This is a rather unique example of a vibrator; in practice one seldom finds such close spacing of the 3-phonon quintuplet.

In the zeorth-order harmonic model, both the state vectors and the multipole operators are treated collectively. The state vector of the N-phonon level of spin J is given by

$$|J_N M\rangle = \frac{1}{\sqrt{N}} \left\{ b^\dagger_{2m_1} \, b^\dagger_{2m_2} \, \ldots \, b^\dagger_{2m_N} \right\}_{JM} |0\rangle \, , \tag{6}$$

where the brackets indicate that the phonon creation operators $b^\dagger_{2m}$ are coupled together to give a resultant J and M, including appropriate angular momentum and parentage coefficients. The collective form of the M1-operator is given by

$$\mathcal{M}(M1,\mu) = \sqrt{\frac{3}{4\pi}}\, g_R \mu_N \, J_\mu \, , \tag{7}$$

$$= \sqrt{\frac{3}{4\pi}}\, g_R \mu_N (\hbar\sqrt{10}) \left\{ (-1)^{m_1} b_{2-m_1} \, b^\dagger_{2m_2} \right\}_{1\mu} . \tag{8}$$

Here μ_N represents the nuclear magneton (= $e\hbar/2Mc$).

If the 2' and 2 levels are interpreted as two- and one-phonon states respectively, it is apparent that 2'→ 2 M1 transition must not exist in this model, since the M1 operator cannot change the number of phonons.

The lowest-order perturbation which can be applied to this model is to allow for configuration mixing of $\Delta N = 1$ phonon levels. For example,

$$|2\rangle = a|2_1\rangle + b\,|2_2\rangle\ ,$$
$$|2'\rangle = a|2_2\rangle - b\,|2_1\rangle\ , \tag{9}$$

where $a^2 + b^2 = 1$, and where the state vector on the right-hand side of Eqs. (9) are the pure-phonon states J_N given by Eq. (6). In this approximation, the static and dynamic properties of the levels may be computed to be

$$\frac{E(2')}{E(2)} = \frac{2-b/a}{1+2b/a}\ , \tag{10a}$$

$$\frac{B(E2,2'\to 2)}{B(E2,2\to 0)} = \frac{2(1-2b^2)^2}{1-b^2}\ , \tag{10b}$$

$$\frac{B(E2,2'\to 0)}{B(E2,2'\to 2)} = \frac{1}{2}\left(\frac{b}{1-2b^2}\right)^2\ , \tag{10c}$$

$$|eQ(2)| = |ab|\sqrt{\frac{256\pi}{35}\,B(E2,2\to 0)}\ . \tag{10d}$$

The deviations of the energy ratios, B(E2) ratios, and quadrupole moments from the predictions of the pure phonon model (b = 0) may be reasonably well accounted for by phonon mixing. Singh et al.[8] have recently done a similar type of calculation based on configuration mixing and have obtained reasonable agreement with experimental values.

However, this type of mixing does not give rise to M1 transitions. It is apparent from the very nature of the M1 operator, that M1 transitions of the type $J \to J \pm 1$ must vanish, since the J_μ operator cannot change the value of J ($J_\mu|JM\rangle \to |JM+\mu\rangle$). Additionally, the vanishing of the M1 component of the $2' \to 2$ transition follows from Eqs. (7) and (9),

$$\langle 2\| \mathcal{M}(M1) \| 2' \rangle \propto ab[g_R(2_2) - g_R(2_1)] \ . \tag{11}$$

As long as the g-factors of the phonon states are identical, the M1 amplitude vanishes.

Nonvanishing M1 transitions may be obtained by introducing noncollective contributions into the state vectors, Eq. (6), or into the M1 operator, Eq. (7). These non-phonon contributions will be discussed in succeeding sections. It should be noted, however, that the reasonable success obtained from a calculation of the B(E2) ratios including phonon mixing suggests that such states may provide a useful basis from which to proceed.

2. Rotational Models

In this section we consider the excited states as members of quasi-rotational bands. Figure 3 illustrates how the multiple-phonon levels may be decomposed into various intrinsic excitations and rotational bands. These rotational bands deviate considerably from the J(J + 1) spacing expected for a rigid rotor, indicating the "softness" of the nuclear deformation. (This is the basis for such considerations as the variable moment of inertia model[9] or the higher-order cranking model.[10]) The 2' excitations may then be considered as states of the quasi-gamma or quasi-beta bands. The E2/M1 mixing ratios then obtain a collective (i.e., E2) character, in agreement

with observations. However, contrary to observation, the crossover $2' \to 0$ E2 transition would not be strongly forbidden by this type of model, and thus it is to be expected that limited success in interpreting E2/M1 mixing would be obtained.

An alternative possibility is to consider an asymmetric rotor model, for example, that of Davydov and Filippov.[11] However, as was shown by Lipas,[12] collective M1 transitions must be identically vanishing in such a model.

3. Phonon-plus-particles Model

As discussed in Sec. III A.1, the phonon model disallows all M1 transitions; in the present approach, we introduce a small admixture of a two-particle state into the state vector. The approximate success of the phonon model is accounting for the lower B(E2) ratios indicates that this admixture may be treated as a perturbation of the phonon state vectors.

In the present calculation we employ the method developed by Tamura and Yoshida.[13] A pair of nucleons in the quasiparticle states $|j_1\rangle$ and $|j_2\rangle$ is excited from the ground state. The coupling of the particle and collective motions is described by the interaction

$$H_{int} = -\frac{\chi}{2} \sum_{\mu} \hat{Q}_{\mu} Q_{\mu}^{*} , \tag{12}$$

where Q_{μ} is the collective quadrupole operator (i.e., the $\alpha_{2\mu}$ in the model of nuclear surface vibrations), and $\hat{Q}_{\mu}$ is a two-quasiparticle quadrupole operator, given by

$$\hat{Q}_{\mu} = \sum_{\substack{jm \\ j'm'}} \langle jm | r^2 Y_{2\mu} | j'm' \rangle \; (u_j v_{j'} + u_{j'} v_j) \; (\alpha_{jm}^{\dagger} \beta_{j'm'}^{\dagger} + \beta_{jm} \alpha_{j'm'}) , \tag{13}$$

where u and v are respectively the usual quasiparticle non-occupation and occupation amplitudes, and $\alpha^\dagger$ and $\beta^\dagger$ (α and β) are the quasiparticle creation (destruction) operators.

Under such an interaction, assumed to be treatable by standard methods of first-order perturbation theory, the state vectors of the 2 and 2' levels can be written as perturbations of the collective states 2_n (with n regarded as a seniority index of the collective states, rather than as a phonon number):

$$|2\rangle = |2_1\rangle + \sum_{j_1 j_2} b(2_1;(j_1 j_2)2)\,|(j_1 j_2)2\rangle ,$$

$$|2'\rangle = |2_2\rangle + \sum_{j_1 j_2} b(2_2;(j_1 j_2)2)\,|(j_1 j_2)2\rangle . \tag{14}$$

We employ a notation slightly different from that of Tamura and Yoshida, but preserve the spirit of their work. The mixing amplitudes b are given by

$$b(2_n;(j_1 j_2)2) = \frac{\chi\,\langle 0\|Q\|2_n\rangle\langle j_1\|\hat{Q}\|j_2\rangle\,(u_{j_1}v_{j_2}+u_{j_2}u_{j_1})}{10\,[E_{j_1}+E_{j_2}-E(2_n)]} \tag{15}$$

where the E_j are the quasiparticle energies. The M1 matrix element may than be computed to be

$$\langle 2\|\mathcal{M}(M1)\|2'\rangle = \frac{\chi^2}{10}\langle 0\|Q\|2_2\rangle\langle 0\|Q\|2_1\rangle \sum_{\substack{j_1 j_2\\ j_1' j_2'}} B_{j_1' j_2';j_1 j_2} . \tag{16}$$

The quantity $B_{j_1'j_2';j_1j_2}$ depends on the couplings of the single-particle states and on the M1-matrix elements between the single-particle states. The dominant contributions to the total M1 transition probability will arise from cases in which $j_1' = j_1$ and $j_2' = j_2$ (including as a special case, $j_1' = j_2' = j_1 = j_2$); that is, the identical configuration (j_1j_2) is admixed into both the 2 and 2' levels. The total M1 transition probability is then proportional to $\langle j_1 \| \mathcal{M}(M1) \| j_1 \rangle$ or $\langle j_2 \| \mathcal{M}(M1) \| j_2 \rangle$, which are at least an order of magnitude larger than the matrix element $\langle j_1 \| \mathcal{M}(M1) \| j_2 \rangle$; that is, empirical values of the former matrix elements do not differ greatly from the single-particle estimate (Schmidt limit) for the static M1-moments, while the empirical M1 transition probabilities are generally retarded by 2-3 orders of magnitude relative to single-particle (Weisskopf) estimates. Thus, the major contribution to the 2' → 2 M1 transition matrix element is proportional to

$$B_{j_1j_2;j_1j_2} = \frac{(u_{j_1}v_{j_2} + u_{j_2}v_{j_1})^2 \, |\langle j_1 \| \hat{Q} \| j_2 \rangle|^2}{[E_{j_1}+E_{j_2}-E(2_2)][E_{j_1}+E_{j_2}-E(2_1)]}$$

$$\times \left\{ \frac{j_1(j_1+1) + 6 - j_2(j_2+1)}{2\sqrt{30 j_1(j_1+1)(2j_1+1)}} \langle j_1 \| \mathcal{M}(M1) \| j_1 \rangle \right.$$

$$\left. + \frac{j_2(j_2+1) + 6 - j_1(j_1+1)}{2\sqrt{30 j_2(j_2+1)(2j_2+1)}} \langle j_2 \| \mathcal{M}(M1) \| j_2 \rangle \right\} . \qquad (17)$$

We expect that the detailed structure of Δ, in particular its phase and most of its variation in magnitude between neighboring even-even nuclei, will be contained in the $B_{j_1j_2;j_1j_2}$ term of Eq. (16). The remaining E2 matrix elements of Eq. (16) and of $\langle 2 \| \mathcal{M}(E2) \| 2' \rangle$ are assumed to be highly collective and thus to vary relatively slowly.

Values of $B_{j_1 j_2; j_1 j_2}$ have been computed from Eq. (17). The single-particle M1 matrix elements have been computed in a manner similar to the Schmidt limits for the magnetic moments, except we have taken $g_s = 0.6(g_s)_{free}$ as giving a more realistic estimate of the empirical moments. For the matrix elements of $\hat{Q}$ we have used the Weisskopf estimate of the E2 transition intensity, modified by taking the neutron and proton effective charges to be 0.7e and 1.7e, respectively. The pairing factors and quasiparticle energies have been computed using the single-particle energies, Fermi energies and gap parameters given by Kisslinger and Sorensen.[14] For each even-even nucleus we have computed the five largest contributions to $B_{j_1 j_2; j_1 j_2}$ from two-neutron and also from two-proton states. The sum of these five values is shown in Table V1.

The tabulated values of $\Sigma\, B_{j_1 j_2; j_1 j_2}$ illustrate the dominance of the contributions from the two-proton configurations over the two-neutron configurations. This dominance follows from four causes: (1) The single-particle matrix elements of $\hat{Q}$ are proportional to the assumed effective charges; the proton contributions would thus be expected to dominate over the neutron contributions by a factor of the square of the ratio of the effective charges, which amounts to a factor of 8. (2) The single-particle M1 moments (i.e., g-factors) are generally larger for protons than for neutrons by a factor of 3. (3) The two-proton excitation energies are generally lower than the two-neutron energies; this produces another factor of at least 2. (4) The single-neutron M1 moment (g-factor) is negative when $j = \ell + 1/2$, leading to cancellations in the summation; this does not occur for protons.

The mixing ratio Δ may be expressed as

$$\Delta = \frac{\langle 2 \| \mathcal{M}(E2) \| 2' \rangle}{\langle 2 \| \mathcal{M}(M1) \| 2' \rangle}$$

$$= \frac{\left(\frac{3}{4\pi} ZeR_0^2\right)^2}{\chi^2/10} \frac{\langle 2 \| \mathcal{M}(E2) \| 2' \rangle}{\langle 0 \| \mathcal{M}(E2) \| 2' \rangle \langle 0 \| \mathcal{M}(E2) \| 2 \rangle} \frac{1}{\sum_{j_1 j_2} B_{j_1 j_2; j_1 j_2}} . \quad (18)$$

We take $R_0 = 1.2A^{1/3}$fm and $\chi = 40$ MeV/R_0^2. The E2 matrix elements may be computed from either of two methods:

<u>Method I.</u> It follows from Eqs. (9) and (10b) that

$$\frac{\langle 2 \| \mathcal{M}(E2) \| 2' \rangle}{\langle 0 \| \mathcal{M}(E2) \| 2 \rangle} = -\sqrt{2} \left(\frac{1-2b^2}{b} \right) , \quad (19)$$

and that

$$eQ(2) = ab\sqrt{\frac{256\pi}{175}} \langle 0 \| \mathcal{M}(E2) \| 2 \rangle , \quad (20)$$

and thus

$$\frac{\langle 2 \| \mathcal{M}(E2) \| 2' \rangle}{\langle 0 \| \mathcal{M}(E2) \| 2' \rangle \langle 0 \| \mathcal{M}(E2) \| 2 \rangle} = \frac{-a(1-2b^2)}{eQ(2)} \sqrt{\frac{512\pi}{175}} . \quad (21)$$

We take $a \approx +1$ and compute b^2 from Eq. (10c) and the crossover-to-cascade B(E2) ratios. If the value of Q(2) is known, the value of Δ may be computed. The sign of Δ is determined by the sign of ΣB as well as that of Q(2).

<u>Method II.</u> In the event Q(2) is unknown, we may obtain a value for the magnitude of the $2 \rightarrow 0$ matrix element as

$$|\langle 0\|\mathcal{M}(E2)\|2\rangle| = \sqrt{5\ B(E2,2\to 0)}\ . \tag{22}$$

We again use Eq. (19) for the ratio of the E2 matrix elements and compute b from Eq. (10c). In this case the signs of the $2 \to 0$ matrix element and of b are undetermined, and hence the sign of Δ is undetermined.

Values of Δ computed according to Methods I and II are shown in Table VI. These values give reasonable order-of-magnitude agreement with the experimental values of Δ. It is expected that a more detailed computation could yield even better agreement by using parameters in Eqs. (17) and (18) more suited to a given nucleus; in the present work we have attempted rather to employ rough estimates for the component E2 and M1 matrix elements so as to indicate the general trend of computed values over the mass region $60 < A \lesssim 150$.

The phase of Δ is not predicted uniquely, but rather is subject to a number of estimates of the E2 matrix elements of Eq. (18). The magnitudes of these E2 matrix elements vary relatively little over the range of even-even isotopes of a given atomic number; it may be assumed that the relative phases of these matrix elements do likewise. We therefore assume that the information on variation in the phase of Δ is contained in the $B_{j_1 j_2; j_1 j_2}$ terms, and from Eq. (17) we see that this in turn depends on the M1 matrix elements and on the energy differences in the denominator. Since the dominant contributions to $B_{j_1 j_2; j_1 j_2}$ come from proton states, the M1 matrix elements (i.e., the g-factors) are all positive, and thus the phase variations will be characterized by the energy differences between the two-quasiparticle states and the unperturbed collective states 2_n. We expect that $(E_{j_1}+E_{j_2})$ will always exceed $E(2_1)$, and so we examine the relationship between $(E_{j_1}+E_{j_2})$ and $E(2_2)$. The

unperturbed energies of the 2_2 states have not been computed, but their relative systematic behavior can be inferred from that of the perturbed 2' states. The energy relationships between the 2' states and the lowest two-quasiparticle states are illustrated in Fig. 4, for a number of sequences of even-even isotopes. In all cases a suitable selection of 2_2 states, highly correlated with the 2' states, could be made to cross with the two-quasiparticle energies at a point corresponding to an observed change in phase of the mixing ratio. It is interesting to note in support of this contention that, in the four cases shown in which this phase change occurs, it always occurs at the beginning of a sequence of isotopes such that only one change of phase of Δ is observed; that is, phase sequences such as (+ - +) or (- + -) do not occur. Furthermore, it is possible that the phase difference of the 2' → 2 mixing ratio between the Te and Xe isotopes may simply result from the relationship between the 2' level and the 2-proton states, as indicated in Fig. 4.

The theory does predict phase changes at $^{134-136}$Ba, $^{100-102}$Pd and $^{94-96}$Ru, while the observed changes occur at $^{132-134}$Ba, $^{104-106}$Pd, and $^{100-102}$Ru. It is indeed possible that a more refined calculation using single-particle energies more suited to each particular isotope (rather than an average set for a larger mass region) could provide more successful predictions of the level crossings and, thus, of the change in phase of Δ. Further conclusions in this respect must await additional measurements of the 2' → 2 mixing ratios, particularly those of the more neutron-deficient isotopes.

An additional point of interest in the comparison of relative phases is the degree to which the phases (and possibly also the magnitudes) of Δ

correlate with Q(2). Unfortunately, the quadrupole moment data available is not sufficient to draw detailed conclusions. It would, for example, be of interest to attempt to account for the difference in the phase of Δ between the Te and Xe isotopes with a change in sign of Q(2), indicating the Xe isotopes may be somewhat oblate. However, the lack of values of Q(2) for the Xe isotopes makes such comparisons impossible at present.

An alternate approach to the phonon-plus-particles model has been given by Korolev.[15] In this approach the nucleus is treated as a core plus one or more zero-spin pairs which excite collective modes of the core. One can then compute, in terms of an interaction constant and a suitable set of unperturbed energy levels, the M1 and E2 matrix elements to be expected for transitions to the first 2^+ state (assumed to be a one-phonon state) from various possible structures of the 2' state (two-phonon, one-phonon plus pair, one "excited" pair, etc.). The success of this model then depends strongly on the interpretation of the physical 2' state, in particular its admixtures of paired states, although Korolev[15] has obtained reasonable agreement with ratios of reduced E2 transition probabilities in the Cd isotopes.

4. Higher-order M1 Operators

In our discussion of the phonon model in Sec. III A.1, it was pointed out that if both the state vectors and the M1 operators are treated in their lowest-order phonon modes, the M1 matrix elements must vanish. In the previous section, the effects of relaxing this restriction for the state vectors was considered; in the present section, we consider the effect of higher-order terms in the M1 operators. Here "higher-order" refers to more sophisticated couplings than that suggested by Eq. (8). (We note that Eq. (7)

contains the implicit assumption that the nuclear mass and charge distributions are identical, and thus we are presently concerned with cases in which the mass and charge distributions differ.)

A generalized M1 operator may be obtained by including higher-order phonon contributions of the form

$$\mathcal{M}(M1,\mu) = \sqrt{\frac{3}{4\pi}}\,\mu_N \{g^{(0)}J_\mu^{(0)} + g^{(1)}J_\mu^{(1)} + \ldots\} , \tag{23}$$

where we can make the identifications [cf.,Eq. (7)]

$$g^{(0)} = g_R ,$$

$$J_\mu^{(0)} = J_\mu , \tag{24}$$

and where we can define a first-order coupling of the form

$$J_\mu^{(1)} = \left\{\alpha_{2m_1} J_{m_2}^{(0)}\right\}_{1\mu} = \sum_{m_1 m_2} \langle 2m_1 1m_2 | 1\mu \rangle\, \alpha_{2m_1}\, J_{m_2}^{(0)} , \tag{25}$$

where α_{2m_1} is the collective quadrupole operator which can be represented as a linear combination of the phonon creation and annihilation operators $b^\dagger_{2m_1}$ and b_{2m_1}. This operator $J_\mu^{(1)}$ can now connect states differing by one phonon number, and thus we expect nonvanishing M1 matrix elements between the 2' and 2 levels. The coefficient $g^{(1)}$ may either be computed on the basis of the interaction which is assumed to give rise to the coupling of Eq. (25), or else may be regarded as a single parameter of the theory to be determined from comparisons with experiment.

Greiner[16] has used such a model to compute g-factors and E2/M1 mixing ratios in vibrational nuclei by assuming that the existence of a stronger pairing force for protons than for neutrons results in a smaller proton deformation, which in turn causes the M1 operator to obtain the tensorial structure described by Eq. (23). The factor $g^{(1)}$ is given by

$$g^{(1)} = -\frac{Z}{A}(1-2f)\frac{2}{3}\sqrt{10}\frac{f}{\beta_0} \tag{26}$$

where β_0 is the root mean square amplitude of the vibration, defined by

$$B(E2,2\to 0) = \frac{1}{5}\left(\frac{3}{4\pi}ZeR_0^2\right)^2 \beta_0^2 , \tag{27}$$

and where f gives the difference in proton and neutron deformations,

$$f \approx \frac{N}{A}\left(\frac{\beta_0(n)}{\beta_0(p)} - 1\right) . \tag{28}$$

We then obtain

$$\Delta = \frac{ZA^{2/3}}{g^{(1)}}(2.3\times 10^{-3}) , \tag{29}$$

or, from Eq. (26),

$$\Delta = -(1.1\times 10^{-3})\frac{A^{5/3}\beta_0}{f(1-2f)} . \tag{30}$$

An alternative formulation of this type has been given in a series of papers by Grechukhin.[17] The result obtained is similar in form to Eq. (29); however, $g^{(1)}$ is ultimately regarded as a free parameter to be determined from comparisons with the empirical Δ values. Grechukhin does attempt a

semi-classical calculation of $g^{(1)}$, which again depends on the difference between the nuclear mass and charge distribtions. Whereas in Greiner's model this difference is simply represented by the parameter f, in Grechukhin's calculations the proton distribution is computed semi-classically by considering the competition between the Coulomb repulsion and the proton-neutron attraction, from which one obtains

$$g^{(1)} = 8\times10^{-3}\,\frac{Ze^2}{R_0\kappa}\,, \tag{31}$$

where κ is the symmetry energy constant in the Weiszacker semi-empirical mass formula ($\kappa \approx 20$ MeV). This leads to values of $g^{(1)}$ of order 5×10^{-3}, whereas from a comparison of Eq. (29) with the tabulated values of Δ, it can be seen that values of $g^{(1)}$ of order 0.5 are required. In Greiner's model, the parameter f may be determined from the difference between the empirical value of g(2), the g-factor of the first 2^+ level, and the hydrodynamical value Z/A. The deduced values of f generally lie in the range 0.1-0.2, and hence in Greiner's model, $g^{(1)}$ is of order Z/A, leading to reasonable agreement between the predicted values of Δ and the measured values. Grechukhin takes $g^{(1)}$ as a parameter of the model, and sets Z/A as a limiting value. In the present calculation we will follow Greiner's method.

In Table VII are shown the predicted values of Δ based on Greiner's[16] parameter f which we have derived from the empirical g(2) values.[18] The data are shown for the medium weight nuclei in the mass range $100 \leqslant A \leqslant 126$, for which the most reliable g(2) values are available. The agreement both in sign and in magnitude is striking, particularly in ^{100}Ru and ^{102}Ru, in which cases the theory correctly predicts the change in sign at ^{100}Ru and the resonance-like

effect observed for ^{102}Ru. (It should be noted that $f < 0$ requires that the neutron pairing force be stronger than the proton pairing force. This can be circumvented by regarding f as a free parameter of the theory, on which g(2) and Δ both depend, but arising from some undetermined facet of the nuclear structure.) Although values of g(2) are available for Ge and Se, the errors are sufficiently large (±50% for Ge, ±25% for Se) to preclude reaching similar conclusions. Nevertheless, the success of this relatively simple model suggests the desirability of obtaining improved g-factors for such calculations.

B. I = 3 Levels

1. Phonon (Vibrational) Model

In this model the 3^+ level is regarded as a member of the 3-phonon quintuplet. The phonon selection rule ($\Delta N = 1$) then forbids the $\Delta N = 2$ $3 \rightarrow 2$ transition. As can be seen from the B(E2) ratios of Table II, the phonon selection rule seems to have approximately the same relative effect on the $3 \rightarrow 2$ transition as on the $2' \rightarrow 0$ transition, indicating that again the phonon model may provide a reasonable set of basis states. The small $3 \rightarrow 2$ transition intensities may be obtained with a configuration mixing calculation identical to that done above [Eq. (9)], and we obtain

$$\frac{B(E2,3 \rightarrow 2)}{B(E2,3 \rightarrow 2')} = \frac{b^2}{1-b^2} . \tag{32}$$

Thus, only a small amplitude b (of one order of 0.1) is necessary to account for the observed crossover intensities.

However, as discussed above, in this model in which both the state vectors and M1-operator are treated collectively, the M1 amplitudes must vanish and hence the mixing ratios would be infinite.

2. Rotational Models

In this type of model, the 3^+ state is regarded as the second excited state of a quasi-gamma band, and thus the $3 \rightarrow 2'$ transition becomes an intraband transition, whose mixing ratio can be expressed in terms of the intrinsic moments of the band as

$$\Delta \approx 0.4 \frac{Q_0}{g_K - g_R} . \tag{33}$$

In view of the small values of Q_0 expected for very "soft" rotors, it is difficult to justify the systematically large value of Δ $(3 \rightarrow 2')$ in this type of model. Additionally, the hindrance of the $3 \rightarrow 2$ transition is likewise difficult to account for within such a model.

3. Phonon-plus-particles Model

In a manner analogous to that used above for the 2' levels, the $I = 3$ level may be treated in terms of a two-particle excitation of the form $|(j_1 j_2)3\rangle$ admixed with the collective state. Since sufficient systematic information regarding the mixing ratios from this level is not available, we will not attempt the type of detailed calculation done above for the 2' levels. However, it is possible to draw some conclusions regarding the relative magnitudes and phases of the $3 \rightarrow 2$ and $3 \rightarrow 2'$ mixing ratios.

In the method of Tamura and Yoshida,[13] the perturbed state vector of the 3^+ level is written [cf., Eq. (14)] as

$$|3\rangle = |3_3\rangle + \sum_{j_1 j_2} b(3_3;(j_1 j_2)3)\, |(j_1 j_2)3\rangle . \tag{34}$$

However, some care must be exercised, since if one assumes the two particles to be excited from the ground state, one obtains matrix elements of the form $\langle 0_0 \| Q \| 3_3 \rangle$, which vanish. Hence, we must select the I = 2 state as the I' state of Tamura and Yoshida.[13] We then obtan an M1 matrix element of the form

$$\langle 2' \| \mathcal{M}(M1) \| 3 \rangle = \frac{\chi^2}{2\sqrt{35}} \langle 2_1 \| Q \| 3_3 \rangle \langle 0 \| Q \| 2_2 \rangle$$

$$\times \sum_{j_1 j_2 j_1' j_2'} B_{j_1 j_2; j_1 j_2} \tag{35}$$

The 3 → 2 M1 matrix element is identical, except that the second matrix element of the operator Q in Eq. (35) contains the state vector 2_1 instead of 2_2. The $B_{j_1 j_2; j_1' j_2'}$ terms are obtained to be similar to Eq. (17); for the above case,

$$B_{j_1 j_2; j_1 j_2} = \frac{(u_{j_1} v_{j_2} + u_{j_2} v_{j_1})^2 \, |\langle j_1 \| \hat{Q} \| j_2 \rangle|^2}{[E(2_1) + E_{j_1} + E_{j_2} - E(3_3)][E_{j_1} + E_{j_2} - E(2_2)]}$$

$$\times (-1) \left[\frac{(j_1+j_2+4)(j_1+j_2-2)(j_1-j_2+3)(j_2-j_1+3)}{105} \right]^{1/2}$$

$$\times \left\{ \frac{\langle j_1 \| \mathcal{M}(M1) \| j_1 \rangle}{\sqrt{2j_1(2j_1+1)(2j_1+2)}} + \frac{\langle j_2 \| \mathcal{M}(M1) \| j_2 \rangle}{\sqrt{2j_2(2j_2+1)(2j_2+2)}} \right\} . \tag{36}$$

For the 3 → 2 case, we again substitute $E(2_1)$ for $E(2_2)$ in the second term of the energy denominator. Here we have assumed, as for the 2' levels, that the most significant terms are those in which $j_1 = j_1'$ and $j_2 = j_2'$.

The E2 matrix elements may be obtained using Eq. (32):

$$\langle 2'\|\mathcal{M}(E2)\|3\rangle = \sqrt{1-b^2}\,\frac{3}{4\pi}\,ZeR_0^2\,\langle 2_2\|Q\|3_3\rangle\ ,$$

$$\langle 2\|\mathcal{M}(E2)\|3\rangle = b\,\frac{3}{4\pi}\,ZeR_0^2\,\langle 2_2\|Q\|3_3\rangle\ . \tag{37}$$

If we take only the single most significant contribution to the summation over the $(j_1\ j_2)$ states, we obtain the ratio

$$\frac{\Delta_{3\to 2}}{\Delta_{3\to 2'}} = \frac{b}{\sqrt{1-b^2}}\,\frac{\langle 0\|Q\|2_2\rangle}{\langle 0\|Q\|2_1\rangle}\,\frac{[E_{j_1}+E_{j_2}-E(2_1)]}{[E_{j_1}+E_{j_2}-E(2_2)]}$$

$$= -\frac{b^2}{1-b^2}\,\frac{E_{j_1}+E_{j_2}-E(2_1)}{E_{j_1}+E_{j_2}-E(2_2)}\ , \tag{38}$$

where we have used Eqs. (9) and (10).

In this approximation we conclude that, in general, the 3 → 2 mixing ratio will be significantly smaller than the 3 → 2' mixing ratio, and that the signs will be opposite when the 2_2 state lies significantly lower in energy than $(E_{j_1} + E_{j_2})$.

Referring to Table III, it can be seen that the magnitudes are generally as predicted by Eq. (38). The phases depend again on the energy factor $[E_{j_1}+E_{j_2} - E(2_2)]$; in the case of ^{74}Ge, which is least likely to have a crossing of the energy levels (Fig. 4), the phase relationship holds as predicted by Eq. (38). The relationship between the 2_2 and two-particle levels is not as clear in Ru and Cd; ^{102}Ru, in fact, seems to exhibit evidence of a level crossing, owing to the nearly resonance-like behavior of $\Delta_{2'\to 2}$

and the change in phase between ^{100}Ru and ^{104}Ru. The ^{132}Xe and ^{134}Ba isotopes show similar behavior of $\Delta_{3\to2}$ and $\Delta_{3\to2'}$ although the phase change of $\Delta_{2'\to2}$ in the Ba isotopes gives evidence for a level crossing not observed in the 3→2 and 3→2' transitions. A more detailed examination of the theory would require additional experimental results for comparison.

4. Higher Order M1 Operators

A calculation similar to Greiner's (Sec. III A.4) can be done for the transitions from the 3^+ level. For the $\Delta N = 1$ transitions we obtain [cf., Eq. (29)]

$$\Delta_{I_i\to I_f} = \frac{ZA^{2/3}}{g^{(1)}} C_{I_i\to I_f} , \tag{39}$$

where $C_{3\to2'} = 2.2\times10^{-3}$ and $C_{3\to4} = 1.6\times10^{-3}$. On this basis we expect $\Delta_{3\to2'} \approx \Delta_{2'\to2}$ and $\Delta_{3\to4} \approx 0.7\Delta_{2'\to2}$, with the phases expected to be identical. We note that values of $\Delta_{3\to2'}$ and $\Delta_{3\to4}$ predicted for ^{102}Ru and ^{110}Cd are in agreement with experimental values, as were the values calculated above for $\Delta_{2'\to2}$. Furthermore, the relationship between $\Delta_{2'\to2}$ and $\Delta_{3\to2'}$ seems to be valid for ^{82}Kr and ^{134}Ba, although it appears to be violated for the relative phases in the cases of ^{74}Ge and ^{132}Xe.

The 3→2 ($\Delta N=2$) transition is forbidden to exist, since both the E2 and first-order M1 operators can only change the phonon number by one unit. If we allow mixing of the one- and two-phonon 2^+ levels, the 3→2 transition would have both the E2 and M1 amplitudes proportional to the phonon mixing parameter b. Hence, the mixing ratio Δ would be independent of b and we would expect $\Delta_{3\to2} = \Delta_{3\to2'}$. From the values tabulated in Table III, it can be seen that, generally, $|\Delta_{3\to2}|<|\Delta_{3\to2'}|$. From this we infer that the first-order expansion

for M1 given by Eq. (23) is not sufficient to explain the 3→2 mixing ratios, and that it is necessary to introduce a term of the form $g^{(2)}J_{\mu}^{(2)}$, with $J_{\mu}^{(2)} = \{\alpha_{2m_1} J_{m_2}^{(1)}\}_{1\mu}$. Such a calculation will not be attempted in this present work, owing to the lack of significant data for comparison, and to the lack of an alternative means to obtain $g^{(2)}$.

C. J = 2" Levels

If the 2" level were to be interpreted as a pure three-phonon level, the 2"→2 and 2"→2' M1 transitions amplitudes must again vanish. However, it is not likely that the general properties of such states may be successfully interpreted in terms of any collective model, since the 2" energies are approaching the pairing gap 2Δ. The B(E2) ratios shown in Table II support this contention; the ΔN = 1 cascade transitions are not strongly favored over the ΔN = 2 and ΔN = 3 crossover transitions. In the two-quasiparticle model, it is possible to construct a state whose primary character is of the form $(j_1 j_2)$, with possibly a small admixture of one- and two-phonon components. Assuming the admixtures to be small, the J = 2 levels may be approximately described by

$$|2\rangle \approx |2_1\rangle + b|2_2\rangle + d|2_{j_1 j_2}\rangle,$$

$$|2'\rangle \approx |2_2\rangle - b|2_1\rangle + c|2_{j_1 j_2}\rangle,$$

$$|2''\rangle \approx |2_{j_1 j_2}\rangle - d|2_1\rangle - c|2_2\rangle, \qquad (40)$$

where $|2_N\rangle$ represents the pure-phonon state and $|2_{j_1 j_2}\rangle$ is a two-quasiparticle state. The amplitudes c and d are related to those given in Eqs. (14) and (15).

The following relationships between the transition probabilities may then be directly obtained:

$$\frac{B(E2,2''\to 0)}{B(E2,2''\to 2)} \approx \frac{d^2}{2(c-bd)^2}\,, \tag{41a}$$

$$\frac{B(E2,2''\to 2')}{B(E2,2''\to 2)} \approx \frac{(bc+d)}{(c-bd)^2}\,. \tag{41b}$$

The relevant E2/M1 mixing ratios then become

$$\Delta_{2'\to 2} \approx \frac{1}{cd}\;\frac{\langle 2_1 \| \mathcal{M}(E2) \| 2_2 \rangle}{\langle 2_{j_1 j_2} \| \mathcal{M}(M1) \| 2_{j_1 j_2} \rangle}\,, \tag{42a}$$

$$\Delta_{2''\to 2} \approx \left(\frac{-c+bd}{d}\right)\frac{\langle 2_1 \| \mathcal{M}(E2) \| 2_2 \rangle}{\langle 2_{j_1 j_2} \| \mathcal{M}(M1) \| 2_{j_1 j_2} \rangle}\,, \tag{42b}$$

$$\Delta_{2''\to 2'} \approx \frac{d + cb}{c}\;\frac{\langle 2_1 \| \mathcal{M}(E2) \| 2_2 \rangle}{\langle 2_{j_1 j_2} \| \mathcal{M}(M1) \| 2_{j_1 j_2} \rangle}\,. \tag{42c}$$

Within these approximations, one may make some comparisons of the deduced Δ values in those regions where sufficient data exist. The lowest 2" states in ^{102}Ru and ^{106,108}Pd appear to be quite similar, both in the B(E2) ratios and in the Δ values. From Eq. (41a), we have (assuming $|bd| << |c|$) $|d/c| \approx 1$, while previously we found from Eq.(10c), $|b| \approx 0.2$. It is thus apparent for this case that the phases will be determined by the d/c term, and we must then have $\Delta_{2'\to 2}$ and $\Delta_{2''\to 2}$ opposite in phase. From the tabulated values in Table II, it can be seen that this is indeed the case. Furthermore, from the ratio $\Delta_{2''\to 2}/\Delta_{2'\to 2}$, we estimate $|c| \approx |d| \approx 0.15$. Similarly for the

Cd isotopes, $|b| \approx 0.2$, and $|d/c|>1$, and the same approximation will be valid; once again the signs of $\Delta_{2'\to 2}$ and $\Delta_{2''\to 2}$ are opposite. For both of these cases we expect $|\Delta_{2''\to 2}|<<|\Delta_{2'\to 2}|$ in order for such a first-order approximation to be valid; this relationship is valid for these Ru, Pd, and Cd isotopes, for the lowest 2" states.

The case of ^{124}Te is the only one in which $\Delta_{2''\to 2}$ and $\Delta_{2''\to 2'}$ have both been determined. Here we compute $|d/c| \approx 0.16$, $|b| \approx 0.12$. We therefore would expect identical phases for $\Delta_{2'\to 2}$ and $\Delta_{2''\to 2'}$, and opposite phases for $\Delta_{2''\to 2}$ and $\Delta_{2'\to 2}$. These expected phase relationships are not in agreement with the observed values for the lower 2" level of ^{124}Te; however, this approximation also fails for the branching ratio given by Eq. (41b), where we would expect a value of 0.02, compared with the experimental value of 40. We would also predict $|\Delta_{2''\to 2}|>|\Delta_{2''\to 2'}|$, in agreement with experiment, although the ratio between the two is predicted to be a factor of five larger than the observed ratio.

Although we obtain reasonable agreement for the 2"→2 mixing ratios within the assumptions of this highly phenomenological calculation, the lack of systematic data prevents the drawing of definite conclusions regarding the nature of the 2" states. A more detailed consideration of the possible two-particle nature of these states and the 2' states will be discussed in a subsequent communication.

D. J = 4', 5, 6' Levels

The transitions depopulating these levels, tabulated in Tables IV and V, in general show sizeable E2 admixtures, frequently greater than 25% of the transition intensity. This indicates that even at these high excitations ($E \gtrsim 2$ MeV) collective effects still appear to be present. Although at present this survey lacks sufficient vertical as well as horizontal systematic information on the mixing ratios, we will briefly indicate how the types of calculations discussed above can be applied to those cases in which information is available.

In the case of ^{132}Xe, for example, we might consider the higher lying 4' state of Table IV to be primarily of two-particle character, and we could then describe the three J = 4 levels by a state vector using the basis state $|4_2\rangle$, $|4_3\rangle$ and $|4_{j_1 j_2}\rangle$, as was similarly done in Eqs.(40). We would then obtain a series of relationships between the transition probabilities and mixing ratios similar to Eqs. (41) and (42), and we would similarly expect that $\Delta_{4''\to 4}$ and $\Delta_{4'\to 4}$ must be opposite in phase and that $|\Delta_{4''\to 4}| << |\Delta_{4'\to 4}|$. Neither of these expectations are in agreement with the data for ^{132}Xe, and we conclude that a simple, first-order approximation such as Eq. (40) is not valid for the 4' states, at least in the case of ^{132}Xe.

We can also extend the computation using the higher-order M1 term to include the higher phonon levels. We would interpret the 4' and 6 levels as 3-phonon states, and the 5 and 6' levels as 4-phonon states. We could then derive a relationship of the form of Eq. (39), and obtain $C_{4'\to 4} = 1.2\times 10^{-3}$, $C_{5\to 4'} = 1.3\times 10^{-3}$, $C_{5\to 6} = 1.1\times 10^{-3}$, $C_{6'\to 6} = 0.85\times 10^{-3}$. The case of ^{110}Cd is the only one in which a systematic comparison of these values may be made,

and, as predicted, $\Delta_{4'\to 4} \approx \Delta_{5\to 4'} \approx \Delta_{5\to 6} \approx \frac{1}{2}\,\Delta_{2'\to 2}$, with identical phases as predicted. It is indeed remarkable that such a simple phenomenological one-parameter model is capable of providing such good agreement with the relative magnitudes and phases of all six measured $\Delta N = 1$ mixing ratios in ^{110}Cd.

IV. CONCLUSIONS

A complete understanding of the E2/M1 mixing ratios of the low-lying even-parity states of even-even nuclei in the mass range $60 \lesssim A \lesssim 150$ obviously requires a detailed calculation which considers all multiple-seniority configurations of a complete set of single-particle basis states. The success of such a calculation for a given nucleus, however, requires knowledge of the appropriate single-particle states and transition probabilities. We have shown in the present work that the systematic behavior of the E2/M1 mixing ratios can be interpreted with reasonable success across the entire mass range by considering only a few low-lying two-particle states as perturbations of the phonon basis states (presumably the correlations among the multiple-seniority configurations are implicitly included in the phonon state vectors such that their effects are included in the single-phonon E2 transition matrix elements). The variations in phase, and, to a lesser extent, the variations in magnitude can be successfully accounted for using a relatively small number of parameters.

Additionally, nearly all of the "single-phonon" transitions in the Ru, Pd, Cd, and Te isotopes may be interpreted in terms of a single-parameter model based on the higher-phonon terms in the magnetic moment operator, where that single parameter is deduced according to the method of Greiner from the difference between the g-factor of the first 2^+ level and its expected hydrodynamical value of Z/A. Such a model not only is successful in predicting the relative magnitudes over a large range of mass numbers, but also seems to hold for transitions from levels of up to four phonons.

The phase of the E2/M1 mixing ratio is a nuclear observable which has in the past not been widely used as a probe of the nuclear structure. This situation has resulted in part from the confusion resulting from the several different phase conventions which can be used to extract the mixing ratio from the angular correlation data (along with a corresponding failure on the part of numerous investigators to specify which convention they have adopted). In the present work these phases have been determined in a consistent manner and can be related to the intrinsic electromagnetic matrix elements. We have shown how this phase can be related in a model-dependent way to details of the nuclear structure. It is hoped that considerations such as these can lead to a better understanding of the phase relationships between nuclear electromagnetic matrix elements; for example, it should be possible to employ the observed mixing ratios and suitably computed (model-dependent) M1 matrix elements to determine the phase of the corresponding E2 matrix element, which might be important in understanding a similar phase-dependent problem such as the interference term which arises from measurements of quadrupole moments by the reorientation effect following Coulomb excitation. In any event, it is apparent that both the phase and the magnitude of the E2/M1 mixing ratio can be successfully employed as a probe of the nuclear structure.

V. APPENDIX

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FOOTNOTES AND REFERENCES

* Work performed under the auspices of the U. S. Atomic Energy Commission.

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Table I. E2/M1 Mixing Ratios of 2' → 2 Gamma Transitions

Nucleus	E(2) [MeV]	B(E2,2→0) [10^{-2}(eb)^2]	E(2')/E(2)	B(E2,2'→0)/B(E2,2'→2)	B(E2,2'→2)/B(E2,2→0)	E_γ [MeV]	δ	Δ [eb/nm]	Reference[a]
$^{58}_{28}Ni_{30}$	1.454	1.3	1.91	0.0018	540	1.321	-1.1(1)	-1.0	71St02,72Va01
$^{60}_{28}Ni_{32}$	1.333	1.8	1.62	0.0015	26	0.826	+1.1(1)	+1.5	72Va01
$^{62}_{28}Ni_{34}$	1.172	1.8	1.96	0.033	320	1.129	+3.2(1)	+3.4	72Va01
$^{64}_{30}Zn_{34}$	0.992	3.4	1.82	0.0062		0.812	-4(1)	-6	63Se02
$^{66}_{30}Zn_{36}$	1.039	3.0	1.80	0.000		0.828	-2.2(2)	-3.1	69Ha46
$^{68}_{30}Zn_{38}$	1.077	2.4	1.75	0.031		0.806	-1.5(1)	-2.3	73La01
$^{70}_{32}Ge_{38}$	1.040	3.6	1.64	0.0086	3.1	0.670	$-(5^{+3}_{-1})$	-9	65Mo01
$^{72}_{32}Ge_{40}$	0.835	4.6	1.75	0.0019	3.6	0.630	$+(5^{+3}_{-1})$	+10	70Ch15
$^{74}_{32}Ge_{42}$	0.596	6.2	2.04	0.023	0.93	0.609	+3.2(7)	+6	69Sc11
$^{76}_{32}Ge_{34}$	0.563	5.2	1.97	0.019		0.546	+3.5(15)	+8	70Ch15
$^{74}_{34}Se_{40}$	0.635	9.2	2.00	0.072		0.635	$-(6^{+2}_{-1})$	-11	72Co32
$^{76}_{34}Se_{42}$	0.559	8.6	2.18	0.034	0.75	0.657	+6.2(4)	+11	60Gr07
$^{78}_{34}Se_{44}$	0.614	7.2	2.13	0.031		0.695	$+(6^{+7}_{-2})$	+11	70Li11
$^{80}_{36}Kr_{44}$	0.618	6.8	2.04	0.013		0.640	$+(17^{+80}_{-9})$	+31	67Ra12
$^{82}_{36}Kr_{46}$	0.777	3.4	1.90	0.018		0.698	+2.0(1)	+3.5	72Sa24
$^{84}_{36}Kr_{48}$	0.883	3.0	2.14	0.10		1.018	$-(40^{+\infty}_{-30})$	-48	65Ro06

(continued)

Table I. continued

Nucleus	E(2) [MeV]	B(E2,2→0) [10^{-2}(eb)^2]	E(2')/E(2)	B(E2,2'→0)/B(E2,2'→2)	B(E2,2'→2)/B(E2,2→0)	E_γ [MeV]	δ	Δ [eb/nm]	Reference[a]
$^{84}_{38}Sr_{46}$	0.795	8.6	1.83	0.0075		0.661	+0.8(2)	+1.4	70Re03
$^{86}_{38}Sr_{48}$	1.078	3.8	1.72	0.14		0.778	+0.30(2)	+0.47	70Ar03
$^{88}_{38}Sr_{50}$	1.836	2.9	1.75	2.7		1.383	-0.04(2)	-0.04	70Ka28
$^{92}_{40}Zr_{52}$	0.934	1.6	1.98	16		0.913	+0.03(1)	+0.04	62Bu16
$^{94}_{42}Mo_{52}$	0.871	4.4	2.14	0.0050	2.7	0.993	-2.0(4)	-2.4	68Ar06,72Ba90
$^{96}_{42}Mo_{54}$	0.778	5.6	1.92	0.068	0.86	0.717	+0.44(4)	+0.73	70He27
			2.09	0.0063	0.80	0.847	-1.1(1)	-1.6	70He27
$^{98}_{42}Mo_{56}$	0.787	5.7	1.82	0.018	2.7	0.645	+0.58(3)	+1.1	71He10,72Ba90
			2.24	<0.006	0.30	0.973	-2.2(2)	-2.6	71He10
$^{100}_{44}Ru_{56}$	0.540	11	2.52	0.059	0.64	0.826	+5(1)	+7	68Ka04
$^{102}_{44}Ru_{58}$	0.475	14	2.32	0.038	0.53	0.628	-60(20)	-110	70Si13
$^{104}_{44}Ru_{60}$	0.358	19	2.50	0.049	0.66	0.535	-9(2)	-19	68Mc08
$^{104}_{46}Pd_{58}$	0.556	10	2.42	0.042		0.786	$+(30^{+\infty}_{-22})$	+46	72Si08
$^{106}_{46}Pd_{60}$	0.512	12	2.20	0.024	1.1	0.616	-8(1)	-16	68Ha35
$^{108}_{46}Pd_{62}$	0.434	15	2.14	0.018	0.77	0.497	-3.1(4)	-7.5	71Ro09
$^{110}_{46}Pd_{64}$	0.374	18	2.18	0.011	1.4	0.440	$-(5^{+2}_{-1})$	-12	69Ro05

(continued)

Table I. continued

Nucleus	E(2) [MeV]	B(E2,2→0) [10^{-2}(eb)^2]	E(2')/E(2)	B(E2,2'→0)/B(E2,2'→2)	B(E2,2'→2)/B(E2,2→0)	E_γ [MeV]	δ	Δ [eb/nm]	Reference[a]
$^{106}_{48}Cd_{58}$	0.633	8.5	2.71	1.4	0.25	1.084	-0.9(2)	-1.0	73Gr16,69Mi07
$^{108}_{48}Cd_{60}$	0.633	8.8	2.53	0.097	0.63	0.973	$-(1.5^{+1.5}_{-0.6})$	-1.8	69Mi07
$^{110}_{48}Cd_{62}$	0.658	9.3	2.23	0.048	1.1	0.818	-1.2(1)	-1.8	70Kr03
$^{112}_{48}Cd_{64}$	0.617	11	2.12	0.032	0.55	0.695	-0.77(6)	-1.3	73Gr16
$^{114}_{48}Cd_{66}$	0.558	12	2.16	0.022	0.70	0.650	$-(1.4^{+0.7}_{-0.3})$	-2.8	73Gr16
$^{116}_{48}Cd_{68}$	0.513	12	2.38	0.052	0.63	0.710	$-(1.5^{+0.9}_{-0.4})$	-2.5	69Mi07
$^{116}_{50}Sn_{66}$	1.293	4.4	1.63	0.015		0.820	-1.7(3)	-2.5	73Gu00
$^{122}_{52}Te_{70}$	0.564	13	2.23	0.011	2.7	0.691	-3.46(5)	-6.0	67Ko13
$^{124}_{52}Te_{72}$	0.603	10	2.20	0.0074	2.0	0.722	-3.4(1)	-5.6	71Gr14
$^{126}_{52}Te_{74}$	0.667	10	2.13	0.0036	3.2	0.754	-5.1(2)	-8.2	71Gr14,71Ta04
$^{126}_{54}Xe_{72}$	0.386	17	2.28	0.020		0.491	+17(3)	+40	71Gr14,71Ta04
$^{128}_{54}Xe_{74}$	0.441	19	2.20	0.013		0.528	+6.4(10)	+14	67Ha44
$^{130}_{54}Xe_{76}$	0.538	14	2.09						
$^{132}_{54}Xe_{78}$	0.668	9	1.94	0.0014		0.630	$+(6^{+2}_{-1})$	+11	71Kr16,71Ta21

(continued)

Table I. continued

Nucleus	E(2) [MeV]	B(E2,2→0) [10^{-2}(eb)^2]	E(2')/E(2)	B(E2,2'→0)/B(E2,2'→2)	B(E2,2'→2)/B(E2,2→0)	E_γ [MeV]	δ	Δ [eb/nm]	Reference[a]
$^{132}_{56}Ba_{76}$	0.464	16	2.24	0.026		0.573	$+(9^{+7}_{-3})$	+19	71Ta21
$^{134}_{56}Ba_{78}$	0.605	13	1.93	0.0063		0.563	-7.4(9)	-16	72Be45
$^{140}_{58}Ce_{82}$	1.596	5.4	1.47	0.0070		0.752	+0.33(3)	+0.53	71Wi23
			1.58	3.7		0.816	-0.15(7)	-0.19	73Sa01
$^{142}_{58}Ce_{84}$	0.642	9.2	2.39	2.9		0.894	-0.08(3)	-0.11	65Pr03
$^{144}_{60}Nd_{84}$	0.695	10	2.24	0.004		0.864	-0.63(15)	-0.9	68Re01
$^{150}_{62}Sm_{88}$	0.334	28	3.13	0.017		0.712	-6(2)	-10	66Sm03,70Gr42
			3.49	0.25		0.860	$+(8^{+5}_{-2})$	+12	69Re11
$^{152}_{64}Gd_{88}$	0.344	24	2.70	0.021		0.586	-3.1(1)	-6.4	72Ka45,71Zo05
			3.23	0.15		0.765	+4.3(7)	+6.7	72Ka45

[a]References are listed in the Appendix.

Table II. E2/M1 Mixing Ratios of Gamma Transitions from 2" Levels

Nucleus	E(2")/E(2)	$\frac{B(E2,2''\to0)}{B(E2,2''\to2)}$	$\frac{T(2''\to2')}{T(2''\to2)}$	E_γ [MeV]	δ	Δ [eb/nm]	Reference[a]
$^{58}_{28}Ni_{30}$	2.09	0.62	0.02	1.584	+0.20(3)	+0.15	69Va24,67Ho15
	2.25	0.42	<0.02	1.809	+0.54(15)	+0.36	69Sc31,67Du07
$^{60}_{28}Ni_{32}$	2.34	0.18	0.007	1.791	-0.22(3)	-0.14	66Sh12,71Mo22
	2.45	0.076	0.48	1.936	+0.7(3)	+0.4	71Mo22
$^{62}_{28}Ni_{34}$	2.46	0.00	<0.04	1.718	$-(4^{+3}_{-1})$	-2.8	70Fa06
	2.61	0.076	0.90	1.886	+0.7(2)	+0.4	70Fa06
	2.70	2.5	0.13	1.985	+0.13(8)	+0.08	70Fa06
$^{68}_{30}Zn_{38}$	2.18	0.025	<0.1	1.261	-0.18(4)	-0.17	71Ot01
	2.62	0.069	<1	1.745	+0.28(5)	+0.19	73La01
$^{76}_{34}Se_{42}$	3.20	80	0.13	1.220	+0.02(2)	+0.02	60Gr07,63Sa26
$^{94}_{42}Mo_{52}$	2.75	0.014	<0.2	1.522	-1.9(5)	-1.6	68Ar06
$^{102}_{44}Ru_{58}$	3.34	0.36	<0.5	1.106	+0.25(3)	+0.24	70Si13
	4.30	0.083	<0.13	1.562	-2.0(4)	-1.6	70Si13
$^{106}_{46}Pd_{60}$	3.06	0.36	0.03	1.050	+0.20(1)	+0.24	68Ha35
	4.28	>10	>120	1.061[b]	-1.20(15)	-1.4	73Av01
	4.39	0.40	6	1.113[b]	$+(1.5^{+1.5}_{-0.6})$	+1.6	73Av01
	4.75	0.17	0.32	1.925	$-(2.4^{+1.1}_{-0.5})$	-1.5	73Av01
	4.88	<0.01	<0.01	1.988	$-(2.3^{+1.7}_{-0.8})$	-1.4	73Av01
$^{108}_{46}Pd_{62}$	3.32	0.57	<0.2	1.007	+0.24(4)	+0.29	71Ok01
$^{110}_{48}Cd_{62}$	2.70	7	<0.3	1.126	$+(0.06^{+0.07}_{-0.12})$	+0.06	72Ka34
$^{112}_{48}Cd_{64}$	2.38	0.9	<0.01	0.851	+0.16(5)	+0.22	72Ka34,73Gr16
$^{114}_{48}Cd_{66}$	2.44	7	0.001	0.805	+0.10(10)	+0.07	73Gr16,69Mi07
$^{124}_{52}Te_{72}$	3.39	0.011	2.4	0.714[b]	+1.0(2)	+1.7	71Gr14
				1.437	+4(2)	+3	72Ba38
	3.47	<0.04	0.06	1.489	-3.3(2)	-2.8	72Ba38

(continued)

Table II. continued

Nucleus	E(2")/E(2)	$\frac{B(E2,2''\to 0)}{B(E2,2''\to 2)}$	$\frac{T(2''\to 2')}{T(2''\to 2)}$	E_γ [MeV]	δ	Δ [eb/nm]	Reference[a]
$^{132}_{54}Xe_{78}$	2.97	1.5	<1	1.317	-0.077(25)	-0.070	62Ro07
$^{152}_{64}Gd_{88}$	3.84	0.11	0.13	0.974	+0.58(7)	+0.71	72Ka45

[a]References are listed in the Appendix.

[b]2" → 2' transitions; all others are 2" → 2 transitions.

Table III. E2/M1 Mixing Ratios of Gamma Transitions from 3^+ Levels

Nucleus	E(3)/E(2')/E(2)	$\frac{T(3\to2')}{T(3\to2)}$	$\frac{B(E2,3\to2)}{B(E2,3\to2')}$	3 → 2			3 → 2'			3 → 4			References[a]
				E_γ [MeV]	δ	Δ [eb/nm]	E_γ [MeV]	δ	Δ [eb/nm]	E_γ [MeV]	δ	Δ [eb/nm]	
$^{72}_{32}Ge_{40}$	2.5/1.8	5.5	0.002	1.231	$-(1^{+3}_{-0.6})$	-1							69Mo23
$^{74}_{32}Ge_{42}$	2.9/2.0	1.0	0.0038	1.102	+0.34(5)	+0.36	0.493	-1.3(4)	-3.1				70Ch15
$^{82}_{36}Kr_{46}$	2.7/1.9	1.5	0.018	1.317	+4.3(2)	+3.8	0.619	+2.1(1)	+4.1				72Sa24
$^{96}_{42}Mo_{54}$	2.5/1.9	0.19		1.200	+1.7(3)	+1.7							70He27
$^{100}_{44}Ru_{56}$	3.5/2.5	0.15		1.347	$+(0.5^{+1.2}_{-0.2})$	+0.5							68Ka04
$^{102}_{44}Ru_{58}$	3.2/2.3	0.32	0.037	1.047	-7.0(6)	-8.0	0.418	-7.2(10)	-20				70Si13
$^{106}_{46}Pd_{60}$	3.1/2.2	0.58		1.045	-0.18(2)	-0.21							68We16
$^{110}_{48}Cd_{62}$	3.3/2.2	0.62	0.0078	1.505	-0.48(3)	-0.38	0.687	-1.80(5)	-3.1	0.620	-0.8(5)	-1.6	73Jo08,70Kr03
$^{132}_{54}Xe_{78}$	2.7/1.9	1.0	0.012	1.136	+0.9(3)	+1.0	0.506	-1.3(4)	-3.1				71Kr16
$^{134}_{56}Ba_{78}$	2.7/1.9	1.4	0.011	1.038	+1.9(2)	+2.1	0.475	$-(10^{+5}_{-2})$	-25				72Be45,73Ga10
$^{138}_{56}Ba_{82}$	1.71/1.54	0.037		1.010	+0.009(18)	+0.011				0.547	-0.039(29)	-0.09	73Si00
$^{140}_{58}Ce_{82}$	1.51/1.47	0.002		0.815	-0.031(5)	-0.046				0.329	-0.041(12)	-0.15	73Sa01,71Wi23
$^{142}_{58}Ce_{84}$	4.2/2.4	0.44		2.060	+0.30(5)	+0.18							65Pr03
$^{150}_{62}Sm_{88}$	3.8/3.1	0.0								0.505	-2.3(6)	-5.5	66Sm03
$^{152}_{64}Gd_{88}$	4.2/2.7	17		1.090	-0.22(3)	-0.24				0.678	>+9 or <-14	>+16 or <-25	70Ba32

[a] References are listed in the Appendix.

Table IV. E2/M1 Mixing Ratios of 4' → 4 Gamma Transitions

Nucleus	E(4')/E(4)/E(2)	$\frac{T(4'-4)}{T(4'-2)}$	$\frac{B(E2,4'-2)}{B(E2,4'-4)}$	E_γ [MeV]	δ	Δ [eb/nm]	Reference[a]
$^{76}_{34}Se_{42}$	3.6/2.4	0.8	0.028	0.695	$+(4^{+3}_{-1})$	+7	70Li11
$^{104}_{46}Pd_{58}$	3.7/2.4	1.0		0.759	-0.05(12) or -0.77(10)	-0.08 or -1.2	72Ok01
	3.9/2.4	1.9	<0.28	0.858	+0.27(10) or -1.7(4)	+0.38 or -2.4	72Ok01
	4.1/2.4	>15	<0.020	0.942	+0.41(13)	+0.53	72Ok01
$^{110}_{48}Cd_{62}$	3.4/2.3	8.9	0.011	0.677	-0.44(5)	-0.78	73Jo08
$^{124}_{52}Te_{72}$	3.2/2.1	1.5	12	0.709	+0.04(4)	+0.07	71Gr14
$^{132}_{54}Xe_{78}$	2.9/2.2 3.6/2.2	8.2 300	0.022 0.008	0.523 0.955	-0.25(15) -0.15(5)	-0.58 -0.19	71Kr16
$^{134}_{56}Ba_{78}$	3.3/2.3	4.1	0.037	0.569	+0.29(2)	+0.62	72Be45
$^{140}_{58}Ce_{82}$	1.6/1.3	1.2	6	0.432	-0.05(3)	-0.14	73Sa01,69Co17
$^{150}_{62}Sm_{88}$	4.3/2.3 4.9/2.3	23 2.6	0.006 0.06	0.675 0.869	-1.4(2) >+2.6	-2.5 >+3.6	66Co00 66Sm03

[a]References are listed in the Appendix.

Table V. E2/M1 Mixing Ratios of Gamma Transitions from Spin 5 or 6 Levels

Nucleus	J_i	$E(J_i)/E(2)$	$\frac{T(J_i \to 4')}{T(J_i \to 4)}$	$\frac{T(J_i \to 6)}{T(J_i \to 4)}$	J_f	E_γ [MeV]	δ	Δ [eb/nm]	Reference[a]
$^{96}_{42}Mo_{54}$	6'	3.54	0.027	0.16	6	0.314	+0.27(7)	+1.0	71Ba59
$^{102}_{44}Ru_{58}$	5[b]	3.95	0.01		4	0.768	+2.3(2)	+3.6	70Si13
	5	4.68	0.10	0.055[b]	4	1.113	-2.7(3)	-2.9	68Ad02
$^{110}_{48}Cd_{62}$	5	4.45	0.9	0.13	6	0.446	-0.45(20)	-1.2	70Kr03
					4"	0.707	-0.58(2)	-0.98	73Jo08
					4	1.384	-0.46(1)	-0.40	73Jo08
$^{130}_{54}Xe_{76}$	5	4.41	<0.3	2.9	6	0.419	-0.40(3)	-1.1	72Ba51
					4	1.162	+0.25(3)	+0.26	68Ho26
$^{148}_{62}Sm_{86}$	6'	3.99	<0.01	0.65	6	0.289	-0.01(8)	-0.04	68Wy00

[a]References are listed in the Appendix.

[b]The angular correlation data are not inconsistent with a 6^+-assignment for the 1.873 MeV level.

Table VI. Computed Two-Particle Contributions to 2' → 2 E2/M1 Mixing Ratios

Nucleus	Δ (eb/nm)	$\sum B_{j_1 j_2; j_1 j_2}$ (fm^4/MeV^2) Protons	Neutrons	Δ_I	$\lvert\Delta_{II}\rvert$
^{58}Ni	-1.0		-0.5	-250	180
^{60}Ni	+1.5		3.9	-150	20
^{62}Ni	+3.4		1.1	-350	20
^{64}Zn	-6	77	1		4
^{66}Zn	-3.1	84	-0.1		10
^{68}Zn	-2.3	88	-0.3		2
^{70}Ge	-9	148	-0.4	-5	2.5
^{72}Ge	+10	105	-0.5		
^{74}Ge	+6	76	-1.4	+5	2.5
^{76}Ge	+8	75	-3	+6	3
^{74}Se	-11	105	-0.4		0.6
^{76}Se	+11	111	-1.5		1.2
^{78}Se	+11	150	-3.7		1.2
^{80}Kr	+31	215	-4		2
^{82}Kr	+3.5	283	-9		2
^{84}Kr	-48	353	-11		
^{84}Sr	+1.4	150	-9		4
^{86}Sr	+0.47	326	27		0.6
^{88}Sr	-0.04	-1027			
^{92}Zr	+0.04	160	45		0.2
^{94}Mo	-2.4	312	44		4
^{96}Mo	+0.73	236	-43		2
^{98}Mo	+1.1	283	-13		3
^{100}Ru	+7	293	-7		1.2

(continued)

Table VI continued

Nucleus	Δ (eb/nm)	$\sum B_{j_1 j_2; j_1 j_2}$ (fm^4/MeV^2) Protons	Neutrons	Δ_I	$\lvert\Delta_{II}\rvert$
^{102}Ru	-110	261	-3		1.2
^{104}Ru	-19	202	-1	+1.4	1.9
^{104}Pd	+46	425	-4	+1.7	1.2
^{106}Pd	-16	371	-1	+1.3	1.9
^{108}Pd	-7.5	283	2	+1.6	2.5
^{110}Pd	-12	237	3	+2.2	4.0
^{106}Cd	-1.0	2500	-2		
^{108}Cd	-1.8	2600	1	+0.1	0.2
^{110}Cd	-1.8	4000	3	+0.1	0.2
^{112}Cd	-1.3	2100	4	+0.5	0.6
^{114}Cd	-2.8	1300	3	+0.6	0.7
^{116}Cd	-2.5	930	1	+0.3	0.7
^{116}Sn	-2.5		-2	+300	160
^{122}Te	-6.0	1500	-3	+0.6	1.2
^{124}Te	-5.6	3200	-6	+2.0	0.7
^{126}Te	-8.2	-5600	-5	-0.6	0.7
^{126}Xe	+40	250	-4		5
^{128}Xe	+14	310	-3		5
^{132}Xe	+11	1800	4		4
^{132}Ba	+19	510	-1		3
^{134}Ba	-16	930	2	+1.2	4
^{140}Ce	+0.53	-1600			3
^{142}Ce	-0.11	1200	108	+1.2	0.2
^{144}Nd	-0.9	2400	650	+1.3	2.5
^{150}Sm	-10	430	0.8	+2.0	5
^{152}Gd	-6.4	230	0.1		12

Table VII. Comparison of Experimental 2' → 2 E2/M1 Mixing Ratios with Computed Values

Nucleus	g(2)	Z/A	f	Δ theory	Δ experiment
^{100}Ru	0.55(7)	0.44	-0.16	+2.5	+7
^{102}Ru	0.41(3)	0.43	0.03	-20	-110
^{104}Ru	0.29(4)	0.42	0.20	-5.8	-19
^{106}Pd	0.38(3)	0.43	0.09	-7.8	-16
^{108}Pd	0.30(4)	0.43	0.20	-5.5	-7.5
^{110}Pd	0.25(3)	0.42	0.25	-5.9	-12
^{110}Cd	0.35(8)	0.44	0.14	-4.8	-1.8
^{112}Cd	0.30(6)	0.43	0.20	-4.5	-1.3
^{122}Te	0.27(5)	0.43	0.22	-4.6	-6.0
^{124}Te	0.29(4)	0.42	0.21	-4.5	-5.6
^{126}Te	0.25(7)	0.41	0.25	-4.5	-8.2

FIGURE CAPTIONS

Fig. 1. Histogram of E2/M1 mixing ratios Δ of $2' \rightarrow 2$ transitions in even-even nuclei $60 \lesssim A \lesssim 150$. The labels + and - refer to the phase of Δ, as defined in the present work.

Fig. 2. E2/M1 mixing ratios of $2' \rightarrow 2$ transitions in even-even nuclei $60 \lesssim A \lesssim 150$. The solid curve indicates the trend of the measured values and shows pronounced minima in the vicinity of closed shells.

Fig. 3. Spectrum of excited states expected on the basis of the phonon vibrational model. The actual spectrum of ^{120}Te is shown to the left. To the right is indicated the decomposition of the phonon states into quasi-rotational bands, which might be identified (in order) as the ground-state, γ, β, $\gamma\gamma$, $\beta\beta$, $\gamma\gamma\gamma$, $\beta\gamma$, and $\beta\beta\beta$ quasi-bands.

Fig. 4. Relationship between energies of 2' states and lowest 2-proton configuration. The 2' states are labelled with + or - to indicate the phase of the $2' \rightarrow 2$ E2/M1 mixing ratio.

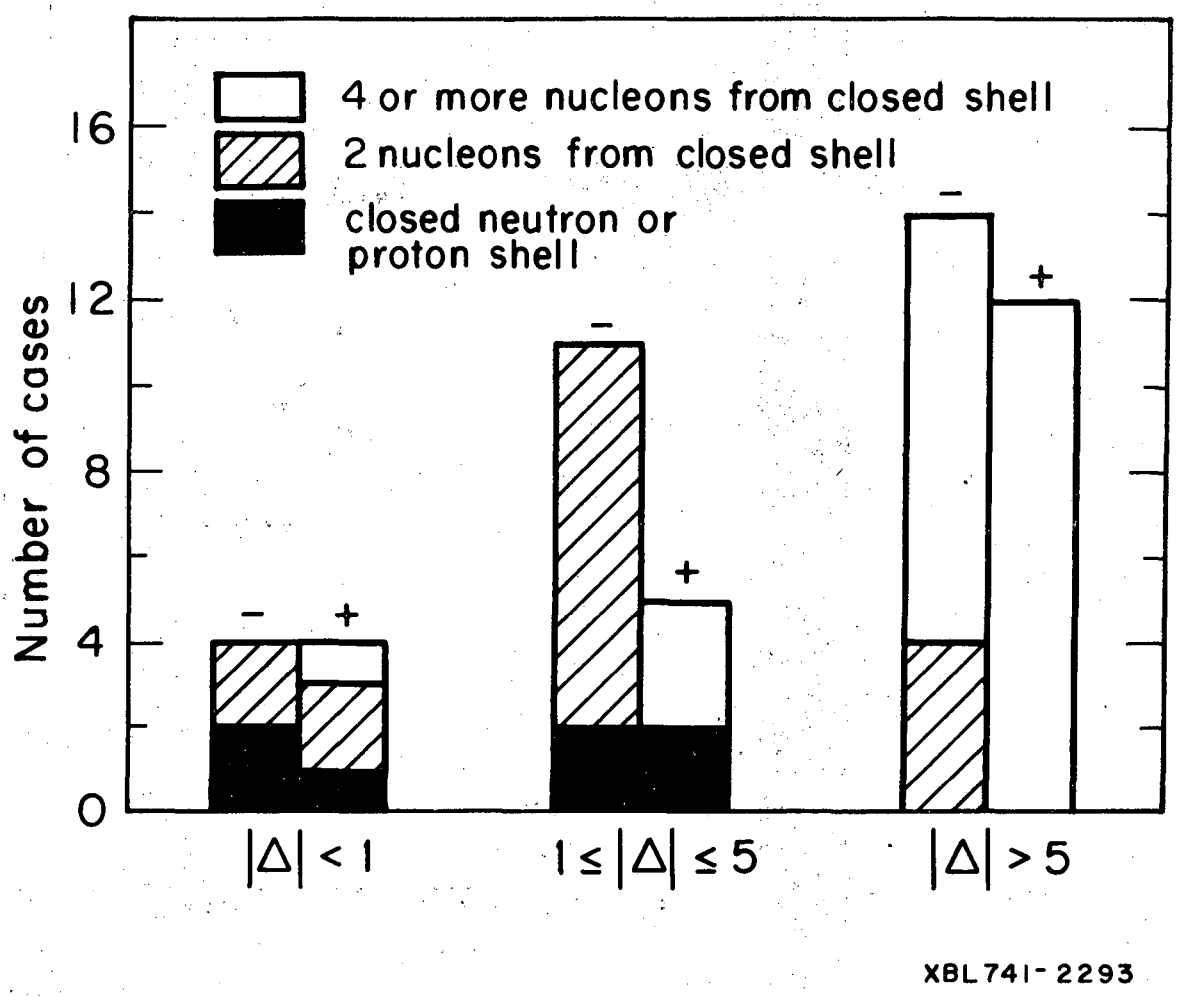

4 or more nucleons from closed shell
2 nucleons from closed shell
closed neutron or proton shell
Number of cases
0
4
8
12
16
−
+
|Δ| < 1
1 ≤ |Δ| ≤ 5
|Δ| > 5
XBL741-2293

Fig. 1

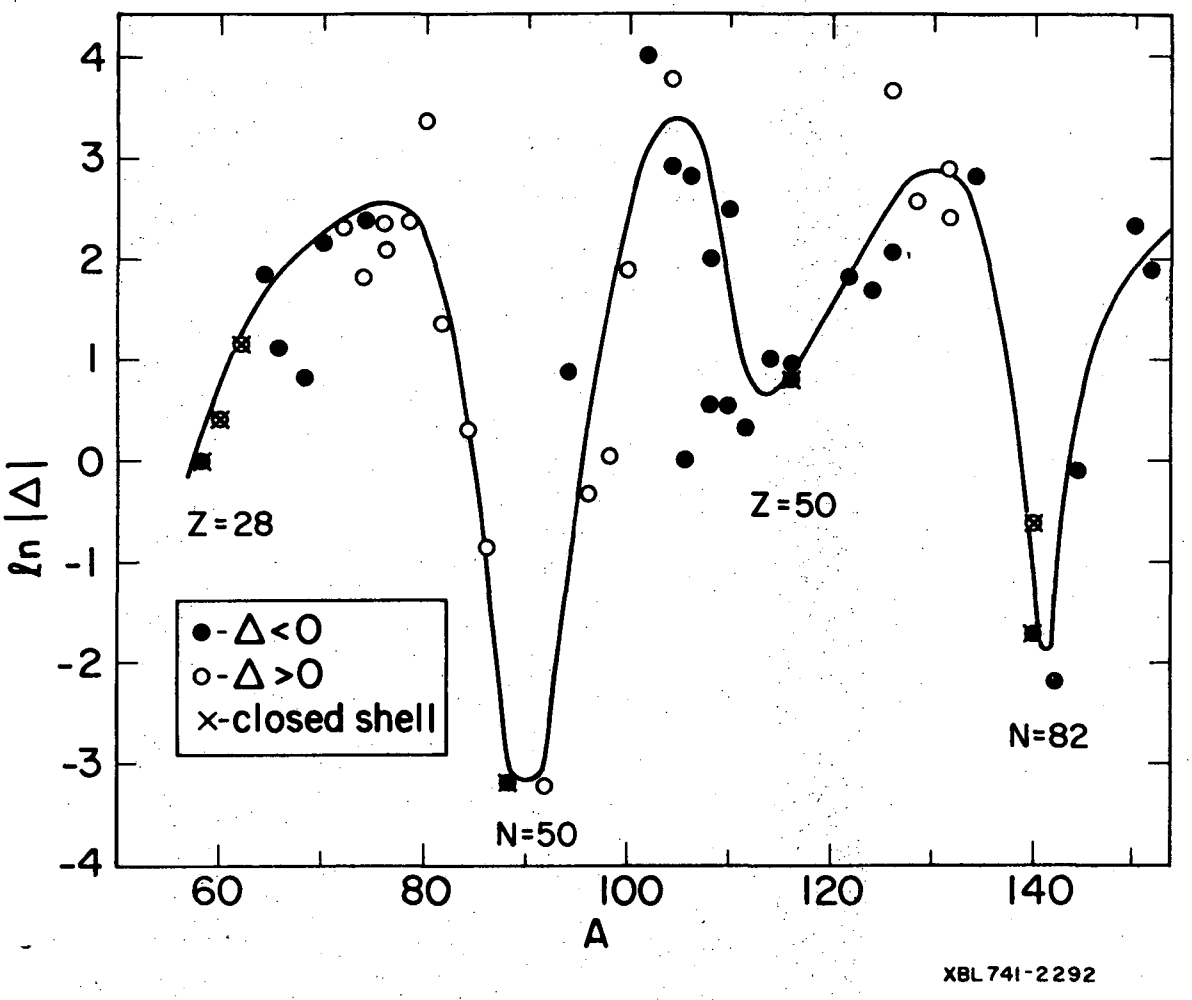

4
3
2
1
0
-1
-2
-3
-4
ln |Δ|
60
80
100
120
140
A
Z=28
N=50
Z=50
N=82
●-Δ<0
○-Δ>0
×-closed shell
XBL741-2292

Fig. 2

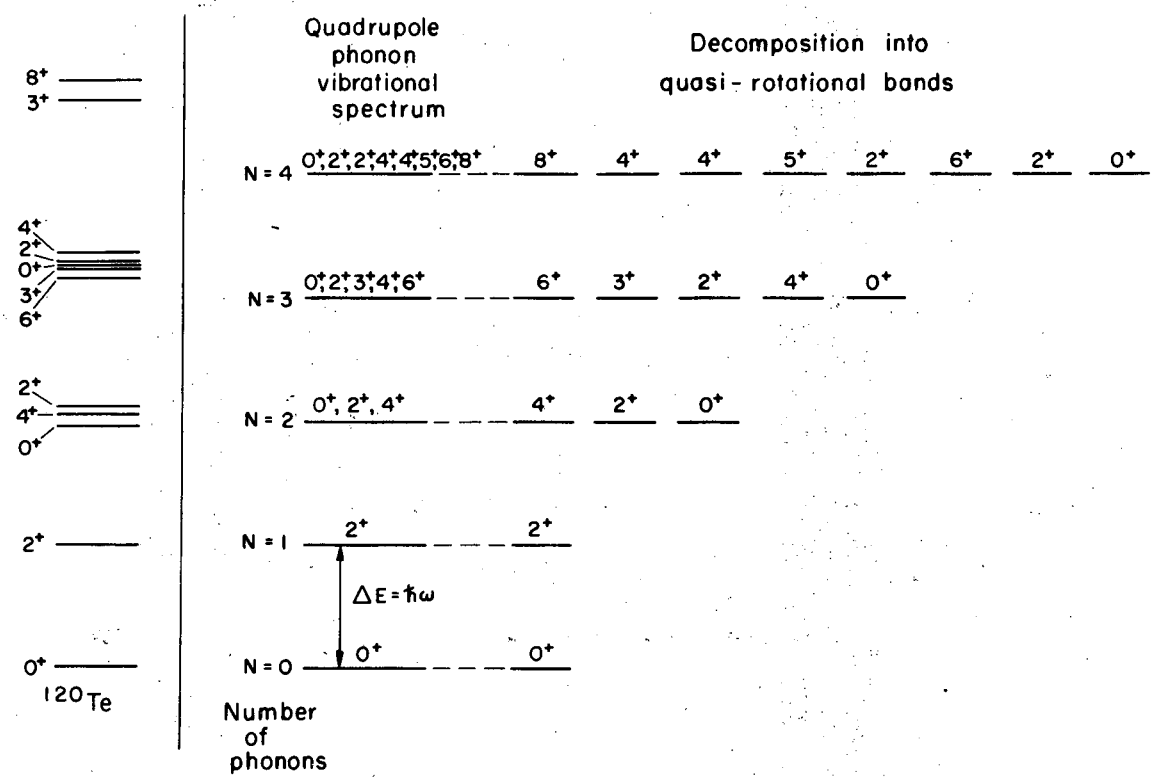
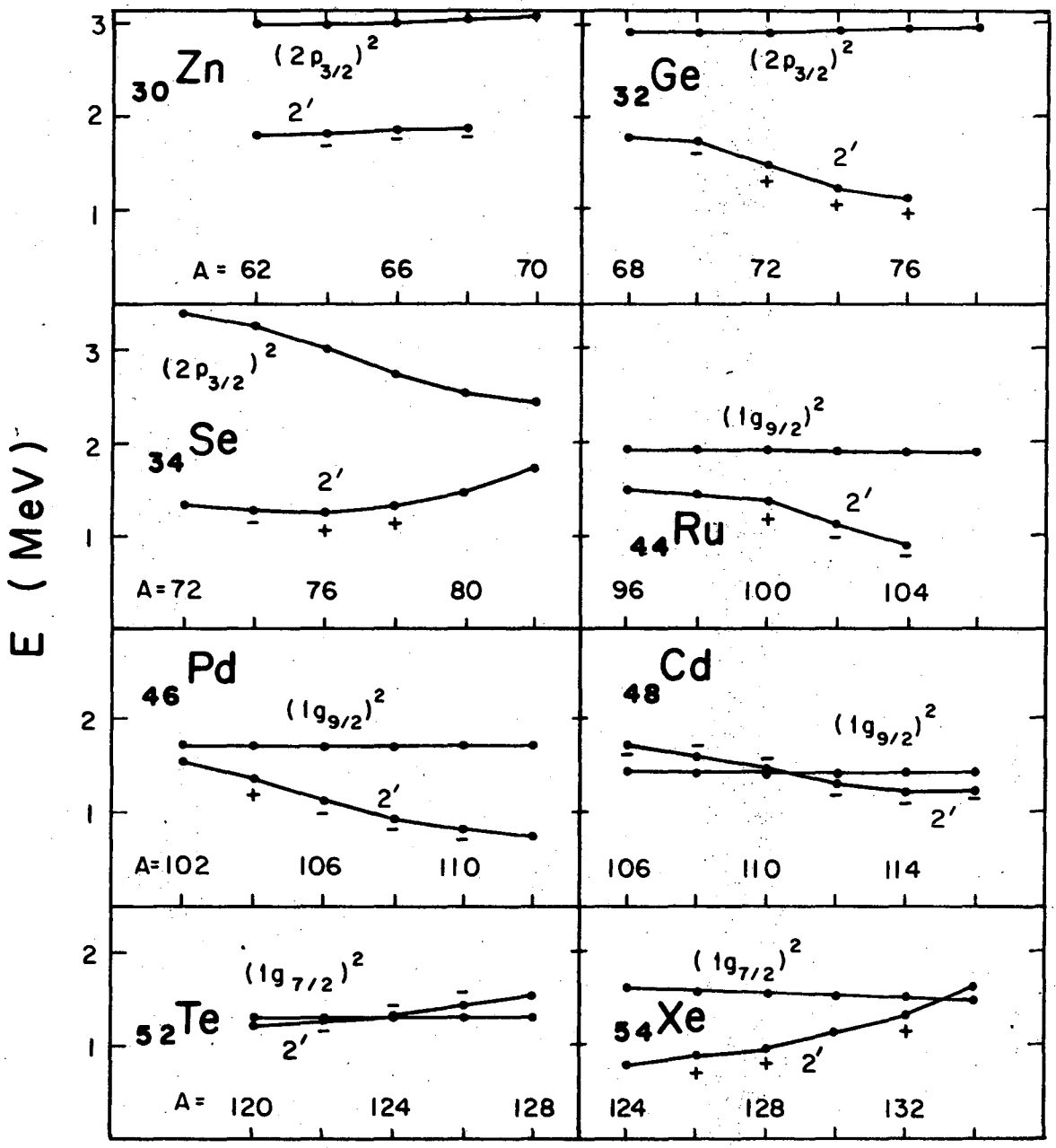

Quadrupole phonon vibrational spectrum
Decomposition into quasi-rotational bands
8+
3+
4+
2+
0+
3+
6+
2+
4+
0+
2+
0+
120Te
N = 4
0+,2+,2+,4+,4+,5+,6+,8+
8+
4+
4+
5+
2+
6+
2+
0+
N = 3
0+,2+,3+,4+,6+
6+
3+
2+
4+
0+
N = 2
0+, 2+, 4+
4+
2+
0+
N = 1
2+
2+
ΔE = ħω
N = 0
0+
0+
Number of phonons
XBL733-2484

Fig. 3



XBL741-2210

Fig. 4

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