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October 1987

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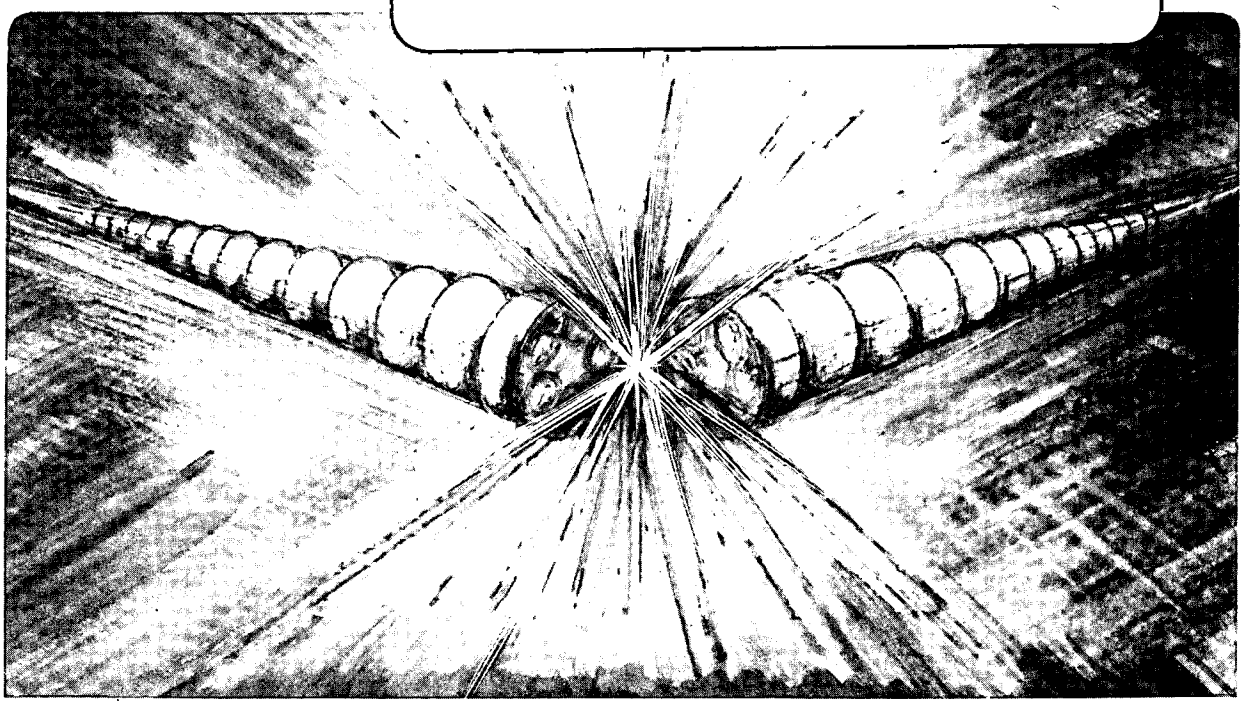

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Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

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LBL-24072

GYRORESONANT ABSORPTION FROM CONGRUENT REDUCTION OF AN ANISOTROPIC PRESSURE FLUID MODEL

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A system of Maxwell, multi-fluid momentum and anisotropic pressure equations for a weakly inhomogeneous magnetized plasma is written in a special Hermitian form. A recently developed congruent reduction technique, for extracting embedded, multi-dimensional, pairwise mode conversion events, is applied in studying the electron gyroresonant absorption problem. The mode conversion from an electromagnetic mode to a fluid pressure mode near the fundamental and second harmonic gyrofrequency is interpreted as gyroresonant absorption. The transmission coefficient is found for the perpendicular propagation case in an arbitrary three-dimensional plasma and magnetic field geometry, demonstrating the potential of the approach for systematically reducing the order in general multi-component wave propagation problems in nonuniform plasmas.

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I. INTRODUCTION

Gyroresonant absorption problems at both the electron and ion cyclotron frequencies and their harmonics in a nonuniform plasma belong to a class of problems typically requiring lengthy and complex calculations.[1] The complexity is due to the fact that the absorption process basically is a thermal effect and therefore the theory necessarily involves solutions of the system of Maxwell and kinetic equations, a difficult task in usually three-dimensional plasma geometries. Even when the spatial variation of the equilibrium plasma parameters is sufficiently weak and the description of the waves via the geometric optics seems to be feasible, one cannot use the conventional geometric optics theory directly, since typically the gyroresonant absorption process is highly localized in space. This fact, in some sense, simplifies the theory, since various slab models can be used in studying the details of the interaction. Nevertheless, even the one-dimensional calculations performed to date had been rather complex and certainly very different in each particular application.

On the other hand, the recent developments of the congruent reduction[2] and multi-dimensional mode conversion[3] theories provide new tools in simplifying multi-component wave propagation problems in plasmas. These theories comprise a systematic method of solving homogeneous integral equations of form

$$\int d^4x' \, \hat{D}_{ij}(x,x') \, Z_j(x') = 0 \qquad (1)$$

for an N-component vector-field $\mathbf{Z}(x)$ on space-time $[x=(\mathbf{r},t)]$, where the dispersion kernel $\hat{\mathbf{D}}$ is Hermitian $[\hat{D}_{ij}(x,x')=\hat{D}_{ji}^{*}(x',x)]$ and known. In cases when $\mathbf{Z}$ describes a perturbation of a smooth, but weakly varying background equilibrium, the dependence of $\hat{\mathbf{D}}$ on $(x+x')/2$ is weak compared to that on $x-x'$ and the eikonal representation $\mathbf{Z}(x)=\mathbf{A}(x)\exp[i\psi(x)]$ with slowly varying amplitude $\mathbf{A}$ and rapidly varying phase ψ is feasible. Then one can rewrite (1) as the slow amplitude transport equation

$$\mathbf{D}\bullet\mathbf{A} = i\left[\frac{\partial \mathbf{D}}{\partial k_\mu}\bullet\frac{\partial \mathbf{A}}{\partial x^\mu} + \frac{1}{2}\frac{d}{dx^\mu}\left(\frac{\partial \mathbf{D}}{\partial k_\mu}\right)\bullet\mathbf{A}\right] + O(\delta^2) \qquad (2)$$

where $k_\mu(x)=\partial\psi/\partial x^\mu$, $\mathbf{D}(k,x)=\int d^4s\,\hat{\mathbf{D}}(x-s/2,x+s/2)\exp(-ik\cdot s)$ and $\delta \ll 1$ is a small

dimensionless parameter associated with the nonuniformity of the background. It is our understanding of the solutions of Eq.(2), which had advanced significantly with the development of the above-mentioned reduction and mode conversion theories. The congruent reduction theory[2] yields an algorithm for eliminating some of the components of **A** from the problem by preserving, at the same time, the form, first differential order and the hermiticity of the *reduced* transport equation, which describes the remaining wave components. This reduced equation still has form (2) with **D** replaced by reduced matrix $\mathbf{D}^r$ of rank $M \leq N$ such that all its elements are of $O(\delta)$. Typically, in non-degenerate plasma regions, where only one of the eigenvalues of $\mathbf{D}(k,x)$ vanishes at a time, one has $M=1$ and thus the system reduces to a single first order PDE for the last remaining component of the amplitude, so that the problem becomes easily integrable. In a more restricted class of problems the final reduced matrix has rank $M=2$ and describes the pairwise mode conversion problem, solution of which for a general geometry was found in Ref.3. The total wave-action flux is still conserved in this type of problems (degenerate plasma regions) but, nevertheless, the flux is redistributed in two distinct channels (modes) automatically prescribed by the reduction procedure. This redistribution phenomenon is typically localized , so that outside the regions of the near-degeneracy of $\mathbf{D}^r$, the reduction to case $M=1$ is again possible and the simple integrability is restored for each of the two channels independently.

Thus if a physical problem is described, in its unreduced form, by Eq.(2), the solution can be found systematically by combining the reduction and multi-dimensional mode conversion theories. We therefore conclude that the problem of finding the solution in practice can be replaced by the question of whether a particular application allows description in form (2). We shall use the term "Hermitian problem" if the answer to this question is positive and show that the multi-species, anisotropic pressure magnetized plasma fluid model, in its unreduced form, comprises a Hermitian problem (see Sec.III). Thus, within this model, *all* wave problems in weakly nonuniform plasmas can be dealt with *systematically* , as described above. As an example, the actual reduction for the fundamental and second harmonic electron gyroresonance cases will be carried out in Sec.V and the results interpreted as the mode conversion from the electromagnetic to plasma pressure-fluid modes.

II. HERMITIAN FORM FOR COLD MULTI-SPECIES PLASMA MODEL

It is instructive to find the Hermitian form (2) for *unreduced* wave problems in the *cold* plasma case first. We start with the linearized Maxwell-momentum equations

$$\nabla x E_1 = -\frac{1}{c}\frac{\partial B_1}{\partial t} \tag{3}$$

$$\nabla x B_1 = \frac{1}{c}\frac{\partial E_1}{\partial t} + \frac{4\pi e}{c}\sum_\alpha Z_\alpha \varepsilon_\alpha N_{\alpha 0} V_{\alpha 1} \tag{4}$$

$$m_\alpha \frac{\partial V_{\alpha 1}}{\partial t} = Z_\alpha \varepsilon_\alpha e(E_1 + \frac{1}{c} V_{\alpha 1} x B_0) \tag{5}$$

where m_α , Z_α and ε_α are the mass, the charge number and its sign, respectively for species α, and B_0 and $N_{\alpha 0}$ are the equilibrium magnetic field and density. We multiply Eq.(5) by $(N_{\alpha 0})^{1/2}$ (note that $\partial N_{\alpha 0}/\partial t=0$) and define vector

$$Z \equiv \begin{bmatrix} c(4\pi)^{-1/2} B_1 \\ c(4\pi)^{-1/2} E_1 \\ (N_{\alpha 0} m_\alpha)^{1/2} V_{\alpha 1} \end{bmatrix} \equiv \mathrm{Re}\left\{ \begin{bmatrix} b \\ a \\ v_\alpha \end{bmatrix} \exp(i\psi) \right\} \tag{6}$$

Now, by inspecting Eqs.(3-5), one finds that indeed, without any further approximation, the amplitude $\mathbf{A} \equiv (\mathbf{b}, \mathbf{a}, \mathbf{v}_\alpha)$ is described by Eq.(2) where the *Hermitian* dispersion matrix is given by

$$
D = \begin{array}{c}
\begin{array}{ccccccccc} b_x & b_y & b_z & a_x & a_y & a_z & v_{\alpha x} & v_{\alpha y} & v_{\alpha z} \end{array} \\
\begin{bmatrix}
\omega & 0 & 0 & 0 & k_z & -k_y & 0 & 0 & 0 \\
0 & \omega & 0 & -k_z & 0 & k_x & 0 & 0 & 0 \\
0 & 0 & \omega & k_y & -k_x & 0 & 0 & 0 & 0 \\
0 & -k_z & k_y & \omega & 0 & 0 & i\omega_{p\alpha} & 0 & 0 \\
k_z & 0 & -k_x & 0 & \omega & 0 & 0 & i\omega_{p\alpha} & 0 \\
-k_y & k_x & 0 & 0 & 0 & \omega & 0 & 0 & i\omega_{p\alpha} \\
0 & 0 & 0 & -i\omega_{p\alpha} & 0 & 0 & \omega & -i\Omega_{\alpha z} & i\Omega_{\alpha y} \\
0 & 0 & 0 & 0 & -i\omega_{p\alpha} & 0 & i\Omega_{\alpha z} & \omega & -i\Omega_{\alpha x} \\
0 & 0 & 0 & 0 & 0 & -i\omega_{p\alpha} & -i\Omega_{\alpha y} & i\Omega_{\alpha x} & \omega
\end{bmatrix}
\end{array}
\begin{array}{c} b_x \\ b_y \\ b_z \\ a_x \\ a_y \\ a_z \\ v_{\alpha x} \\ v_{\alpha y} \\ v_{\alpha z} \end{array}
\qquad (7)
$$

Here a *fixed* cartesian coordinate system is used and the corresponding amplitude components are shown above and beside the matrix for easier identification of various matrix components. Also we use definitions $c\omega=\partial\psi/\partial t$, $c\omega_p=\pm(4\pi e^2 Z_\alpha{}^2 N_{\alpha 0}/m_\alpha)^{1/2}$ and $c\underline{\Omega}_\alpha=\pm e Z_\alpha B_0/m_\alpha c$, where the sign is defined by the charge sign of the corresponding species . Note that all the frequencies differ by a factor of 1/c from the conventional definitions, which is convenient since now ω and **k** have the same dimensions. Thus we have demonstrated that the unreduced cold plasma case comprises a Hermitian problem and therefore, within the model, various wave problems can be systematically reduced as described above and therefore solved in principle. Examples of such a reduction can be found elsewhere.[2]

At this point we shall discuss applications with the background magnetic field B_0 which defines a preferred direction in the plasma. The dispersion matrix **D** has the form given by Eq.(7) in any *fixed* cartesian coordinate system. We can ask the question about the effect on the form of the transport equation, if, instead of constant base vectors e_x, e_y, e_z, one chooses a different orthogonal representation, say e_1,

e_2, e_3 ($e_i \bullet e_j = \delta_{ij}$) associated, for example, with the direction of the magnetic field and thus dependent on position. The answer to this question can be found in Ref.2. Indeed, the use of the new representation is equivalent to introducing a linear transformation of the amplitude **A** , i.e.

$$\mathbf{A}(x) = \mathbf{Q}(x) \bullet \overline{\mathbf{A}}(x) \tag{8}$$

where **Q** is the transformation matrix. It was shown in Ref.2 that then, to $O(\delta)$, the transformed amplitude $\overline{\mathbf{A}}$ is described by the transport equation of the same form as (2) with **D** replaced by $\overline{\mathbf{D}}$ given by

$$\overline{D}_{ij}(z) = \dot{Q}_{mi} D_{mn} Q_{nj} + \frac{i}{2}[\{\dot{Q}_{mi}, D_{mn}\} Q_{nj} + \dot{Q}_{mi}\{D_{mn}, Q_{nj}\} + \{\dot{Q}_{mi}, Q_{nj}\} D_{mn}] \tag{9}$$

where $z=(k,x)$ and $\{F,G\} = (\partial F/\partial x^{\mu})(\partial G/\partial k_{\mu}) - (\partial G/\partial x^{\mu})(\partial F/\partial k_{\mu})$ is the conventional Poisson bracket. Now we can finally answer the question about the effect of introducing a preferred coordinate system. Since transformation **Q** in (8) does not depend on k , the nontrivial Poisson brackets in Eq.(9) vanish and $\overline{\mathbf{D}}$ is simply the congruent transformation of **D**:

$$\overline{\mathbf{D}} = \mathbf{Q}^{+} \bullet \mathbf{D} \bullet \mathbf{Q} \tag{10}$$

Therefore, in conclusion, Eq.(2) with **D** given by (7) holds in *any slowly varying* cartesian coordinate system.The convenient choice is the system with $\mathbf{e}_z$ along the direction of the background magnetic field $\mathbf{B}_0$. One can, of course, use a non-cartesian coordinate system, such as the conventional base vectors $\mathbf{e}_{\pm} = (\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}$ and $\mathbf{e}_z = \mathbf{B}_0/|\mathbf{B}_0|$. In this case we write $\mathbf{S} = S_+\mathbf{e}_+ + S_-\mathbf{e}_- + S_z\mathbf{e}_z$, where **S** is either **b**, **a**, $\mathbf{v}_{\alpha}$ or **k**. Then, in the new representation, the evolution of the amplitude $\mathbf{A} = (b_+, b_-, b_z, a_+, a_-, a_z, v_{\alpha+}, v_{\alpha-}, v_{\alpha z})$ is described again by Eq.(2) with the dispersion matrix given by

$$
D = \begin{array}{c}
\begin{array}{ccccccccc} b_+ & b_- & b_z & a_+ & a_- & a_z & v_{\alpha+} & v_{\alpha-} & v_{\alpha z} \end{array} \\
\begin{bmatrix}
\omega & 0 & 0 & -ik_z & 0 & ik_+ & 0 & 0 & 0 \\
0 & \omega & 0 & 0 & ik_z & -ik_- & 0 & 0 & 0 \\
0 & 0 & \omega & ik_- & -ik_+ & 0 & 0 & 0 & 0 \\
ik_z & 0 & -ik_+ & \omega & 0 & 0 & i\omega_{p\alpha} & 0 & 0 \\
0 & -ik_z & ik_- & 0 & \omega & 0 & 0 & i\omega_{p\alpha} & 0 \\
-ik_- & ik_+ & 0 & 0 & 0 & \omega & 0 & 0 & i\omega_{p\alpha} \\
0 & 0 & 0 & -i\omega_{p\alpha} & 0 & 0 & \omega+\Omega_\alpha & 0 & 0 \\
0 & 0 & 0 & 0 & -i\omega_{p\alpha} & 0 & 0 & \omega-\Omega_\alpha & 0 \\
0 & 0 & 0 & 0 & 0 & -i\omega_{p\alpha} & 0 & 0 & \omega
\end{bmatrix}
\end{array}
\begin{array}{c} b_+ \\ b_- \\ b_z \\ a_+ \\ a_- \\ a_z \\ v_{\alpha x} \\ v_{\alpha y} \\ v_{\alpha z} \end{array}
\qquad (11)
$$

where $\Omega_\alpha = \pm|\underline{\Omega}_\alpha|$ with the signs again defined by the charge sign of species α.

Finally, we observe that as the result of the Hermiticity of **D**, Eq.(2) yields the conservation law, i.e.,

$$\frac{d}{dx^\mu} J^\mu = 0 \quad , \quad J^\mu = \mathbf{A}^* \bullet \frac{\partial \mathbf{D}}{\partial k_\mu} \bullet \mathbf{A} + O(\delta) \qquad (12)$$

In the cold plasma case described by dispersion matrix (7), Eq.(12) becomes

$$\frac{\partial U}{\partial t} + \nabla \bullet \mathbf{G} = 0 \qquad (13)$$

where

$$U = (\langle B_1^2 \rangle + \langle E_1^2 \rangle)/8\pi + N_{\alpha 0} m_\alpha \langle V_1^2 \rangle/2 \quad ; \quad \mathbf{G} = c\langle E_1 x B_1 \rangle/16\pi$$

and $\langle ... \rangle$ describes the averaging over the rapid phase ψ. Thus, we simply obtained the energy conservation law for the perturbed fluid.

III. THE ANISOTROPIC PRESSURE MODEL

Addition of thermal effects modifies the momentum equations for various species, which now become

$$m_\alpha(\partial/\partial t + V_\alpha \bullet \nabla)V_\alpha = \pm eZ_\alpha(E + \frac{1}{c} V_\alpha x B) - \frac{1}{N_\alpha} \nabla \bullet P_\alpha \tag{14}$$

where the anisotropic pressure tensor **P** is described by: [4]

$$\partial P_\alpha/\partial t + P_\alpha \bullet (\nabla V_\alpha) + (P_\alpha \bullet \nabla V_\alpha)^T + \underline{\Omega}_\alpha x P_\alpha - P_\alpha x \underline{\Omega}_\alpha = 0 \tag{15}$$

We neglected the heat flux in the last equation, i.e. assumed a sufficiently low temperature case. The effect of collisions was also neglected for simplicity.

At this point we shall choose the most simple equilibrium, i.e. assume the Boltzmann distribution of the unperturbed density

$$N_{\alpha 0}(r) = n_{\alpha 0} \exp[\pm eZ_\alpha \Phi(r)/T_\alpha] \tag{16}$$

with temperature T_α constant throughout the volume. Also we shall use an isotropic pressure model in the equilibrium

$$P_{\alpha 0} = N_{\alpha 0}(r) T_\alpha I \tag{17}$$

and assume no streaming in the fluid

$$V_{\alpha 0} = 0 \tag{18}$$

It can be easily verified that Eqs.(16-18) indeed describe a good equilibrium of Eqs.(14) and (15).

Now we shall linearize the momentum and pressure equations

$$m_\alpha \partial V_{\alpha 1}/\partial t = \pm Z_\alpha e(E_1 + \frac{1}{c} V_{\alpha 1} x B_0) - \frac{T_\alpha^{1/2}}{N_{\alpha 0}} \nabla \bullet P'_{\alpha 1} \tag{19}$$

$$\frac{\partial P'_{\alpha 1}}{\partial t} + N_{\alpha 0} T_\alpha^{1/2} I \nabla \bullet V_{\alpha 1} + N_{\alpha 0} T_\alpha^{1/2} [(\nabla V_{\alpha 1}) + (\nabla V_{\alpha 1})^T] + \underline{\Omega}_{\alpha 0} x P'_{\alpha 1} - P'_{\alpha 1} x \underline{\Omega}_{\alpha 0} = 0 \tag{20}$$

Here we defined $P_{\alpha 1}' = P_{\alpha 1}/(T_\alpha)^{1/2}$. It can be seen from Eq.(20) that $P_{\alpha 1}'$ is of $O(T_\alpha^{1/2})$. Let us make an additional ordering assumption, i.e. we shall treat the objects $kv_{th\alpha}/\omega c \equiv (k/\omega c)(T_\alpha/m_\alpha)^{1/2}$ as being of $O(\delta)$. Then, to the lowest order, we can replace the operator ∇ in (19) and (20) by ik and $\partial/\partial t$, in (20), by $-i\omega$. The result is

$$m_\alpha \frac{\partial V_{\alpha 1}}{\partial t} = \pm Z_\alpha e(E_1 + \frac{1}{c} V_{\alpha 1} x B_0) - i \frac{T_\alpha^{1/2}}{N_{\alpha 0}} k \bullet P'_{\alpha 1} \tag{21}$$

$$\omega P'_{\alpha 1} + i(\underline{\Omega}_{\alpha 0} x P'_{\alpha 1} - P'_{\alpha 1} x \underline{\Omega}_{\alpha 0}) = N_{\alpha 0} T_{\alpha}^{1/2} (k \bullet V_{\alpha 1} I + k V_{\alpha 1} + V_{\alpha 1} k) \qquad (22)$$

In dealing with Eq.(22), at this stage, we replace the vector product in the left hand side by the equivalent matrix operations, i.e., write

$$i\,(\underline{\Omega}_{\alpha 0} x P'_{\alpha 1} - P'_{\alpha 1} x \underline{\Omega}_{\alpha 0}) = \underline{\underline{\Omega}}_{\alpha} \bullet P'_{\alpha 1} + P'_{\alpha 1} \bullet \underline{\underline{\Omega}}_{\alpha}^{T} \qquad (23)$$

where the matrix $\underline{\underline{\Omega}}_{\alpha}$ is defined via

$$\underline{\underline{\Omega}}_{\alpha} = \begin{bmatrix} 0 & -i\Omega_{\alpha 0z} & i\Omega_{\alpha 0y} \\ i\Omega_{\alpha 0z} & 0 & -i\Omega_{\alpha 0x} \\ -i\Omega_{\alpha 0y} & i\Omega_{\alpha 0x} & 0 \end{bmatrix} \qquad (24)$$

It is convenient, at this stage, to introduce the representation in which $\underline{\underline{\Omega}}_{\alpha}$ is diagonal. The desired base vectors are $e_{\pm}$ and e_z as defined above. In terms of these vectors

$$\underline{\underline{\Omega}}_{\alpha} = \Omega_{\alpha}(e_+ e_+^* - e_- e_-^*) = \Omega_{\alpha}(e_+ e_- - e_- e_+) \qquad (25)$$

where $\Omega_{\alpha} = \pm|\underline{\Omega}_{\alpha 0}|$. In this representation Eq.(22) is particularly simple and becomes, written in components (we shall omit the subscript α for simplicity)

$$\begin{aligned} (\omega-\Omega)P'_{z+} &= N_0 T^{1/2}(k_z v_- + k_- v_z) \\ (\omega+\Omega)P'_{z-} &= N_0 T^{1/2}(k_z v_+ + k_+ v_z) \\ (\omega-2\Omega)P'_{-+} &= 2N_0 T^{1/2} k_- v_- \\ (\omega+2\Omega)P'_{+-} &= 2N_0 T^{1/2} k_+ v_+ \\ \omega P_{++} &= N_0 T^{1/2}[2(k_+ v_- + k_- v_+) + k_z v_z] \\ \omega P'_{zz} &= N_0 T^{1/2}(k_+ v_- + k_- v_+ + 3k_z v_z) \end{aligned} \qquad (26)$$

and for the remaining tensor elements

$$P_{-z} = P_{z+}\ ;\ P_{+z} = P_{z-}\ ;\ P_{--} = P_{++} \qquad (27)$$

Finally, we introduce new variables $\tilde{P}_{++} = P_{++}/\sqrt{2}$ and $\tilde{P}_{zz} = (2P_{zz} - P_{++})/\sqrt{10}$ and, similar to the cold plasma case (see Eq.(6)) use the eikonal representation, i.e. define the wave field $\mathbf{Z}$ via

$$\mathbf{Z} \equiv \begin{bmatrix} cB_1/(4\pi)^{1/2} \\ cE_1/(4\pi)^{1/2} \\ (N_0 m)^{1/2} V_1 \\ (TN_0)^{-1/2} P_{z+} \\ (TN_0)^{-1/2} P_{z-} \\ (TN_0)^{-1/2} P_{-+} \\ (TN_0)^{-1/2} P_{+-} \\ (TN_0)^{-1/2} \tilde{P}_{++} \\ (TN_0)^{-1/2} \tilde{P}_{zz} \end{bmatrix} \equiv \mathrm{Re} \left[\begin{pmatrix} b \\ a \\ v \\ p_{z+} \\ p_{z-} \\ p_{-+} \\ p_{+-} \\ p_{++} \\ p_{zz} \end{pmatrix} \exp(i\psi) \right] \equiv \mathrm{Re}[A\exp(i\psi)] \qquad (28)$$

Eqs. (3), (4), (21) and (26) can now be written in form (2) for the amplitude $\mathbf{A}=(\mathbf{b}, \mathbf{a}, \mathbf{v}_\alpha, p_{z+}, p_{z-}, p_{-+}, p_{+-}, p_{++}, p_{zz})$ with the Hermitian dispersion matrix given by (compare with Eq.(11) for the cold plasma case)

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_{bb} & \mathbf{D}_{ba} & 0 & 0 \\ \mathbf{D}^{\bullet}_{ba} & \mathbf{D}_{aa} & \mathbf{D}_{av} & 0 \\ 0 & \mathbf{D}^{\bullet}_{av} & \mathbf{D}_{vv} & \mathbf{D}_{vp} \\ 0 & 0 & \mathbf{D}^{\bullet}_{vp} & \mathbf{D}_{pp} \end{bmatrix} \qquad (29)$$

where the various matrixes are

$$\mathbf{D}_{bb}=\mathbf{D}_{aa}=\omega\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \; ; \; \mathbf{D}_{vv}=\begin{bmatrix} \omega+\Omega & 0 & 0 \\ 0 & \omega-\Omega & 0 \\ 0 & 0 & \omega \end{bmatrix} \qquad (30)$$

$$\mathbf{D}_{av}= i\omega_p\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \; ; \; \mathbf{D}_{ba}=\begin{bmatrix} -ik_z & 0 & ik_+ \\ 0 & ik_z & -ik_- \\ ik_- & -ik_+ & 0 \end{bmatrix} \qquad (31)$$

$$D_{vp}=\begin{bmatrix} 0 & \beta k_z & 0 & \beta k_- & \sqrt{2}\,\beta k_+ & 0 \\ \beta k_z & 0 & \beta k_+ & 0 & \sqrt{2}\,\beta k_- & 0 \\ \beta k_+ & \beta k_- & 0 & 0 & \beta k_z/\sqrt{2} & \sqrt{5/2}\beta k_z \end{bmatrix} \tag{32}$$

$$D_{pp}=\begin{bmatrix} \omega-\Omega & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega+\Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & (\omega-2\Omega)/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & (\omega+2\Omega)/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega \end{bmatrix} \tag{33}$$

Here $\beta=v_{th}/c$ and we observe that the thermal effects are introduced via the coupling matrix D_{vp} between the fluid velocity and pressure tensor perturbations. This completes the derivation of the Hermitian form (2) for the anisotropic pressure fluid model.

We shall conclude this section by the following two remarks. The first is regarding the validity of the model. We expect the fluid description to be valid in cases when the phenomenon of interest involves interactions with the *bulk* of particle distributions. Therefore, such effects as the Landau damping by the distribution tails are clearly omitted from the fluid description. On the other hand, in the vicinity of the cyclotron resonances, where $\omega-n\Omega_\alpha\sim O(\delta)$, the bulk of the distribution of the corresponding species is in the resonance, since, by our ordering assumption $kv_{th}/\omega c\sim O(\delta)$. Therefore, the fluid description of the resonant cyclotron interaction can be expected to be valid. This indeed will be demonstrated in the examples in Sec.VI by a direct comparison with the results of the kinetic theory.

The second remark is related to the energy conservation. Due to the hermiticity of D, as in the cold plasma case, the anisotropic pressure model yields the conservation

law of form (13). This is of course an expected feature of the theory, since the collisional dissipation was neglected. The energy conservation is especially important in the context of possible localized linear mode conversion events in our multi-degree of freedom system. The energy is conserved globally, still allowing the energy flux *redistribution* in various channels (modes) when the local linear mode conversion takes place.

IV. CONGRUENT REDUCTION

Before proceeding to various examples, let us briefly describe the details of the reduction procedure.[2] The method is applied as follows. Consider an unreduced, Hermitian, NxN dispersion matrix **D** characterizing a Hermitian problem, i.e. the case described by Eq.(2). If any *diagonal* element of **D** , say D_{kk} , is of O(1), then the k-th wave component A_k can be eliminated from the problem and the remaining N-1 components are again described by Eq.(2), with **D** replaced by the reduced (N-1)x(N-1) dispersion matrix $\mathbf{D}^r$ given by

$$D^r_{ij} = D_{ij} - D_{ik}D_{kj}/D_{kk} \quad ; \quad i,j\neq k \tag{34}$$

This is the essence of the the Reduction Theorem proven in Ref.2. The case, when all the diagonal elements of **D** are of O(δ), but a non-diagonal element is of O(1), still allows the reduction of order (even from N to N-2, in this case[2]). This situation, however, is very rare and will not be discussed here.

Eq.(34) describes a single reduction step. The procedure is then repeated for the reduced problem described by $\mathbf{D}^r$ and so on, until one arrives at the final reduced dispersion matrix $\mathbf{D}^f$ with all elements of O(δ) . The final transport equation still has form (2) but is irreducible within the geometric optics approximation. In the most simple, doubly degenerate case, the rank of $\mathbf{D}^f$ is 2 and the transport equation describes the pairwise mode conversion problem[3]. Examples of this type of problem will be given in the next section.

We notice that according to Eq.(34), D_{ij}^r (i,j≠k) differs from the original dispersion matrix element only if *both* coupling elements D_{ik} and D_{kj} do not vanish. This is an important observation because of the sparsity of many unreduced dispersion matrixes (see, for example, Eq(29)). Indeed, we can shorten the reduction procedure, at each

reduction step, by eliminating the amplitude component (say A_k) characterized by the smallest number of the non-vanishing coupling elements D_{ik} ($i \neq k$). Three cases of increasing complexity are then encountered frequently:

Case 1. $D_{ki}=0$ ($i \neq k$) . None of the components of the dispersion tensor are modified as the result of the reduction in this case, i.e. $D_{ij}^r = D_{ij}$ ($i,j \neq k$), and component A_k simply *drops* from the problem [$A_k \sim O(\delta)$].

Case 2. $D_{kl} \neq 0$, $D_{ki}=0$,$i \neq l$ ($i,l \neq k$). Only *one* matrix element in changes in this case as the result of the elimination of A_k, i.e., the *diagonal* element $D_{ll}^r = D_{ll} - |D_{kl}|^2/D_{kk}$. All the remaining elements $D_{ij}^r = D_{ij}$ ($i,j \neq l$).

Case 3. $D_{kl} \neq 0$, $D_{km} \neq 0$, $D_{ki}=0$, $i \neq l,m$ (i, l, $m \neq k$). In this case *four* elements of **D** are effected by the reduction, i.e., D_{ll} , D_{mm}, D_{lm} and D_{ml}

Thus, in conclusion, the reduction algorithm is relatively simple for sparse matrixes and the algebraic complexity increases rapidly when the matrix becomes increasingly non-sparse as the result of the reduction. At this point we proceed to examples in which Cases 1 and 2 above are the only ones encountered in the step by step reduction process.

V. REDUCTION OF THE ANISOTROPIC PRESSURE MODEL AND GYRORESONANT ABSORPTION

We return now to the anisotropic pressure fluid case and consider the electron cyclotron frequency range, neglecting the ion contribution. Also, Eqs.(26) show that thermal effects are most important in the vicinity of the fundamental or second harmonic gyroresonances (i.e., where $\omega+\Omega$, or $\omega+2\Omega$ are of $O(\delta)$) since one expects components p_{z-} or p_{+-} to be relatively large in these regions. Consequently, we shall neglect all the components of **p** but p_{z-} (or p_{+-} for the second harmonic case) in the wave amplitude [Eq.(28)]. We shall consider the fundamental resonance first, so that the amplitude vector is $\mathbf{A}=(\mathbf{b}, \mathbf{a}, \mathbf{v}, p_{z-})$, where **v** represents the perturbed electron fluid velocity. Additional simplification is achieved by restricting the treatment to the

perpendicular incidence case, i.e., $k_x=k$, $k_y=k_z=0$, described by the unreduced dispersion matrix (see Eq.(29))

$$
D = \begin{array}{c}
\begin{array}{cccccccccc} b_+ & b_- & b_z & a_+ & a_- & a_z & v_+ & v_- & v_z & p_{z-} \end{array} \\
\begin{bmatrix}
\omega & 0 & 0 & 0 & 0 & ik/\sqrt{2} & 0 & 0 & 0 & 0 \\
0 & \omega & 0 & 0 & 0 & -ik/\sqrt{2} & 0 & 0 & 0 & 0 \\
0 & 0 & \omega & ik/\sqrt{2} & -ik/\sqrt{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -ik/\sqrt{2} & \omega & 0 & 0 & i\omega_p & 0 & 0 & 0 \\
0 & 0 & ik/\sqrt{2} & 0 & \omega & 0 & 0 & i\omega_p & 0 & 0 \\
-ik/\sqrt{2} & ik/\sqrt{2} & 0 & 0 & 0 & \omega & 0 & 0 & i\omega_p & 0 \\
0 & 0 & 0 & -i\omega_p & 0 & 0 & \omega+\Omega & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -i\omega_p & 0 & 0 & \omega-\Omega & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -i\omega_p & 0 & 0 & \omega & \beta k/\sqrt{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta k/\sqrt{2} & \omega+\Omega
\end{bmatrix}
\end{array}
\begin{array}{c} b_+ \\ b_- \\ b_z \\ a_+ \\ a_- \\ a_z \\ v_+ \\ v_- \\ v_z \\ p_{z-} \end{array}
\qquad (35)
$$

At this point we start the reduction procedure, by focusing our attention on the vicinity of the fundamental cyclotron resonance, $\omega+\Omega \sim O(\delta)$. We eliminate the components b_+, b_-, and v_- first. These reduction steps correspond to Case2 in Sec.IV (only one non-vanishing coupling element in the dispersion matrix). As was shown earlier, the reduction affects only the corresponding diagonal elements of the dispersion matrix. The reduced amplitude becomes $\mathbf{A}=(b_z, a_+, a_-, a_z, v_+, v_z, p_{z-})$ and the corresponding reduced dispersion matrix at this stage is

$$
D=\begin{array}{c}
\begin{array}{cccccccc} b_z & a_+ & a_- & a_z & v_+ & v_z & p_{z-} & \end{array} \\
\begin{bmatrix}
\omega & -ik/\sqrt{2} & ik/\sqrt{2} & 0 & 0 & 0 & 0 \\
ik/\sqrt{2} & \omega & 0 & 0 & i\omega_p & 0 & 0 \\
-ik/\sqrt{2} & 0 & \omega-\omega_p^2/(\omega-\Omega) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \omega-k^2/\omega & 0 & i\omega_p & 0 \\
0 & -i\omega_p & 0 & 0 & \omega+\Omega & 0 & 0 \\
0 & 0 & 0 & -i\omega_p & 0 & \omega & \beta k/\sqrt{2} \\
0 & 0 & 0 & 0 & 0 & \beta k/\sqrt{2} & \omega+\Omega
\end{bmatrix}
\end{array}
\begin{array}{c} \\ b_z \\ a_+ \\ a_- \\ a_z \\ v_+ \\ v_z \\ p_{z-} \end{array}
\qquad (36)
$$

Now we can eliminate a_z , again Case 2 in Sec.IV. We assume, of course, that the corresponding diagonal element $(\omega-k^2/\omega)$ is of O(1). The resulting dispersion matrix describes amplitude $A=(b_z, a_+, a_-, v_+, v_z, p_{z-})$ and has the form

$$
D=\begin{bmatrix}
\omega & -ik/\sqrt{2} & ik/\sqrt{2} & 0 & 0 & 0 \\
ik/\sqrt{2} & \omega & 0 & i\omega_p & 0 & 0 \\
-ik/\sqrt{2} & 0 & \dfrac{\omega^2-\omega\Omega-\omega_p^2}{\omega-\Omega} & 0 & 0 & 0 \\
0 & -i\omega_p & 0 & \omega+\Omega & 0 & 0 \\
0 & 0 & 0 & 0 & \dfrac{\omega(\omega^2-k^2-\omega_p^2)}{\omega^2-k^2} & \beta k/\sqrt{2} \\
0 & 0 & 0 & 0 & \beta k/\sqrt{2} & \omega+\Omega
\end{bmatrix}
\qquad (37)
$$

We observe now that the system splits into two *decoupled* problems for amplitudes $A_1=(b_z, a_+, a_-, v_+)$ and $A_2=(v_z, p_{z-})$ characterized by dispersion matrices

$$D_1=\begin{bmatrix} \omega & -ik/\sqrt{2} & ik/\sqrt{2} & 0 \\ ik/\sqrt{2} & \omega & 0 & i\omega_p \\ -ik/\sqrt{2} & 0 & \dfrac{\omega^2-\omega\Omega-\omega_p^2}{\omega-\Omega} & 0 \\ 0 & -i\omega_p & 0 & \omega+\Omega \end{bmatrix} \; ; \; D_2=\begin{bmatrix} \dfrac{\omega(\omega^2-k^2-\omega_p^2)}{\omega^2-k^2} & \beta k/\sqrt{2} \\ \beta k/\sqrt{2} & \omega+\Omega \end{bmatrix} \tag{38}$$

It can be easily verified that D_1 represents the usual cold plasma extraordinary mode, which, obviously, is not influenced by thermal effects at the resonance. No coupling to fluid-pressure modes takes place in this case. On the other hand, complete near degeneracy of D_2 requires

$$\omega(\omega^2-k^2-\omega_p^2)/(\omega^2-k^2) \sim O(\delta) \tag{39}$$

since βk and $\omega+\Omega$ are of $O(\delta)$ by assumption. When Eq.(39) is satisfied, further reduction of D_2 yields singular coefficients and thus is forbidden within the geometric optics approximation. The final near-degenerate dispersion matrix has the characteristic form

$$D_2=\begin{bmatrix} D_a & \eta \\ \eta^* & D_b \end{bmatrix} \tag{40}$$

and the corresponding transport equation (2) serves as the system of coupled mode equations[3] describing two weakly coupled modes, $D_a=\omega(\omega^2-k^2-\omega_p{}^2)/(\omega^2-k^2)\approx 0$ (obviously the cold plasma ordinary mode) and $D_b=\omega+\Omega\approx 0$, which is the fluid pressure mode, carried by the component p_{z-} of the pressure tensor perturbation. The coupling is due to the thermal effects and $\eta=\beta k/\sqrt{2}$ serves as a weak, almost constant, troughout the degenerate region (normal degeneracy[2]), coupling coefficient .

Now we can find the transmission coefficient of the ordinary mode through the mode coupling region. The transmission, for general geometry, is given by[3]

$$T = \exp(-2\pi|\eta|^2/|B|) \tag{41}$$

where

$$B = \{D_a, D_b\}_{x_0,k_0} = \left(\frac{\partial D_a}{\partial x^\mu} \frac{\partial D_b}{\partial k_\mu} - \frac{\partial D_b}{\partial x^\mu} \frac{\partial D_a}{\partial k_\mu} \right)_{x_0,k_0} \tag{42}$$

and is evaluated at the *crossing point* $\mathbf{x}_0, \mathbf{k}_0$ ($D_a=D_b=0$ at $\mathbf{x}_0, \mathbf{k}_0$) which is defined along the geometric optics ray generated by the dispersion $D_a=0$ (in our case the ray for the ordinary cold plasma mode). Evaluation of T in the case of interest yields

$$T = \exp\left(- \frac{\pi \beta^2 k^2 \omega_p^2}{2\omega |\mathbf{k} \bullet \nabla \Omega|} \right) \tag{43}$$

This expression, for the perpendicular stratification of the magnetic field ($\nabla\Omega \perp \mathbf{B}_0$), coincides with the result predicted by a more elaborate kinetic theory.[5] Note that Eq.(43) was derived by using the multi-dimensional mode conversion theory[3] and, therefore, is not limited to the case of the perpendicular stratification and can be used in plasmas of arbitrary geometry.

At this stage we proceed to the second harmonic case and thus retain the component p_{+-} in the amplitude instead of p_{-z}. Thus the unreduced amplitude becomes $\mathbf{A}=(\mathbf{b}, \mathbf{a}, \mathbf{v}, p_{+-})$ and the corresponding dispersion matrix is [see Eq.(29)]

$$
D = \begin{array}{c}
\begin{array}{cccccccccc} b_+ & b_- & b_z & a_+ & a_- & a_z & v_+ & v_- & v_z & p_{+-} \end{array} \\
\begin{bmatrix}
\omega & 0 & 0 & 0 & 0 & ik/\sqrt{2} & 0 & 0 & 0 & 0 \\
0 & \omega & 0 & 0 & 0 & -ik/\sqrt{2} & 0 & 0 & 0 & 0 \\
0 & 0 & \omega & ik/\sqrt{2} & -ik/\sqrt{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -ik/\sqrt{2} & \omega & 0 & 0 & i\omega_p & 0 & 0 & 0 \\
0 & 0 & ik/\sqrt{2} & 0 & \omega & 0 & 0 & i\omega_p & 0 & 0 \\
-ik/\sqrt{2} & ik/\sqrt{2} & 0 & 0 & 0 & \omega & 0 & 0 & i\omega_p & 0 \\
0 & 0 & 0 & -i\omega_p & 0 & 0 & \omega+\Omega & 0 & 0 & \beta k/\sqrt{2} \\
0 & 0 & 0 & 0 & -i\omega_p & 0 & 0 & \omega-\Omega & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -i\omega_p & 0 & 0 & \omega & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta k/\sqrt{2} & 0 & 0 & (\omega+2\Omega)/2
\end{bmatrix}
\end{array}
\begin{array}{c} b_+ \\ b_- \\ b_z \\ a_+ \\ a_- \\ a_z \\ v_+ \\ v_- \\ v_z \\ p_{z-} \end{array}
\qquad (44)
$$

As before, for simplicity, we restricted the treatment to the perpendicular incidence case ($k_x=k$, $k_y=k_z=0$). Now we observe that elimination of components b_+, b_-, v_- and v_z, again corresponds to Case 2 in Sec. IV. The resulting matrix

$$
D=\begin{array}{c}
\begin{array}{cccccc} b_z & a_+ & a_- & a_z & v_+ & p_{+-} \end{array} \\
\begin{bmatrix}
\omega & ik/\sqrt{2} & -ik/\sqrt{2} & 0 & 0 & 0 \\
-ik/\sqrt{2} & \omega & 0 & 0 & i\omega_p & 0 \\
ik/\sqrt{2} & 0 & \omega-\dfrac{\omega_p^2}{\omega-\Omega} & 0 & 0 & 0 \\
0 & 0 & 0 & \omega-\dfrac{k^2+\omega_p^2}{\omega} & 0 & 0 \\
0 & -i\omega_p & 0 & 0 & \omega+\Omega & \beta k/\sqrt{2} \\
0 & 0 & 0 & 0 & \beta k/\sqrt{2} & \dfrac{\omega+2\Omega}{2}
\end{bmatrix}
\end{array}
\begin{array}{c} b_z \\ a_+ \\ a_- \\ a_z \\ v_+ \\ p_{+-} \end{array}
\qquad (45)
$$

describes the amplitude $A=(b_z, a_+, a_-, a_z, v_+, p_{+-})$. Now , obviously, component a_z can be simply omitted (Case 1 in Sec.IV) and a_- can be reduced as Case 2. The result is

$$
D=\begin{array}{c}
\begin{array}{cccc} b_z & a_+ & v_+ & p_{+-} \end{array} \\
\begin{bmatrix}
\dfrac{(\omega-\Omega)(2\omega^2-k^2)-2\omega\omega_p^2}{2(\omega^2-\omega\Omega-\omega_p^2)} & ik/\sqrt{2} & 0 & 0 \\
-ik/\sqrt{2} & \omega & i\omega_p & 0 \\
0 & -i\omega_p & \omega+\Omega & \beta k/\sqrt{2} \\
0 & 0 & \beta k/\sqrt{2} & \dfrac{\omega+2\Omega}{2}
\end{bmatrix}
\end{array}
\begin{array}{c} b_z \\ a_+ \\ v_+ \\ p_{+-} \end{array}
\qquad (46)
$$

Next, we reduce b_z, which results in

$$
D=\begin{bmatrix}
\dfrac{(\omega^2-k^2)[2\omega(\omega-\Omega)-\omega_p^2]-\omega_p^2\omega^2}{(\omega-\Omega)(2\omega^2-k^2)-2\omega\omega_p^2} & i\omega_p & 0 \\
-i\omega_p & \omega+\Omega & \beta k/\sqrt{2} \\
0 & \beta k/\sqrt{2} & \dfrac{\omega+2\Omega}{2}
\end{bmatrix}
\qquad (47)
$$

describing $A=(a_+, v_+, p_{+-})$. Finally, reduction of a_+ (again Case 2) yields the final characteristic matrix

$$D^f = \begin{bmatrix} \dfrac{2\omega[(\omega^2-k^2)(\omega^2-\Omega^2-\omega_p^2)-\omega_p^2(\omega^2-\omega_p^2)]}{(\omega^2-k^2)[2\omega(\omega-\Omega)-\omega_p^2]-\omega_p^2\omega^2} & \beta k/\sqrt{2} \\ \beta k/\sqrt{2} & \dfrac{\omega+2\Omega}{2} \end{bmatrix} \equiv \begin{bmatrix} D_a & \eta \\ \eta^* & D_b \end{bmatrix} \qquad (48)$$

describing the remaining amplitude components $\mathbf{A}=(v_+, p_{+-})$. Similar to the ordinary mode case, one finds a possibility of a complete near-degeneracy of this matrix when

$$D_a = \frac{2\omega[(\omega^2-k^2)(\omega^2-\Omega^2-\omega_p^2)-\omega_p^2(\omega^2-\omega_p^2)]}{(\omega^2-k^2)[2\omega(\omega-\Omega)-\omega_p^2]-\omega_p^2\omega^2} \sim O(\delta) \qquad (49)$$

$$D_b = (\omega+2\Omega)/2 \sim O(\delta) \qquad (50)$$

Condition $D_a=0$ yields the cold plasma extraordinary mode dispersion relation, so that, in the degenerate region, (48) describes the coupling between the extraordinary mode and the the fluid pressure mode ($D_b \sim 0$), associated with the excitation of component p_{+-} of the perturbed pressure tensor. The element $\eta=\beta k/\sqrt{2}$ in (48) serves as a weak [$O(\delta)$] coupling coefficient.

At this stage we can find the transmission of the extraordinary mode at the second harmonic through the mode conversion region. Again we shall use expression (41) for the transmission coefficient, yielding

$$T=\exp\left\{-\frac{\pi\beta^2k^2\omega_p^2}{4|\mathbf{k}\bullet\nabla\Omega|}\,\frac{(\omega^2-k^2)[2\omega(\omega-\Omega)-\omega_p^2]-\omega^2\omega_p^2}{\omega(\omega^2-\Omega^2-\omega_p^2)}\right\} \qquad (51)$$

Note that since Eq.(51) is evaluated at the crossing point, where $D_a=D_b=0$, we can substitute $\omega=-2\Omega$ and

$$\omega^2-k^2=\omega_p^2(\omega^2-\omega_p^2)/(\omega^2-\Omega^2-\omega_p^2) \qquad (52)$$

Then, after some algebra, the transmission coefficient becomes

$$T=\exp\left[-\frac{\pi\beta^2k^2\omega_p^2}{|\mathbf{k}\bullet\nabla\Omega|\omega}\left(\frac{3\omega^2-2\omega_p^2}{3\omega^2-4\omega_p^2}\right)^2\right] \qquad (53)$$

again in a full agreement with the results of the kinetic theory[5] in the case of the perpendicular stratification of the background magnetic field ($\mathbf{B}_0 \perp \nabla\Omega$).

VI. CONCLUSIONS

i. It was shown here that the unreduced anisotropic pressure, multi-fluid plasma model can be formulated in the Hermitian form (2), characterized by dispersion matrix (29). Consequently, within the model, all waves in a weakly varying plasma can be dealt with within the framework of the congruent reduction method.[2] This method *automatically* yields either a single PDE describing one of the wave components in the non-degenerate plasma case, or a system of coupled mode equations (two coupled PDE's) in nearly degenerate plasma regions, where the mode conversion takes place. Thus the degenerate case reduces to the multi-dimensional mode conversion problem, solution of which for general geometry is already known.[3]

ii. Examples of application of the reduction algorithm in the vicinity of the electron fundamental and second harmonic gyroresonances, in Sec. V, demonstrate our systematic approach to the reduction of order. The procedure yields the characteristic coupled mode system at the resonances suggesting the interpretation of the gyroresonant absorption as the mode conversion from the electromagnetic mode to a fluid-pressure mode. The transmission coefficients thus obtained were found in a complete agreement with the predictions of the fully kinetic theory.[5]

iii. We have considered the normal incidence case ($k \perp B_0$) and single species plasma in the examples in Sec.V . Nevertheless, since the method is general, it can be automated by using a computer, thus allowing the study of much more complicated mode interactions such as those in multi-species plasmas of arbitrary geometry, where the usual theories are typically restricted to slab models and become extremely elaborate and non-trivial.

iv. The possibility of formulating both the cold plasma and the anisotropic pressure fluid models in the Hermitian form suggests that higher moment equations, and possibly the kinetic problem in the absence of collisions, also comprise the Hermitian case. We shall address this problem in our future studies.

ACKNOWLEDGEMENT

The major part of this work was performed at the Lawrence Berkeley Laboratory during the author's sabbatical leave from the Hebrew University of

Jerusalem. The research was supported by the U.S. Dept. of Energy under Contract No.DE-AC03-76SF00098 and in its final stage by the U.S.-Israel Binational Science Foundation. The author is grateful to Allan Kaufman and Ira Bernstein for valuable discussions and encouragement in the course of the research. Also the author wishes to express his deep appreciation to Allan Kaufman, Wulf Kunkel and the rest of the MFE staff at LBL for their hospitality, patience and help, which made the sabbatical leave especially pleasant and fruitful.

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