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A Mechanical Analog of the Synchrotron, Illustrating Phase Stability and Two-Dimensional Focusing*

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ABSTRACT

A steel ball bounces in synchronism with a vertically oscillating piston. The piston surface is a hardened steel disk on which the ball bounces; two-dimensional horizontal focusing is provided by the concavity of the surface. The period of oscillation can be varied over a 3:1 range, with the amplitude kept constant. As the period is increased, the ball bounces higher. As the period is decreased, the ball bounces lower, contrary to the intuition of most observers. The model illustrates the important properties of synchrotron accelerators.

INTRODUCTION

The principle of phase stability, first formulated by Veksler[1] and McMillan,[2] transformed the cyclotron, designed by E. O. Lawrence, into other more versatile forms of accelerators. We describe a working mechanical synchrotron analog that demonstrates quite clearly the principle of phase stability. The model also demonstrates the important focusing properties of a practical synchrotron.

THE PHASE STABLE CONDITION

A hardened-metal piston moves up and down in sinusoidal fashion with fixed amplitude and frequency. If a small steel ball is dropped from the proper height above the piston and at the appropriate time with respect to the phase of the piston's motion, the ball will bounce in a stable mode. Each impact of the ball on the top of the piston will occur during the time when the piston is passing from maximum upward velocity at the center of its stroke toward zero velocity at the top of its stroke.

Phase stability, with the ball striking the piston at nearly the same time in each piston cycle, is maintained for the following reasons:

1) Suppose that immediately after an impact the steel ball has *insufficient* kinetic energy for its time of flight in the air to match the piston's period of oscillation. The next impact will then occur *earlier* in the piston cycle, when the piston velocity is *higher*, so that more kinetic energy will be transferred to the ball upon impact than for the preceding impact; hence the ball will rise *higher than before* and the succeeding time of flight will be slightly longer than before.

2) If, by contrast, the steel ball has *too much* kinetic energy so that its time of flight is longer than the piston's period of oscillation, the next impact will take place *later* in the piston cycle and consequently the ball's kinetic energy will be smaller immediately

after impact and the ball's succeeding time of flight shorter than the previous one.

A remarkable phenomenon will occur if the ball is bouncing in a phase stable mode and if the frequency of oscillation of the piston is then gradually reduced. The condition of phase stable bouncing continues, because the time of flight increases and correspondingly the ball rises to a greater maximum height than before (see Fig. 1). (This behavior is contrary to what one's intuition would suggest; a lower frequency usually implies a lower energy, so most observers expect the ball to bounce less high when the frequency is lowered.) This phenomenon can be understood in terms of the following considerations.

Suppose the ball is bouncing in a phase stable mode. An impact occurs. Then the period of the piston oscillation is made to increase. This means that the next impact will occur when the piston velocity is slightly higher, since one full cycle of the piston motion will not quite be completed when the next impact occurs. This, in turn, will effect a lengthening of the time of flight, since more kinetic energy will be transferred to the ball upon impact than before. In the simplest theory the phase of the bounce would oscillate forever about the "stable phase." But in practice, the inherent friction of the bouncing process quickly damps out the oscillation, and the ball bounces higher than before, and close to the new stable phase.

In a similar fashion, increasing the frequency of piston oscillation or shortening the period will lead to shorter time of flight and lesser maximum bounce heights for the ball.

OPTIMUM INITIAL CONDITIONS

Some interesting quantitative considerations can be made. Assuming negligible air friction, a coefficient of elasticity e, a

piston angular frequenncy ω, and a piston motion amplitude A, the optimum ball height and timing for initiating phase stable bouncing can be calculated. Since the piston motion is sinusoidal, the position of the top surface may be described as $y_p = A \sin(\omega t - \pi)$. The question, then, is what are the appropriate initial values of y_b and t for dropping the ball. Impact will occur, of course, when $y_b = y_p$.

By insisting that the kinetic-energy gain upon impact equal the loss due to inelasticity, the result is readily obtained that the piston velocity at the instant of impact must be $v_p = [(1 - e)/(1 + e)]\, v_b'$, in which v_b' is the velocity of the ball immediately after impact. And if the time of flight is to equal the period of the piston oscillation, T, then the magnitude of the velocity of the ball immediately before and after impact must be $|v_b| = v_b' = (1/2)gT$, in which g is the acceleration of gravity, taken as 980 cm/sec^2. This, in turn, implies that the ball must be released at a time T/2 before impact and at a height $(1/2)g(T/2)^2$ above the position of impact.

Defining the piston position $y_p = 0$, the piston velocity $v_p = -A\omega$ at time $t = 0$, and $K = [(1 - e)/(1 + e)]\,(g/A\omega^2)$, the following results obtain:

1) The ball must be released from a height $y_b = (1/2)\, g\, (T/2)^2 + A\,(1 - K^2)^{1/2}$ at time $t = (T/2\pi)\cos^{-1}K$, when the position of the piston is $y_p = -A\,(1 - K^2)^{1/2}$ and the velocity of the piston is $v_p = -A\omega K$.

2) The position of the ball and plate upon the first impact is $y_p = A\,(1 - K^2)^{1/2}$ and the impact occurs at time $t = (T/2) + (T/2\pi)\cos^{-1}K$. Succeeding bounce heights will be the same as the initial drop height.

3) Since $K < 1$, the amplitude is subject to the condition

$$A > \frac{\pi g (1 - e)}{\omega^2 (1 + e)} .$$

Supposing $e = 0.95$ and $T = 1.00$ sec, then $A > 2.00$ cm. If $A = 5.00$ cm then the ball must be released from a height of 127.1 cm at $t = 0.185$ sec, when $y_p = -4.58$ cm and $v_p = -12.56$ cm/sec. At impact $y_p = 4.58$ cm, $v_p = +12.56$ cm/sec, $v_b' = +490$ cm/sec, and $t = 0.685$ sec. Succeeding bounce heights will also be 127.1 cm above the zero position of the piston.

THE ANALOGY

Even though no analogy is perfect, the parallels between this mechanical device and certain forms of accelerator are instructive.

(1) There is a strong parallel in the phenomenon of synchronism or phase stability, with stability obtaining when impacts occur during the quarter cycle that the piston velocity decreases from its maximum value toward zero. If there were no energy losses, viz., no air resistance and perfectly elastic impacts, the synchronous impacts would occur just as the piston reached zero velocity every cycle and the total energy of the ball, kinetic plus potential, would be maintained at a constant value.

In circular accelerators, the phase stable condition obtains when particles are accelerated during the quarter cycle for which the electric field falls from maximum value to zero.

(2) As the frequency of the piston oscillation is lowered (the period increased) bounce heights and the total energy of the ball increase as the square of the period.

In the synchrocyclotron the particle energy is likewise increased by lowering the frequency of the accelerating field and causing the particle to travel in larger orbits.

TWO-DIMENSIONAL FOCUSING

If the piston had a plane upper surface, the ball position would be unstable, in so far as motion in the two orthogonal horizontal directions is concerned. The piston is therefore given a concave spherical surface, with a radius of curvature of about 4 m. This shape insures that the ball is given a radial focusing force, to keep it from drifting away from the finite area of the piston. Practical accelerators use similar magnetically derived focusing forces to keep the particles from hitting the magnetic pole pieces and the circumferential walls of the vacuum chamber.

The machine is designed to be operated, safely, by inexperienced museum visitors. All the mechanism is therefore enclosed and methods are provided for retrieving stray balls and for injecting the ball into the oscillating mode.

Operating controls are a motor start button, a speed (frequency) change knob, and a ball-start button. A pressure pad on the floor stops the machine when the experimenter leaves.

The cabinet (Fig. 2) is 0.9 m wide, 2.4 m high, and 1.3 m deep and is made of panels attached to a welded frame of square steel tubing. It is divided into three compartments which will be called the base, the stage, and the back volumes.

The base is about 0.9 m cube and contains the variable frequency plate drive mechanism and plate sweep system. Projecting from the top of the base into the stage volume is the column which supports and moves the piston. Behind the base is the rear volume which is ∿0.9 m wide

like the base, 0.4 m deep and 2.4 m high. It contains the "loading mechanism" (a motor-operated system for raising and dropping the ball onto the oscillating piston), a relay system for controlling this mechanism, and an illuminated box for a written description of the display.

The stage volume is above the base and in front of the back volume and is ∿0.9 m square by 1.5 m high. It is enclosed on two sides and the front by 6-mm-thick transparent acrylic plastic sheets. The top is open to permit the ball to bounce higher than the restricted height of the machine. Two illumination boxes in wings to either side of the stage volume contain vertically mounted 40-watt fluorescent lights to illuminate the stage. The "floor" of the stage volume is sloped to a common "pick-up" point in rear center behind the oscillating plate.

MECHANISM - OSCILLATOR

The "oscillator" consists of a fairly constant speed (3/4 hp capacitor start) motor, a variable-ratio[3] belt drive, a gear-and-chain speed reduction to a crankshaft, and connecting rods to a variation of a "Stephenson's-link" for fixing the stroke of a vertical column which supports the piston.

The mechanism is made mostly of commercial components mounted between two vertical 12-mm-thick steel plates welded to a similar horizontal bottom plate. No attempt was made to reduce the ∿120-kg weight of the mechanism, because no attempt was made to counterbalance the oscillating plate.

The speed range is limited between 38 and 114 rpm and the amplitude is adjustable from 50 to 150 mm (peak to peak).

The amplitude is selected and fixed such that oscillations (at the fundamental frequency) will no longer be sustained at a bounce height of over ∿3 m (38 rpm). Even at this reduced amplitude, the peak acceleration at the highest frequency approaches 1 G, and with the oscillating components weighing about 30 kg the remaining stationary weight of 90 kg is not excessive.

Many possible harmonic modes of bouncing are possible at the highest plate frequency, and it is possible to accelerate balls in these modes to heights that severely dent the piston plate. To suppress these modes, the injection phase requirements are rather stringent, and the rate of change of frequency is limited by requiring many turns of the speed-change knob to cover the frequency range.

The injector is a rod with an electromagnet on the end (see Fig. 3). It lifts the ball from the stage to above the piston and drops it at the proper time. It is moved by the injector motor.

The injector motor is a reversible 6 rpm gear-motor with an electric brake. It is connected via a slip clutch to an axle piercing the back wall of the stage at an angle of (-)30° to the horizontal. A rod or "arm" of ∿0.4 m length is attached to the axle on the stage side of the wall at an angle of 60° to the axle. It sweeps a conical path from vertically downward to 30° above horizontal as the axle is turned. At the end of the arm is an electromagnet with its core parallel to the axle.

Another arm is also attached to the axle (on the back side of the stage wall) to counterbalance the magnet arm and to operate limit switches.

One half-turn of the axle moves the magnet tip from a position 1 mm above the ball (at the lowest point on the stage floor) to a

position ∿0.4 m above the center of the oscillating piston.

PISTON-SWEEP SYSTEM

If the machine is abandoned with the ball bouncing, the motor stops and the ball ultimately comes to rest near the center of the piston. The ball must be moved off the piston to pick-up position on the stage before injection can begin. A tank of ∿2 liter volume is charged with compressed air at 80 psig whenever the oscillator motor stops. When the start button is pressed this air is exhausted through a series of nozzle holes in a block in front center stage and thus blows the ball off the piston and on to the stage. The ball then rolls to the lowest point of the stage and on to a small photocell.

RELAY SYSTEM

A pressure-pad switch on the floor in front of the machine provides "hold" power for operating relays and thus shuts off the machine when the experimenter leaves.

A cadmium-sulfide photocell[4] in the stage floor at the ball pick-up position uses the general stage illumination to detect the ball and otherwise inhibit the operation of the injector arm.

The requirements for starting the injection cycle are: "Start ball" button pushed, oscillator motor running, injector arm down, injector arm motor stopped, ball at pick-up position, and speed (frequency) maximum. Limit stops are set on the maximum and minimum available frequencies. The maximum frequency (114 rpm) gives a bounce height slightly less than the injector arm height.

The requirements for dropping the ball and starting the injector arm down are: Frequency maximum, injector arm up, injector motor stopped, and oscillator crankshaft phase correct. (The phase switch is adjusted to make the first bounce of the ball equal to the

height of the injector arm. These requirements prevent the arm's passage into the path of the bouncing ball.

PISTON DISK

The piston disk was made of "KETOS"[5] oil hardening tool steel 260 mm diam by 60 mm thick. The front surface was ground concave to a 4 m spherical radius, and the back surface was tapped with three mounting holes on a 75-mm-diam circle. The plate was heat treated to Rockwell C-60 hardness.

Some Brinelling (denting) is observed with each bounce of the ball from heights above 3 m, so the minimum frequency of oscillation is limited to ∿38 rpm.

The coefficient of elasticity is a function of the height of drop of the ball, changing from 0.97 at 15 cm to ∿0/95 at 3 m.

The balls used are chrome-alloy steel, composition 1, grade 50 ball bearings, 16 mm diam. Carbide balls damage the place too much, quartz balls shatter, and ceramic balls don't bounce well.

A sheet-metal pipe and cloth webbing strip are clamped around the piston disk with a metal band to form the piston skirt. The skirt keeps stray balls out of the operating mechanism, and the webbing absorbs energy from a "grazing miss" ball so it can't come through or over the acrylic windows of the stage volume.

SPEED (FREQUENCY) CHANGE MECHANISM

The speed change mechanism uses a V-belt between two pairs of conical half-pulleys.[3] The two cones of the motor pulley can be squeezed together by the experimenter by means of a screw mechanism. As the motor pulley is squeezed the belt is forced to run at a larger radius. The belt will not stretch, so it forces the two cones of the driven pulley apart (they are squeezed together by a spring), reducing

the effective radius of the driven pulley and thus increasing the speed.

The screw mechanism has limit stops to set the maximum and minimum frequency, a slip clutch so the stops can't be broken, a microswitch to detect maximum speed setting and permit injection, and a sprocket-and-chain connection to a knob on the experimenter's control panel.

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FOOTNOTES AND REFERENCES

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†(Rev.) Gerard Senecal was appointed President of Benedictine College on Feb. 12, 1972.

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2. E. M. McMillan, Phys. Rev. *68*, 143 (1945).
3. Maurey-Matic model 6600 (modified), Maurey Mfg. Corp., 2901 S. Wabash Avenue, Chicago, Illinois 60616.
4. Clairex CL 603, Clairex Electronics, Mt. Vernon, New York.
5. Ketos oil hardening tool steel, Crucible Steel Co. of America, Four Gateway, P. O. Box 88, Pittsburg, Pennsylvania.

FIGURE CAPTIONS

Fig. 1. Bounce height vs frequency.

Fig. 2. Mechanical synchrotron analog.

Fig. 3. Stage region showing piston, injector arm, piston sweep nozzle block, and a ball.

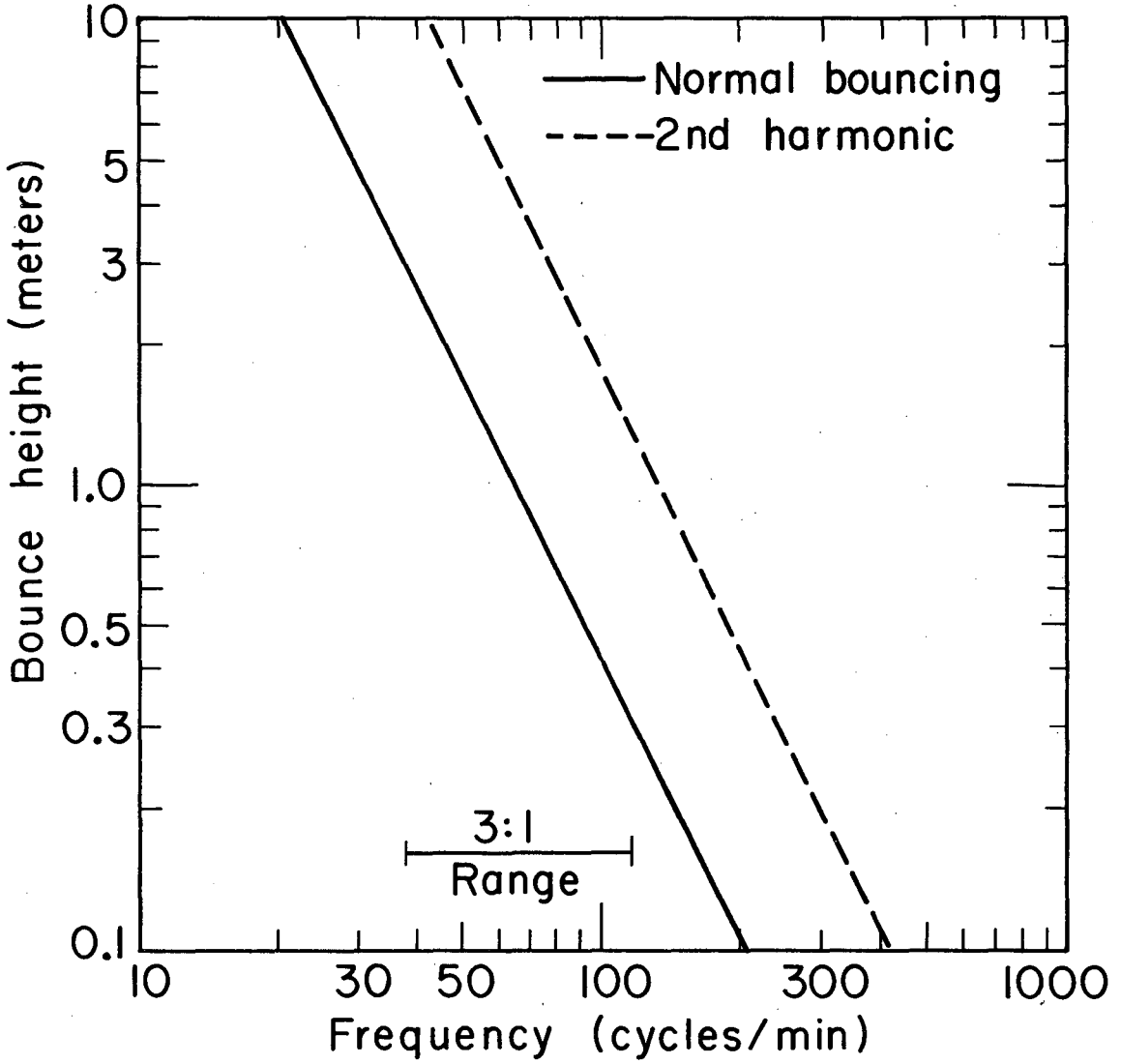

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Fig. 1

Fig. 2

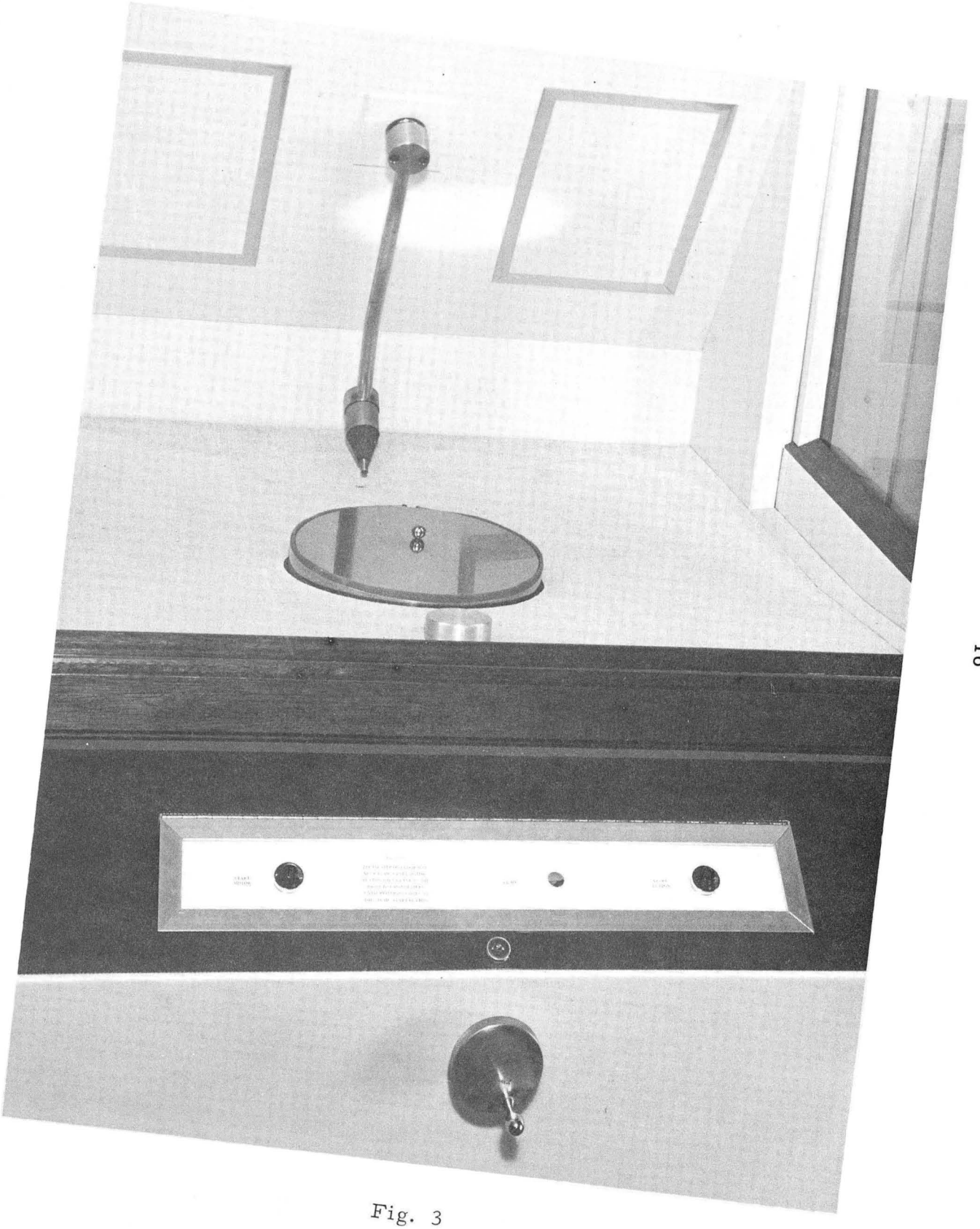

Fig. 3

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