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The Elasticity of Appliance Demand for Energy with Respect to Efficiency

January 1988

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INTRODUCTION

This paper presents a practical method for estimating the elasticity of demand for energy with respect to a change in appliance efficiency. The first step in this direction was taken by J. Daniel Khazzoom (1980), who challenged the idea that mandating more efficient appliances would lead to a decline in energy demand. To do so he criticized those who ignored the consequences of reduced service cost and who took account only of the mechanical effects of efficiency improvements.

With hindsight it seems remarkable that he did not take the next step in bringing economic rationality to the question of energy demand elasticity. That step is to include the effect on energy demand of the cost of purchasing and maintaining the appliance. That appliance cost is closely tied to the appliance's efficiency can hardly be doubted. The loud protests of the appliance manufacturers whenever Congress or DOE considers a new energy-efficiency standard is ample testimony to this connection. But these protests tell us more; the manufacturers would not worry about the additional costs if they did not believe that passing them on would adversely effect appliance demand. That this effect could be significant, may be appreciated by considering that one appliance-efficiency option is the heat-pump electric clothes dryer. This option roughly doubles both the cost and efficiency of the electric dryer.

Henly *et al.* (1987) have taken the next step and corrected Khazzoom's equation to make it account for the cost of owning and operating the appliance. (As explained later this cost will be referred to as the rental cost of the appliance.) While Henly's result is exact, it is restrictive in a way the mirrors Khazzoom's. Khazzoom's result holds for rents that depend on the appliance's capacity but not its efficiency, while Henly's result holds for rents that depend on efficiency but not capacity. Even this limited version of Henly's result is impractical, and when it is extended to cover appliance rents that depend on both capacity and efficiency the situation deteriorates.

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This is explained in the section that derives and extends Henly's result.

To remedy this predicament, this paper derives an approximation to Henly's equation that is practical in much the same way that Khazzoom's equation is practical. That is, the new equation shifts the information requirements from a knowledge of the consumer's response to a complex change in *appliance prices* to a knowledge of the consumer's response to a change in *energy price* coupled with the consumer's discount rate. After presenting the new approximation, a simple example is used to demonstrate that the new effects captured by it could be large enough to reverse Khazzoom's "rebound" effect. Thus any serious estimation of the elasticity of energy demand with respect to efficiency must rely on the new equation.

The last section derives both an exact formula, and a practical approximation for energy elasticity when it is know that no "renters" will stop "renting" when confronted with new efficiency regulations. The Summary and Conclusion contains a table showing the evolution of the formula for energy elasticity, including this paper's main result.

PRELIMINARY CONSIDERATIONS

Rent

In order to simplify analysis we will convert the purchase cost of an appliance into an equivalent rent via the consumer's discount rate (Ruderman, Levine, and McMahon, 1987).[1] This is a standard trick and puts the three costs--energy, maintenance, and acquisition--on a comparable footing. Of course, we could have converted all three to present values with the same effect, but since this is simply a matter of multiplying each by the same constant it would in no way change the analysis. So from now on a consumer who owns an appliance is called a "renter" and is said to pay a "rent" R, which may depend on efficiency, e, and/or service, s, and which covers the costs of purchase and maintenance.

A Benefit-Cost Approach

A proper analysis of consumer behavior would start with the following utility function:

$$U = U(y - C(s), s),$$

where y is income,

1. In practice, measuring the consumer's discount rate is perhaps the most difficult step in estimating η_e^E.

$$C(s) = \frac{p}{e}s + R(s,e),$$

is the cost of service, p is the price of energy, e is efficiency, and $R(s,e)$ is the rental cost. (This is exactly analogous to the utility function used by Henderson and Quandt (1971, p.29) to analyze the income-leisure tradeoff; service being analogous to leisure, and y—C being analogous to income given leisure.)

However if we believe that income effects are small, then the above approach may legitimately be simplified by assuming them to be zero. By this I mean that the two derivatives of the utility function must change little when income is changed by the annual cost of an appliance. Given that a more efficient appliance costs *at most* an additional few hundred dollars and saves at most a few hundred dollars in energy costs, it seems that in most cases it would be safe to assume that income effects were small. In this case, benefit-cost analysis shows that consumers maximize

$$B(s) - C(s),$$

where $B(s)$ is consumer benefit as a function of service s.

The Population of Consumers

Consumers differ in both their benefit and cost functions. They differ in C(s) because of their differing discount rates, while they differ in B(s) because of differences in U(). Clearly if the cost curves shift down, more consumers will rent and the stock of appliances will go up. If the cost curves get steeper, those who rent will choose less service and thus will use less energy. While a parallel shift of the cost curve does not change marginal conditions and thus does not effect usage by those who rent, any change in the slope of $C(s)$ will effect total cost for some, and therefore will effect the choice of whether or not to rent. This distinction between shifts in $C(s)$ and changes in the slope of $C(s)$ will play a useful role in the analysis.

Notation

Throughout the rest of the paper, elasticities will be denoted by a super and subscripted η; for instance, η_e^E means the elasticity of energy with respect to efficiency.

KHAZZOOM'S ANALYSIS

Khazzoom's formulation of the problem in 1980 eliminated R(s,e) entirely from the analysis, yet his result can be derived when R(s,e) is only restricted to R(s). This restriction yields our first and most simple formulation of the cost assumption:

$$C(s) = \frac{p}{e}s + R(s). \tag{1A}$$

Now when a consumer maximizes $B - C$ over s the result will clearly depend only on

the ratio $\frac{p}{e}$, and this holds even when $B - C$ is maximized by $s = 0$, that is, by not renting. Since $\frac{p}{e}$ determines s for every consumer, it also determines S, the *average* service demand. (This is an average over renters and non-renters alike.) Thus we can write the average service demand as

$$S = S(\frac{p}{e}). \tag{1B}$$

We now derive Khazzoom's formula for η_e^E . To do this we define F to be the cost of fuel per unit of service, which is just $\frac{p}{e}$,[2] and write

$$S = S(F). \tag{1B*}$$

Using the chain rule for elasticities, which operates exactly like that for derivatives, we have

$$\eta_e^S = \eta_F^S \eta_e^F .$$

We now need a simple pair of results that will be used repeatedly and that follow from the definition of efficiency that is implicit in the equation $S = e \cdot E$, where E is average energy use counting renters and non-renters alike. Calculating the elasticity of both sides of this equation first with respect to e and then with respect to p, we have

$$\eta_e^S = 1 + \eta_e^E \qquad \text{and} \qquad \eta_p^S = \eta_p^E . \tag{1C}$$

This gives us the theoretical version of Khazzoom's result:

$$\eta_e^E = -1 - \eta_F^S \eta_e^F$$

Using $F = \frac{p}{e}$, find that $\eta_e^F = -\eta_p^F$, and apply the chain rule to equation (1B*) to arrive at

$$\eta_p^S = \eta_F^S \eta_p^F .$$

Together these two results transform the theoretical formula into Khazzoom's actual result:

$$\eta_e^E = -1 - \eta_p^E . \tag{1E}$$

Since η_p^E is negative, $-\eta_p^E$ constitutes a "rebound" effect relative to the mechanical elasticity of -1. The misleading implication of Khazzoom's equation is that energy saving due to increased efficiency will definitely be less than is implied by the mechanical effect.[3]

HENLY'S ANALYSIS

Henly, *et al.* have extended Khazzoom's equation to cover appliance rent that is a function of efficiency. This leads to the cost function, $C(s) = \frac{p}{e}s + R(e)$. As noted in the introduction, this mirrors Khazzoom's assumption by leaving service capacity out just as Khazzoom left out efficiency. When consumers maximize $B - C$, this leads to the average optimal service function:

$$S = S\big(F,\ R(e)\big), \tag{2B}$$

where again $F = \frac{p}{e}$. Using the chain rule for elasticities, notice that

$$\eta_e^S = \eta_F^S \eta_e^F + \eta_R^S \eta_e^R .$$

Now transforming the first part of the equation just as before, we have Henly's result.

$$\eta_e^E = -1 - \eta_p^E + \eta_R^S \eta_e^R . \tag{2E}$$

From here the direction to go is clear, Henly's rent term must, if possible, be transformed in a practical way similar to Khazzoom's fuel cost term. But before doing this it will prove useful to generalize Henly's result. This will help explain the impracticality of the "theoretical" version and guide the development of its transformation.

The above derivation unrealistically restricted rent to be a function only of efficiency. For most appliances, the economical, if not the only way, to achieve a significantly higher service level is to rent a more expensive version of that appliance, e.g. rent a bigger refrigerator. Thus, rent should depend on both efficiency and service.[4] This leads to the general cost function:

$$C(s) = \frac{p}{e}s + R(s,e)$$

Clearly one need only know $\frac{p}{e}$ and e to determine an individual's optimal s. As argued above these also determine the average optimal service function,

$$S = S_e(\frac{p}{e},\ e).$$

Next define a function similar to Henly's R(e),

$$\overline{R}(e) = R(\overline{s},\ e),$$

where $\overline{s}$ is any fixed amount of service. Now define another optimal total service

2. The use of F in place of $\frac{p}{e}$ allows us to differentiate with respect to F and find elasticities with respect to F. Later in the paper when it no longer simplifies derivations and notation it will be dropped.

3. As recently as October 1987 in the Energy Journal (p. 87) Khazzoom asks "How do Lovins' saving estimates fare when juxtaposed against Lovins' estimates of η [in this paper's notation, η_p^E] and answers: "Paradoxically, Lovins does not claim the price elasticity of demand is zero."

function which yields the same value for s, but gets it via a more familiar looking route.

$$S\left(\frac{p}{e}, \overline{R}(e)\right) \triangleq S_e\left(\frac{p}{e}, e\right)$$

The above derivation of Henly's formula can now be carried through with no change except to substitute $\overline{R}$ for R. This gives

$$\eta_e^E = -1 - \eta_p^E + \eta_{\overline{R}}^S \eta_e^{\overline{R}},$$

which has the same form but a more subtle interpretation. $\eta_{\overline{R}}^S$ is the elasticity of market service demand with respect to the change in the rent of the appliance that is used to obtain service $\overline{s}$. But of course when $\overline{R}(e)$ changes, it means e has changed for all appliances, so all other rents will change as well, and change in a very particular way. They change exactly in accordance with the costs incurred in changing e for each size of appliance. In other words, for each e a new rent function of the form $R_e(s)$ can be defined, and it is this whole function, not just one rent value, that is really changing with e.[5] The elasticity $\eta_{\overline{R}}^S$ must be evaluated as $R_e(s)$ changes in precisely this way. Unfortunately, rent is only likely to change in this way when the efficiency standard under consideration is actually implemented. At that time it would make more sense just to measure η_e^E directly than to calculate it from this formula.

Before going on to develop a more practical formula it should be noted that the term Henly has added to Khazzoom's equation is, as Henly points out, sure to be negative. Thus it reinforces the mechanical effect and counteracts Khazzoom's rebound effect.

A PRACTICAL APPROXIMATION TO HENLY'S FORMULA

It is now necessary to retreat from the generality of the rent function just analyzed to a second order approximation of that function.

$$R(s,e) = R_1(s) + R_2(e) + s \cdot R_3(e) \tag{3A}$$

Although each of the three functions, $R_1()$, $R_2()$, and $R_3()$, can be chosen arbitrarily, this form is not completely general since it cannot include terms such as $s^2 \cdot e$. This is the only missing third order term, and all possible second order terms are included.

4. Note that even though we are assuming that rent varies with the capacity of the appliance, rent is *not* defined as a function of the capacity (or size) of an appliance since this would cause rent to be a multivalued function. By defining it as a function of service and efficiency the consumer's rationality will cause it to be single valued. If two different appliances (or combinations thereof) provide the same service at the same efficiency, the consumer will choose the one with the lower rent. The rent function is then defined to take this lower value.

5. It is assumed throughout that any change in e applies to all sizes of appliance; otherwise η_e^E would not would not be well defined. Other assumptions can be handled by treating different size appliances as if they were different kinds of appliance.

Thus the approximation can be used to construct a second order approximation of an arbitrary $R(s,e)$. Though still somewhat restrictive, this approximation allows the slope of R , $\frac{\partial R(s,e)}{\partial S}$, to change when e changes. Changing the slope of R must have the same impact as a change in slope caused by a change in the fuel cost $\frac{p}{e}s$. Thus changing e will change the utilization of an appliance by its renter in a way that can be measured by η_p^E.

As always, the rent function generates the average service demand function, which is now

$$S = S\left(\frac{p}{e} + R_3(e),\ R_2(e)\right) \tag{3B}$$

The derivation of the new elasticity formula, is consigned to the appendix, and the result simply presented here.

$$\eta_e^E = -1 - \eta_p^E\left(1 - \frac{X}{pE}\eta_S^X\,\eta_e^X\right) + \eta_p^A\,\frac{T}{pE_r}\,\eta_e^T \tag{3E}$$

where

$$T = R_1(s) + R_2(e) \quad \text{and} \quad X = s \cdot R_3(e),$$

A is the total number of appliance renters, and E_r is the average energy use of *renters*. Notice that there are two new terms. One is proportional to Khazzoom's rebound effect (η_p^E), but has opposite sign, while the other is proportional to the elasticity of appliance ownership with respect to energy. It too contradicts the rebound effect. In some cases, even though Khazzoom's effect is unknown, it may be possible to evaluate the first term, using price and engineering data, and show that the total rebound effect is zero or negative.

AN EXAMPLE

For a very simple example of the interpretation and use of the new η_e^E term, we turn to furnace data from the Sears Catalog.[6] This example demonstrate a partial application of the new formula, allows us to make several points about its implications, and even provides one example of when its use is inappropriate. Consider the four most extreme furnaces in the upflow natural gas category.

Rent has been computed from price by assuming a consumer discount rate of 10%. Although this rate is above a real mortgage interest rate, it is typical to observe consumer discount rates for appliance purchases in the neighborhood of 100%.[7] In fact, most furnaces are bought by contractors whose effective discount rate is thought to be much higher, but this brings up a difficulty that is discussed at the end of this section.

6. The 1987 Annual Home, Hardware & Leisure Catalog
7. See Ruderman, Levine, and McMahone, 1987.

Table 1. Data on Four Gas Furnaces.

Price	R[a]	s[b]	e[c]	$s \cdot e$
$300	$34	39	0.78	30.4
$450	$51	96	0.77	73.7
$1,300	$148	48	0.96	46.1
$1,600	$182	113	0.90	102.2

(a) Assumes a 20 year furnace life and a 10% consumer discount rate.

(b) Heating capacity in thousands of Btuh.

(c) Heating capacity in Btuh / Btuh input.

To evaluate this data we must fit a rent function to it. Choosing the simplest linear function:

$$R = T(s,e) + X(s,e)$$

$$R = -285 - 3.14s + 386e + s \cdot 4.64e$$

Now it is easily shown that if $R_3(e) = k \cdot e$, then $\eta_S^X \eta_e^X = 1$. Thus the second term in equation (3E) reduces to $\eta_p^E(1 - \frac{X}{pE})$. For the above data, the average of s is 74, and of e is .85; thus $X = \$293$. Now the typical energy cost of a 74 kBtu/h furnace is about $550 per year.[8] This means $\frac{X}{pE}$ is just over .5, which is quite a significant correction to Khazzoom's term. If the effective consumer discount rate were 30% instead of 10%, not an impossibility, Khazzoom' term would have been more than eliminated.

This "back-of-the-envelope" example is in no way meant to be an accurate estimate, but it does demonstrate a few key points. First, the new η_p^E term requires only a consumer discount rate, and engineering and pricing data, unlike Henly's term which involves information about the consumer response to appliance price increases. Second, it is plausible that in some circumstances the effect it measures is quite significant; and therefore should not be ignored out of hand. Third, it is *possible* for the new η_p^E term to more than offset Khazzoom's term. Thus more energy may be saved by raising e than would be implied by the mechanical effect alone.

Now a word of caution. In the case of furnaces, the purchaser (usually a builder) is general distinct from the user (home owner or apartment renter). Thus η_p^E for the appliance user may not capture correctly the behavior of the appliance purchaser (renter) if there is a principal-agent problem.[9] For replacement furnaces bough by a home owner, there would be no such difficulty.

ENERGY DEMAND WITH CONSTANT SATURATION

Let us now consider a more simple situation in which no consumer stops renting because of the mandated efficiency change.[10] In this case, the only change in energy demand is due to the change in usage, and this means that the rent function affects demand only through its change in slope. Using now familiar techniques it can be shown that if

$$C(S) = \frac{p}{e} S + R(S,e)$$

then

$$\eta_e^E = -1 - \eta_p^E \left(1 - \frac{e^2}{p} \frac{\partial R}{\partial S \partial e}\right)$$

This is a more general result than equation (3E). However it is less transparent. By restricting R() a little, the similarities with (3E) can be made evident. So let

$$R(s,e) = f(e) \cdot g(s).$$

Then

$$\eta_e^E = -1 - \eta_p^E \left(1 - \frac{R}{pE} \eta_s^R \eta_e^R\right).$$

Also note as before, that if R(s,e) takes the form $R_0 \cdot s \cdot e$, then $\eta_e^R \eta_s^R = 1$, and the coefficient of η_e^E simplifies to $(1 - \frac{R}{pE})$.

SUMMARY AND CONCLUSIONS

The top part of Table 2. shows the evolution of expressions for the elasticity of energy demand with respect to efficiency. In the lower left corner is Khazzoom's original formula. This formula is practical because it uses η_p^E to evaluate the more difficult term in the equivalent formula given in the upper left corner. Henly, *et al.* have shown that Khazzoom's formula, though easy to apply, is not complete, and added a third term to account for the effect of the interaction between efficiency, the rental cost, and energy use. Henly's term is even more difficult to estimate than the term Khazzoom simplified, so a transformation of it is needed before estimation is feasible.

8. This is rough estimate bases on data from the Lawrence Berkeley Laboratory Residential Energy Model.
9. In simple theory the builder should act as the agent for the eventual owner (principal), but in practice the furnace industry reports that builders behave as if they had a very high discount rate. In fact the average effective discount rate for the efficiency/first-cost decision was estimated at 56% by (Levine et al., 1986).

10. Although we are about to assume the opposite, this may be an appropriate place to explain why some "renters" would be expected to stop renting because of a mandated efficiency change. Recall that the term "renter" is being used for owner as well.

Table 2. Elasticity of Energy Use with Respect to Efficiency

	With Energy Costs	... Plus Rental Costs
Theoretical	$\eta_e^E = -1 - \eta_F^S \eta_e^F$	$\eta_e^E = -1 - \eta_p^E + \eta_R^S \eta_e^R$
Applicable	$\eta_e^E = -1 - \eta_p^E$	$\eta_e^E = -1 - \eta_p^E (1 - \frac{X}{pE} \eta_e^X \eta_S^X) + \eta_p^A \frac{T}{pE_r} \eta_e^T$

E (E_r)	Avg. energy demand (of renters)	e	Efficiency
p	Price of energy	F	Fuel cost / unit service
S	Service generated = eE	A	# of consumers renting
T	$= R_1(s) + R_2(e)$	X	$= s \cdot R_3(e)$
R	Rental cost / appliance $= R_1(s) + R_2(e) + s \cdot R_3(e) = T + X$		

This paper presents a practical version of Henly's formula based on a second order approximation to the rent function. Although the new "practical" equation appears more complex, for the following reason it is actually much easier to apply than Henly's theoretical equation. The new equation relies on no knowledge of consumer behavior except for a knowledge of consumer response to changes in energy price and the consumer discount rate. The elasticities with respect to energy price should be easier to estimate than elasticities with respect to appliance cost because energy prices are much more prone to rapid fluctuation and to regional variation. In some cases, as shown by an example, the new η_p^E term alone may be sufficient to negate Khazzoom's "rebound" effect. The other elasticities (not with respect to energy-price) in the new equation can all be evaluated from engineering and price data, without any reference to consumer behavior.

In the special case where no consumer will stop "renting" an appliance due to standards, Henly's formula may be generalized without resorting to an approximation. This result and some more tractable results based on approximations are presented in the next to last section.

In conclusion one must agree with Khazzoom, that the mechanical effect may well be a poor estimator of energy savings, and that it is likely to estimate badly when energy demand for appliance usage is sensitive to its own price. However, it now must be admitted that *a priori* it cannot be said whether a large sensitivity to energy price will bias the mechanical estimate upward or downward.

Consider a "renter" whose appliance (say a room air conditioner) fails after the imposition of an efficiency standard. Because the cheaper model of the type that failed is no longer available, the renter might have to choose between a new more expensive model (i.e. one with higher "rent") and not renting (owning). In this case he may well choose not to rent.

Equation (3E) provides the only current practical method for taking these diverse effects into account. Although it requires an estimate of the effect of *energy* price on appliance ownership, it does not require estimates of the effect of *appliance* price on ownership by appliance size, as direct use of Henly's equation would.

APPENDIX

In the appendix a short-hand notation for partial derivatives is used; examples of this notation follows. $\partial_1 S$, $\partial_2 S$, and $\partial_{12} R$ represent first partial derivatives with respect to the first and second arguments of S(.,.) and the second (cross) partial derivative of R(.,.).

Starting with the consumer's optimal average service demand, given by equation (3B) we derive Eq. (3E).

$$S = S\left(\frac{p}{e} + R_3(e),\ R_2(e)\right) \tag{3B}$$

Differentiate with respect to e and multiply by $\frac{e}{S}$.

$$\eta_e^S = \partial_1 S \cdot \left(\frac{-p}{eS} + \frac{e}{S} R_3'\right) + \frac{e}{S} \partial_2 S \cdot R_2' \tag{A1}$$

Differentiate equation (3B) with respect to p and multiply by $\frac{p}{S}$.

$$\eta_p^S = \frac{p}{eS} \partial_1 S \tag{A2}$$

Use equation (A2) to simplify (A1); then use equations (1A).

$$\eta_e^E = -1 - \eta_p^E + \frac{e}{S} \cdot R_3' \cdot \partial_1 S + \frac{e}{S} \partial_2 S \cdot R_2' \tag{A3}$$

Now relate $\partial_1 S$ to η_e^E by noting that $\frac{dS}{dp} = D1S \cdot \frac{1}{e}$, then use the second of equations (1A).

$$\eta_e^E = -1 - \eta_p^E + \frac{e^2}{p} R_3' \cdot \eta_p^E + \frac{e}{S} \partial_2 S \cdot R_2' \tag{A4}$$

Next we must relate $\partial_2 S$ to η_p^A. To this end note that

$$S = \frac{1}{N} \sum_{j=1}^{J} s_j \cdot A_j \tag{A5}$$

where N is the total number of consumers (including non-renters), the s_j are the centers of a sequence of small intervals along the service axis, and the A_j are the numbers of renters in these intervals.[11] The lowest interval, at s_j does not include

11. This part of the derivation could be accomplished more elegantly, if less transparently, using integration.

$s = 0$, and with it those who do not rent. Now differentiate with respect to R_2 to find:

$$\frac{\partial S}{\partial R_2} = \frac{1}{N}\sum_{j=1}^{J} s_j \frac{dA_j}{dR_2} \tag{A6}$$

For a small change in p the cost curve appears locally to be translated up by an amount $\frac{e}{s}$ (i.e., $dC = \frac{e}{s} \cdot dp$), while R_2 translates C one for one (i.e. $dC = dR_2$). Therefore

$$\frac{\partial A_j}{\partial R_2} = \frac{e}{s_j} \cdot \frac{dA_j}{dp} \tag{A7}$$

Substituting equation (A7) into (A6) we have

$$\frac{\partial S}{\partial R_2} = \frac{1}{N}\sum_j s_j \frac{e}{s_j} \cdot \frac{dA_j}{dp} = \frac{e}{N} \cdot \frac{dA}{dp}. \tag{A8}$$

In the last term in equation (A4), our current result, substitute eE for S, and use equation (A7) to substitute for $\partial_2 S$. This gives a perfectly usable version of the main result.

$$\eta_e^E = -1 - \eta_p^E + \frac{e^2}{p} R_3' \cdot \eta_p^E + \frac{e}{pE_r} \cdot \eta_p^A \cdot R_2'. \tag{A9}$$

The final transformation makes the equation appear more "natural". In particular, it shows that the result does not depend on the choice of units. Divide the rent function into an "interaction" part, X, and a "translation" part, T, so that

$$X = S \cdot R_3(e) \qquad \text{and} \qquad T = R_1(S) + R_2(e). \tag{A10}$$

Note that

$$\eta_S^X \eta_e^X = \frac{Se}{X} \cdot R_3' \qquad \text{and} \qquad \eta_e^T = \frac{e}{T} R_2'. \tag{A11}$$

Now use equation (A11) to transform (A9) into its final form which was equation (3E) and the goal of this derivation.

$$\eta_e^E = -1 - \eta_p^E \left(1 - \frac{X}{pE} \eta_S^X \eta_e^X\right) + \eta_p^A \frac{T}{pE_r} \eta_e^T \tag{A12}$$

Note that because S was used in the definitions of X and T, they are approximately the average costs for the entire consumer population, and not just appliance renters.

REFERENCES

Henly, John., Henry Ruderman, and Mark D. Levine (forthcoming). "Energy Saving from More Efficient Appliances: Another View." Energy Journal.

Khazzoom, J. Daniel (1980). "Economic Implications of Mandated Efficiency Standards for Household Appliances." Energy Journal (October), pp. 21-40.

_____ (1987). "Energy Saving Resulting from the Adoption of More Efficient Appliances." Energy Journal (October), pp. 85-89.

Ruderman, Henry, Mark D. Levine, and James E. McMahon (1987). "The Behavior of the Market for Energy Efficiency in Residential Appliances Including Heating and Cooling Equipment. Energy Journal (January), pp. 101-124.

Henderson, James M. and Quandt, Richard E., *Microeconomic Theory, A Mathematical Approach*, New York: McGraw-Hill, 1971.

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