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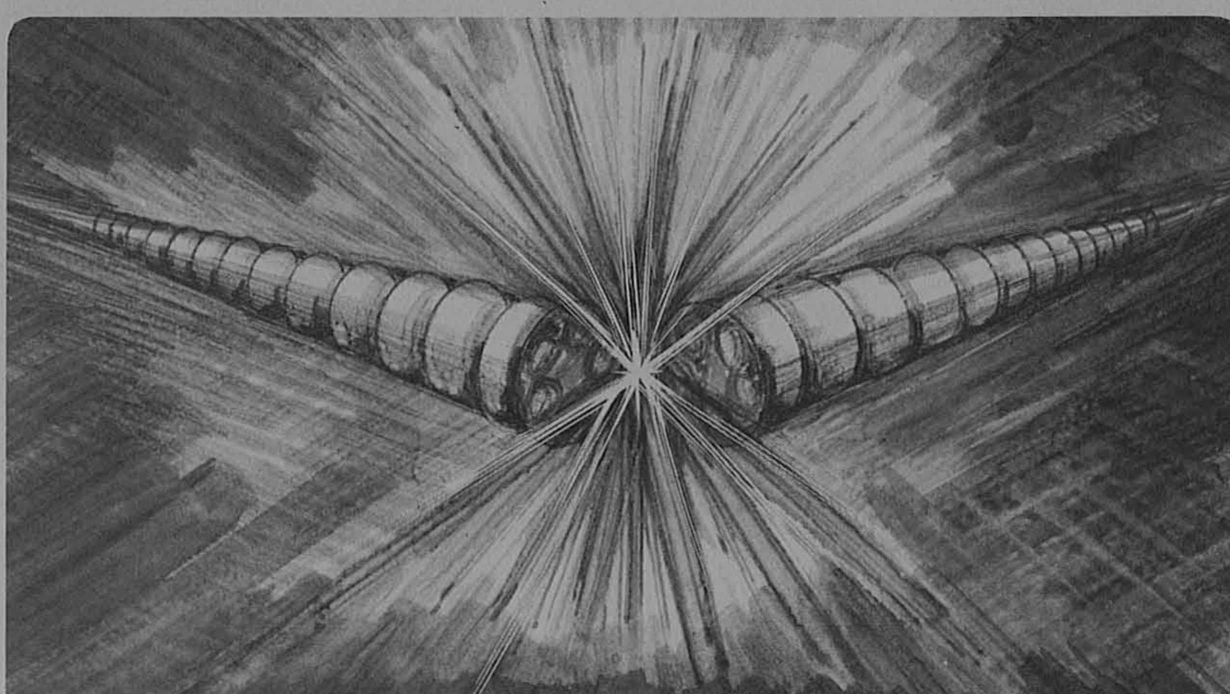

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A STRATEGY FOR A THOROUGH BEAM STABILITY AND APERTURE ANALYSIS FOR A STORAGE RING FROM DESIGN CONSIDERATIONS*

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ABSTRACT

An outline of a possible approach to understanding and analyzing beam stability and related aperture as thoroughly as one can using tracking and all the other available analytic tools to date is presented in the context of designing any particular storage ring.

We attempt in this report to outline a reasonable approach to understanding and analyzing beam stability and related aperture as thoroughly as one can, using tracking and all the available analytic tools mentioned in a companion summary report [1], while designing any particular storage ring. The approach might consist of the following sequential stages:

1. Determine the "Needed Aperture": Estimate the size of stable aperture of "good behavior", (x_o, y_o), needed for beam injection, operation, lifetime etc. in both planes - usually prescribed as certain specifications by the users and builders of beam optical systems. Often one may want to specify the maximum tolerable optical aberrations (geometric and chromatic) and dynamical distortions at the border of the needed aperture as well. We note that the needed aperture is a machine and technology dependent concept.

2. Quickly estimate the short-term "Dynamic Aperture" by tracking over a few hundred turns (typically 400) with a good tracking code, both for the ideal machine and then with realistically "guessed" magnet errors. This Dynamic Aperture (A_o, B_o) better be much larger than the "Needed Aperture" (x_o, y_o) and preferably comparable to or larger than the physical size of the beam chamber. See Fig. 1.

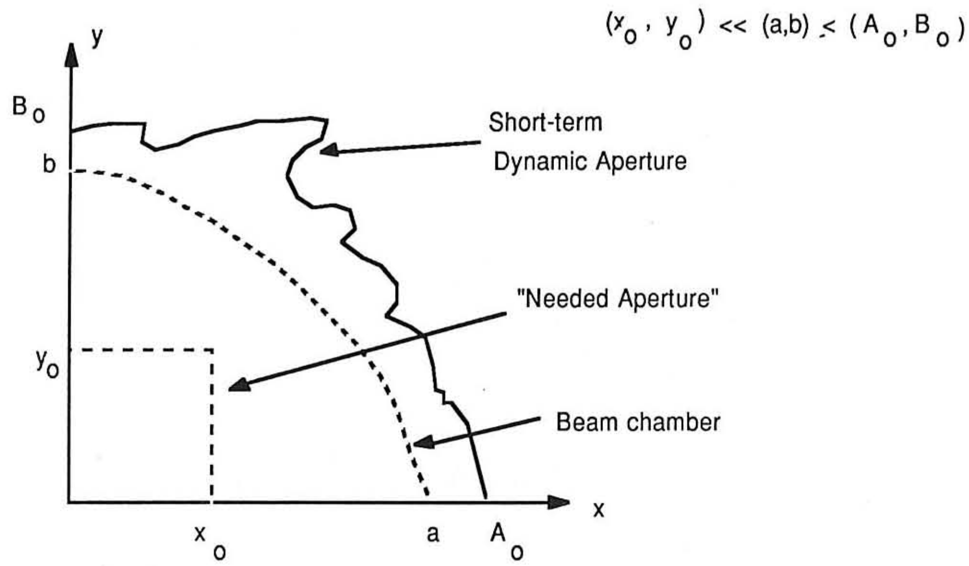

Fig. 1

The 400-turns "Dynamic Aperture" is admittedly a rather elusive concept, with very little to do with long-time stability in most cases. However, ensuring the above hierarchy of magnitudes in apertures is a necessary, although not sufficient, condition for beam stability. For small electron machines, tracking over a few "radiation damping times" may be sufficient for estimating Dynamic Aperture for long-time stability. For example, for synchrotron storage rings like the one being designed at Berkeley, the damping time $\tau_D \simeq 10$ ms corresponds to approximately 10,000 turns. Tracking over twenty or thirty thousand turns is still feasible. Long term tracking for large hadron machines is however almost impractical due to the hundreds of thousands of turns required for that purpose.

3. Once the "needed aperture" has been defined and short-and/or long-term "Dynamic Aperture" has been established by tracking, systematically explore the phase space structure and topology from small amplitudes to as large an amplitude as possible, in order to establish what we call the "Understandable Aperture". The "Needed Aperture" must lie within the "Understandable Aperture". The "Understandable Aperture" must be not only smaller than the "Dynamic Aperture", however determined by tracking, but must enclose a region of phase space where one or more of the relevant dynamical distortions (e.g., SMEAR, tune shifts, resonance widths, etc.), must stay bounded and reasonably finite and exhibit relatively understandable stable trends under small perturbations of the machine or the initial conditions. All the words "small perturbations", "relevant dynamical distortions", "bounded and reasonably finite", "relatively understandable

stable trends", etc. are machine dependent and ought to be clearly understood and defined, tailored to a specific machine. For example, the concept of "Linear Aperture", defined as that within which the famous "SMEAR" must be less than a certain percentage (< 10 %), was elected as the definition of "understandable aperture " for the design of the SSC. It is a conservative and safe criterion and has tremendous practical value in the design of any machine. One should remember however that the prescription of a maximum percentage of fluctuation in the linear invariants is only a necessary but insufficient criterion for the long-term stability in any machine.

Instead of "linear", we have used the word "understandable". This is to imply that unlike the SSC at injection, many machines (all third generation synchrotron radiation sources and damping rings for example) have a nonlinear ideal sextupole lattice as the unperturbed starting point. Nevertheless, thanks to some powerful newly-developed computational tools as outlined in the summary report on theoretical and analytical studies [1], we are in a position to perform nonlinear analysis, perturbative or otherwise, at impressively large amplitudes, on any lattice for which a tracking code exists, without compromising on accuracy and faithfulness.

To perform such analysis, one first constructs the one-turn map "M" of the machine as precisely as one can to the desired level of accuracy (computer precision) either via the differential algebraic technique of M. Berz [1,2] or via a few-turn tracking data a la Warnock, et al. [1,3]. If successful, one has done most of the work. One can then analyze these maps either through Normal Form perturbative analysis [1,2,4,5] or through the nonperturbative iterative solution of the functional equation for the invariant surface itself [3,6], to compute quantities containing the detailed information of the nonlinear optics. Most of it is contained in the resonance strengths (or widths). Knowing the strengths of all resonances of all orders, in principle, implies knowing the entire nonlinear optical map. In practice however, one is limited to only a small number of figures of merit to be computed to characterize the nonlinear machine e.g. $\sigma_n(A)$ = sum of the strengths of all n-th order resonances at amplitude A, $\sigma_n^*(A)$= contribution of all n-th order resonances to pure distortion [1], nonlinear tune shift $\Delta\nu(A)$, distortion of invariant tori or SMEAR $\sigma(A)$, etc. Using these, one can try to 'creep into' the nonlinear region as far as one can.

4. One can then look for trends in these nonlinear optical distortions as a function of amplitude A and resonance order 'n'. Criteria for "good behavior" should then be decided upon by imposing judicious constraints on the upper bounds and patterns of these quantities, specific to any particular machine. Possible criteria could be various combinations of the following:

(i) $|\Delta\nu(A)| \leq .01$ or .001, etc.

(ii) convergence and/or regularity of $\sigma_n(A)$

(iii) sensitivity of $\Delta\nu(A)$ and $\sigma_n(A)$ to amplitude (i.e., $\sigma_n'(A) \equiv d\sigma_n(A)/dA$, $\Delta\nu'(A)$, etc).

(iv) $|\sigma_n| \leq ..., n > n_0$.

(v) SMEAR = $\sigma(A)$ < (certain percentage) ~ 10% (imposes constraint on $\sum_n \sigma_n^*(A)$).

Pattern of the resonance strengths $\sigma_n(A)$ in tune units as a function of order 'n' and amplitude A (.001 to .005 in suitable units) expected for the bare lattice of the Advanced Light Source being designed at Berkeley, for example, is shown in Fig. 2. It shows the usual convergence pattern of falling off by an order of magnitude per each increasing order of the resonance at small amplitudes (.001). Pattern gets worse with higher amplitudes, with no systematic trend or convergence expected close to the Dynamic Aperture.

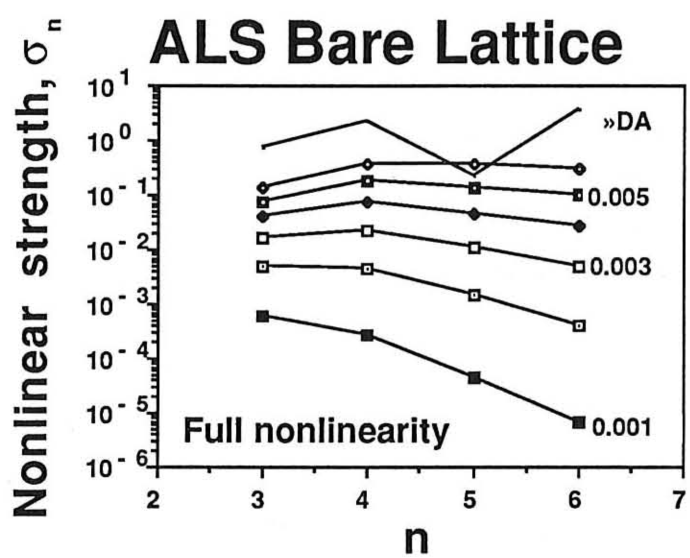

Fig. 2

5. Finally, one would want to compare the "Needed Aperture" with these behavior criteria. Hopefully, one can correlate the storage ring behavior with these nonlinear optical distortions as outlined above. The final aim is some kind of visualization of contours of constant "dynamical quality" in the amplitude plane, as sketched in Fig. 3 and choice of a certain nonlinearly distorted but understandable and controllable amplitude contour A_c as the aperture limit. Once chosen, $\Delta\nu(A_c)$, $\sigma(A_c)$, $\sigma_n(A_c)$, $\sigma_n^*(A_c)$, $\sigma_n'(A_c)$ etc. will then dictate tolerances on magnetic multipole components of the machine.

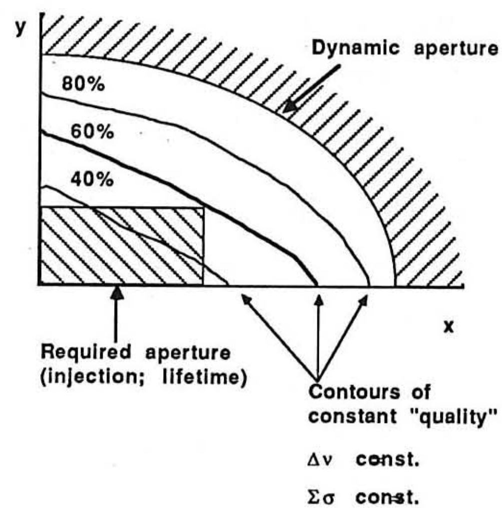

Fig. 3

Further details of chosing the aperture criteria for various machines are exposed in detail in the Criteria Working Group Summary by A. Chao [7] in these proceedings.

6. Now then, how about ultimate long-term stability (after 10^{10} turns, say)? Both the short-term Dynamic Aperture and the above analysis have little connection with long-term stability. What is worse, no precise statement in this regard can be made, since no general theorem exists. The KAM theorem only predicts bounded motion for "small" amplitudes in one dimension. In practice, this small amplitude is so small as to be useless for stability considerations in storage rings. Moreover, particles really move in three dimensions and due to Arnold diffusion and other semi-quantitative concepts, it is clear that some particles very close to the origin will nevertheless escape. The position in phase space of these particles is extremely machine dependent and the most insignificant

change in modeling a machine sometimes may turn an unstable particle into a stable one and vice versa. Nothing, not even the powerful mathematical theorems like the KAM, seems to rescue particles from possible long term loss, except electrons where there is radiation damping. But then for electrons one can track over thirty thousand turns or so to determine numerically the Dynamic Aperture. This leads us to long term tracking for protons (hundreds of thousands of turns), which may not be feasible.

For protons, a relevant question to ask instead is: what is the expected lifetime of a beam having a certain maximum amplitude of motion? For this purpose, one may attempt to estimate, in the spirit of Nekhoroshev's theorem [1,5], the "remainder" at that amplitude, the weak diffusion rate due to these stochastic layers and their enhancement by external noise, modulation by synchrotron oscillation, etc. One can use any or all of these qualitative methods including Arnold diffusion rate, Chirikov criterion, modulation diffusion, etc. to arrive at some estimate of beam lifetime τ_L expected from nonlinear dynamics. This has to be compared with expected beam lifetime τ_M from other considerations by carefully looking at mechanisms of other beam losses and their various time scales: noise, beam-beam, Touschek and Intrabeam scattering, Luminosity etc. All one has to ensure is that $\tau_L > \tau_M$.

We note that it is nontrivial, if not impossible, to compute τ_L accurately. Qualitative estimates may be off from reality by several orders of magnitude owing to arbitrary constant parameters appearing in the remainder estimates [1,5]. Considerations along these lines and the possibility of simulating long-term tracking with short-term tracking using finitely many particles and imposed noise, is exposed in detail in the contribution of Heifets [8], in these proceedings.

Given the importance of this issue in the design of a real storage ring and the difficulty and lack of any quick sound method of determining the border of stability directly in one step, the above systematic, lengthy and somewhat painful process seems to be the only approach to a comfortable design of a storage ring, safeguarding the understandibility and controllability of the beam dynamics, even under extremely nonlinear situations.

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REFERENCES:

[1] S. Chattopadhyay, "Theory and Analysis of Nonlinear Dynamics and Stability in Storage Rings - A Working Group Summary," these proceedings.

[2] E. Forest, M. Berz, and J. Irwin," Exact Computation of Derivatives with Differential Algebra and Applications to Beam Dynamics," SSC-166, submitted to *Particle Accelerators*.

[3] R. L .Warnock, W. Gabella and R. D. Ruth, "Construction of Symplectic Full-turn Maps by Application of an Arbitrary Tracking Code," manuscript under preparation.

[4] A. Bazzani et al., "Normal Forms for Hamiltonian Maps and Nonlinear Effects in a LHC Model," CERN SPS/88-2 (AMS), LHC Note 66.

[5] A. Bazzani and G. Turchetti, "Asymptotic Properties of Birkhoff Normal Forms," these proceedings.

[6] W. E. Gabella, R. D. Ruth, and R. L. Warnock, "Periodic Solutions of the Hamilton-Jacobi Equation by the Shooting Method: A Technique for Beam Dynamics," these proceedings.

[7] A. Chao, "The Criteria Working Group Summary," these proceedings.

[8] S. A. Heifets, "Beam Dynamics and Tracking," these proceedings.