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Transverse Mis-Alignments in a Driver

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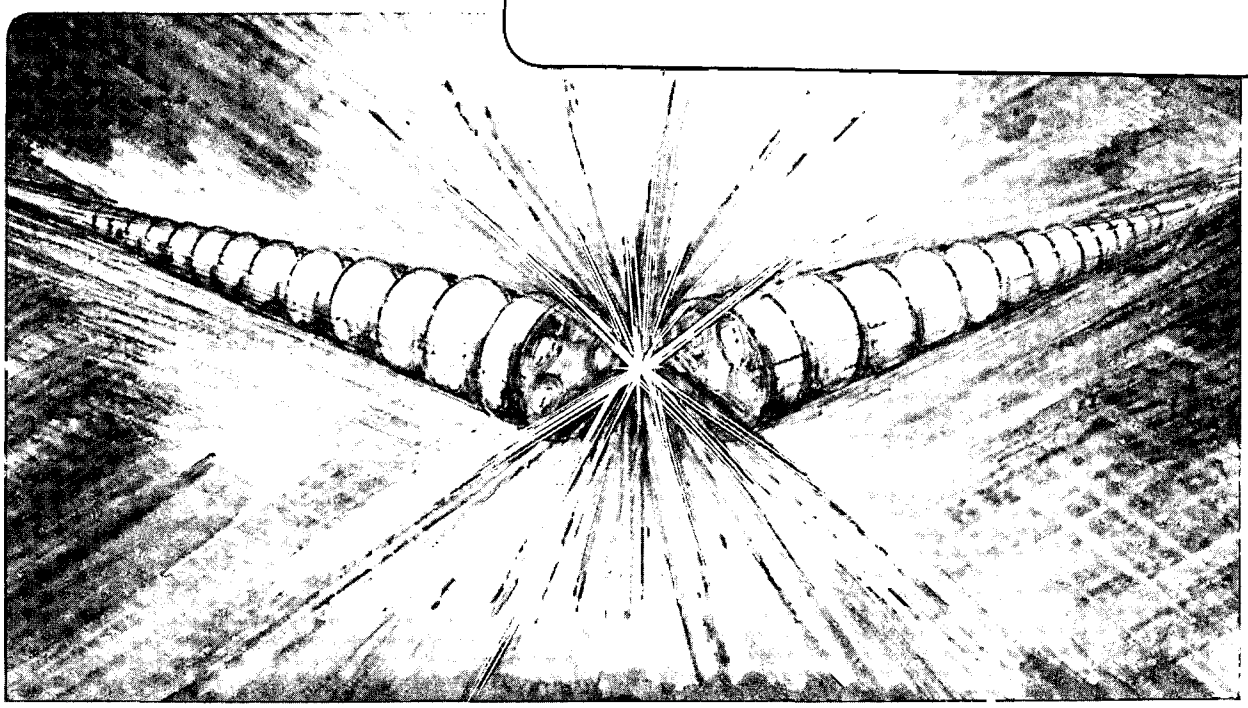

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Transverse Mis-Alignments in a Driver

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Transverse mis-alignments in a driver†

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The transverse errors of the beam lines are usually corrected by an appropriate feed back to bring the beam back on axis. In an induction linac, however, the head and tail of the bunch differ substantially in momentum at a given lens location. As a result, the correction has to be time dependent. Such a correction becomes increasingly difficult as the beam energy increases and the time duration of the bunch decreases. As a step towards an understanding of the problem, we have analyzed the extreme case of applying no correction. Since the lattice configuration changes and the transverse oscillations are damped as the ions are accelerated, the rms amplitude does not increase simply as the square root of the number of periods, as one would expect for constant velocity in a uniform channel.

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1. Introduction

The focusing system for a driver is taken to be a simple FODO lattice in which cell length, lens strength, and filling factor (fraction of cell length occupied by lenses) are determined as a function of kinetic energy by a cost optimization procedure for the entire system. The effect of transverse mis-alignment of a quadrupole is to excite a coherent oscillation of the beam as a whole; the motion of the centroid is not affected by the space charge forces except for a small correction to the oscillation frequency due to image forces, which is neglected in this paper. Thus the beam dynamical problem can be treated as the motion of a single particle through a randomly mis-aligned system.

2. Calculation of the amplitude after N periods

After traversing a mis-aligned focussing element of strength k with a lateral displacement δ_+, the central orbit acquires an additional displacement and velocity given by*

$$\mathbf{x}_a = \begin{pmatrix} x \\ \dot{x} \end{pmatrix} = \delta_+ \begin{pmatrix} 1-\cos 2\theta \\ \sqrt{k}\,\sin 2\theta \end{pmatrix} \tag{1}$$

where $\theta = \sqrt{k}\,\eta\,\frac{T}{4}$ with η the filling factor and T the time taken to traverse a complete cell.

The resulting oscillation is the same as though the oscillation started in the middle of the lens with a displacement and velocity given by operating on $\mathbf{x}_a$ with the inverse of the matrix representing a half-lens:

$$\mathbf{x}_+ = \begin{pmatrix} \cos\theta & -\frac{\sin\theta}{\sqrt{k}} \\ \sqrt{k}\,\sin\theta & \cos\theta \end{pmatrix} \mathbf{x}_a = 2\,\delta_+ \begin{pmatrix} 0 \\ \sqrt{k}\,\sin\theta \end{pmatrix} \tag{2}$$

Similarly, for a defocussing lens,

$$\mathbf{x}_- = -\,2\,\delta_- \begin{pmatrix} 0 \\ \sqrt{k}\,\sinh\theta \end{pmatrix} \tag{3}$$

The contribution of the nth cell to the subsequent oscillation is then given by:

$$x_n(t) = 2\sqrt{\beta(t)}\sqrt{k_n}\,\left[\delta_{+n}\sqrt{\beta_{+n}}\,\sin\theta_n\,\sin(\psi-\psi_n) - \delta_{-n}\sqrt{\beta_{-n}}\,\sinh\theta_n\,\sin(\psi-\psi_n+\frac{\sigma_n}{2})\right] \tag{4}$$

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* We use time as the independent variable instead of the customary distance in order to extract the velocity dependence in equation (7) more easily.

where β is the Courant-Snyder β-function [1] and:

$\beta_{\pm n}=\beta_{\substack{max\\min}}$ in the nth cell.

σ_n= zero intensity phase advance, σ_0, in the nth cell.

$$\psi-\psi_n=\int_{t_n}^{t}\frac{dt}{\beta}$$

t_n= time of arrival at the center of the nth focussing element.

At the center of the Nth focussing element, the displacement and velocity due to mis-alignment of lenses are given by the sums of the individual contributions of the preceding cells:

$$X_N = 2\sqrt{\beta_{+N}}\sum_{n=1}^{N}\sqrt{k_n}\,[\delta_{+n}\sqrt{\beta_{+n}}\,\sin\theta_n\,\sin(\psi-\psi_n) - \delta_{-n}\sqrt{\beta_{-n}}\,\sinh\theta_n\,\sin(\psi-\psi_n+\tfrac{\sigma_n}{2})]$$

$$\dot{X}_N = \frac{2}{\sqrt{\beta_{+N}}}\sum_{n=1}^{N}\sqrt{k_n}\,[\delta_{+n}\sqrt{\beta_{+n}}\,\sin\theta_n\,\cos(\psi-\psi_n) - \delta_{-n}\sqrt{\beta_{-n}}\,\sinh\theta_n\,\cos(\psi-\psi_n+\tfrac{\sigma_n}{2})]$$

The final amplitude square is defined as follows as a convenient measure of the error:

$$A^2 = X_N^2 + \beta_N^2\dot{X}_N^2 \tag{5}$$

Assuming errors in different lenses are independent and are randomly distributed, the ensemble average of final amplitude A is given as,

$$\overline{A^2} = 64\,\beta_N\sum_{n=1}^{N}\frac{\theta_n^4}{\eta_n^2\,T_n^2}\,(\beta_{+n}+\beta_{-n})\,\overline{\delta^2} \tag{6}$$

where only the leading term in $\theta_n = \frac{1}{4}\sqrt{k_n}\,\eta_n\,T_n << 1$ is used.

It is more convenient to use σ_0, cell length, L, and ion velocity, V instead of β, T, and θ. To a good approximation for $\sigma_0 < 90^o$ ($\theta<1$), the relations between these parameters are given as follows:

$$\theta^4 = \frac{3\eta^2}{4(3-2\eta)}\sin^2\frac{\sigma_0}{2}$$

$$\beta_\pm = \frac{L}{V\sin\sigma_0}\left[\,1 \pm (2-\eta)\sqrt{\frac{3}{4(3-2\eta)}}\sin\frac{\sigma_0}{2}\right]$$

Thus the final amplitude can be expressed by

$$\overline{A^2} = [48\,\overline{\delta^2}]\,\frac{1+(2-\eta_N)\sqrt{\frac{3}{4(3-2\eta_N)}}\sin\frac{\sigma_N}{2}}{\sin\sigma_N}\sum_{n=1}^{N}\frac{\tan\frac{\sigma_n}{2}}{(3-2\eta_n)}\left(\frac{V_n}{V_N}\right)\left(\frac{L_N}{L_n}\right) \qquad (7)$$

Note the explicit adiabatic damping term $(\frac{V_n}{V_N})$; the contribution of a transverse error to $\overline{A^2}$ is reduced accordingly as the particle speed is raised.

3. Model driver [2]

The lowest cost drivers for charge state 3 are typically divided into a low energy section and a high energy section. In the low energy section, starting at 9 MeV, the accelerating gradient increases as the cube of the velocity to an assumed maximum achievable value of 1 MV/meter at a kinetic energy of 180 MeV. The cell length is one meter, η=0.5 and σ_0=72° throughout. Thereafter, the accelerating gradient is constant but the cell length increases according to: $L=L_{180}\dfrac{\frac{w}{180}}{2\sqrt{\frac{w}{180}}-1}$

where w is the kinetic energy of the ion(MeV).

Again, σ_0=72° and we take η=0.25. Figure 1 shows the variation of these quantities along the accelerator. There are approximately 400 periods in the low energy section and 1200 in the high energy section. In the low energy section, the head-tail velocity difference is $(\frac{\Delta V}{V})$=0.3, which causes the bunch to shorten and incidentally causes the coherent oscillations to get rapidly out of phase. In the high energy section, the residual $(\frac{\Delta V}{V})$ causes errors accumulated at less than 5 GeV to differ in phase by more than 90° at 10 GeV. The summation in equation (7) can be well approximated by integrals, leading to the result shown in figure 2. In practise, the mean square error is not very useful because the alignment procedure can only guarantee that the individual

errors are less than a certain value and the distribution of errors within that limit is unknown. We have assumed that the distribution is uniform between $\pm\Delta_m$, leading to $\Delta_m^2=3\overline{\delta^2}$, and plot the ratio, $\frac{A_{rms}}{\Delta_m}$. It is interesting to note that the ratio actually decreases around the end of the low energy section. This occurs because the rapidly increasing velocity damps the earlier contributions faster than the later ones accumulate.

4. Conclusion

Consideration of the final focusing system suggests a tolerable coherent amplitude of ~1-2 mm. With present alignment techniques, corrections along the way would be required. However, the technology of precision alignment is improving rapidly and it is conceivable that a precision of $(\frac{1mm}{100})=10\ \mu m$, at acceptable cost, could be available by the time a real driver might be built. Because of the beneficial effect of damping, this tolerance could be relaxed at the low energy end if the aperture is adequate.

References

[1] E.D. Courant and H.S. Snyder, *Annals of Physics*, **3**, 1 (1958).

[2] E.P. Lee, private communication

Figure captions

Figure 1. Accelerating gradient, ion energy and cell length versus z. The broken line represents the boundary between the low and high energy sections.

Figure 2. The ratio of central orbit error amplitude (A_{rms}) and maximum displacement of the randomly distributed mis-aligned lenses ($\Delta_m=\sqrt{3}\ \delta_{rms}$) versus energy.

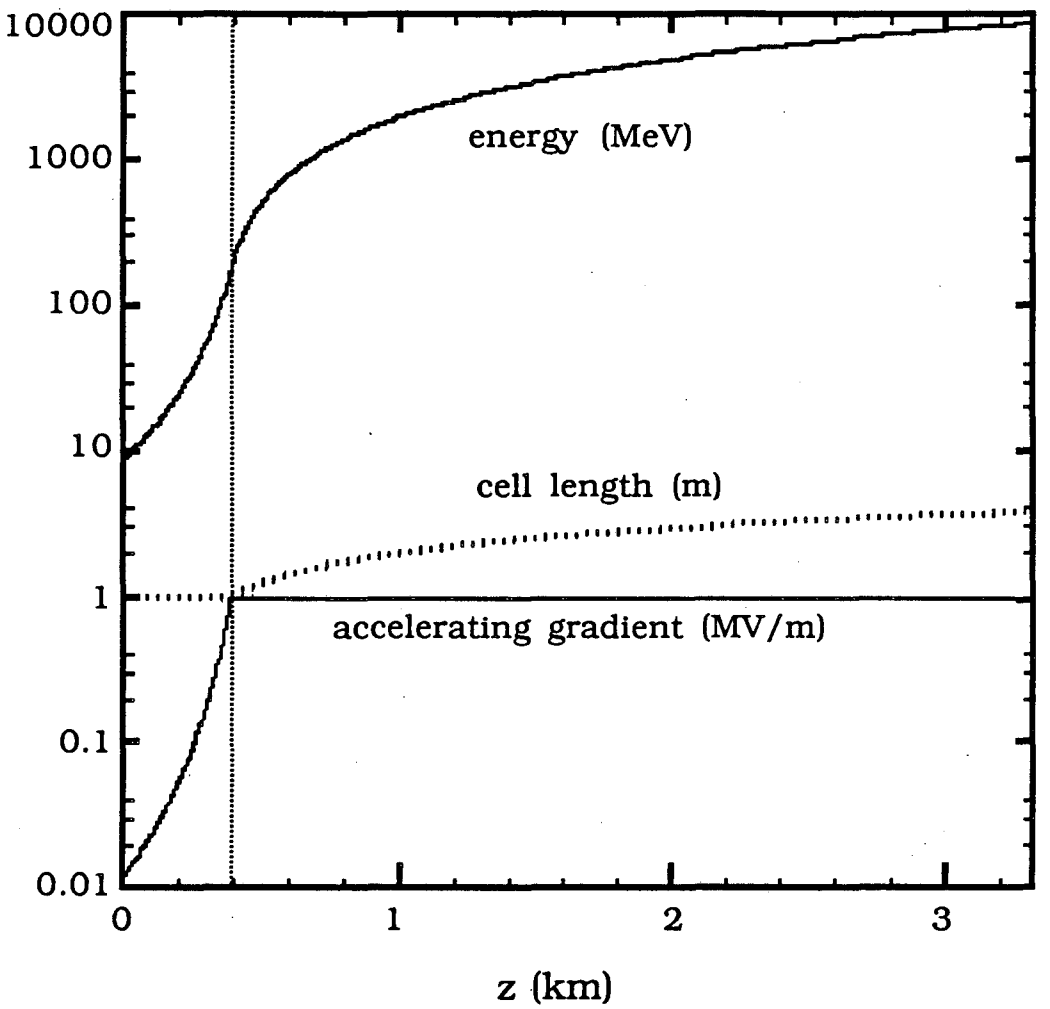
10000
1000
100
10
1
0.1
0.01
energy (MeV)
cell length (m)
accelerating gradient (MV/m)
0
1
2
3
z (km)

Figure 1

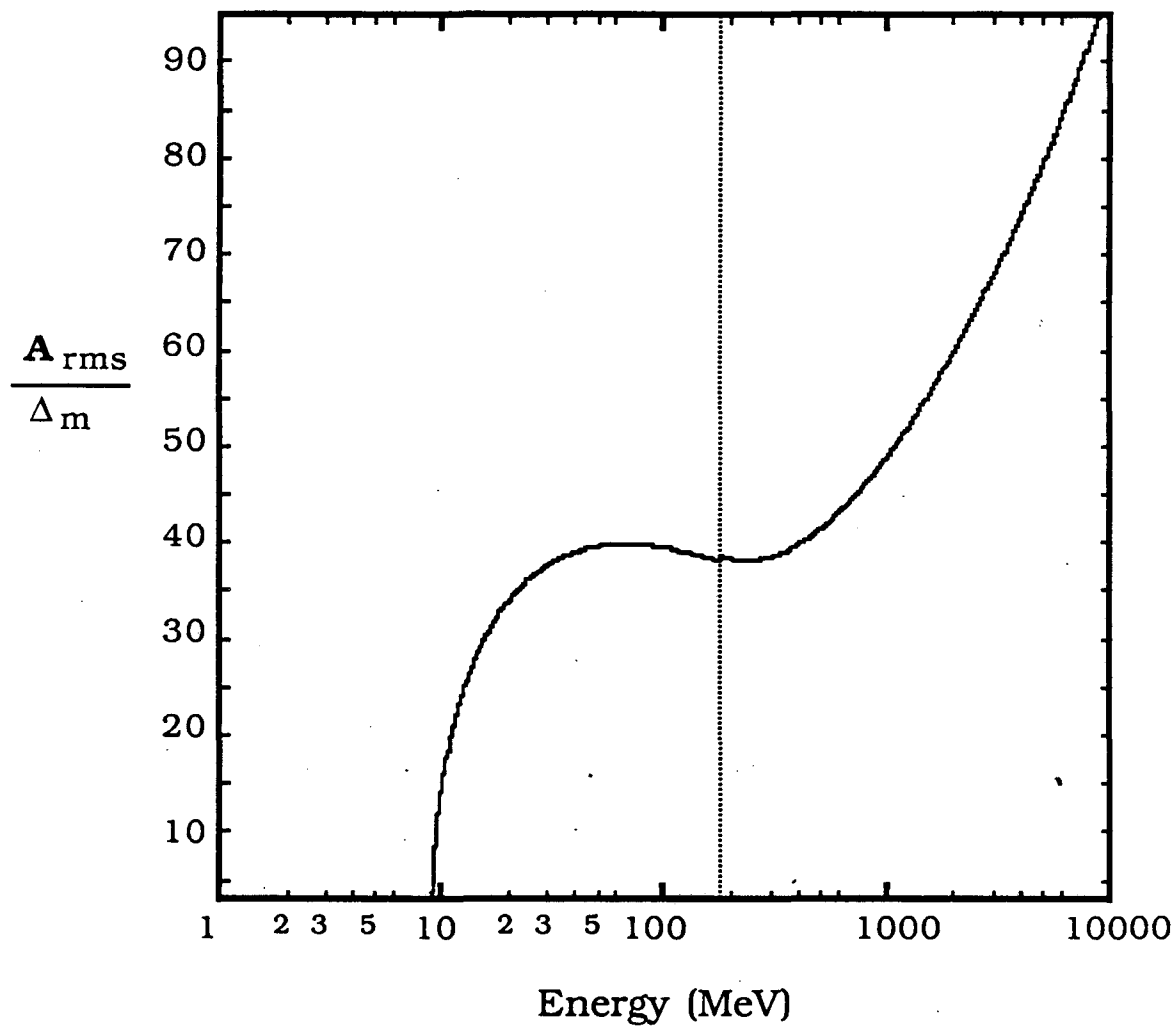
90
80
70
60
50
40
30
20
10
A rms / Δ m
1
2
3
5
10
2
3
5
100
1000
10000
Energy (MeV)

Figure 2

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