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GAMMA AND OCTUPOLE VIBRATIONS IN DEFORMED NUCLEI*

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Abstract

The gamma (2+) and octupole (0-, 2-) states of deformed even nuclei in the rare earth region are calculated in the Fermi liquid theory. The relation of RPA and schematic model calculations to the more general formulation is discussed. Off-diagonal matrix elements are found to peak at the nuclear surface. The vibration modes studied are found to be only weakly dependent on the spin and isospin dependent terms in the interaction. An example is presented to show the necessity for including a long-ranged term in the renormalized interaction.

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1. Introduction

The microscopic structure of the non-rotational collective states in deformed even-mass nuclei has been the subject of many theoretical investigations. These studies fall into two rather general categories. In the first attempts are made to solve the phenomenological Bohr[1]) hamiltonian for quadrupole motion. The inertial and potential parameters are either chosen empirically[2]) or are calculated from microscopic models[3]). The second group includes the completely microscopic approaches.

The coexistence of single particle and collective motion in nuclei has been understood by performing microscopic calculations using modern two nucleon interactions[4]). A parallel theoretical approach is a phenomenological theory of finite Fermi systems[5]) where the parameters are chosen on the basis of a microscopic analysis of the many body problem. Most microscopic calculations have been performed for nuclei near closed shells where the single particle level density is small and the numerical problems associated with the theory are tractable. In the rare earth and actinide regions of deformation, the single particle level density is very large, so that to obtain a manageable numerical problem theories have been simplified to a point where some variant of the schematic model could be used[6]). Bes[7]) and Soloviev[8]) have used a pairing plus a quadrupole or an octupole force, while Faessler[9]), although using a surface delta interaction, made the necessary approximations to insure the separability of matrix elements. He was thus able to apply the schematic model. More recently Pyatov[10]) has attempted to explain certain features of higher 2+ and 0+ states by adding a spin-quadrupole interaction. Schematic model calculations give an incite into the microscopic structure of the gamma quadrupole and octupole vibrations and reproduce the

qualitative features observed experimentally. Nevertheless it is difficult to generalize these theories or to understand how their results would be obtained using more rigorous many-body theory.

In this paper we wish to utilize a microscopic description of gamma and octupole vibrational states which does not depend on the schematic model. As a first approximation we solve the Bethe-Salpater equation using the deformed quasiparticle approximation. In particular we ignore for the present non-adiabatic rotational effects and the effects of phonon-particle coupling. These will be discussed in subsequent publications.

In sec. 2 the theoretical considerations are developed and in sec. 3 a description of the calculation is presented. Technical details too lengthy to be presented in a journal article are contained in an appendix to ref. [11]). Section 4 contains the results and a discussion. A summary and conclusions are presented in sec. 5.

2. Theory

The response of a nucleus to a weak external field is given to first order in the coupling constant by the linear response function[12]). It is often convenient to define new quantities which contain the same information as the linear response function but which describe the effective field produced in the nucleus under the influence of the external field. In a superfluid nucleus there are four such fields, V, V^h, d^1, and d^2. V and V^h cause particle-particle and hole-hole transitions respectively, while d^1 and d^2 connect the particle component of the initial state to the hole component of the final state and vice versa. Migdal and Larkin[13]) have derived the Bethe-Salpater equations for these fields. In deriving them, it is assumed that the Bogolon approximation† for single particle propagators is valid and that the irreducible amplitudes describing particle-particle, (pp), and particle-hole, (ph), scattering are not retarded.

The Migdal-Larkin equations can be put in a compact form by defining[14]) new effective fields $V^{\pm} = \frac{1}{2}(V \pm V^h)$, $d^{\pm} = \frac{1}{2}(d^1 \pm d^2)$. Denoting matrix elements with respect to single particle states of the set ϕ_λ by subscripts these equations become

$$V^{\pm}_{12} = (e_q V^o)^{\pm}_{12} + \sum_{34} F_{1\bar{4}\bar{2}3}\, \eta^{\pm}_{34}\, Z^{\pm}_{34}$$

$$d^{\pm}_{12} = (e'_q d^o)^{\pm}_{12} - \sum_{34} F^{\xi}_{1234}\, \xi^{\pm}_{34}\, Z^{\pm}_{34} \quad . \tag{2.1}$$

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†By Bogolon approximation we understand a BCS transformation between Landau quasiparticles and -holes (hereafter simply referred to as quasiparticles and quasiholes).

$Z^{\pm}$ are the changes in the Bogolon density matrix due to the field and are given by

$$Z^{\pm}_{12} = \frac{1}{E_{12}-\omega^2} \{-E_{12}\, \eta^{\pm}_{12}\, V^{\pm}_{12} + \omega\, \eta^{\mp}_{12}\, V^{\mp}_{12} + E_{12}\, \xi^{\pm}_{12}\, d^{\pm}_{12} - \omega\, \xi^{\mp}_{12}\, d^{\mp}_{12}\} \ . \quad (2.2)$$

In the above expressions, $\bar{\lambda}$ denotes the state which is time conjugate to ϕ_{λ},† and $\eta^{\pm}_{12} = u_1 v_2 \pm v_1 u_2$, $\xi^{\pm}_{12} = u_1 u_2 \mp v_1 v_2$, $E_{12} = E_1 + E_2$. The frequency of the external field is denoted by ω while u^2 specifies non-occupation and v^2 the occupation of the quasiparticle pairs in single particle states and E denotes the energy of the Bogolon. These are related to the pairing potential Δ and to the single particle energy ε by the usual BCS formalism. V^{o} and d^{o} are external fields which do not change and change the number of quasiparticles in the nucleus by two, e_q and e_q' are the local charges for these fields, and F and F^{ξ} are renormalized irreducible amplitudes describing (ph) and (pp) scattering of the quasiparticles. There are simple poles in these external fields corresponding to excited states in the system and so one can obtain energies and transition probabilities by solving the homogeneous set of equations corresponding to (2.1)[5]. By substituting (2.1) into (2.2) and setting the inhomogeneous terms to zero, we obtain the equations for the change of the Bogolon density matrices at frequencies corresponding to excited states in the nucleus.

$$\omega_K Z^{\mp}_{12}(\omega_K) = E_{12} Z^{\mp}_{12}(\omega_K) + \sum_{34} [\eta^{\pm}_{12}\eta^{\pm}_{34} F_{1\bar{4}\bar{2}3} + \xi^{\pm}_{12}\xi^{\pm}_{34} F^{\xi}_{1234}] Z^{\pm}_{34}(\omega_K) \ . \quad (2.3)$$

If the following normalization is used

$$\sum_{12>0} Z^{+}_{12}(\omega_K) Z^{-}_{12}(\omega_K) = 1 \quad , \quad (2.4)$$

† For deformed but axially symmetric nuclei, $\phi^{-}_{\lambda} = (-1)^{1/2-\Omega_{\lambda}}\, \phi_{-\lambda}$ where Ω_{λ} is the projection of the angular momentum on the symmetry axis.

the probability for an electric transition is given by

$$M_{0 \to K} = \sum_{12>0} \eta^{+}_{12} \, Z^{+}_{12}(\omega_K)(e_q V^o)_{12} \quad . \tag{2.5}$$

The symbol 12 > 0 means that only one of the two admitted combinations (1,2) and $(\bar{2},\bar{1})$ should be included.

Following Kamerdzhiev[15]) we note that for external fields whose frequencies are small compared to the summation intervals in (2.1), it is convenient to separate in the sums those terms which depend little on the frequency and on the degree of filling of the subshells. Let those terms be put into Σ_0. The terms depending on the frequency and filling of the subshell are placed in the sum Σ_1. Then (2.1) may be written

$$V^{\pm}_{12} = (e_{eff} V^o)^{\pm}_{12} + \sum_{34}{}_1 \, \Gamma'_{1\bar{4}\bar{2}3} \, \eta^{\pm}_{34} \, Z^{\pm}_{34}$$

$$d^{\pm}_{12} = (e'_{eff} d^o)^{\pm}_{12} - \sum_{34}{}_1 \, \Gamma''_{1234} \, \xi^{\pm}_{34} \, Z^{\pm}_{34} \quad , \tag{2.6}$$

provided that the effective charges and renormalized amplitudes are defined by the equations

$$(e_{eff} V^o)^{\pm}_{12} = (e_q V^o)^{\pm}_{12} - \sum_{34}{}_0 \, F_{1\bar{4}\bar{2}3} \, (\eta^{\pm}_{34})^2 \, K_{34} \, (e_{eff} V^o)^{\pm}_{34}$$

$$(e'_{eff} d^o)^{\pm}_{12} = (e'_q d^o)^{\pm}_{12} - \sum_{34}{}_0 \, F^{\xi}_{1234} \, (\xi^{\pm}_{34})^2 \, K_{34} \, (e'_{eff} d^o)^{\pm}_{34}$$

$$\Gamma'_{1\bar{4}\bar{2}3} = F_{1\bar{4}\bar{2}3} - \sum_{56}{}_0 F_{1\bar{6}\bar{2}5}(\eta^+_{56})^2 K_{56} \Gamma'_{5\bar{4}\bar{6}3}$$

$$\Gamma''_{1234} = F^{\xi}_{1234} - \sum_{56}{}_0 F^{\xi}_{1256}(\xi_{56})^2 K_{56} \Gamma''_{5634}$$

$$K_{12} = E_{12}/(E^2_{12} - \omega^2) \quad . \tag{2.7}$$

Equations analogous to (2.3) and (2.5) may be derived from (2.6)

$$\omega_K Z^{\mp}_{12}(\omega_K) = E_{12} Z^{\pm}_{12}(\omega_K) + \sum_{34}{}_1 [\eta^{\pm}_{12}\eta^{\pm}_{34}\Gamma'_{1\bar{4}\bar{2}3} + \xi^{\pm}_{12}\xi^{\pm}_{34}\Gamma''_{1234}] Z^{\pm}_{34}(\omega_K)$$

$$\equiv \sum_{34}{}_1 (P \pm Q)_{1234} Z^{\pm}_{34}(\omega_k) \quad , \tag{2.3'}$$

$$M_{0 \to K} = \sum_{12>0}{}_1 \eta^+_{12} Z^+_{12}(\omega_k)(e_{eff}V^0)_{12} \quad , \tag{2.5'}$$

provided the normalization given in (2.4) is used. These equations are formally identical to those derived in the random phase approximation (RPA)[16]). However, the interpretation of the amplitudes Γ', Γ'', and $Z^{\pm}$ is different. In the theory of finite Fermi systems[5]) the quasiparticle approximation is used for the single particle propagators so that even to lowest order in the interaction among the quasiparticles, the scattering amplitudes are renormalized by a factor, a^2, where a is the residue of the dominant pole term and is assumed to be independent of the single particle states. Furthermore, F and F^{ξ} contain the sum of all the perturbation diagrams irreducible in the (ph) and (pp) channels respectively.

$Z^{\pm}$ describe the change in the Bogolon density matrix. Only in the BCS approximation using Hartree-Fock for the single particle propagators and keeping the first order terms in F and F^{ξ} do equations (2.3) correspond to those of ref. [16]). It should be noted that in general F is not antisymmetric. However, if one assumes a bare interaction or more generally a sum of ladder diagrams for F, then it is antisymmetric.

The equations of the schematic model[9]) can be obtained from (2.3') by ignoring the (pp) amplitude Γ'' and parameterizing the (ph) amplitude Γ' as the collective multiple of the direct matrix element of a spin-independent surface delta interaction (SDI). It is further assumed that there exists an effective charge. This corresponds to assuming that in (2.5')

$$(e_{eff}V^{o})_{12} = e_{eff}V^{o}_{12} \quad .$$

The derivation of these results is usually accomplished using the RPA.

3. Description of the Calculation

In the Fermi liquid theory, account is taken of the fact that nuclear motions are highly correlated. However, for excitation energies which are small compared to the Fermi energies, it is assumed there exists a branch of single particle excitations. These are called quasiparticles and have energies which are approximately those of a one-body Hamiltonian $p^2/2m^* + U(\varepsilon,\underset{\sim}{r})$ where the effective mass m^* and the potential $U(\varepsilon,\underset{\sim}{r})$ are determined from experiment. For deformed nuclei perhaps the best parameterization for $U(\varepsilon,\underset{\sim}{r})$ is that of Vogeler[17]). However, in the calculations reported here, the Nilsson[18]) single particle basis set prescribed by Soloviev[8]) and modified by Faessler[9]) has been used. There are 32 proton orbitals and 36 neutron orbitals in this set. The single particle energies of these orbitals were chosen to fit the experimental energy level data of odd mass nuclei[19]) and are listed in units of $\hbar\omega$ in Table 1 of ref. [9]). This set, used both for the gamma and octupole states, facilitates comparisons with earlier calculations. The wave functions are obtained by diagonalizing a deformed harmonic oscillator hamiltonian with $\beta = 0.3$. The strengths of the spin-orbit $(\underset{\sim}{\ell}.\underset{\sim}{s})$ and centrifugal $(\underset{\sim}{\ell}^2)$ terms varied for different principal quantum numbers and are also given in ref. [9]).

In constructing the configuration space from the Bogolon basis set, only those configurations have been kept whose energy E_{12} is less than 1 $\hbar\omega$. It is of interest to study how this affects the renormalization of the interaction in the (pp) and (ph) channels, and a more extensive calculation in several nuclei has been carried out to determine this effect. The results will be discussed in the next section.

For the nucleon-nucleon interaction in a RPA calculation and for the irreducible block in a Fermi liquid calculation, the volume delta potential with the following parameterization is employed:

$$\Gamma'' = \left[\frac{4}{3}\frac{\varepsilon_o}{\rho_o}\right] \nu^{3/2}\, \xi\delta(\underset{\sim}{r}_1 - \underset{\sim}{r}_2)$$

$$\Gamma' = \left[\frac{4}{3}\frac{\varepsilon_o}{\rho_o}\right] \nu^{3/2}\, (f + f'\, \underset{\sim}{\tau}_1 \cdot \underset{\sim}{\tau}_2 + (g + g'\, \underset{\sim}{\tau}_1 \cdot \underset{\sim}{\tau}_2)$$

$$\underset{\sim}{\sigma}_1 \cdot \underset{\sim}{\sigma}_2)\, \delta(\underset{\sim}{r}_1 - \underset{\sim}{r}_2) \quad . \tag{3.1}$$

$\left[\frac{4}{3}\ \frac{\varepsilon_o}{\rho_o}\right]$ is a constant equal to 386.04 MeV-fm^3, where ε_o and ρ_o are the Fermi energy and density of nuclear matter. This constant is chosen so that the strength parameters will be dimensionless and of the order unity. The dimensional dependence of the radial matrix elements has been explicitly factored out in the constant $\nu = m\omega/\hbar$ where $\hbar\omega = \frac{41}{A^{1/3}}$. Thus the A dependence of the matrix elements is proportional to $A^{-1/2}$. It is shown in the appendix of ref. [11]) that for the (ph) antisymmetric RPA matrix elements, g and g' may be set equal to zero. Also in that appendix, various relations between the (ph) and (pp) matrix elements are derived and a discussion of the numerical procedure for evaluating delta force integrations is presented.

From (3.1), the schematic model (SDI) interaction can be obtained by setting $[\frac{4\varepsilon_o}{3\rho_o}]\ \nu^{3/2} = 4\pi(\frac{26 \text{ to } 28}{A})$ MeV. Further, the radial part of the matrix element is taken to be unity, and only one multipole is evaluated in the angular integration of $\frac{1}{2\pi}\ \delta(\cos\omega_{12} - 1)$. Finally, f = -1, while the other force parameters are set equal to zero.

The pairing parameters u_i and v_i of each level should be determined from the BCS gap equation using Γ''. However the finite basis truncation of the pairing problem involves a different renormalization than the truncation of the configuration space so that the BCS problem requires the introduction of a different parameter ξ in the definition of Γ''. Instead the state-independent Δ's and λ's employed by Soloviev[8]) have been adopted. Not only are the empirically fit u's and v's of this scheme nearly the same as those determined by using the gap equation and a new phenomenological interaction, but they also permit comparison with earlier schematic model calculations.

In the course of this work, a very rapid method has been discovered for diagonalizing the non-hermitian RPA problem to obtain the lowest eigenvalues and eigenvectors. Making the definitions $A = (P-Q)(P+Q)(P-Q)$, $B = (P-Q)$, and $\lambda = \omega^2$, eq. (2.3) may be put in the form $AZ^- = \lambda BZ^-$ where matrices A and B are symmetric. Solutions to this equation may be obtained using the vibrational method of steepest descents[20,21]). For matrices of large dimension (N), this method has the advantage that one obtains one eigenvector in N^2 operations rather than all the eigenvectors in N^3 operations.

4. Numerical Results and Discussion

4.1. 2+ STATES

The energies and two quasiparticle amplitudes of the gamma vibrations in the rare earth region have been calculated both in the RPA and the theory of finite Fermi systems. We discuss first the RPA calculations and show how they compare to the schematic model and the specificity of components in the effective nuclear interaction.

Figure 1 displays the energies of 2+ states calculated using various approximations made in deriving the schematic model. The parameters of the force are given in Table 1. There are four approximations to the RPA in the schematic model. These are that the radial matrix element peaks in the surface region, that the spin independent quadrupole-quadrupole component of the interaction predominates, and that the particle-hole exchange and particle-particle matrix elements may be neglected.

We have tested the last two assumptions directly using the VDI. The exchange matrix elements are uniformly repulsive and only renormalize the strength of the effective interaction. Examination of the wave function shows that this approximation has little effect on the microscopic character of the gamma vibration. It is surprising to find that interaction in the (pp) channel not only has no substantial effect on the gamma vibrational states but only renormalizes the (ph) force by a few percent. An earlier estimate of Faessler[9]) based on a degenerate model, gave reduction of 33%. In more realistic calculations, the (pp) and (ph) contributions are to a large extent uncoupled, the (ph) channel being predominant. A large increase in Γ'' can be offset by a much smaller decrease in Γ'. In spherical Sn and Zr isotopes, Vogel[22]) finds that the interaction in the (pp) channel is necessary to obtain even qualitative agreement with the

experimental trends for the 2+ states. A possible reason for this difference is that the spherical nuclei considered were single-closed-shell. Since there is no proton-neutron interaction in the (ph) channel, this channel no longer predominates over the (pp) channel, as in the deformed case.

The comparison of the schematic model and the direct VDI in Fig. 1 supports another of the model approximations viz. the contribution of the interaction inside the nucleus where the radial functions are incoherent is of considerably less consequence in determining collective properties than in the surface region where the radial functions are all positive in sign. The close correspondence of the same graphs also supports the sidely held notion that only the quadrupole component of the force is responsible for coherent 2+ states. This is always true for spherical nuclei because only the quadrupole-quadrupole component survives the Racah transform of a (ph) matrix element. In deformed nuclei, other multipoles are also non-zero. However, on the average, the quadrupole-quadrupole component appears to be predominate when the state calculated is characterized by a fluctuation in the quadrupole field. Since this is the case for the gamma vibration, the separable potential works. The matrix elements of the VDI and the separable interaction are quite often different in magnitude and occasionally in sign. An important example of this will be discussed later in this section. However, for states which are collective, these seemingly random differences appear to average out. The most compelling argument for the use of the separable potential has been its convenience. The calculations here show that for coherent states the approximations involved have been justified.

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†The situation for 0^+ states is more complicated since radial integrals such as $(1\bar{1}|2\bar{2})$ will be coherently positive inside the nucleus as well.

Figure 2 shows the results for calculations of the 2+ states using the Fermi liquid theory parameterization given in Table 2. We have used a (ph) mixture similar to those quoted in ref. [23]). The experimental curve is also shown. The agreement with the experimental trend is in general reproduced quite nicely. Even accepting the validity of the assumptions in the theory of finite Fermi systems, quantitative agreement could be expected only if the single particle energies for each nucleus and blocking were more exactly treated.

There is one major disagreement with experiment in the theoretical spectra. It is observed that the isotopes ^{172,174,176}Yb and ^{174,176}Hf have a higher energy than both experiment and schematic model calculations. The reason for this is found by examining the (ph) matrix elements. The VDI, unlike the quadrupole-quadrupole type forces, uncouples the 5/2(512)-1/2(521) two quasiparticle state from the other two quasiparticle configurations. This has the affect of pushing up the coherent sum of other two-quasiparticle configurations relative to the lowest state which is almost pure. The same feature was exhibited by a SDI which included all multipole components. For most other nuclei disagreements between the VDI and separable matrix elements just average out because the vibration is the coherent sum of many configuration amplitudes. Only when one configuration is completely uncoupled and lies lower than any other is the difference between the interactions observable. With incoherent states, and 0^+ states in particular, the results of a calculation will be more sensitive to the choice of an interaction.

This discrepancy suggests that the irreducible block cannot be accurately approximated by a short range interaction. We plan to discuss this further in a future publication, but note here that effects of configuration space renormalization on the analytic form of Γ' and Γ'' have not been taken into account.

Also, phonon exchange contributes to the irreducible (ph) amplitude (see ref. [24,25]) for details of the Feynman graph), and its effect is both long-ranged and energy dependent[26]). This was not noticed before because the states were so collective that a truly microscopic effect was simply renormalized and averaged out.

It is also interesting that the Fermi liquid parameterization yields results almost identical to the RPA calculation which did not take isospin or spin exchange forces into account. This supports the idea that gamma vibrations are analogous to acoustic waves[4]) and are uncoupled from spin waves or motion of neutrons relative to protons. Thus the parameterization of f', g, and g' cannot be tested by the experimental properties of gamma vibrations.

We have also calculated the second 2+ states using the RPA and Fermi liquid parameterizations. The result is that the spin-dependent Migdal parameterization did not bring down these states from their positions as calculated using the spin-independent force. We further attempted to induce this spin-dependence by changing the parameters. Again no effect from spin dependence on the second lowest state was found. The lowest state remained unaltered as well. These results indicate that the 2+ spin-quadrupole mode postulated on the basis of schematic model ($\sigma Q_2 \cdot \sigma Q_2$) calculations[10]) is inconsistent with the spin dependence of the (ph) interaction obtained from magnetic moment data[5,27]) or from splittings in particle-hole nuclei[23,28]).

Figure 3 shows the B(E2) reduced transition probabilities calculated for the Fermi liquid theory parameters in Table 2. The result is given only for this one set of parameters because, as expected, the various force mixtures generally yield the same results. The disagreement between the theoretical transition probabilities and those of experiment is due to the dependence of the

strength on the collectivity of the state, i.e., whenever $\omega_{th} \gtrless \omega_{exp}$, $B(E2)_{th} \lessgtr B(E2)_{exp}$. Considerably better agreement could be obtained by fitting the excitation energy to experiment before evaluating the B(E2), as in ref. [8,9]).

Concerning configuration space truncation discussed in sec. 2, the renormalization of the interaction for 2+ states is not very large because most of the collectivity results from low-lying $\Delta N = 0$ configurations which are counted whether we truncate at 1 $\hbar\omega$ or 2 $\hbar\omega$. This is observed to be the case. Truncation of configurations at 1 $\hbar\omega$ yields RPA matrices of dimensionality 85-95 whereas in our single particle basis there are 232 possible configurations with no truncation. The value of f (using direct matrix elements only) which fit a limited number of cases for the expanded dimensionality was -1.125 compared to -1.226 in the truncated basis. The B(E2)'s did increase slightly (< 20%). This small change of the strength parameter in a considerably expanded configuration space indicates that the single value of each parameter used to fit the rare earth data is approximately the best value for this region.

4.2. THE OCTUPOLE STATES

Calculations have been carried out for the octupole states. In contrast to the quadrupole case, the effects of truncation are expected to be quite significant for the negative parity states. Within the single particle basis set employed, there are at least two sources of truncation error.[†] The first

[†]We wish to thank P. Vogel for discussing with us the truncation problems for the octupole states.

is due to truncation too soon on the high energy side, that is, not enough states above the Fermi surface are considered for the heavier rare earths. To construct a two-quasiparticle octupole state, Nilsson levels must be considered with $\Delta N = 1$. It is probably necessary to include all Nilsson levels within at least $2\hbar\omega$ on either side of the Fermi surface. Since not enough Nilsson levels were used above the Fermi level, it is to be expected that the strength parameters should be increased as A increases. Another truncation error results from configuration space limitations. For octupole states of higher K values, it is increasingly important to couple downward sloping single particle states with principal quantum number N to upward sloping N + 1 states. Owing to the energy systematics of the single particle levels in the deformed potential well, the average 2- (ph) state of this type has more excitation energy than the average 0- configuration. Truncation at $1\hbar\omega$ which was arbitrary, excludes a greater percentage of the $\Delta N = 1$, 2- configurations than 0-. It is very natural that the strength parameters should have to increase in order to fit the experimental data.

Our purpose in the remainder of this section will be to evaluate the role of the delta force in bringing about coherent nuclear motion and secondarily, to note the effects of the truncations discussed above. It should be pointed out that schematic model studies have been carried out in a manner which avoids these problems and which include the effects of Coriolis coupling[29]). The agreement with experiment is, therefore, much better than will be obtained here.

Figure 4 compares 0- energy levels calculated using the Fermi liquid parameterization of Table 2 with the bandhead deduced from experimental data in the rare earth region. Calculations were also performed in the RPA antisymmetric

(ph) approach with the parameters given in Table 1. The results were almost identical to those of Fig. 4. These results follow the same trend as those of the earlier schematic models which used the same single particle basis. This indicates, as for the quadrupole states, that the octupole-octupole component of the (ph) interaction evaluated at the nuclear surface is predominant in determining collective behavior. For the 0- predictions in nuclei above A = 170, the "vibration" is essentially a two-quasiparticle state according to our calculations. The steep ascent of the theoretical curve and the related disagreement with the sparse experimental data result from truncation of the single particle states above the Fermi surface, as discussed earlier.

Figure 5 shows the theoretical and experimental excitation energies for the 2- states. Again, the theoretical curve is calculated using a Fermi liquid type parameterization (Table 2), while spin independent RPA type matrix elements give almost identical results. The theory yields energies in good agreement with experiment. However, it should be noted that in order to obtain this fit the (ph) strength has been increased substantially over what was used before. This renormalization is due to the more severe truncation of the configuration space for higher K values. As for the 2+ and 0- states, the exchange (ph) matrix elements, the (pp) and spin-dependent (ph) matrix elements do not alter the theoretical trend.

6. Summary

In this report we have solved the Bethe-Salpater equation for superfluid systems using the quasiparticle approximation and assuming the irreducible amplitudes for (pp) and (ph) scattering are δ-like in both space and time. In solving these equations the density dependence of the irreducible amplitudes has been ignored and it was found there will only be contributions to the two body matrix elements from regions near the nuclear surface. The amplitude for (ph) scattering is in general not antisymmetic. However, in first order in the nucleon-nucleon interaction or in the ladder approximation this amplitude is antisymmetric. We have found the properties of our collective states to be completely insensitive to this sublety in the parameterization. Further the 2^+, 0^-, and 2^- states were found to be spin and isospin independent suggesting that they are acoustic waves with neutrons and protons moving in phase.

The agreement of our calculations with both experiment and with the results of the schematic model is in general quite good. The reason for this agreement is the collectivity of the states. Since many configurations contribute to the states, differences in individual matrix elements are averaged out. However, there are cases where individual matrix elements are important in determining the properties of even the lowest states with a given spin and parity. In these cases the different theoretical calculations give disparate results.

The quadrupole states are quite insensitive to truncation in the single particle basis provided all configurations with diagonal energies less than $1\hbar\omega$ are included in the spectrum. This result occurs since, with this truncation, most of the quadrupole strength of two quasiparticle states near the Fermi surface is included. However this is not true for the octupole states with the result that schematic model calculations are much less satisfactory for these states.

Extensions of these calculations will include the consideration of phonon-contributions to the dominant pole terms of the single particle propagator and the inclusion of phonon exchange scattering in the irreducible amplitudes.

APPENDIX I. Relations Between Matrix Elements of the Volume Delta Interaction

In the conventional superfluid RPA calculation, (pp) and (ph) matrix elements are calculated using the VDI and are antisymmetric. For single particle states α and β with γ and δ coupled to $K \neq 0$, six matrix elements need to be calculated for the Wigner term of the VDI. In order to distinguish a Nilsson state, and a spherical basis state, we abandon the $\langle 12|34 \rangle$ notation of the main text. Using the notation $\langle \alpha\beta|\gamma\delta \rangle = \int d^3x_1 d^3x_2 \psi_\alpha(1)\psi_\beta(2)\, \delta(\underset{\sim}{r}_1-\underset{\sim}{r}_2)\, \psi_\gamma(1)\psi_\delta(2)$, these six elements are $\langle \alpha\beta|\gamma\delta \rangle$, $\langle \alpha\beta|\delta\gamma \rangle$, $\langle \alpha-\delta|-\beta\gamma \rangle$, $\langle \alpha-\delta|\gamma-\beta \rangle$, $\langle \alpha-\gamma|-\beta\delta \rangle$, and $\langle \alpha-\gamma|\delta-\beta \rangle$. Only the first two need to be calculated. In order to demonstrate this we start by noting that the last three matrix elements are related to the first three as follows:

$$\begin{aligned}
\langle \alpha-\delta|\gamma-\beta \rangle &= S_\beta S_\delta \langle \alpha\beta|\gamma\delta \rangle \\
\langle \alpha-\gamma|-\beta\delta \rangle &= -S_\gamma S_\delta \langle \alpha-\delta|-\beta\gamma \rangle \\
\langle \alpha-\gamma|\delta-\beta \rangle &= S_\beta S_\gamma \langle \alpha\beta|\delta\gamma \rangle \quad .
\end{aligned} \tag{I.1}$$

Here S_α is the phase $(-1)^{1/2-\Omega_a}$. Though these relations are not new (see, e.g., ref. [9]), the derivation follows.

Define $$\Xi = \sum_{a,b,c,d} A^{\alpha}_{m_a\Sigma_a} A^{\beta}_{m_b\Sigma_b} A^{\gamma}_{m_c\Sigma_c} A^{\delta}_{m_d\Sigma_c} \quad ,$$

where the coefficients are for the uncoupled Nilsson basis.

$$\langle \alpha\beta|\gamma\delta \rangle = \Xi \langle ab|cd \rangle \tag{I.2}$$

$$\langle ab|cd \rangle = \sum_k f_k F^k \delta_{\Sigma_a\Sigma_c} \delta_{\Sigma_b\Sigma_d} \quad , \tag{I.3}$$

where

$$F^k = \int U_a(1)\, U_b(2)\, V_k(r_1 r_2) U_c(1) U_d(2)\, r_1^2\, r_2^2\, dr_1\, dr_2$$

$$V_k(r_1, r_2) = \frac{2k+1}{2} \int V(|\underset{\sim}{r}_1 - \underset{\sim}{r}_2|)\, P_k(\cos\omega_{12}) d(\cos\omega_{12})$$

$$f_k = \langle \ell_a\, m_a\, \ell_b\, m_b | \frac{4\pi}{2k+1} Y^k(1) \cdot Y^k(2) | \ell_c\, m_c\, \ell_d\, m_d \rangle \quad , \qquad \text{(I.4)}$$

clearly, $F^k(abcd) = F^k(cbad) = F^k(adcb) = F^k(cdab)$, f_k is explicitly separable. If we adopt the notation

$$\hat{\ell} = 2\ell + 1 \quad \text{and} \quad \begin{bmatrix} \ell_c & k & \ell_a \\ m_c & q & m_a \end{bmatrix} = \text{Clebsch-Gordan coefficient, we may write}$$

$$f_k(abcd) = \frac{\sqrt{\hat{\ell}_a\, \hat{\ell}_b\, \hat{\ell}_c\, \hat{\ell}_d}}{(\hat{k})^2} (-1)^{m_c + m_d} \quad .$$

$$\begin{bmatrix} \ell_a & \ell_c & k \\ o & o & o \end{bmatrix} \begin{bmatrix} \ell_b & \ell_d & k \\ o & o & o \end{bmatrix} \begin{bmatrix} \ell_a & \ell_c & k & \\ -m_a & m_c & m_c & -m_a \end{bmatrix} \begin{bmatrix} \ell_b & \ell_d & k & \\ -m_b & m_d & m_d & -m_b \end{bmatrix}$$

$$= \frac{\sqrt{\hat{\ell}_a\, \hat{\ell}_d\, \hat{\ell}_c\, \hat{\ell}_b}}{(\hat{k})^2} (-1)^{(\Omega_\beta + \Omega_\delta \pm 1) + (m_c - m_b)}$$

$$- \begin{bmatrix} \ell_a & \ell_c & k \\ o & o & o \end{bmatrix} \begin{bmatrix} \ell_d & \ell_b & k \\ o & o & o \end{bmatrix} \begin{bmatrix} \ell_a & \ell_c & k & \\ -m_a & m_c & m_c & -m_a \end{bmatrix} \begin{bmatrix} \ell_d & \ell_b & k & \\ +m_d & -m_b & m_d & -m_b \end{bmatrix}$$

$$= S_\beta S_\delta\, f_k\, (a\ -d\ c\ -b) \quad . \qquad \text{(I.5)}$$

Since $\langle ab|cd \rangle = S_\beta S_\delta \langle a\ -d\ c\ -b \rangle$ term-by-term we have $\langle \alpha\beta|\gamma\delta \rangle = S_\beta S_\delta \langle \alpha-\delta|\gamma-\beta \rangle$. The other equalities are obtained from this one.

The third of our six matrix elements may be shown to be equal to the difference of the first two. This property holds true only for delta interactions and depends on the expansion of the integral over one co-ordinate of the product of four single-particle functions.

$$\langle \alpha-\delta|-\beta\gamma \rangle = -S_\beta S_\gamma [\langle \alpha\beta|\gamma\delta \rangle - \langle \alpha\beta|\delta\gamma \rangle] \quad . \tag{I.6}$$

The volume-delta-function (VDI) may be written

$$\delta(\underset{\sim}{r}_1 - \underset{\sim}{r}_2) = \frac{1}{r_1\ r_2}\ \delta(r_1 - r_2)\ \delta(\cos\theta_1 - \cos\theta_2)\ \delta(\phi_1 - \phi_2)$$

$$\langle \alpha\beta|\gamma\delta \rangle = \sum \delta_{\Sigma_a \Sigma_c}\ \delta_{\Sigma_b \Sigma_d} * \langle U_a Y_a U_b Y_b | \delta(\underset{\sim}{r}_1 - \underset{\sim}{r}_2) | U_c Y_c U_d Y_d \rangle \quad , \tag{I.7}$$

where U_a is a harmonic oscillator wave function and Y_a is a spherical harmonic. Clearly $\langle ab|cd \rangle = \int Y_a^* Y_b^* Y_c Y_d \int U_a U_b U_c U_d\ d\tau$. From (I.7) and the Slater integral definition (I.3), we have

$$\int dr\ r^2\ U_a\ U_b\ U_c\ U_d = \frac{4\pi}{2k+1} F^k = 4\pi\ F^o\ ^{30)} \quad .$$

Note that $F^k(abcd)$ is independent of the order of the indices for a delta function. The integral over spherical harmonics (note it is only a one-body integral) is

$$\frac{\sqrt{\hat{\ell}_a\ \hat{\ell}_b\ \hat{\ell}_c\ \hat{\ell}_d}}{4\pi}\ \delta_{m_a + m_b,\ m_c + m_d}\ *$$

$$\sum_k \begin{bmatrix} \ell_a & \ell_b & k \\ o & o & o \end{bmatrix} \begin{bmatrix} \ell_a & \ell_b & k \\ m_a & m_b & m_a + m_b \end{bmatrix} *$$

$$\begin{bmatrix} \ell_c & \ell_d & k \\ o & o & o \end{bmatrix} \begin{bmatrix} \ell_c & \ell_d & k \\ m_c & m_d & m_c + m_d \end{bmatrix} \Big/ (2k + 1)$$

$$= (-1)^{m_c - m_b} \frac{2k + 1}{4\pi} f_k(a\text{-}d\text{-}b\ c) = (-1)^{m_d - m_b} \frac{2k + 1}{4\pi} f_k(a\text{-}c\text{-}b\ d)\ . \quad (I.8)$$

Now to obtain the desired relation (I.6):

$$\langle \alpha\text{-}\delta | -\beta\gamma \rangle = \equiv \delta_{\Sigma_a - \Sigma_b} \sum_k f_k(a - d - b\ c)\ F^k \quad ,$$

by (I.2) and (I.3), but

$$\langle \alpha\beta | \gamma\delta \rangle - \langle \alpha\beta | \delta\gamma \rangle = \equiv (\delta_{\Sigma_a \Sigma_c}\ \delta_{\Sigma_b \Sigma_d} - \delta_{\Sigma_a \Sigma_d}\ \delta_{\Sigma_b \Sigma_c})(-1)^{m_c - m_b}$$

$$\sum_k f_k(a\text{-}d\text{-}b\ c)\ F^k \quad .$$

By (I.8). It may be shown that

$(\delta_{\Sigma_a \Sigma_c}\ \delta_{\Sigma_b \Sigma_d} - \delta_{\Sigma_a \Sigma_d}\ \delta_{\Sigma_b \Sigma_c}) = (-1)^{1 + \Sigma_b - \Sigma_c}\ \delta_{\Sigma_a - \Sigma_b}\ \delta_{\Sigma_c - \Sigma_d}$ and $\Sigma_b + m_b = \Omega_b$ by the axial symmetry of the Nilsson problem so that the two matrix elements differ only by a factor of $(-1)^{1 + \Omega_b - \Omega_c} = -S_\beta S_\gamma$.

For $K = 0$, $\alpha \neq \beta$, $\gamma \neq \delta$, we need six more integrals. They are $\langle \alpha\beta|-\delta-\gamma \rangle$, $\langle \alpha\beta|-\gamma-\delta \rangle$, $\langle \alpha\gamma|-\beta-\delta \rangle$, $\langle \alpha\gamma|-\delta-\beta \rangle$, $\langle \alpha\delta|-\beta-\gamma \rangle$, $\langle \alpha\delta|-\gamma-\beta \rangle$. Of these, it is only necessary to calculate $\langle \alpha\beta|-\delta-\gamma \rangle$. $\langle \alpha\gamma|-\beta-\delta \rangle$ equals one of the first six integrals, namely $\langle \alpha-\delta|-\beta\gamma \rangle$ by the same argument as for (I.1). And $\langle \alpha\beta|-\gamma-\delta \rangle = \langle \alpha\beta|-\delta-\gamma \rangle - S_\beta S_\delta \langle \alpha\gamma|-\beta-\delta \rangle$ by the argument of (I.6). The last three can be expressed in terms of the first three as before.

The isospin dependent matrix elements may be evaluated as the product of a Wigner matrix element and an integral over isospin co-ordinates. In particular, $\int d\tau_1\, d\tau_2\, T_\alpha(1)\, T_\beta(2)\, P_\tau\, T_\gamma(1)\, T_\delta(2) = \delta_{\tau_\alpha}(Z),\ {}_{\tau_\delta}(Z)\ \delta_{\tau_\beta}(Z),\ {}_{\tau_\gamma}(Z)$, where P_τ is the isospin exchange operator. It is not necessary to calculate spin dependent matrix elements because of the Pauli exclusion principle, $P_\sigma P_\tau P_x = -1$, and the fact that the matrix element of the delta force times P_x, the space exchange operator, is identically one. That is,

$$P_\sigma = 1/2\,(1 + \underset{\sim}{\sigma}_1 \cdot \underset{\sim}{\sigma}_2) = -P_\tau P_x = -P_\tau$$

$$= -1/2\,(1 + \underset{\sim}{\tau}_1 \cdot \underset{\sim}{\tau}_2) \quad . \tag{I.9}$$

In a Fermi liquid RPA calculation, the particle-hole matrix elements are not antisymmetrized. For the Wigner term, this has the effect that we do not need $\langle \alpha-\delta|\gamma-\beta \rangle$ and $\langle \alpha-\gamma|\delta-\beta \rangle$. It also means that $P_x P_\sigma P_\tau = P_{exchange} \neq -1$. Hence, for $K \neq 0$, two spin dependent integrations are required, $\langle \alpha-\delta|P_\sigma|-\beta\gamma \rangle$ and $\langle \alpha-\gamma|P_\sigma|-\beta\delta \rangle$. The first of these is evaluated as follows

$$\langle \alpha-\delta|P_\sigma|-\beta\gamma \rangle = \langle \alpha-\delta|P_\sigma\, P_\tau^2\, P_x^2|-\beta\gamma \rangle\, (P_\tau^2 = P_x^2 = 1)$$

$$= \langle \alpha-\delta | P_\tau \; P_x | \gamma-\beta \rangle \quad (P_\tau \; P_x \; P_\sigma = P_{exchange})$$

$$= \langle \alpha-\delta | P_\tau | \gamma-\beta \rangle \quad (P_x = 1 \text{ for } \delta\text{-force})$$

$$= \langle \alpha-\delta | \gamma-\beta \rangle \quad (\tau_\alpha = \tau_\beta \text{ for an even-even nucleus}) \quad . \qquad \text{(I.10)}$$

This is one of the spin-independent matrix elements evaluated earlier. Finally, we have that

$$\Gamma'_{\alpha-\delta-\beta\gamma} = \langle \alpha - \delta | f + f' \; \underset{\sim}{\tau}_1 \cdot \underset{\sim}{\tau}_2 + (g + g' \; \underset{\sim}{\tau}_1 \cdot \underset{\sim}{\tau}_2) \; \underset{\sim}{\sigma}_1 \cdot \underset{\sim}{\sigma}_2 | -\beta \; \gamma \rangle$$

$$= [f - f' - g \; + g' + 2 \; (f' - g') \; \delta_{\tau_\alpha}(Z), \; {}_{\tau_\gamma}(Z)] \; \langle \alpha-\delta | -\beta\gamma \rangle$$

$$+ [2 \; (g - g') + 4g' \; \delta_{\tau_\alpha}(Z), \; {}_{\tau_\gamma}(Z)] \; \langle \alpha-\delta | \gamma-\beta \rangle \quad . \qquad \text{(I.11)}$$

$\Gamma'_{\alpha-\gamma-\beta \; \delta}$, $\Gamma'_{\alpha \; \gamma-\beta-\delta}$ $(K = 0)$, and $\Gamma'_{\alpha \; \delta-\beta-\gamma}$ $(K = 0)$ are evaluated in the same manner and involve no new integrations.

APPENDIX II. The Numerical Evaluation of Matrix Elements

In calculating matrix elements of the volume delta interaction, it is convenient to use the expression

$$\langle \alpha\beta | \gamma\delta \rangle = \delta_{\Sigma_a \Sigma_c} \delta_{\Sigma_b \Sigma_d} \int Y_a^* Y_b^* Y_c Y_d \int U_a U_b U_c U_d d\tau \quad ,$$

where the radial integral is $4\pi F^o$, F^o the usual Slater integral and the integral over spherical harmonics is given by expression (I.8).

Apart from phase factors due to complex conjugation of the spherical harmonics, the integrals are independent of ordering of the basis functions. In fact for N (principal quantum number) $\leqslant 7$, there are just 4096 radial integrals possible and the number of different integrals over spherical harmonics is 26126. Matrix elements were thus calculated by using prestored integral tables. For the 2+ states, for example, approximately 54000 two-body integrals using Nilsson wave functions were calculated in 7 minutes on a CDC 7600 using only small core. The preparation of the integral tables themselves takes very little time.

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Table 1. Strength Parameters of Phenomenological RPA Calculations. The particle-particle, ξ, and particle-hole, f, strengths.

K Π	Description	ξ	f
2+	(direct ph matrix elements)	0.	-1.226
2+	(include ph exchange)	0.	-1.722
2+	(include pp interaction)	-0.9	-1.674
2+	(direct ph, no truncation)	0.	-1.125
0-	(pp + ph + exchange)	-1.125	-1.760
2-	(pp + ph + exchange)	-1.125	-2.885

Table 2. Strength Parameters of Finite Fermi Calculations. The particle-particle, ξ, and particle-hole, f, f', g, g', constants as in eq. (3.1) for Γ'' and Γ'. The particle-hole numbers have been divided by a common number to show their relation to parameterizations of other authors. The second set of numbers for the 2+ states was used only to test further possible spin dependence in the states.

$K\pi$	ξ	$\frac{f}{r}$	$\frac{f'}{r}$	$\frac{g}{r}$	$\frac{g'}{r}$	r
2+	-0.9	-0.9	0.2	0.5	0.5	1.38
2+	-0.9	-0.9	0.6	0.5	1.0	1.46
0-	-0.9	-0.9	0.2	0.5	0.5	1.46
2-	-0.9	-0.9	0.2	0.5	0.5	2.39

Figure Captions

Fig. 1. The 2+ (gamma) vibration energies in the rare earth region ($152 \leqslant A \leqslant 190$). The results of the schematic model[9]) are compared to various components of a spin independent (ph) and (pp) interaction.

Fig. 2. The 2+ (gamma) vibration energies calculated using the Fermi liquid type parameterization of the irreducible interactions. Experimental results are shown for comparison.

Fig. 3. The B(E2, 0+ - 2+) transition probabilities calculated for the rare earth nuclei. An effective charge of 0.4 has been used and the 2+ state is the same as in fig. 2.

Fig. 4. The 0- (octupole) vibration energies.

Fig. 5. The 2- (octupole) vibration energies.

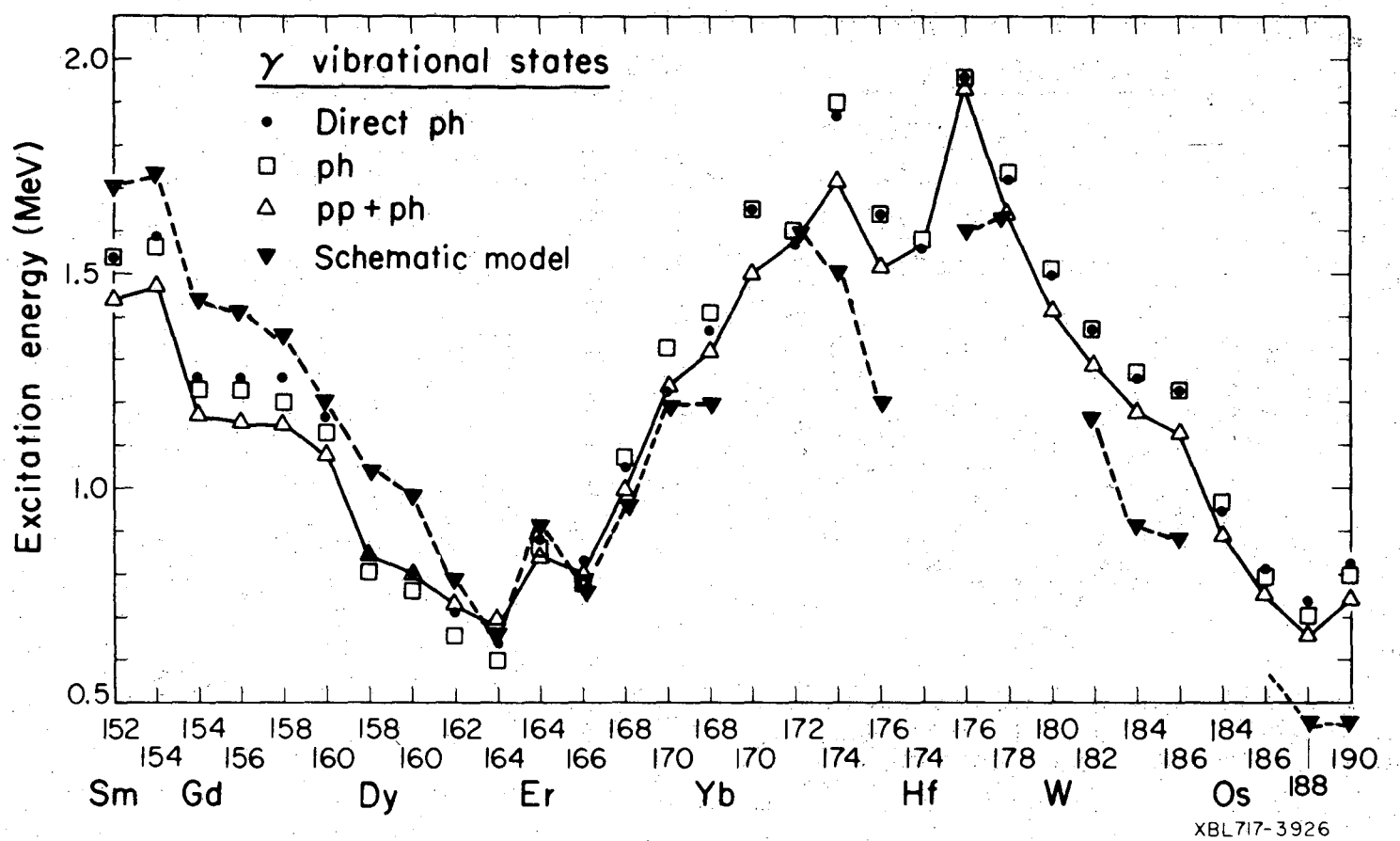
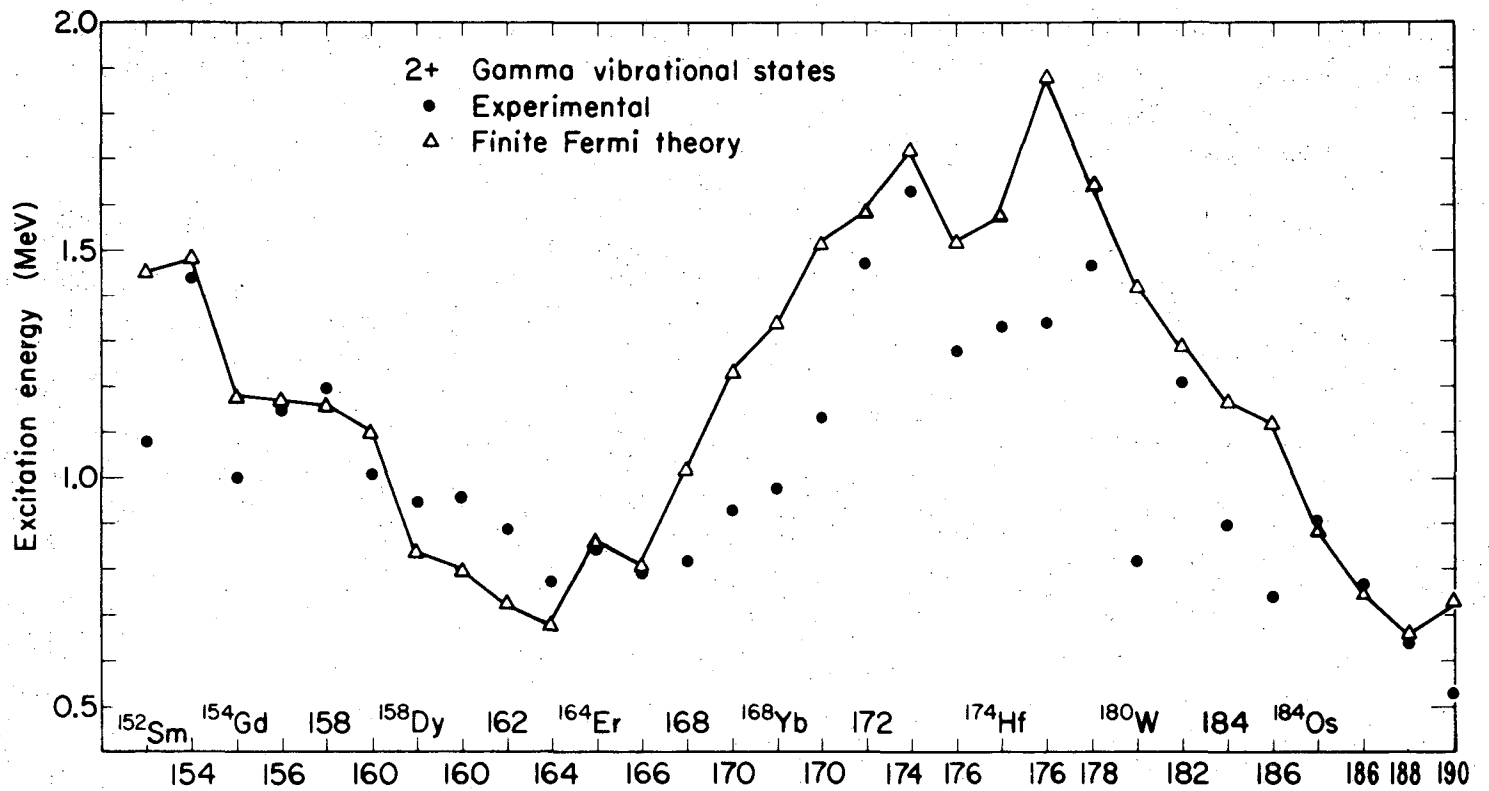

γ vibrational states
Direct ph
ph
pp + ph
Schematic model
Excitation energy (MeV)
2.0
1.5
1.0
0.5
152 154 158 158 162 164 168 168 172 176 176 180 184 184
154 156 160 160 164 166 170 170 174 174 178 182 186 186 190
188
Sm Gd Dy Er Yb Hf W Os
XBL717-3926

Fig. 1



XBL717-3925

Fig. 2

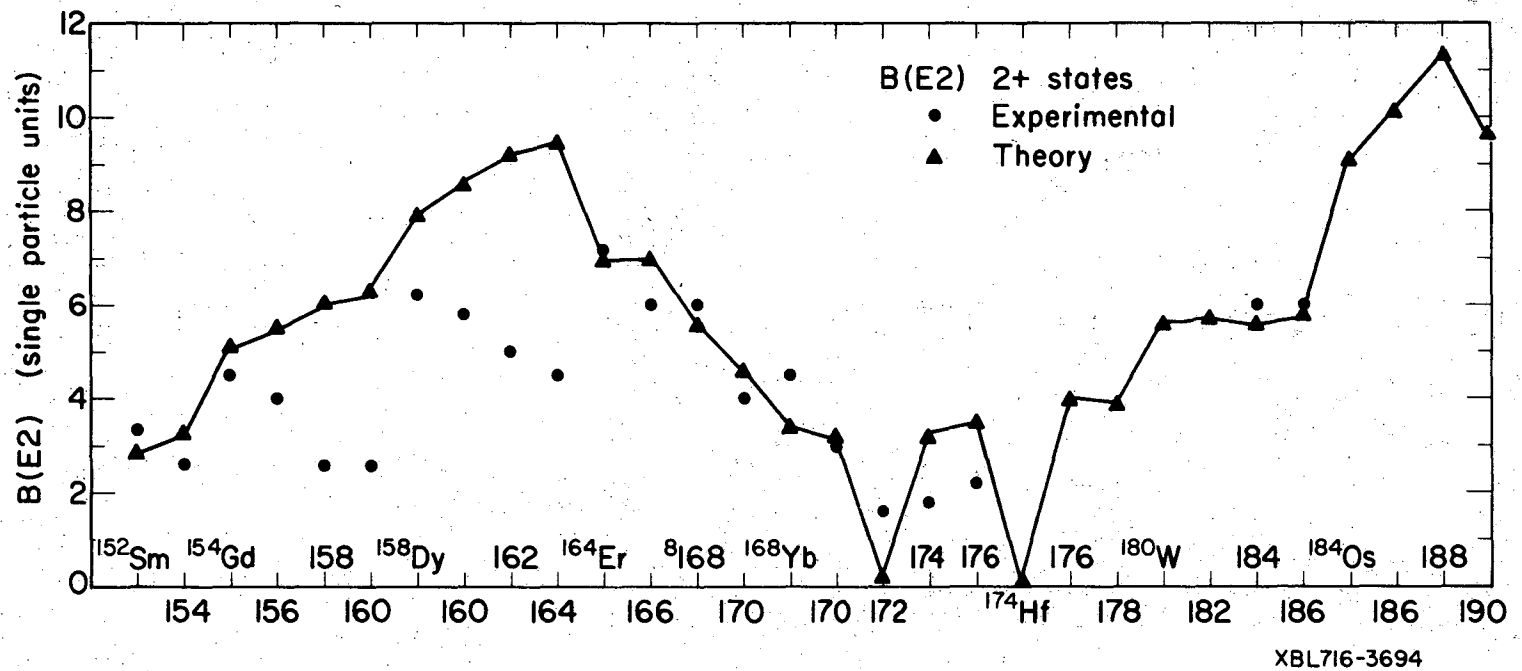

B(E2) (single particle units)
B(E2) 2+ states
Experimental
Theory
0
2
4
6
8
10
12
152Sm
154Gd
158
158Dy
162
164Er
8168
168Yb
174
176
176
180W
184
184Os
188
154
156
160
160
164
166
170
170
172
174Hf
178
182
186
186
190
XBL716-3694

Fig. 3

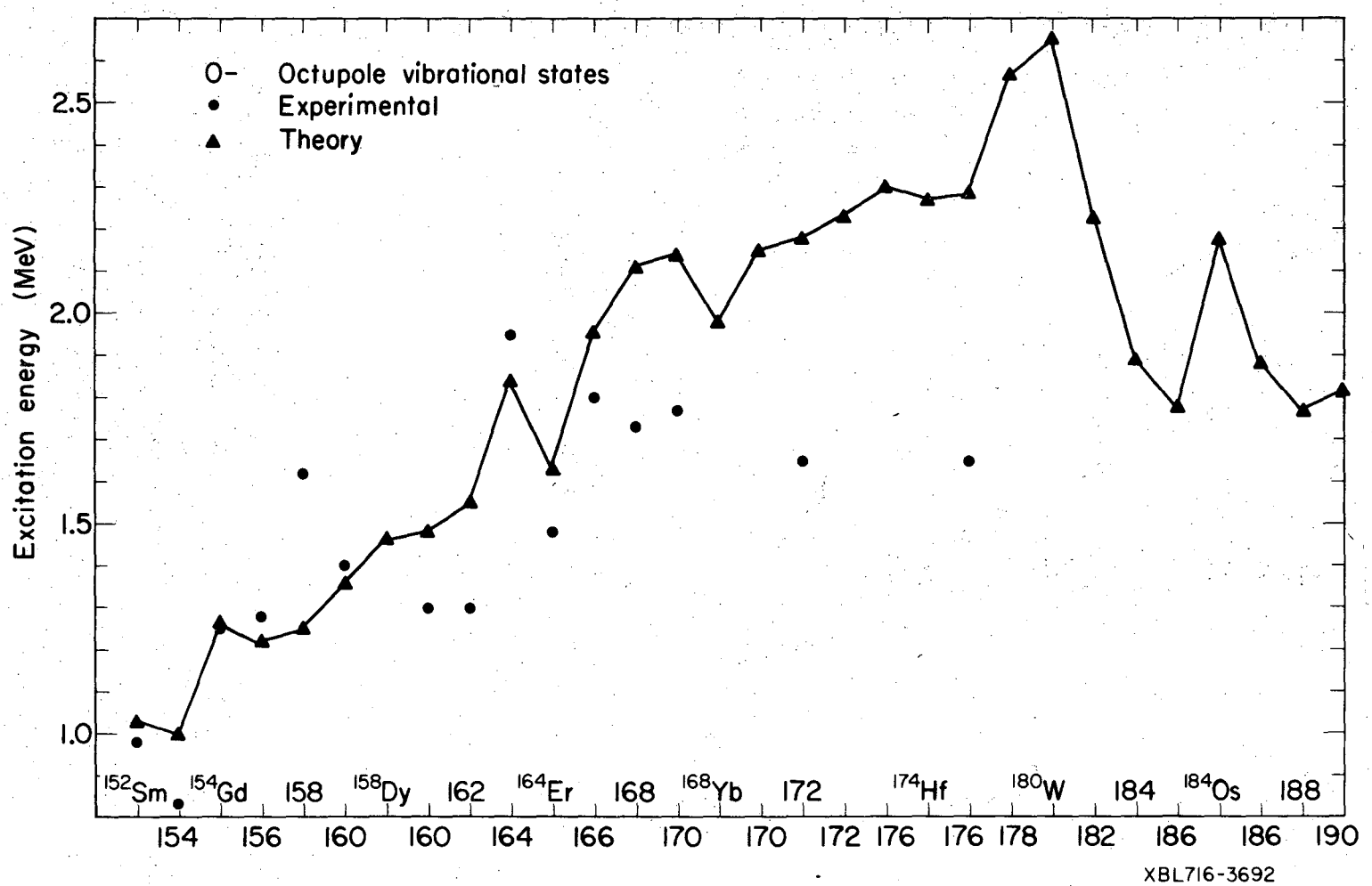

O- Octupole vibrational states
Experimental
Theory
Excitation energy (MeV)
2.5
2.0
1.5
1.0
152Sm
154Gd
158
158Dy
162
164Er
168
168Yb
172
174Hf
180W
184
184Os
188
154
156
160
160
164
166
170
170
172
176
176
178
182
186
186
190
XBL716-3692

Fig. 4

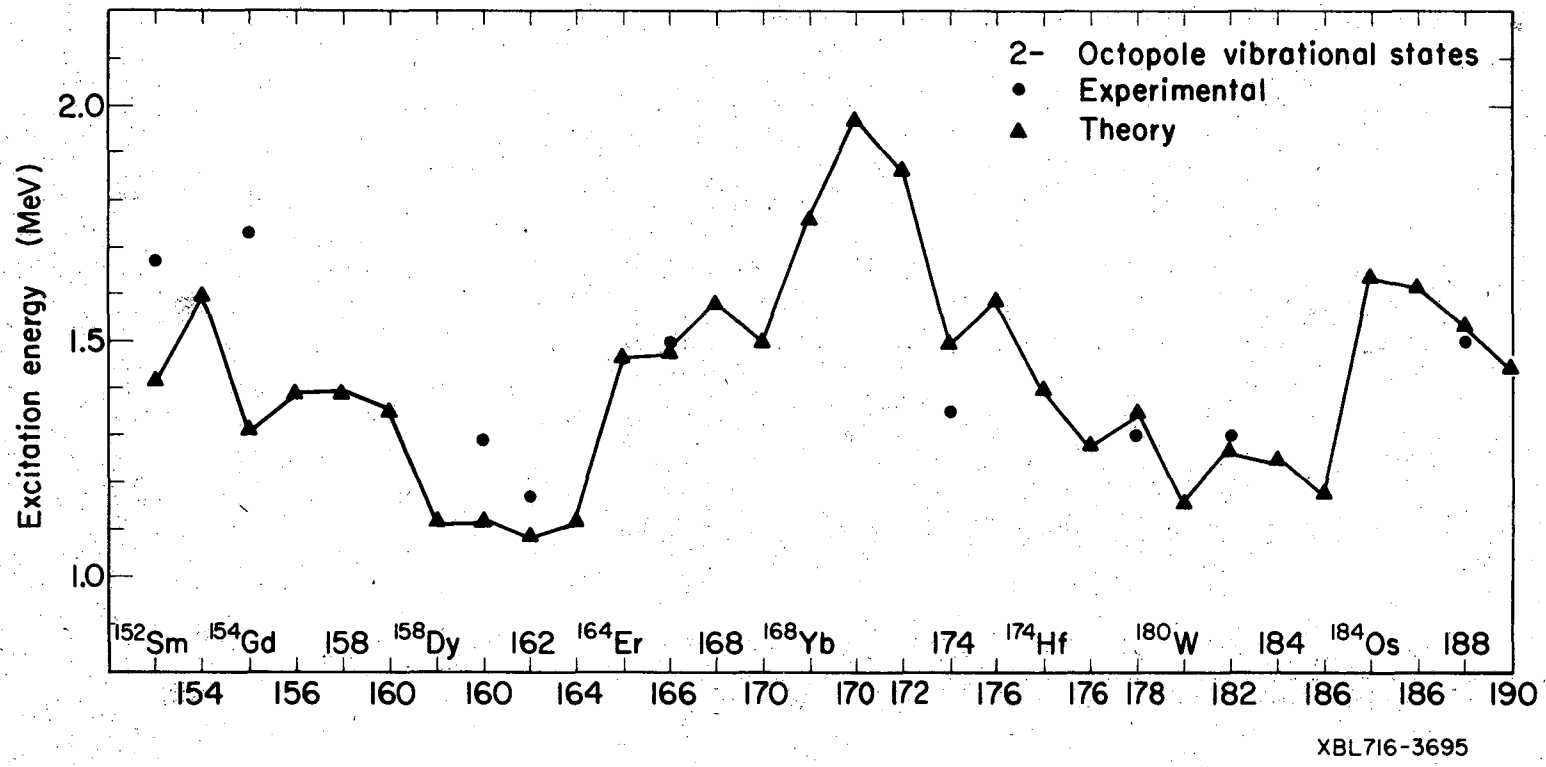

2- Octopole vibrational states
Experimental
Theory
Excitation energy (MeV)
2.0
1.5
1.0
152Sm 154Gd 158 158Dy 162 164Er 168 168Yb 174 174Hf 180W 184 184Os 188
154 156 160 160 164 166 170 170 172 176 176 178 182 186 186 190
XBL716-3695

Fig. 5

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