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1/f Noise from the Diffusion of Particles or Heat*

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ABSTRACT

1/f-like number fluctuations of particles in Brownian motion, observed by light scattering, and temperature fluctuations in thin metal films, observed as 1/f noise in resistance, agree quantitatively with the theory for diffusive systems. The characteristic frequency dependent correlation length is observed in the metal films.

In the previous letter,[1] we showed that fluctuations in a diffusive medium lead to a 1/f-like frequency spectrum. In this paper we show that the theory gives quantitative predictions in good agreement with experiment. We first present results for a light scattering experiment on particles in Brownian motion, and then show that 1/f noise in metal films can be explained by the modulation of resistivity by temperature fluctuations.

Light scattering is a common experimental method of observing Brownian motion. Using a monochromatic source, one can determine the diffusion constant, D, from the fluctuation spectrum of the intensity of the side scattered light.[2] These fluctuations are a two-particle interference effect: as any two particles move relative to each other, the relative phases of their electric fields at the detector vary, causing a change in intensity. The spectrum is a Lorentzian and is independent of frequency, f, for $f \ll DK^2/\pi$, where K is the amplitude of the scattering vector for the light. If the incident beam illuminates only a subvolume of the sample cell, the number of particles in the beam, N, fluctuates, with a predicted 1/f-like spectrum.[1] One should be able to observe this effect in the side scattered light at low enough frequencies where the 1/f-like behavior dominates the Lorentzian. However, it is simpler to observe the effect in the forward direction where the 1/f-like behavior dominates the entire spectrum. Here, to lowest order, movement of a particle does not change the phase of the scattered light at the detector, and the interference terms vanish.

Consider a beam of light of intensity I_o illuminating a small area, A, of a sample cell of thickness ℓ. If σ represents the cross section for scattering light out of the beam for each particle, the transmitted intensity is $I = I_o \exp(-N\sigma/A)$. For a small fluctuation, ΔN, in N, the fluctuation in intensity is $\Delta I = -I\sigma\Delta N/A = -[\ell n(I_o/I)]I\ \Delta N/N$, so that

$$S_I(f) = [\ell n(I_o/I)]^2 I^2\ S_N(f)/N^2, \qquad (1)$$

where $S_I(f) \equiv \langle \Delta I(f)\Delta I^*(f)\rangle$ and $S_N(f)$ is the 1/f-like spectrum for fluctuations in N.

With a photomultiplier, we measured the intensity of light from an incandescent bulb transmitted through a sample cell of thickness $\ell = 0.2$ cm. A circular aperture of diameter d $\sim 100\mu$ limited the beam area. The cell contained 0.34μ diameter polystyrene balls suspended in water ($D \approx 10^{-7}$ $cm^2 sec^{-1}$). The natural frequency of the system was $f_o = D/\pi d^2 \approx 3\times10^{-4}$ Hz . We used a PDP-11 computer to measure the autocorrelation function for the intensity, and by calculating its transform, to determine the spectrum in the range 4×10^{-2} Hz to 10 Hz. $S_I(f)/I^2$ for two samples with different concentrations and apertures is shown dotted in Fig. 1. The values of I_o and fluctuations in I_o (which were orders of magnitude below the spectra shown) were determined with an empty sample cell.

Since $f \gg f_o$, the spectrum is in the high frequency limit. Inserting $A = \pi d\ell$ in Eq.(16) of the previous letter[1], and $N = \pi n_o d^2\ell/4$ in Eq.(1), we find

$$\frac{S_I(f)}{I^2} = \frac{8D^{1/2}}{\pi^{5/2} n_o \ell d^3 f^{3/2}} \; [\ell n(I_o/I)]^2. \qquad (2)$$

The solid lines in Fig. 1 are a plot of Eq. (2) for the values used in the experiments. The agreement in slope is excellent and the scaling with relative changes in d and n_o is correct. The systematic error in the magnitude almost certainly arises from the factor of 3 uncertainty in the value of n_o.

A quite different class of diffusive system involves heat flow, and as an example, we consider 1/f noise in thin metal films. Hooge and Hoppenbrouwers[3] have studied 1/f noise in continuous metal films, mostly Au, and found

$$S_V(f)/V^2 \approx 2.4\text{x}10^{-3}/Nf \qquad (3)$$

for the voltage noise spectrum $S_V(f) \equiv \langle \Delta V(f) \Delta V^*(f) \rangle$. N is the total number of carriers in the sample, and V is the average voltage across the sample. Although both the numerical coefficient and the exact 1/f law represent a rather approximate fit for a large number of samples, Eq. (3) is a useful basis for order-of-magnitude comparison with theory.

We have repeated the experiments using evaporated metal films. We obtained order-of-magnitude agreement with Eq. (3) for films of Cu, Au, Ag, and Pb with $N \sim 10^{12}$, but the observed noise for Bi, where $N \sim 10^8$, was much smaller than predicted. All of these results, however, would be consistent with Eq. (3) if N were defined as the total number of atoms

in the sample.

We postulate that temperature fluctuations[4] account for the 1/f noise in thin metal films. As a simple model, we consider an electrically conducting bar of length $2\ell_1$ and rectangular cross section $2\ell_2 x 2\ell_3$ immersed in an insulating medium with the same thermal diffusivity, D. We neglect any boundary resistance. If $T(\vec{x},t)$ is the local temperature, the spatially averaged temperature of the bar $T(t) = (8\ell_1\ell_2\ell_3)^{-1}\int T(\vec{x},t)d^3x$. Since T(t) is analogous to N(t) for Brownian motion, the spectrum of the temperature fluctuations, $S_T(f) \equiv \langle \Delta T(f)\Delta T^*(f)\rangle$ can be immediately obtained from the Brownian motion spectrum. This spectrum is well approximated by a white spectrum at low frequencies, a 1/f-like intermediate region, and an $f^{-3/2}$ high frequency behavior. We assume, therefore, that $S_T(f)$ follows a model spectrum with three distinct regions: $S_T(f) = C_o/f_1$ for $f<f_1$; $S_T(f) = C_o/f$ for $f_1<f<f_3$; and $S_T(f) = C_o f_3^{1/2}/f^{3/2}$ for $f>f_3$; $f_1 = D/4\pi\ell_1^2$, and $f_3 = D/4\pi\ell_3^2$ are the lowest and highest natural frequencies of the system. Since $\langle(\Delta T)^2\rangle = \int_0^\infty S_T(f)df$,

we find that $C_o = \langle(\Delta T)^2\rangle/[3+\ln(f_3/f_1)]$.

If the bar has a temperature dependent resistivity and is carrying a constant current of uniform density, a temperature fluctuation, ΔT, will produce a voltage change $\Delta V = V\beta\Delta T$, where V is the average voltage and $\beta = \rho^{-1}\,\partial\rho/\partial T$ is the temperature coefficient of resistivity. The spectrum can then be written as $S_V(f) = V^2\beta^2 S_T(f)$. For a system of heat capacity C_V in thermal contact with a reservoir,[5] $\langle(\Delta T)^2\rangle = kT^2/C_V$. Using our model spectrum for $S_T(f)$, we find in the intermediate region $f_1<f<f_3$

$$S_V(f) = V^2\beta^2 kT^2/C_V[3+\ln(f_3/f_1)]f. \tag{4}$$

For most metals at room temperature, $C_V \approx 3Nk$ where N is the number of atoms in the sample, $\beta \approx 0.003$, and $D \approx 0.5\ cm^2 sec^{-1}$. Using these values for a typical sample of Hooge and Hoppenbrouwer[3], 800μ x 10μ x 500Å, we find

$$S_V(f)/V^2 \approx 11x10^{-3}/Nf \tag{5}$$

for $6x10^{-4}$ Hz<f<$1.6x10^9$ Hz. In view of the scatter of the experimental results, we feel that this value is in excellent agreement with Eq. (3) (with N = number of atoms).

This simple model in which 1/f-like temperature fluctuations produce voltage fluctuations when a constant current flows through a temperature dependent resistor is a considerable simplification of a complicated system. Metal film, glass substrate, and ambient atmosphere do not form a uniform system for heat conduction. This lack of inhomogeneity may be expected to smear out any changes in slope and to introduce a surface dependence to the noise when the coupling between various elements changes. Since a resistivity fluctuation across a given cross section changes the current path, the assumption of uniform current density is not strictly valid. This approximation should be reasonably accurate for small fluctuations in a long narrow bar. It is also important to note that the sample is not in thermodynamic equilibrium. Up to 1 kW cm^{-2} of surface area may be dissipated, raising the sample

temperature above ambient temperature. This effect is of particular importance when the film is immersed in a cryogenic fluid, and may contribute to the observed temperature independence of the noise.[3] Further, a resistivity fluctuation causes a change in the power dissipated in the region of the fluctuation and a consequent feedback effect. This effect would be expected to change the magnitude of the noise as well as the dependence on V. However, the fact that $S_V(f)$ is experimentally closely proportional to V^2 indicates that feedback is not generally a dominant effect.

Fluctuations in a diffusive medium involve a frequency-dependent correlation length, $\lambda(f) \equiv (D/\pi f)^{1/2}$. We have observed spatial correlations in the noise using the experimental configuration of Fig. 2(a). A Bi film of thickness 1000Å was cut as shown into two long strips each of length ℓ and width 12μ. Separate batteries and large resistors, R_o, were used to supply a constant current to each strip. The two noise voltages $V_1(t)$ and $V_2(t)$ were separately amplified and the spectrum of their sum or difference measured by a series of tuned circuits modeled in a PDP-11 computer. If $S_+(f)$ and $S_-(f)$ are the spectra of $V_1(t) + V_2(t)$ and $V_1(t) - V_2(t)$, the fractional correlation between the strips is

$$C(f) = [S_+(f) - S_-(f)]/[S_+(f) + S_-(f)]. \tag{6}$$

When the two voltages are independent, $S_+(f) = S_-(f)$ and $C(f) = 0$. When the two strips are completely correlated, $V_1(t) = V_2(t)$, $S_-(f) = 0$, and

$C(f) = 1$. For temperature fluctuations at high f, $\lambda \ll \ell$ and $C(f) \to 0$, while at low f, $\lambda \gg \ell$, and $C(f) \to$ constant. The changeover should occur when $\lambda \approx \ell$, the separation between the centers. Results for two different ℓ are shown in Fig. 2(b). For $D \sim 3$ cm^2 sec^{-1} we expect the changeovers at 4 Hz and 0.4 Hz. The data are in good agreement with our expectations.

The magnitude of the noise at a given frequency often changed with time, probably because of irreversible changes in the films caused by the high current densities ($\sim 10^5$ A cm^{-2}). Since we could not measure $S_+(f)$ and $S_-(f)$ simultaneously, this variation sometimes added a frequency independent shift to $C(f)$ (see Fig. 2(b), $\ell = 2.5$ mm). For the same reason, each spectrum was limited to the range of frequencies over which the computer could take data simultaneously ($\sim$ 2 decades). We could observe $C(f)$ only for a very limited range of ℓ. The 3-dimensional frequency-dependent correlation function is proportional to $\ell^{-1} \exp(-\ell/\lambda)$ The $1/\ell$ dependence implies that the noise in the two strips does not become completely correlated even though $\lambda \gg \ell$. The low frequency limit of $C(f)$ decreases for increasing ℓ. For $\ell \gtrsim 7.5$ mm it became impossible to distinguish a correlated region from scatter in the data. For $\ell \lesssim 2.5$ mm we were unable to attach the required number of leads.

Diffusion implies 1/f-like spectra and frequency dependent correlations. We have shown two examples of how these theoretical ideas can be simply applied to vastly different physical situations to yield quantitative predictions in good agreement with experiment. Diffusion of particles or diffusion of heat occurs in most experimental systems and each can be a cause of 1/f noise. In semiconductors, for example, we

expect both diffusion of impurities or lattice defects and temperature fluctuations to contribute to the noise. Although the theoretical ideas were developed for equilibrium systems, they provide an excellent first approximation for non-equilibrium situations.

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FOOTNOTES AND REFERENCES

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†Alfred P. Sloan Foundation Fellow.

††NSF and IBM Graduate Fellow.

1. Previous letter.

2. N.A. Clark, J.H. Lunacek, and G.B. Benedek, Am. J. Phys. 38, 575 (1970).

3. F.N. Hooge and A.M.H. Hoppenbrouwers, Physica 45, 386 (1969).

4. The entire discussion may also be phrased in terms of energy (rather than temperature) fluctuations.

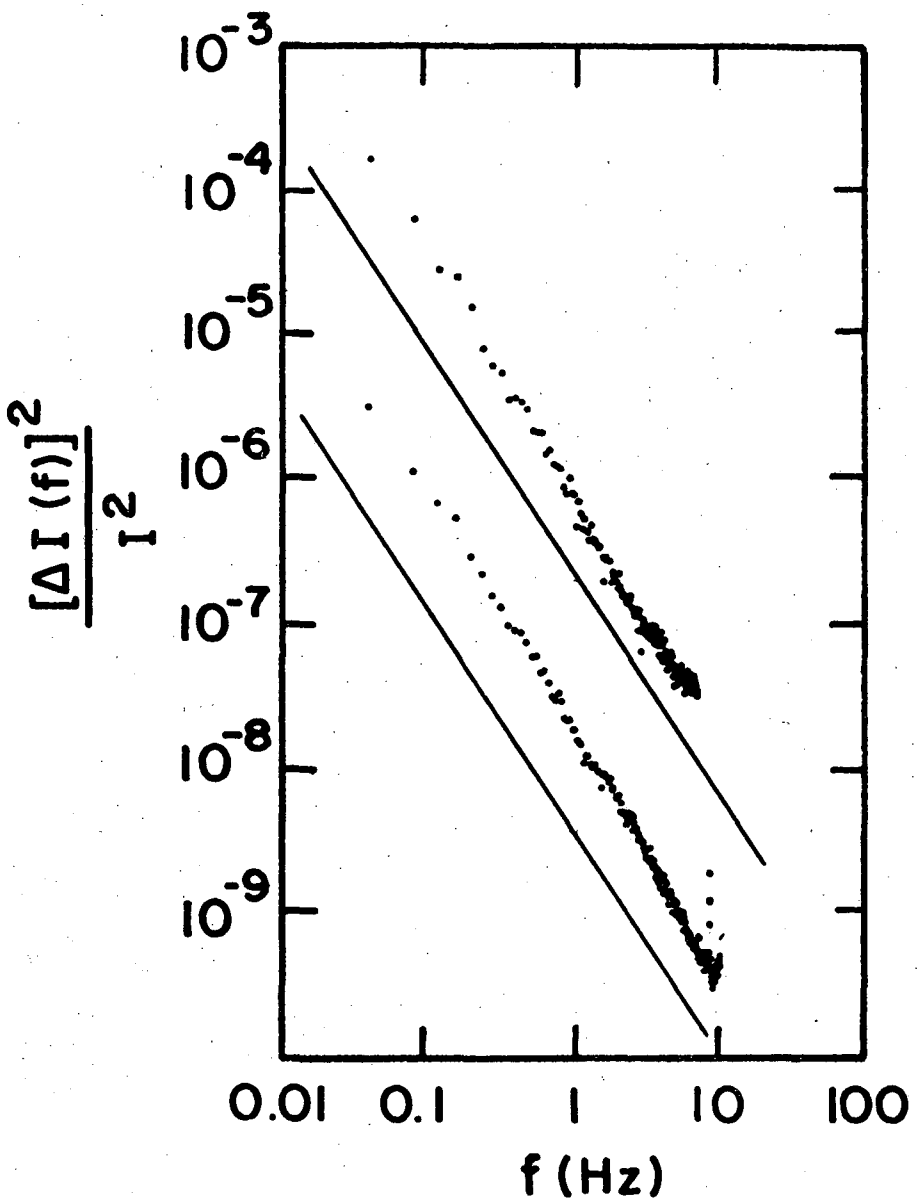
5. L.D. Landau and E.M. Lifshitz, Statistical Physics, (Addison-Wesley, 1969, 2nd edition), p. 350.

FIGURE CAPTIONS

Fig. 1 Two intensity fluctuation spectra for experiment (dotted) and theory (solid).

Fig. 2 (a) Experimental configuration for correlation measurement.

(b) Fractional correlation for two samples.



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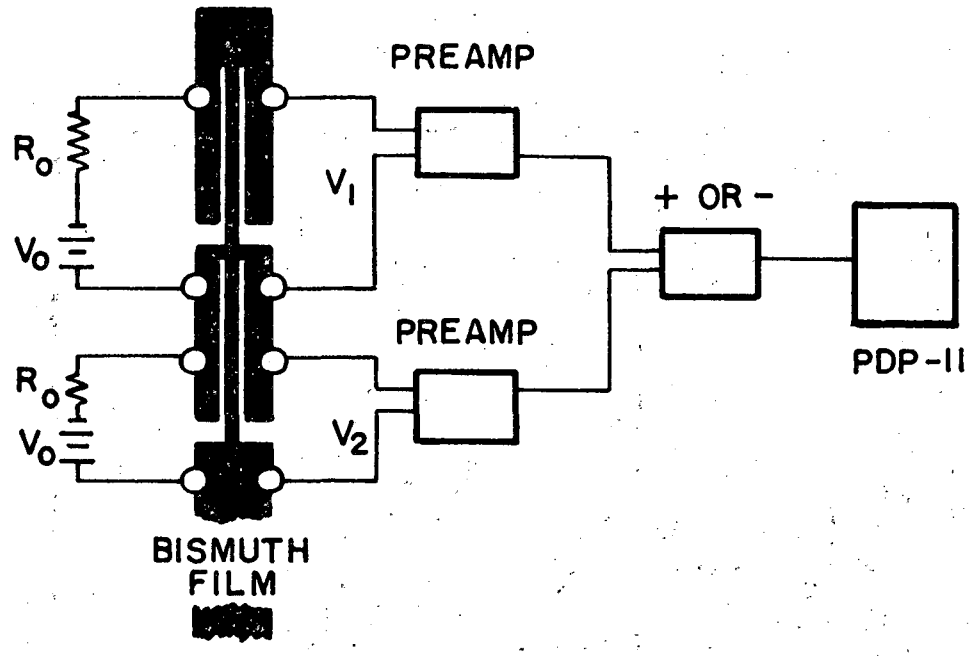
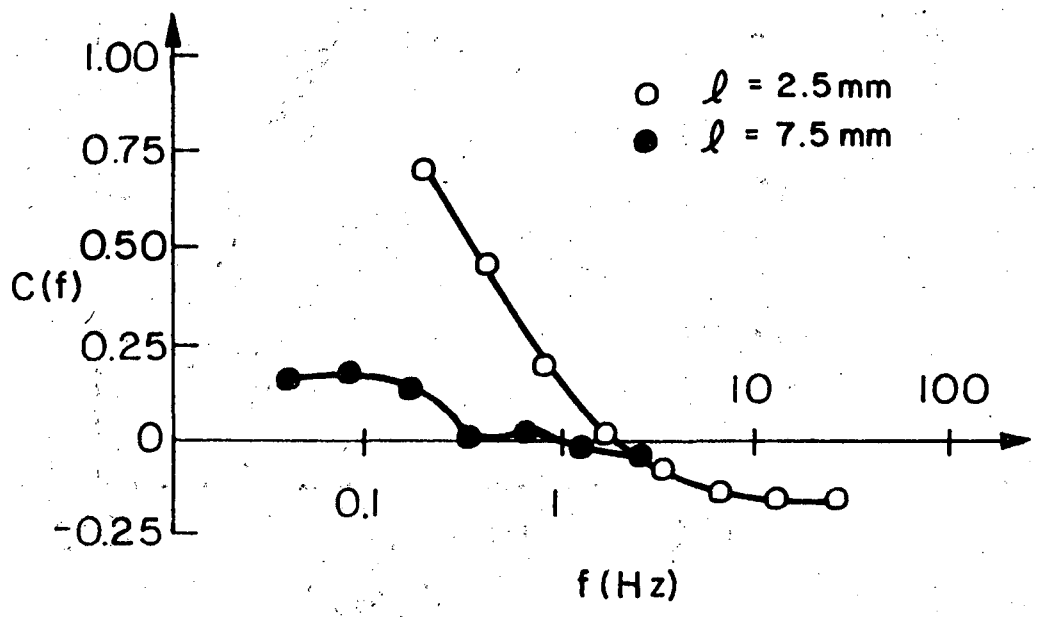


(a)



(b)

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