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The Chromo-Dielectric Model: Confinement, Effective Coupling and Chiral Invariance

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Abstract

The chromo-dielectric mechanism of absolute confinement is studied in the nontopological soliton model. The model Lagrangian is chirally invariant, since it contains no direct coupling between the quarks and the scalar field. The static chromo-electric gluon propagator is calculated in medium in the one-loop approximation, and the ultraviolet divergence in the self-energy of fixed quarks is regulated by a form factor. Effective quark-scalar coupling emerges through the self-energy of the quarks in the dielectric medium, which is a function of the scalar field.

The description of hadrons in terms of quark-soliton models [1–3] will continue to play an important role in our understanding of hadronic properties at least until these properties can be calculated in QCD with great accuracy. In the Friedberg-Lee nontopological soliton model [1], confinement is effected by a color-dielectric function $\kappa(\sigma)$ and through a direct coupling term $g(\sigma)\bar{\psi}\psi$ between the scalar field σ and the quarks. The coupling $g(\sigma)$ acts as an effective mass for the quarks in the region of confinement, and breaks chiral invariance even for massless quarks. Various attempts have been made to derive the chromo-dielectric properties of the system and the effective quark-sigma coupling within the framework of the

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nontopological soliton model [4–6]; see also [7]. A qualitatively similar approach is provided by the dual superconductivity model [8–10].

The starting point of chromo-dielectric soliton models is the covariant, gauge-invariant Lagrangian density

$$\mathcal{L} = \bar{\psi}[i\gamma_\mu\partial^\mu - m - g_s\gamma_\mu\frac{\lambda}{2}A^\mu - g(\sigma)]\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \frac{1}{4}\kappa(\sigma)F_{\mu\nu}F^{\mu\nu} \tag{1}$$

where ψ is the Dirac field, m is the current quark mass matrix, $U(\sigma)$ is a fourth-order polynomial in the chiral singlet scalar field σ with an absolute minimum at σ_v, and $F_{\mu\nu}$ is the gauge field tensor; the color and flavor indices have been suppressed for simplicity. In the original Friedberg-Lee model [1] $\kappa(0) = 1$, $\kappa(\sigma_v) = 0$, and the coupling is linear in the σ-field, $g(\sigma) = g_0\sigma$. Nielsen and Patkós [4] assume the form $\kappa(\sigma) = (1 - \sigma/\sigma_v)^4$, and $g_{eff}(\sigma) = g_0\kappa^{-1/4}$. Bayer, Forkel and Weise [6] use various forms for κ with $g_{eff}(\sigma) = g_0\sigma\kappa^{-1/2}$. For our numerical calculations, we set

$$\kappa(\sigma) = 1 + \Theta(x)x^n[nx - (n+1)], \tag{2}$$

with $x = \sigma/\sigma_v$. The form (2) is a simple generalization of the expression used in Ref. [11] (where $n = 3$), and satisfies the conditions imposed [1] on the function $\kappa(\sigma)$, and also the necessary condition [11] $\kappa'(\sigma_v) = 0$. (The particular choice of n does not influence the results strongly.)

In the present work we focus on the chromo-dielectric confinement mechanism. We assume that the model contains no direct quark-sigma coupling, so that the entire burden of confinement rests on the color-dielectric function $\kappa(\sigma)$. Confinement is the result of nonperturbative gluon self-interactions. Once the gluons are confined, no further mechanism is required to confine the quarks, as we shall show. This idea is not entirely new, and has been discussed by several authors before [12,13], but, to our knowledge, has never been implemented.

One of the main virtues of this approach to confinement is that, for massless quarks, the model is chirally invariant. Evaluation of the quark chromo-electric self-energy leads to an *effective* quark-sigma coupling, which, in the case of a short-range cutoff, can be expressed as a local function of $\kappa(\sigma)$. The effective coupling we derive has a form different from the ones used by others [1],[4–6,14], but such differences can be absorbed into the functional form of $\kappa(\sigma)$. Although the effective coupling breaks chiral invariance of the baryon, this must be restored by the emergence of a massless Nambu-Goldstone boson (pion) coupled to the baryon [15]. Furthermore, this pion will have to be formed from the fundamental degrees of freedom introduced; that is, the pion will be a confined quark-antiquark pair to lowest order.

We first review how a chromo-dielectric function $\kappa(\sigma)$ which decreases sufficiently rapidly as $r \to \infty$ leads to absolute color confinement. The self-energy of a fixed (i.e. massive) quark in an MIT-type cavity is then calculated and it is shown that the numerical result is close to the one obtained in Refs. [16,17] for light quarks in the MIT bag model. Encouraged by this result, we next consider the self-energy of a fixed quark in a smooth chromo-dielectric medium in the one-loop approximation, in order to obtain the form of the effective coupling as a function of $\kappa(\vec{r})$. Finally, we illustrate the properties of the soliton states obtained selfconsistently with the derived effective coupling.

Consider, for simplicity, a spherically symmetric static charge density, $Q\,\delta^3(\vec{r})$. Assume that $\kappa \to 0$ as $r \to \infty$ at least as rapidly as r^{-1}. The chromo-electrostatic field equations are then the familiar

$$\vec{\nabla} \cdot \vec{D} = \rho \qquad \vec{\nabla} \times \vec{E} = 0$$
$$\vec{D} = \kappa \vec{E} \qquad \vec{E} = -\vec{\nabla} A_0 \,. \tag{3}$$

Here, however, all quantities are operators in color space. From Gauss's law we find immediately that

$$\vec{D} = \frac{Q}{4\pi r^2}\hat{r} \tag{4}$$

where $\hat{r}$ is a unit vector in the direction of $\vec{r}$. The energy is

$$\mathcal{E} = \frac{1}{2}\int d^3r\, \vec{D} \cdot \vec{E} = \frac{1}{2}\int d^3r\, \frac{D^2}{\kappa(\vec{r})} = \frac{Q^2}{8\pi}\int_0^\infty \frac{dr}{r^2}\,\frac{1}{\kappa(r)}, \tag{5}$$

where, in the last equation we have used spherical symmetry. The divergence at the lower limit of integration is associated with the usual self-energy problem. However, the integral also diverges at the upper limit if $\kappa \to 0$ at least as rapidly as r^{-1}. Then the energy is infinite unless the total internal charge vanishes. The argument does not depend on spherical symmetry. Any net charge gives rise to a $\vec{D}$-field that falls off at large distances as r^{-2}. Therefore the charge within the 'cavity' must be in a color singlet state. This simple argument indicates that the dielectric properties of the cavity and vacuum assure absolute color confinement: an isolated structure not in a color singlet state has infinite energy.

By Gauss's law, the field energy, $\mathcal{E}$, associated with a charge distribution, is equal to 1/2 the integral of the charge distribution times the potential. Thus, for the self-energy of a quark fixed at $\vec{r}$, we find (using the color charge operator $g_s\lambda/2$ and $\alpha_s = g_s^2/(4\pi)$, and taking into account $<\lambda \cdot \lambda>= 16/3$), that

$$E_{self}(r) = \frac{1}{2} <(\frac{1}{2}g_s\lambda)^2> \lim_{\vec{r}\,'\to\vec{r}} G(\vec{r},\vec{r}\,') = \frac{8\pi}{3}\alpha_s \lim_{\vec{r}\,'\to\vec{r}} G(\vec{r},\vec{r}\,') . \tag{6}$$

We now consider a fixed quark at an arbitrary point in an MIT-type cavity of radius R, with dielectric constant $\kappa = 1$ inside and $\kappa = \epsilon$ outside. Eventually we want the limit $\epsilon \to 0$.

The static chromo-electric Green's function is

$$G(\vec{r},\vec{r}\,') = \frac{1}{4\pi}\sum_{\ell} P_\ell(\cos\theta)\left[\frac{r_<^\ell}{r_>^{\ell+1}} + \frac{(rr')^\ell}{R^{2\ell+1}}\frac{(1-\epsilon)(\ell+1)}{\epsilon(\ell+1)+\ell}\right], \qquad r, r' < R . \tag{7}$$

A calculation of the self-energy (6) gives

$$E_{self}(r) = \frac{2}{3}\alpha_s\left\{\frac{1}{r}\sum_{\ell} 1 + \frac{(1-\epsilon)}{R\epsilon} + \frac{1}{R}\left[\frac{r^2}{R^2-r^2} - \log(1-r^2/R^2)\right]\right\} . \tag{8}$$

The first term on the RHS of (8) is to be identified with the usual self-energy of an isolated quark in a medium of $\kappa = 1$. The second term in (8) is a monopole ($\ell = 0$) contribution arising from the second term of (7). It is independent of position but becomes infinite as $\epsilon \to 0$. It is cancelled by mutual interaction with other quarks in the cavity if the total color wave function is in a singlet state. The remaining terms ($\ell \geq 1$) in (8) are finite as $\epsilon \to 0$ and are summed to give the terms in square brackets. Note that as $r \to R$, these yield $\frac{2}{3}\alpha_s/2(R-r)$ corresponding to the interaction of a quark with an image charge of the same sign [12].

Since the self-energy appears as a mass term for the quark, the term in square brackets in Eq. (8) can be identified as a scalar confinement potential

$$V_{conf}(r) = \frac{2\alpha_s}{3R}\left[\frac{r^2}{R^2-r^2} - \log(1-r^2/R^2)\right] . \tag{9}$$

It is worth mentioning that even though the single quark potential (9) is more strongly confining than linear, one gets a linear confinement potential when the confinement region is allowed to deform [18]. A calculation of the expectation value of V_{conf}

$$\int \bar{\psi}_0 V_{conf} \psi_0 d^3 r \,, \tag{10}$$

for the lowest $s_{1/2}$ quark state in the MIT bag model yields the value $0.932\alpha_s/R$, compared with the results of Goldhaber, Hansson and Jaffe [16,17], who obtained $0.903\alpha_s/R$ for light quarks. The similarity in the numerical results indicates that it may be reasonable to use the massive quark potential results for light quarks. This is because confinement imparts to the quarks an effective mass.

Let us now turn to the general case of a smooth $\kappa(\vec{r})$. The quark self-energy diagram in the one-loop approximation is depicted in Fig. 1a. For an infinitely massive quark, the quark propagator shrinks to a delta function, Fig. 1b. In the Abelian approximation, the self-energy of a fixed quark is given by (6). The infrared, or long range, divergence in this quantity is associated with color confinement, as was discussed above. An infrared regularization (e.g. $\kappa \to \epsilon > 0$ as $r \to \infty$) permits the calculation, for example, of fissioning bags, where quarks confined to separated sections must be in singlet states (the results are independent of ϵ in the limit $\epsilon \to 0$).

There is also an ultraviolet divergence in the Abelian approximation, although we know that QCD is free of such divergences because of asymptotic freedom. To remedy this problem it is not sufficient to subtract the free ($\kappa = 1$) self-energy. The term in the self-energy which diverges when $\vec{r}\,' \to \vec{r}$ depends on $\kappa(\vec{r})$. The divergence is thus spatially dependent and cannot be removed by renormalization of the quark mass.

In order to regularize the short distance behavior, a form factor $f(|\vec{r} - \vec{r}\,'|)$ is introduced. (The function $f(r)$ is normalized to unity; $\int d^3r f(r) = 1$.) For example, if we choose

$$f(r) = \frac{e^{-r^2/a^2}}{a^3 \pi^{3/2}}, \tag{11}$$

then the divergent term $\lim_{r\to 0} 2\alpha_s/3\kappa r$ is replaced by

$$\frac{2}{3}\alpha_s \frac{1}{\kappa} \int d^3r f(r)/r = \frac{4\alpha_s}{3\sqrt{\pi}\, a\, \kappa}\,. \tag{12}$$

The divergence is proportional to a^{-1} when $a \to 0$ and would be linear in a momentum cut-off. If the quark is described by a finite mass Dirac propagator, as in QED, the divergence is logarithmic. In either case we expect the self-energy to be dominated by a term proportional to $1/\kappa$. We now show this in detail for a fixed quark in the case of spherical symmetry using the Gaussian form factor (11).

We consider

$$E_{self}(r) = \frac{2}{3} g_s^2 \int d^3r' G(\vec{r}, \vec{r}\,') \frac{e^{-|\vec{r}-\vec{r}\,'|^2/a^2}}{a^3\pi^{3/2}}, \tag{13}$$

where [11]

$$G(\vec{r}, \vec{r}\,') = \frac{1}{r\, r' \sqrt{\kappa(r)\kappa(r')}} \sum_\ell \frac{(2\ell+1)}{4\pi} \frac{f_\ell(r_<)g_\ell(r_>)}{w(g_\ell, f_\ell)} P_\ell(\cos \widehat{r r'}). \tag{14}$$

Here, f_ℓ and g_ℓ are the solutions of the differential equation

$$\left[-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} + \frac{1}{4} \mid \vec{\nabla} \ln \kappa \mid^2 + \frac{1}{2} \nabla^2 \ln \kappa\right] \left\{ \begin{matrix} f_\ell(r) \\ g_\ell(r) \end{matrix} \right\} = 0 \tag{15}$$

that are regular at $r = 0$ and $r \to \infty$, respectively; $w\{g, f\} = gf' - g'f$ is the Wronskian.

Substitution into (13) gives

$$E_{self} = \frac{4\pi\alpha_s}{3} \sum_\ell (2\ell+1) \int_0^\infty \frac{r'^2 dr'}{r\, r'} \frac{1}{\sqrt{\kappa(r)\kappa(r')}} \frac{f_\ell(r_<)g_\ell(r_>)}{w(g_\ell, f_\ell)} \int_{-1}^1 dx \frac{e^{-|\vec{r}-\vec{r}\,'|^2/a^2} P_\ell(x)}{a^3\pi^{3/2}}, \tag{16}$$

where $x = \cos \widehat{r r'}$. We evaluate

$$\frac{e^{-(r^2+r'^2)/a^2}}{a^3\pi^{3/2}} 2\pi \int_{-1}^1 dx e^{2rr'x/a^2} P_\ell(x) = \frac{4}{a^3\pi^{1/2}} e^{-(r^2+r'^2)/a^2} i_\ell(2rr'/a^2), \tag{17}$$

where $i_\ell(z)$ is the modified spherical Bessel function of the first kind [19]:

$$i_\ell(z) \equiv \sqrt{\frac{\pi}{2z}} I_{\ell+1/2}(z) = i^{-\ell} j_\ell(i\, z), \tag{18}$$

with the limiting forms

$$i_\ell(z) \to \left\{ \begin{array}{ll} z^\ell/(2\ell+1)!! & \text{for } z \to 0, \\ e^z/2z & \text{for } z \to \infty. \end{array} \right. \tag{19}$$

Then

$$E_{self}(r) = \frac{8\alpha_s}{3a^3\pi^{1/2}\sqrt{\kappa(r)}} \sum_\ell \frac{(2\ell+1)}{w(g_\ell, f_\ell)} \int \frac{r' dr'}{r\sqrt{\kappa(r')}} f_\ell(r_<) g_\ell(r_>)\, e^{-(r^2+r'^2)/a^2} i_\ell(2rr'/a^2). \tag{20}$$

For the spherically symmetric case under consideration, we define the effective potential to be the expression above, up to a (finite) mass renormalization term. Since only energy differences have physical meaning for a single quark, and our potential is flat in the interior, the self-energy $E_{self}(r)$ minus $E_{self}(r=0)$, is calculated. We first estimate the effective coupling, and then evaluate it numerically.

The behavior of $i_\ell(z)$ divided by its limiting form $e^z/2z$ is displayed in Fig. 2 as a function of ℓ for several values of the argument z. For sufficiently small values of the argument, $i_\ell(z)$ decreases rapidly as a function of ℓ. To obtain a qualitative understanding of the behavior of the effective potential, we set

$$2z\, e^{-z} i_\ell(z) \approx \theta(\ell_c - \ell)\,, \tag{21}$$

where

$$\ell_c = \xi\sqrt{z} \tag{22}$$

with ξ a number of the order of unity. Then

$$E_{self}(r) \approx \frac{2}{3}\alpha_s \frac{1}{r^2\sqrt{\kappa(r)}} \sum_{\ell=0}^{\xi\sqrt{2rr'/a^2}} \frac{(2\ell+1)}{w(g_\ell, f_\ell)} \int dr' \frac{f_\ell(r_<) g_\ell(r_>)}{\sqrt{\kappa(r')}} \frac{e^{-(r-r')^2/a^2}}{\pi^{1/2} a}\,. \tag{23}$$

For a small compared to the characteristic distance of the variation in κ, we can treat the Gaussian factor as a δ-function:

$$E_{self}(r) \approx \frac{2}{3}\alpha_s \frac{1}{r\kappa(r)} \sum_{\ell=0}^{\xi\sqrt{2}r/a} \frac{(2\ell+1)}{w(g_\ell, f_\ell)} f_\ell(r) g_\ell(r)\,. \tag{24}$$

Finally, since the cut-off in ℓ is very large for small a, we can use the asymptotic forms for f_ℓ and g_ℓ, which give

$$\frac{f_\ell(r)\, g_\ell(r)}{w(g_\ell, f_\ell)} \to \frac{r}{(2\ell+1)}\,, \tag{25}$$

so that we obtain

$$E_{self}(r) \approx \frac{2}{3}\alpha_s \frac{\xi\sqrt{2}}{a\,\kappa(r)}\,. \tag{26}$$

Comparison with Eq. (12) suggests that $\xi = \sqrt{2/\pi}$.

On the basis of Eqs. (12) and (26) we take the asymptotic ($a \to 0$) effective quark-sigma coupling to be

$$g_{eff}(\sigma) = g_0\sigma_v \left[\frac{1}{\kappa(\vec{r})} - 1\right] . \tag{27}$$

with the identification

$$g_0\sigma_v = \frac{4\alpha_s}{3\sqrt{\pi}a} . \tag{28}$$

The term -1 in the brackets is the mass renormalization of the quarks: $g_{eff}(\sigma)$ vanishes for $\sigma = 0$, where $\kappa = 1$. The quantity σ_v has been introduced in (27) to display the dimensionality of the effective coupling and to facilitate comparison with earlier work [1,4,6].

We present here numerical calculations of $E_{self}(r)$, as given by Eq. (20), using an assumed chromo-dielectric function

$$\kappa(r) = \frac{1}{1 + e^{(r-R)/s}} , \tag{29}$$

for $R = 1.0$ fm and $s = 0.3$ fm. The results are displayed in Fig. 3 for three values of the form factor width a (with the value at the origin subtracted). The asymptotic ($a \to 0$) effective coupling (27) is also shown, as is $\kappa(r)$ (right scale). It can be seen that the confinement potential rises more rapidly with decreasing value of the width a, and the results of the form-factor calculation approach the result of the δ-function approximation, g_{eff}.

The effective selfconsistent nucleon (3 quarks) mean field equations can be summarized by the following coupled nonlinear integro-differential equations:

$$\left[\vec{\alpha} \cdot \vec{p} + \beta \left(m + E_{self}(r) - \frac{4\alpha_s}{3\sqrt{\pi}a}\right) - \epsilon\right] \psi(r) = 0 , \tag{30}$$

$$-\nabla^2\sigma + U'(\sigma) + 2g_s^2\kappa'(\sigma) \left[\int \vec{\nabla}G(\vec{r},\vec{r}\,') \, |\psi(r')|^2 d^3r'\right]^2 = 0 , \tag{31}$$

where E_{self} is given by (20) and $G(\vec{r},\vec{r}\,')$ by (14). The term $4\alpha_s/(3\sqrt{\pi}a)$ is the quark mass renormalization. The energy of the nucleon is

$$\mathcal{E} = 3\epsilon - 2g_s^2 \int |\psi(\vec{r})|^2 G(\vec{r},\vec{r}\,')|\psi(\vec{r}\,')|^2 f(|\vec{r} - \vec{r}\,'|) d^3r d^3r' . \tag{32}$$

We here present the results of a simpler calculation, using the asymptotic for the self-energy, i.e. we solve

$$[\vec{\alpha} \cdot \vec{p} + \beta(m + g_{eff}(\sigma)) - \epsilon]\psi = 0 \tag{33}$$

$$-\nabla^2\sigma + U'(\sigma) + 3g'_{eff}(\sigma)\bar{\psi}\psi = 0 \tag{34}$$

with $m = 0$ and $\kappa(\sigma)$ as given in (2) with $n = 3$. For

$$U(\sigma) = a\frac{\sigma^2}{2} + b\frac{\sigma^3}{6} + c\frac{\sigma^4}{24} + B\,, \tag{35}$$

the parameter values $a = 7.212$ fm^{-2}, $b = -805.64$ fm^{-1}, $c = 10,000.$ and $B = 0.2793$ fm^{-4} = 55 MeV/fm^3. have been used [20]. This gives $\sigma_v = 0.2222$ fm^{-1} for the vacuum value of the σ-field.

Fig. 4 illustrates the properties of the selfconsistent bag states with the coupling (27). The upper and lower components ($u(r)$ and $v(r)$) of the quark wave function of the lowest energy are plotted together with the sigma field and the effective coupling g_{eff}. Observe the rapid rise of the potential. The corresponding eigenvalue is $\epsilon = 0.36$ GeV, while the total energy of the three-quark system is 1.46 GeV. (This value does not include the recoil correction discussed in Ref. [21].) The glueball mass [2] is $\sqrt{U''(\sigma_v)} = 1.71$ GeV.

We have shown in this note how confinement is brought about by the self-energy of a quark in a chromo-dielectric medium. Upon introducing a form factor to regularize the short distance behavior, the result can be formulated in terms of an absolute confinement potential. (In QCD, asymptotic freedom guarantees that there is no need to introduce a short-distance cutoff.) We wish to emphasize that the g_{eff} calculated here is to be regarded as a useful approximation for spherical, color-singlet systems. More generally, for nonspherical systems, and to make color-confinement explicit, one must use the self-consistent propagator, as described in Ref. [11] to calculate E_{self} according to Eq. (13), or with another form factor.

The actual calculation of the self-energy was carried out in the one-loop approximation, in the limit of fixed quarks. The computational problem is substantially more difficult for light quarks, and is being pursued by Tang and Williams [22]. However, qualitatively similar results are expected on the basis of a comparison to the results of calculations for light quarks in the MIT bag model [16,17]. Our model Lagrangian contains no direct quark-sigma coupling, and is chirally invariant for massless quarks. Chiral invariance is broken by the effective coupling, and should be restored, in a full theory, by a Goldstone boson. The question of how such a boson is dynamically generated in the model, is left for further investigation.

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Figure Captions

Fig. 1: (a) The quark self-energy diagram in the one-loop approximation. (b) The quark self-energy diagram in the one-loop approximation for a fixed (infinitely massive) quark.

Fig. 2: The normalized modified spherical Bessel function as a function of the order for several values of the argument.

Fig. 3: The regularized self-energy (with the value at the origin subtracted) for several choices of the width of the form factor and the asymptotic effective coupling (solid line). The chromo-dielectric function is also shown (short dashes, right scale). The self-energy for $a = 0.125$ fm was obtained by shifting the result calculated with $R = 0.5$ fm to assure convergence.

Fig. 4: Properties of the selfconsistent bag states with the asymptotic effective coupling. The upper (u) and lower (v) components of the quark wave function and the effective coupling (g_{eff}, right scale) are plotted. The shape of the σ-field is indicated for completeness (arbitrary units).

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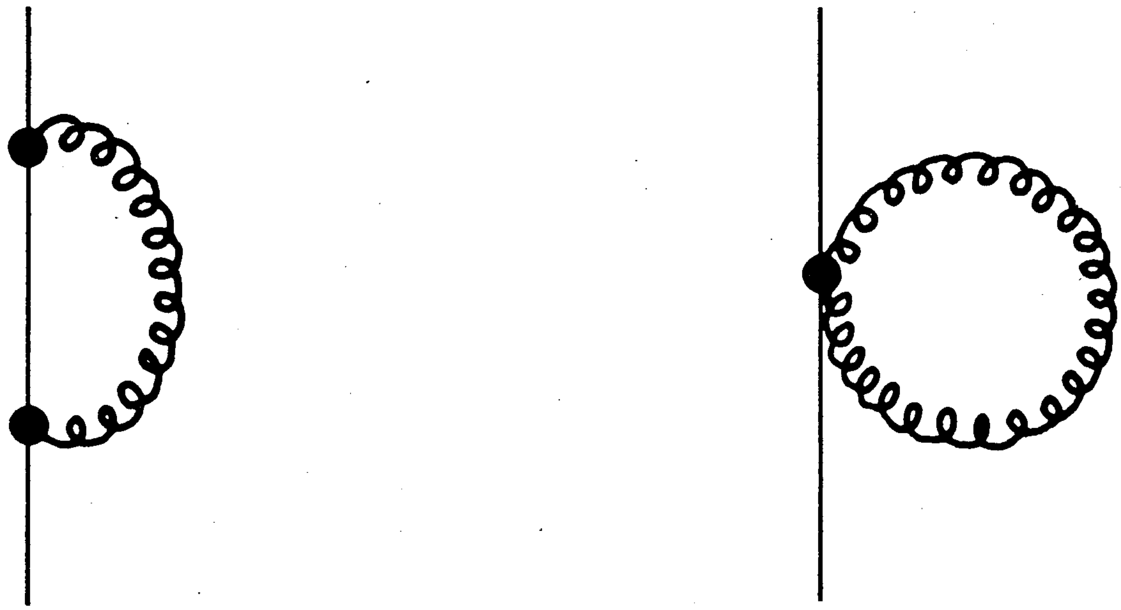
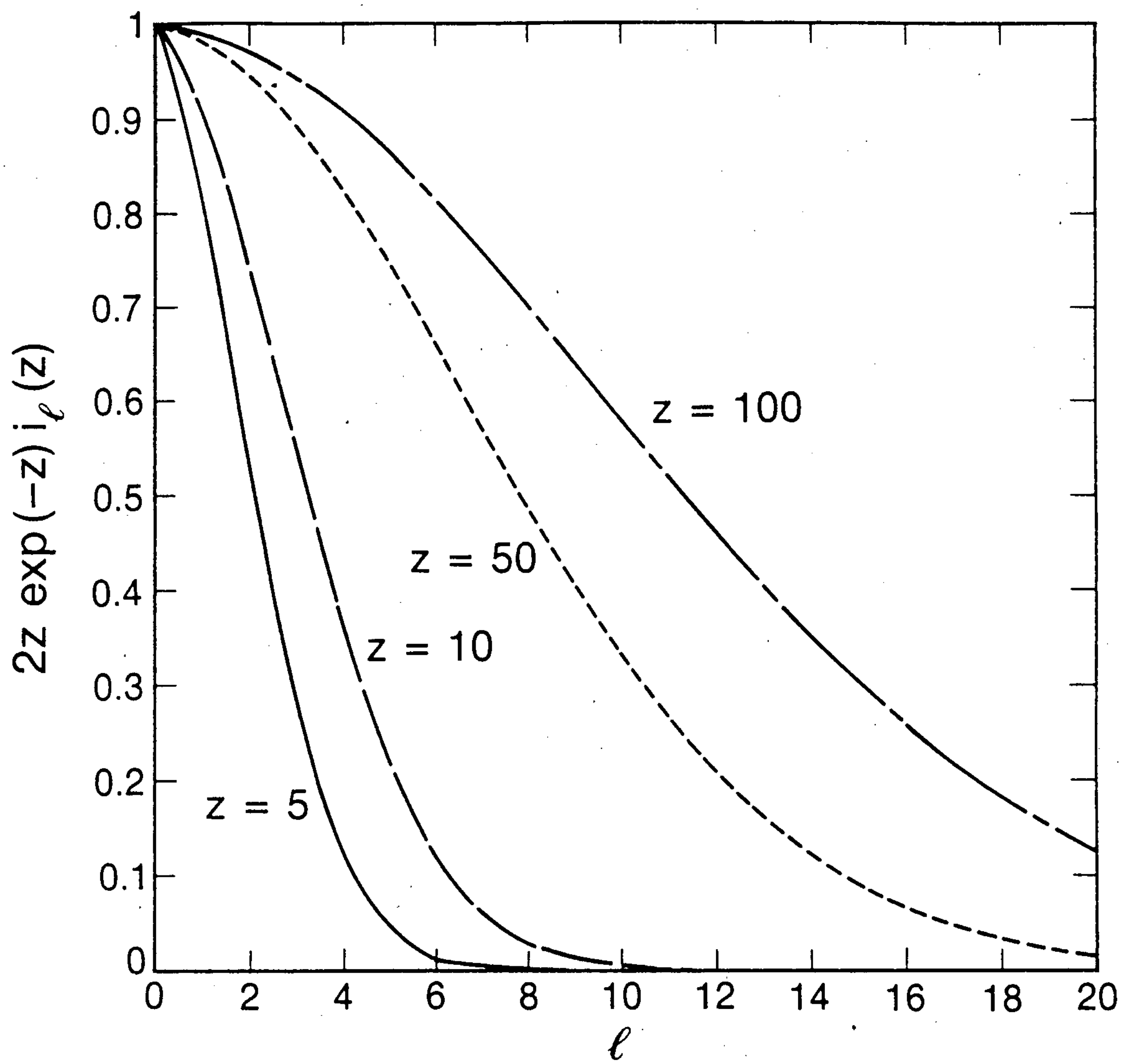


(a) (b)

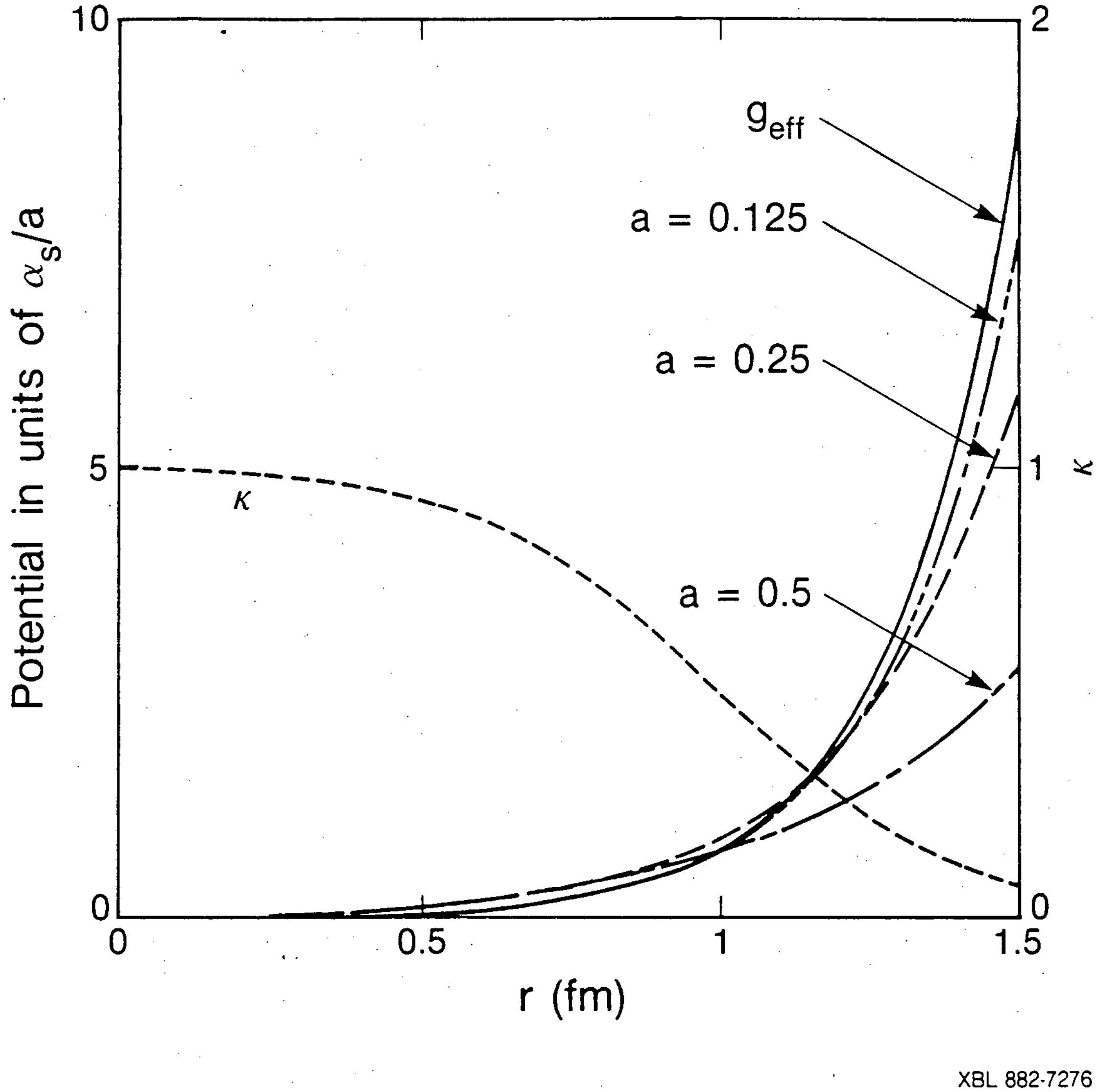
XBL 882-7274

Fig. 1



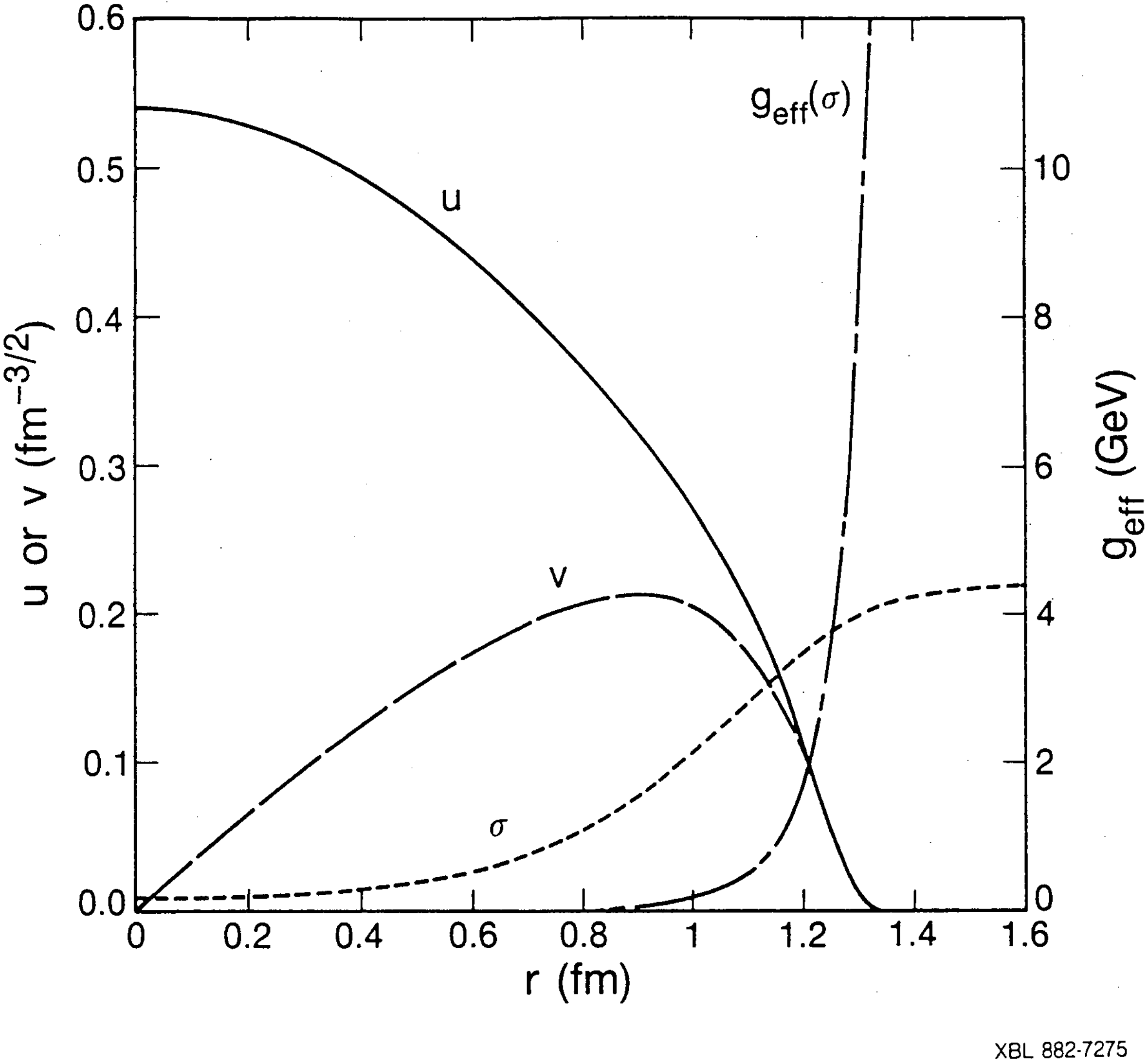
XBL 882-7277

Fig. 2



Potential in units of α_s/a
10
5
0
0
0.5
1
1.5
r (fm)
2
1
0
κ
g_{eff}
a = 0.125
a = 0.25
a = 0.5
κ
XBL 882-7276

Fig. 3



u or v (fm$^{-3/2}$)
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$g_{eff}(\sigma)$
u
v
σ
10
8
6
4
2
0
g_{eff} (GeV)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
r (fm)
XBL 882-7275

Fig. 4

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