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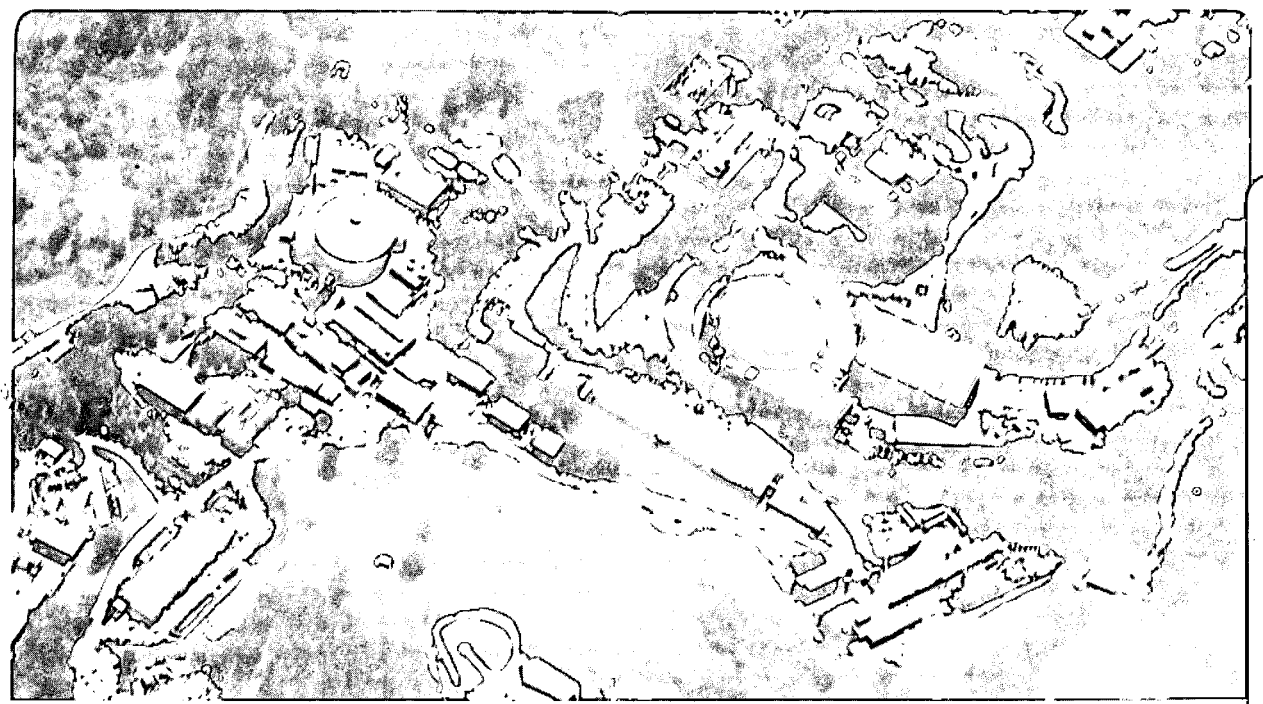

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TWOS IN TWO PHOTON PHYSICS: A CONVENTION FOR THE $\gamma\gamma^*$ WIDTH OF A SPIN-ONE PARTICLE *

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Abstract

The two conventions for the $\gamma\gamma^*$ width of of a spin-one resonance are discussed. It is shown that the more reasonable one is the one that gives the larger experimental value.

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The cross section to produce a spin-zero C-even particle or resonance in e^+e^- collisions generated by electron and positron beams of energy E is related to the $\gamma\gamma$ width of the particle by Low's formula[1]

$$\sigma_{e^+e^-\to Re^+e^-} = \eta^2 \frac{8\pi^2\Gamma(R\to\gamma\gamma)}{m_R s} f\left(\frac{m_R^2}{s}\right) \tag{1}$$

where

$$f(\omega) = \frac{1}{\omega}\left[(2+\omega)^2 \ln\frac{1}{\omega} - 2(1-\omega)(3+\omega)\right] \tag{2}$$

and $\eta = (\alpha/\pi)\ln(2E/m_e)$. (Of course this is only an approximation, especially because it does not include suppression by form factors when the Q^2 of the virtual photons is large.) Because the bulk of the "two photon" events have both photons near mass shell, there is a direct connection between the production rate in the e^+e^- process and the actual decay width of the resonance into $\gamma\gamma$.

The situation for spin-one resonances is quite different. Yang's theorem[2] forbids the decay of a spin-one particle into two real photons. On the other hand, the decay into one real and one virtual photon is allowed. The virtual photon might materialize as an e^+e^- pair. Of course, as a consequence of Yang's theorem we cannot write an effective Lagrangian for $V\to\gamma\gamma$. We can write, for example, if V has $J^{PC} = 1^{++}$

$$\mathcal{L}_{eff} = g\epsilon_{\alpha\beta\gamma\delta}V^\alpha F^{\beta\gamma}\Box A^\delta \tag{3}$$

where by $\Box A^\delta$ we mean j^δ, the e.m. current, which includes terms like

$$j^\delta = e\overline{\psi}\gamma^\delta\psi. \tag{4}$$

This effective Lagrangian would allow us to calculate both production of the spin-one particle through the e^+e^- process and its decay into one real and one longitudinal photon or an electron-positron pair.

There has been some controversy about the normalization to be used for the $\gamma\gamma^*$ width of a spin-one particle as measured in e^+e^- collisions. This width occurs in the formula for the cross section for $e^+e^-\to e^+e^-V$ and different groups use formulas that differ by a factor of two. (In addition, they differ by terms of order Q^2/m_R^2, where Q^2 is the mass of the virtual photon and m_R is the mass of the resonance. This difference is not addressed here.)

In fact, the factor of two in dispute for the production of spin-one particles in e^+e^- collisions comes from a very simple consideration that has nothing to do with spin *per se*. Let us suppose we have a flux of scalar particles a from the left and a flux from the right of b particles. We may suppose each beam to be monoenergetic. The a and b scatter resonantly, through a scalar resonance c that decays to d and e. If the coupling constants are g_{abc} and g_{dec} (so the couplings are $g_{abc}\phi_a\phi_b\phi_c$ and $g_{dec}\phi_d\phi_e\phi_c$) the scattering amplitude is

$$\mathcal{M} = \frac{g_{abc}g_{dec}}{(s - m_c^2) + i\Gamma m_c}. \tag{5}$$

The partial decay rates are

$$\Gamma(c \to ab) = \frac{g_{abc}^2}{8\pi}\,\frac{p_{cm}^a}{m_c^2}, \quad \Gamma(c \to de) = \frac{g_{dec}^2}{8\pi}\,\frac{p_{cm}^d}{m_c^2}. \tag{6}$$

The cross section is

$$d\sigma_{ab\to de} = \frac{(2\pi)^4\delta^4(p_a + p_b - p_d - p_e)}{4\sqrt{(p_a\cdot p_b)^2 - m_a^2 m_b^2}}\,\frac{d^3p_d}{(2\pi)^3 2E_d}\,\frac{d^3p_e}{(2\pi)^3 2E_e}$$
$$\times\,\frac{g_{abc}^2 g_{dec}^2}{(s - m_c^2)^2 + \Gamma^2 m_c^2} = A\,g_{abc}^2. \tag{7}$$

We can eliminate g_{abc} through $g_{abc}^2 = 8\pi m_c^2\,\Gamma(c \to ab)/p_{cm}^a$,

$$d\sigma = A\frac{8\pi m_c^2}{p_{cm}^a}\,\Gamma(c \to ab). \tag{8}$$

Now suppose a and b are in fact the same spin-0 boson and write the coupling as g_{aac} (*i.e.* field theoretically the coupling is $g_{aac}\phi_a^2\phi_c/2$ since there are two ways to couple the ϕ_a field to the particles). The only thing that changes is that

$$\Gamma(c \to aa) = \frac{g_{aac}^2}{16\pi}\,\frac{p_{cm}^a}{m_c^2} \tag{9}$$

because we must integrate only over 2π not 4π solid angle of the final state lest we double-count. Thus

$$d\sigma_{aa\to de} = A\frac{8\pi m_c^2}{p_{cm}^a}\,2\Gamma(c \to aa). \tag{10}$$

This slippery factor of two actually turns up in the ordinary derivation of Low's formula. The flux of (transverse) photons from an electron beam is approximately

$$\frac{dN}{dx} = \eta \frac{1+(1-x)^2}{x} \tag{11}$$

so the resonance production cross section is

$$\sigma = \eta^2 \int dx_1\, dx_2 \frac{1+(1-x_1)^2}{x_1} \frac{1+(1-x_2)^2}{x_2} \hat{\sigma}(x_1 x_2 s) \tag{12}$$

where $\hat{\sigma}(\hat{s})$ is the resonant cross section evaluated at $\hat{s}$. Now the proper form of the Breit-Wigner formula is

$$\hat{\sigma}_{TT} = \frac{4\pi}{k^2} \frac{2J+1}{(2S_1+1)(2S_2+1)} \frac{2\Gamma(R \to \gamma\gamma)\Gamma m_R^2}{(\hat{s}-m_R^2)^2 + \Gamma^2 m_R^2} \tag{13}$$

Here an explicit factor of two multiplies the formula that would be proper for scattering of distinguishable particles, just as the factor of two must occur in Eq. (10) relative to the expression in Eq. (8). Now if the resonance is narrow,

$$\frac{\Gamma m_R^2}{(\hat{s}-m_R^2)^2 + \Gamma^2 m_R^2} \simeq \pi m_R \delta(\hat{s} - m_R^2) \tag{14}$$

so

$$\hat{\sigma}_{TT} \simeq \frac{8\pi^2 m_R}{k^2} \frac{2J+1}{(2S_1+1)(2S_2+1)} \Gamma(R \to \gamma\gamma) \delta(\hat{s} - m_R^2) \tag{15}$$

and

$$\begin{aligned} \sigma_{e^+e^- \to e^+e^- R} &\simeq \eta^2 \frac{32\pi^2}{m_R} \frac{2J+1}{4} \Gamma(R \to \gamma\gamma) \\ &\times \int dx_1\, dx_1 \frac{1+(1-x_1)^2}{x_1} \frac{1+(1-x_2)^2}{x_2} \delta(s x_1 x_2 - m_R^2) \\ &\approx \eta^2 \frac{8\pi^2(2J+1)}{m_R^3} \Gamma(R \to \gamma\gamma) \left[(2+\tau)^2 \ln \frac{1}{\tau} - 2(1-\tau)(3+\tau) \right] \end{aligned} \tag{16}$$

with $\tau = m_R^2/s$. Had we failed to put the factor of two in Eq. (13) we would not have gotten Low's formula, Eq. (1).

How does this reflect on the question of spin-one resonances? The formula for the cross section to produce them via one longitudinal and one transverse

photon differs from that for production of a spin $J \neq 1$ resonance by two transverse photons by a factor two quite aside from the $(2J+1)/[(2S_1+1)(2S_2+1)]$ statistical factor. This is a consequence of the identical particles in the transverse-transverse state. For the transverse-longitudinal cross section we write [3]

$$\begin{aligned}\sigma_{TL}(\hat{s}) &\approx \frac{4\pi^2}{k^2}\,\frac{2J+1}{(2S_1+1)(2S_2+1)}\,\Gamma_{TL} m_R \delta(\hat{s}-m_R^2) \\ &\approx \frac{16\pi^2}{m_R}\,\frac{3}{2\cdot 1}\,\Gamma_{TL}\delta(\hat{s}-m_R^2) \qquad (17)\end{aligned}$$

This gives for the cross section

$$\begin{aligned}d\sigma_{e^+e^-\to e^+e^-R} &= dn_{T1}dn_{L2}\sigma_{TL} \\ &\cong \frac{dx_1}{x_1}\,\frac{dk_1^2}{k_1^2}\,\frac{dx_2}{x_2}\,\frac{dk_2^2}{k_2^2}\left(\frac{\alpha}{\pi}\right)^2\frac{16\pi^2}{m_R}\,\delta(sx_1x_2-m_R^2) \\ &\times \frac{3}{2}\,\Gamma_{TL}(1-x_2)(1-x_1+x_1^2/4). \qquad (18)\end{aligned}$$

This formula is the one used by the Mark II Collaboration [4] and differs from the one used by the TPC/Two Photon Collaboration [5]. I believe Eq. (18) is the more sensible choice. The factor of two simply reflects the non-identical nature of the longitudinal and transverse photons.

We interpret the production of resonances in $e^+e^- \to e^+e^-R$ in terms of physical decay widths. When it is the decay into two real, transverse photons that is extracted it is necessary to take into account the identical particle nature of those photons. When it is the decay into one transverse and one longitudinal photon that is extracted it is logical to treat them as not identical. It is this that is responsible for the factor of two difference relating the observed cross section to the inferred width. It is the nature of the photons in the real or imagined decay of the resonance that counts, not the nature of the virtual photons in the e^+e^- production process.

One other minor detail should be mentioned. If for the spin-1 decay we take as the amplitude[3]

$$\mathcal{M} = \mathcal{A}(k_1^2,k_2^2)\epsilon_{\alpha\beta\gamma\delta}(k_2^2k_1^\alpha - k_1^2k_2^\alpha)\xi^\beta\epsilon_1^\gamma\epsilon_2^\delta \qquad (19)$$

where k_1 and ϵ_1 are the momentum and polarization vector of photon 1, *mutatis mutandis* for photon 2, and ξ is the polarization vector of the decaying spin-one particle, then for photon 1 real, with the magnitude of the photon three-momentum in the decay rest frame being k,

$$\begin{aligned}\Gamma_{T_1L_2} &= \frac{k^3\,|k_2^2|\,\mathcal{A}(0,k_2^2)^2}{12\pi} \\ \Gamma_{T_1T_2} &= \frac{k^3\,|k_2^2|\,\mathcal{A}(0,k_2^2)^2}{12\pi}\,\frac{|k_2^2|}{m^2}\end{aligned} \tag{20}$$

where we have treated the two photons as distinguishable *in both cases.* This is reasonable because there is no limit to consider in which the two transverse photons both become real and thus identical. The decay width vanishes in that limit. Now because we treated them as distinguishable, we write

$$\begin{aligned}\sigma_{T_1T_2}(\hat{s}) &\approx \frac{4\pi^2}{k^2}\,\frac{2J+1}{(2S_1+1)(2S_2+1)}\,\Gamma_{T_1T_2}m_R\delta(\hat{s}-m_R^2) \\ &\approx \frac{16\pi^2}{m_R}\,\frac{3}{4}\,\Gamma_{TT}\delta(\hat{s}-m_R^2)\end{aligned} \tag{21}$$

and

$$\begin{aligned}d\sigma_{e^+e^-\to e^+e^-R} &= dn_{T1}dn_{L2}\sigma_{T_1L_2} + dn_{T1}dn_{T2}\sigma_{T_1T_2} \\ &\cong \frac{dx_1}{x_1}\,\frac{dk_1^2}{k_1^2}\,\frac{dx_2}{x_2}\,\frac{dk_2^2}{k_2^2}\,\left(\frac{\alpha}{\pi}\right)^2\,\frac{16\pi^2}{m_R}\,\delta(sx_1x_2-m_R^2) \\ &\quad\times\left[\frac{3}{2}\,\Gamma_{T_1L_2}(1-x_2)+\frac{3}{4}\,\Gamma_{T_1T_2}(1-x_2+x_2^2/2)\right] \\ &\quad\times(1-x_1+x_1^2/4)\end{aligned} \tag{22}$$

Using Eq. (20) we can eliminate $\Gamma_{T_1T_2}$ to obtain

$$\begin{aligned}d\sigma_{e^+e^-\to e^+e^-R} &\cong \frac{dx_1}{x_1}\,\frac{dk_1^2}{k_1^2}\,\frac{dx_2}{x_2}\,\frac{dk_2^2}{k_2^2}\,\left(\frac{\alpha}{\pi}\right)^2\,\frac{16\pi^2\Gamma_{T_1L_2}}{m_R}\,\delta(sx_1x_2-m_R^2) \\ &\quad\times\left[\frac{3}{2}\,(1-x_2)+\frac{3}{4}\,\frac{|k_2^2|}{M^2}(1-x_2+x_2^2/2)\right] \\ &\quad\times(1-x_1+x_1^2/4)\end{aligned} \tag{23}$$

Now the convention for $\Gamma_{T_1T_2}$ has disappeared. We needed only to be consistent in treating the two transverse photons here as distinguishable in all of Eqs. (19)–(21).

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