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THE DECAY OF THE STRING IN THE SKYRME MODEL†

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Abstract: We discuss the decay of an unstable string-like solution in the Skyrme model. Previously, we found this solution to have a string tension of approximately 0.85 GeV/fm. Here we show that configurations of this kind occur wherever a fundamental Q.C.D. string would be expected. We show that one always produces decaying tubes of energy density when pieces of a Skyrmion are separated in space. The string solution is unstable, we construct an homotopy that deforms it to the vacuum while monotonically decreasing its energy. During the decay a baryon current flows along the string, producing half a baryon and half an anti-baryon. Long strings can decay via many different decay modes, some producing baryon anti-baryon pairs. Finally, we solve the equation of motion for the initial infinitesimal fluctuations that cause the decay, finding the form and growth rate of the unstable mode.

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1. Introduction.

The Skyrme model provides a qualitatively appealing description of many of the properties of the nucleon [1,2,3]. It has been used as a tool to investigate many interesting theoretical questions [4], such as the nucleon nucleon interaction [5], pion nucleon scattering [6], chiral phase transitions in dense matter [7], the strangeness content of the proton [8], and the structure of strange baryons [9]. In this paper, we shall demonstrate that string-like configurations in the model have interesting properties in common with the Q.C.D. string. Since the effective theory contain no explicit quarks, it would seem difficult to describe a string resulting say, from the separation of a quark from a proton in a collision. Nevertheless, baryon density exists explicitly in the theory, and one can describe, for example, a baryon being torn in half in a high energy collision by separating two half Skyrmions. Continuity of the Skyrme field then demands that some configuration interpolates the gap between these separated halves and to do so string-like configurations are natural candidates.

In a previous paper [10], we studied a string-like solution to the equations of motion in the Skyrme model. There we showed that the equations of motion of the Skyrme model with a pion mass term posses a solution with pseudo-cylindrical symmetry, translational symmetry along the cylindrical axis, and baryon density that vanishes everywhere along its length. The solution has the form

$$U = \cos(f(\rho)) + i\tau^i\hat{\rho}^i \sin(f(\rho)) \tag{1.1}$$

where ρ is the radial coordinate from the cylindrical axis and $\hat{\rho}^i$ is the corresponding unit vector. The equation of motion for f is easily solved, and the solution is found to have a string tension of 0.85 GeV/fm and an rms radius of about 1 fm, for the parameters used by Jackson and Rho [3] to describe f_π and $g_{\pi NN}$. This is reviewed in section 2.

While fields of the form (1.1) do solve the static equations of motion, in section 3 we show that the solution is unstable, and demonstrate that the decay proceeds by the generation of a baryon current along the string. The current gives rise to the production of regions of baryon or anti-baryon density and these regions terminate the string. This process is startlingly reminiscent of the decay of Q.C.D. strings via the production of quark antiquark pairs. The two directions in

which baryon current can flow along the string give rise to a variety of decay mechanisms. If a long string decays at several places independently along its length, then either mesons or baryons can result, depending on the directions of the flow of baryon current at adjacent decay sites.

We want to stress the topological nature of our result, production of baryon anti-baryon density is the inevitable result of the decay of a surface to a point in internal space. Recently Ellis and Kowalski [11] have used the topological structure of the Skyrme model and thermodynamic assumptions to deduce qualitative features of baryon production in jets. The thermodynamic methods used there are not employed here, but in both cases baryon production arises topologically from the decay of exited chiral fields.

In section 4, we deduce the specific form of the instability causing the decay of the string by deriving and solving the equation of motion for such modes. The form of this mode is found to be very similar to that of the homotopies constructed in section 3. The growth time for the unstable mode is found to vary with the length of the string, about 0.5 fm/c for infinitely long strings, about 1 fm/c for strings 2 fm long. Strings with length below about 1.6 fm cannot decay at all by this mechanism.

In section 5, we discuss the results, draw our conclusions and indicate directions for future work.

2. The String Solution

In a previous paper, we found a static string-like solution to the equations of motion of the Skyrme model. We will briefly review the structure of this solution.

The static energy density arising from Skyrme's Lagrangian [1] with a mass term is

$$\mathcal{E} = \frac{f_\pi^2}{2} B^i_\mu B^i_\mu + 2\epsilon^2 C^i_{\mu\nu} C^i_{\mu\nu} + m_\pi^2 f_\pi^2 \frac{Tr}{2}(1-U) \ , \tag{2.1}$$

where

$$U = \cos(f) + i\tau^i \hat{\pi}^i \sin(f) \ , \quad U^\dagger U = 1 \ , \tag{2.2}$$

$$B^i_\mu = -i\frac{Tr}{2}(\tau^i U^\dagger \partial_\mu U) \ , \tag{2.3}$$

$$C^i_{\mu\nu} = \epsilon^{ijk} B^j_\mu B^k_\nu \ , \tag{2.4}$$

and, f_π and m_π are the decay constant and mass of the pion, respectively.

The equations of motion for static fields arising from (2.1) are

$$\partial_\mu [a B^i_\mu + b B^j_\nu (B^i_\mu B^j_\nu - B^j_\mu B^i_\nu)] - m_\pi^2 f_\pi^2 sin(f) \hat{\pi}^i = 0 \ , \tag{2.5}$$

where $a = f_\pi^2$ and $b = 8\epsilon^2$.

In [10] we showed that the cylindrical form

$$U = \cos(f(\rho)) + i\tau^i \hat{\rho}^i \sin(f(\rho)) \ , \tag{2.6}$$

subject to the boundary conditions

$$f(\rho) \to 0, \quad \rho \to \infty; \quad f(0) = \pi \ , \tag{2.7}$$

satisfies the equations of motion (2.5), provided f satisfies

$$f''(a + b\frac{\sin^2(f)}{\rho^2}) + \frac{f'}{\rho}(a - b\frac{\sin^2(f)}{\rho^2}) - \frac{\sin(f)\cos(f)}{\rho^2}(a - b(f')^2) - m_\pi^2 a \sin(f) = 0 \ . \tag{2.8}$$

Using (2.6), the energy density (2.1) becomes

$$\mathcal{E} = \frac{a}{2}(f'^2 + \frac{\sin^2(f)}{\rho^2}) + \frac{b}{2} f'^2 \frac{\sin^2(f)}{\rho^2} + m_\pi^2 f_\pi^2 [1 - \cos(f)] \ . \tag{2.9}$$

We then solved equation (2.8) using the parameters of Jackson and Rho [3], $a = 93^2$ MeV^2, $b = 8 \times 0.00552$ and $m_\pi = 137$ MeV. Asymptotically, the solution is a bessel function, $f \to 1.2\, K_1(m_\pi \rho)$. The cylindrical form of the ansatz (2.6) is illustrated in Fig. 1, and the full solution for f is shown in Fig. 2. The string-tension, σ, of the solution is 858 MeV/fm, and the r.m.s. string tension radius, $< r^2 >_\sigma^{1/2} = 1.05$ fm. Further details of the solution are presented in [10].

3. Production and Decay Properties of Skyrme Strings

We now argue that tubular configurations similar to those discussed in section 2 may be expected to occur in many familiar circumstances. To do this, we investigate separated pieces of

the same baryon, and consider how the pion field interpolates between them. Similar arguments can be made for for more general circumstances where non-integral baryon charges are separated in space. For example, in mesons with high angular momentum, the quark and anti-quark are separated from each other.

We assume that by some mechanism (such as, *e.g.*, a high-energy collision) a fraction of a baryon, f_A, has been separated from the remaining $(1-f_A)$. We therefore assume that space can be separated into three regions A, B and C, by two plane surfaces, Σ_A and Σ_B. Behind these surfaces we find the fraction f_A, and $f_B = (1 - f_A)$ in A and B, respectively. We now want to determine the structure of the pion field in C (between Σ_A and Σ_B). We will now show that unless there is non-zero baryon density between Σ_A and Σ_B, *there must be*, minimally, a tube of energy density connecting A to B of the same general type as the solution outlined in section 2. We will later show that the alternative possibility (non-zero baryon density) is just what is produced by the decay of the tube.

It is easier to see these results if we exploit the fact that, topologically, the internal space is equivalent to a ball with points on the surface identified (this way of looking at the target space is naturally suggested by the structure of the hedgehog). We shown this in Fig. 3. For simplicity, we study the case where the separated fractions of baryon number, f_A and f_B cover simply connected pieces of the ball, A' and B', respectively. The more complicated cases where these pieces are not simply connected can be handled in essentially the same way. A' and B' are *complementary*, and separated by a common surface, Σ'. We illustrate this in Fig. 4.

Points in the internal space represented by the surface Σ' must be covered on Σ_A, since these must be continuously connected to points *not* covered in A. The same holds for the surface Σ_B.

We now make a crucial distinction amongst the forms of the pion field that interpolate between Σ_A and Σ_B. We divide these fields into two classes: those that have non-zero baryon density between Σ_A and Σ_B, and those that do not. For the moment we concentrate on the latter.

This restriction implys that, on any spatial surface, Σ_c, between Σ_A and Σ_B, the internal space surface, Σ', is covered. To see this it is sufficient to notice that if some part of Σ is not covered, *e.g.* if σ_c is mapped into the surface Σ'_c in Fig. 5, then some net volume of the ball must have been

covered between Σ_A and Σ_c, and uncovered between Σ_c and Σ_B. But of course, covering a volume of internal space is, by definition, producing baryon or anti-baryon number density. Considering the integral over Σ_c of the energy density of the tube, we can use the Skyrme-Bogomol'nyi bound, [1,12], to show that this is bounded below by $f_\pi^2 \mathcal{A}_{\Sigma'}$, where $\mathcal{A}_{\Sigma'}$ is the area of Σ' (for the form (1.1) this bound yields about 550 MeV/fm). In the presence of the mass term in (2.1), we expect this energy to be localized in a tube, similar in form to that of the solution (1.1), (notice that this form is one example of the interpolating field we are describing, if we have half Skyrmions behind the planes Σ_A and Σ_B). We have established that, in the absence of baryon density between regions A and B, we expect, minimally, a tube of energy density between them. We will show below, that interpolating fields in the alternative class (those with non-zero winding density) are produced when the tube decays.

Witten [13] first demonstrated that flux tubes in effective field theories from underlying SU(N) gauge theories are not topologically stable. Therefore there must exist an homotopy that deforms the solution (1.1) to the vacuum, $U = 1$ everywhere. Furthermore, although the form (1.1) provided us with a genuine solution to the static equations of motion, it is not a stable solution. We now provide an explicit homotopy which deforms the solution to the vacuum (we call this the "umbrella" homotopy, for reasons that will become obvious). The homotopy is constructed in order to display, as clearly as possible, that the energy density decreases at every stage of the process. It is clear that we can lower the energy of the solution we have found by rotating the pion field into the $\hat{z}^i$ direction, since then all gradients of the field resulting from the turning of the vector $\hat{\rho}^i$ around the z axis vanish. Once this is accomplished, the function $f(\rho)$ may be smoothly deformed to a constant.

First we fold up the pion field vectors, like the spokes of an umbrella. For $0 < \mathcal{T} < 1/2$ we use

$$U_{\mathcal{T}} = \cos(f(\rho)) + i\tau^i \left[\hat{\rho}^i \cos(\mathcal{T}\pi) + \hat{z}^i \sin(\mathcal{T}\pi)\right] \sin(f(\rho)) \ . \tag{3.1}$$

Now we decrease the chiral angle to zero. For $1/2 < \mathcal{T} < 1$ we put

$$U_{\mathcal{T}} = \cos(\tilde{f}) + i\tau^i \hat{z}^i \sin(\tilde{f}) \ , \tag{3.2}$$

where $\tilde{f} = f(\rho)\sin(\pi\mathcal{T})$. It is easy to see that, while the pion field vectors are being folded up,

(2.1) gives, for $0 < \mathcal{T} < 1/2$

$$\mathcal{E}_2(\mathcal{T}) = \frac{f_\pi^2}{2}(f'^2 + \cos^2(\mathcal{T}\pi)\frac{\sin^2(f)}{\rho^2}) \ , \quad \mathcal{E}_4(\mathcal{T}) = 4\epsilon^2 f'^2 \frac{\sin^2(f)}{\rho^2}\cos^2(\mathcal{T}\pi) \ , \quad \mathcal{E}_{m_\pi}(\mathcal{T}) = \mathcal{E}_{m_\pi}(0) \ , \tag{3.3}$$

and while the chiral angle is decreased to zero, for $1/2 < \mathcal{T} < 1$,

$$\mathcal{E}_2(\mathcal{T}) = \frac{f_\pi^2}{2} f'^2 \sin^2(\mathcal{T}\pi) \ , \quad \mathcal{E}_4(\mathcal{T}) = 0 \ , \quad \mathcal{E}_{m_\pi} = m_\pi^2 f_\pi^2[1 - \cos(f(\rho)\sin(\mathcal{T}\pi))] \ , \tag{3.4}$$

where we have split up the contributions of the second and fourth order, and pion mass terms. We have neglected any kinetic terms that would arise if such a process occurred in time.

Although we have demonstrated that the solution we have found is unstable, this is not really an objection, since in Q.C.D., it is well known that strings break. Indeed, many of the applications of Q.C.D. string models consist in describing how strings break. We shall therefore concentrate our attention on the decay of the string. As we will find, this process proceeds via the production of baryon anti-baryon number density. One easy way to see this is to notice that, if one interprets $\mathcal{T}$ in the homotopy (3.1,3.2) as parametrizing a time variable, then as the decay of the string proceeds, a baryon number current flows along the string. The gradients of (3.1) can be written

$$B_\mu^a = f'\hat{\rho}_\mu \hat{B}(1)^a + \frac{\sin(f)}{\rho}\cos(\mathcal{T}\pi)\hat{\phi}_\mu \hat{B}(2)^a + \pi\sin(f)\frac{d\mathcal{T}}{dt}\hat{t}_\mu \hat{B}(3)^a \ , \tag{3.5}$$

where $\mathcal{T}$ is regarded as an arbitrary parametrization of time, t, and

$$\hat{B}(1)^a = \cos(\mathcal{T}\pi)\hat{\rho}^a + \sin(\mathcal{T}\pi)\hat{z}^a \ , \quad \hat{B}(2)^a = \cos(f)\hat{\phi}^a + \sin(f)[\cos(\mathcal{T}\pi)\hat{z}^a - \sin(\mathcal{T}\pi)\hat{\rho}^a] \ ,$$

$$\text{and } \hat{B}(3)^a = \epsilon^{abc}\hat{B}(1)^b\hat{B}(2)^c \ , \tag{3.6}$$

are orthonormal unit vectors in iso-space. The baryon number that crosses the surface Σ_z, between $\mathcal{T} = 0$ and $\mathcal{T} = 1/2$ is then

$$\mathcal{B} = \int_{t_0}^{t_{1/2}} dt \int_{\Sigma_z} dS B^\mu \hat{z}_\mu \ , \tag{3.7}$$

where the baryon current B^μ is given by

$$B^\mu = -\frac{\epsilon^{\mu\nu\alpha\beta}}{12\pi^2}\epsilon^{abc}B_\nu^a B_\alpha^b B_\beta^c \ , \tag{3.8}$$

and t_0 and $t_{1/2}$ are the times corresponding to $\mathcal{T} = 0$ and $\mathcal{T} = 1/2$, respectively. Putting (3.5) in (3.7,3.8), it is then easy to show that

$$\mathcal{B} = -\int_0^{1/2} d\mathcal{T} \cos(\mathcal{T}\pi) \int_0^{\pi} df \sin^2(f) = -1/2 \ . \tag{3.9}$$

One can easily convince one's self that between $\mathcal{T} = 1/2$ and $\mathcal{T} = 1$, no baryon current flows. The result, (3.9), is independent of the exact form of the homotopy used to deform the string to the vacuum. The only exception to this is the sign of $\mathcal{B}$, which may be changed by choosing *e.g.*, $-\hat{z}^i$ instead of $\hat{z}^i$ in (3.1). This sign ambiguity has important consequences, as we will see.

An easier and more intuitive way of seeing this result is to use the picture of the internal space as a ball. At any given time t, Σ_z is mapped into a surface $\Sigma'(t)$ in the ball. As the decay proceeds via (3.1,3.2), $\Sigma'(t)$ is deformed from an equatorial slice through the ball to a point on its surface. This is shown in Fig. 6. The quantity of baryon number crossing Σ_z during the decay is then just the fractional volume of the ball swept out by the surface $\Sigma'(t)$. It is then clear that, *independent* of the details of the homotopy used, the net quantity of baryon number that crosses the surface Σ_z is one half. Further, since we have transferred baryon number one half across Σ_z, we must have created a deficit of one half on the other side of Σ_z.

The homotopy (3.1) and (3.2) demonstrates the classical instability in the solution, but it does not deal with the boundary conditions at the ends of the tube. We will now study this problem. For the sake of definiteness, we will consider the case where two half Skyrmions are separated by a distance $2l$ along the z axis. We therefore hold the field fixed in the form (2.16) on two surfaces $\Sigma_\pm$ at $z = \pm l$ respectively. Between these surfaces the field is free to decay. However, with these boundary conditions, the decay cannot proceed uniformly along the length of the string, since then we would encounter discontinuities on $\Sigma_\pm$, where the tube joins the half Skyrmions.

As noted above, there are two possible directions in which the baryon current can flow, $\pm\hat{z}^i$. If the string is long enough, there is no reason to suppose that this direction need be chosen uniformly along the length of the string. However, we will consider the simplest cases first, where there is only one direction for the whole string. After we have dealt with these, the other cases, where different directions are chosen at several places along the string, will be constructed from them by patching copies of them together in a chain.

When the decay direction is the same along its length, $-\hat{z}^i$, the string can decay as a whole with the exception of the ends. We now provide an interpolating form that gives an explicit picture of what happens there. Since we are chiefly interested in the baryon number flow, we shall not attempt to optimize its dynamical structure. To perform the interpolation, use the following trick:

Imagine a skyrmion at $z = -l$, and then stretch it out more and more violently in the z direction, as the field stretches it becomes more and more cylindrical in form. One can picture the interpolation as merely a less and less violent stretching until a complete half Skyrmion is pulled back between $z = -l$ and $z = 0$. Note that this naturally incorporates the condition that one half unit of baryon number flows across the surface $z = 0$. To achieve this is simple, take $0 < \mathcal{T} < 1$ and now write for $0 > z > -l$

$$U'(x, y, z, \mathcal{T}) = \cos(f(r'(\mathcal{T}))) + i\tau^i \hat{r}'^i(\mathcal{T}) \sin(f(r'(\mathcal{T}))) \ , \tag{3.10}$$

where we define $r'^i(\mathcal{T})$ as

$$r'^i(\mathcal{T}) = x\hat{x}^i + y\hat{y}^i + z'(\mathcal{T})\hat{z}^i \ , \quad \text{with} \quad z'(\mathcal{T}) = \tanh^{-1}(\mathcal{T}(z + l)/l) \ , \tag{3.11}$$

and f is the chiral angle function having the property $f(X) \to 0$, as $X \to \infty$; and $f(X) \to \pi$, as $X \to 0$. Notice that, for $\mathcal{T} = 0$, $z'(0) = 0$ for all $0 > z > -l$, and the field is the cylindrical one (1.1), whereas for $\mathcal{T} = 1$, the field is that of a distorted half Skyrmion

$$U'_{s-}(x, y, z, 1) = \cos(f(r'(1))) + i\tau^i \hat{r}'^i(1) \sin(f(r'(1))) \ , \tag{3.12}$$

where now $z'(1) = \tanh^{-1}((z + l)/l)$. Note what happens as $\mathcal{T} \to 1$. At $z = -l$, $z'(\mathcal{T}) = 0$ for all $\mathcal{T}$, but at $z = 0$, we find instead $z' = \tanh^{-1}(\mathcal{T}) \to \infty$ as $\mathcal{T} \to 1$. In particular, at $z = 0$, the vacuum state ($U = +1$) reaches in all the way to the z axis. If we follow the evolution of the pion field on the plane $z = 0$, we see that due to the definition of $\hat{z}'^i$ in (3.11), the pion field vectors rotate into the $\hat{z}^i$ direction, and eventually shrink in size to zero magnitude.

Now we specify how to handle the region $l > z > 0$. The simplest thing to do is write for $l > z > -l$

$$U'(x, y, z, \mathcal{T}) = U'(x, y, -z, \mathcal{T}) \ , \tag{3.13}$$

i.e. use reflection about $z = 0$. Note that internal space vectors do not change sign under this, but derivatives of the field in the z direction do. In consequence we guarantee both continuity at $z = 0$ and the change of sign of the Jacobian across the plane $z = 0$. Thus we create half an anti-Skyrmion between $z = 0$ and $z = +l$. The geometry of the decay process is illustrated in Fig. 7.

While (3.10,11,13) are sufficient for our purpose, the gradients are not optimal. In particular, there is a discontinuity in the gradient at $z = 0$, this can be removed by using *e.g.*, $z'(\mathcal{T}) = \tanh^{-1}\left[\mathcal{T}\sin\left(\pi(z+l)/2l\right)\right]$ in (3.11).

Given the original half Skyrmions on the ends of the string we can expect the half anti-Skyrmion and half Skyrmion at $z = +l$ to annihilate to produce pions. At the other end of the tube, there is now a whole Skyrmion at $z = -l$. Therefore this case corresponds to the decay of the string into mesons. Note that decay cannot proceed until there is at least enough energy in the string to create a half Skyrmion and a half anti-Skyrmion, *i.e.* about the Skyrmion mass. This puts a lower limit on the length of the string of about 1.7 fm using the values of the string tension and Skyrmion mass obtained from the Jackson-Rho parameters.

The case where the direction of the baryon current is $+\hat{z}^i$ can be obtained from (3.10,11,13) by using

$$z'(\mathcal{T}) = -\tanh^{-1}(\mathcal{T}(z+l)/l), \quad 0 < \mathcal{T} < 1 \,, \tag{3.14}$$

in (3.11). This just exchanges the positions of the half Skyrmion and the half anti-Skyrmion.

To discuss more complicated decays, that will be relevant for long strings, we will introduce an abbreviated diagramatic notation to characterize the possibilities. In this notation, the diagrams

$$(+|\underline{\overline{---}}|+) \to (+|+)(-|+) \,, \quad \text{or} \quad (+|\underline{\overline{---}}|+) \to (+|-)(+|+)$$

represent the simplest possibilities: the decay of the string, $\underline{\overline{---}}$, with half Skyrmion endcaps, $(+|$ and $|+)$, via production of a half Skyrmion and a half anti-Skyrmion, $|+)(-|$ or $|-)(+|$. The end result is a Skyrmion, $(+|+)$ and mesons *e.g.* $(-|+)$.

For more complex decays, we can patch together chains of the elements $|+)(-|$ and $|-)(+|$. The four diagrams corresponding to choosing two decay directions along the length of the string

are shown below.

$$(+|\overline{}|+) \rightarrow (+|-)(+|+)(-|+) \tag{3.16a}$$

$$(+|\overline{}|+) \rightarrow (+|+)(-|-)(+|+) \tag{3.16b}$$

$$(+|\overline{}|+) \rightarrow (+|+)(-|+)(-|+) \tag{3.16c}$$

$$(+|\overline{}|+) \rightarrow (+|-)(+|-)(+|+) \tag{3.16d}$$

Three of these diagrams, (3.16a,c,d) correspond to producing a Skyrmion plus mesons, with the skyrmion produced at different places along the string. However, diagram (3.1bb) is different, and corresponds to the production of an anti-Skyrmion in the center of the string, and one Skyrmion at each end. Note that such a mode is not possible unless there is at least enough energy in the string to produce the mass of the Skyrmion anti-Skyrmion pair. Using the parameters in (2.1) adopted here [3], this requires the string to be at least 3.3 fm long. From (3.16a-d), it should be clear that there are many possible decay modes of the string, corresponding to the production of different numbers of mesons and baryon anti-baryon pairs at different places along the string. Baryons and anti-baryons are correlated in an alternating sequence down the string for all decay modes, but elements in the sequence can be separated by any number of mesonic objects, *e.g.*,

$$(+|\overline{}|+) \rightarrow (+|-)(+|+)(-|-)(+|-)(+|+)\,. \tag{3.17}$$

We note in passing that an uninterupted alternating sequence of baryons and antibaryons is essentially similar to the ansatz of [14], putting $m = 1$ and removing the rather un-natural modulus sign in equation (9) there.

We can make contact with the program of reference [11], by estimating the probability of a long string producing baryon anti-baryon pairs. If we assume that such a string decays, independently, at n_d sites along its length, and that strings of the form (1.1) decay with equal probability for the baryon current to flow in either direction along the string, then the probability of producing a baryon or anti-baryon between any two decay sites is the 1/2. The average number of baryons plus anti-baryons produced between n_d decay sites is therefore $(n_d - 1)/2$. For string-like configurations whose form is not that of (1.1) the probability for the baryon current to flow in either direction

need not be symmetrical, and the probability of producing a baryon between any two decay sites is decreased. If the probability to produce baryon current flowing in the two directions are $P_+ = 1/2+\epsilon$ and $P_- = 1/2 - \epsilon$, then the probability to produce a baryon between two decay sites is $1/2 - 2\epsilon^2$. We will comment further on this matter in section 5.

Examining Fig. 7, we can see that reversing the order of the sequence, we have an explicit set of configurations that lead to the annihilation of baryon density. We expect that when a $B - \overline{B}$ pair with the appropriate relative rotation (π about the separation axis) approach to a distance on the order of a fermi, the string-like configurations discussed here become relevant. The sequence (d,c,b,a) in Fig. 7 shows clearly how the gradients in the z direction are removed by the formation of a string. Thus, string-like configurations may be important intermediate stages in $B - \overline{B}$ annihilation processes.

4. Fluctuations around the Skyrme String.

In the last section, we looked at the decay properties of the Skyrme string using very general arguments. This was possible since we were chiefly interested in demonstrating the instability of the string and its connection with the production of baryon density. In this section we investigate the exact form of the fluctuations around the solution that cause the decay of the string. Whilst we can make more precise statements concerning this problem, they are of less general relevance, since they are only applicable to the string solution discussed in section 2, and not to the more generic tubular configurations. We first find the most general form of infinitesimal fluctuations about the string solution, and then write the linearized equations of motion for them. This system is block diagonal in the basis we choose, consisting of one matrix governing modes of translation, rotation and vibration of the string, and one other isolated equation. Through analogy with the two dimensional skyrmion, we argue that fluctuations that correspond to vibrations of the string cannot lower its energy. The remaining equation posses' an unstable mode similar in form to the homotopies of section 2.

The equations of motion for the Skyrme model can be written

$$\partial_\mu[aB^i_\mu + bB^j_\nu(B^i_\mu B^j_\nu - B^j_\mu B^i_\nu)] + m_\pi^2 f_\pi^2 \frac{Tr}{2}[-i\tau^i U] = 0 \ . \tag{4.1}$$

where now greek indices indicate four vectors. We now substitute the form

$$B^i_\mu = B^i_{\mu s} + \eta^i_\mu \ , \tag{4.2}$$

where $B^i_{\mu s}$ are the gradients of the string, and η^i_μ are the gradients of an infinitesimal fluctuation around the string. If the fluctuation of U is considered to be

$$U = U_s + \Delta \ , \tag{4.3}$$

where U_s is the string solution, and Δ the fluctuation, then to first order in Δ

$$i\tau^i \eta^i_\mu = U^\dagger \partial_\mu \Delta + \Delta^\dagger \partial_\mu U \ . \tag{4.4}$$

We will consider the field to be fixed on plane surfaces at $\pm L/2 = \pm l$, and at infinity, so that $\Delta = 0$ on these surfaces. Substituting (4.2-4) into (4.1) [15], we find the linearized equation of motion for the fluctuations

$$\partial_\mu[a\eta^i_\mu + b\eta^j_\nu(B^i_{\mu s}B^j_{\nu s} - B^j_{\mu s}B^i_{\nu s}) + bB^j_{\nu s}(\eta^i_\mu B^j_{\nu s} + B^i_{\mu s}\eta^j_\nu - \eta^j_\mu B^i_{\nu s} - B^j_{\mu s}\eta^i_\nu)] + m_\pi^2 f_\pi^2 \frac{Tr}{2}[-i\tau^i \Delta] = 0 \ . \tag{4.5}$$

The most general form of the fluctuation is constrained by the condition (2.2). Using (4.3) gives, to first order in Δ

$$U_s^\dagger \Delta + \Delta^\dagger U_s = 0 \ . \tag{4.6}$$

This expresses the fact that we can only fluctuate in the tangent volume to the sphere in internal space. We can satisfy this if we choose Δ to be of the form

$$\Delta = \delta_1 \Delta_1 + \delta_2 \Delta_2 + \delta_3 \Delta_3 \ , \tag{4.7}$$

where δ_1, δ_2 and δ_3 are arbitrary infinitesimal continuous functions obeying the boudary conditions, and

$$\Delta_1 = \sin(f) + i\tau^i \hat{\rho}^i \cos(f) \ , \quad \Delta_2 = i\tau^i \hat{\phi}^i \ , \text{ and } \ \Delta_3 = i\tau^i \hat{z}^i \ . \tag{4.8}$$

These are explicitly constructed to lie in three orthogonal directions in the tangent volume. Notice that Δ_1 produces local changes in the chiral angle, Δ_2 rotates the pion vector about the z axis, and Δ_3 has the same general character as the homotopy (3.1).

Substituting (4.7-8) into (4.5) reveals, (after a lot of algebra) a remarkable result. The equations of motion for δ_3 are completely decoupled from those for δ_1 and δ_2. We argue below that fluctuations of the form

$$\overline{\Delta} = \delta_1 \Delta_1 + \delta_2 \Delta_2 \ , \tag{4.9}$$

can only raise the energy of the configuration. Therefore we neglect them and concentrate on fluctuations of the form

$$\tilde{\Delta} = \delta_3 \Delta_3 \ . \tag{4.10}$$

This vastly simplifies matters, since we need consider only one equation for one function, rather than a coupled set of three equations for three functions.

To see that the fluctuations $\overline{\Delta}$ raise the energy, recall the picture of the internal space as a ball. The solution (1.1) covers an equatorial surface in the ball. Then it is clear that fluctuations like $\overline{\Delta}$ consist of distortions of (1.1) that still cover the same surface. Using the arguments of section 3, we can see that such fluctuations maintain the tubular structure of the field. For the moment consider the energy density in any plane at constant z. Let us divide the contributions into two pieces, those coming from the z component of gradients of the field, and those that do not. With the exception of the gradients in the z direction, this problem reduces to that of the two-dimensional Skyrme model with a mass term, where one expects the $2-D$ skyrmion (whose form is essentially identical to (1.1)) to be the lowest energy static winding number one solution. Neglecting the contribution of terms depending on the gradients in the z direction, the lowest energy configuration would be given by $\overline{\Delta} = 0$. Now consider the effect of gradients in the z direction. The gradients of the field contribute to the energy density in the form of positive definite squares of combinations of them. Hence any additional gradients raise the energy. Thus, taking into account the gradients of the field in the z direction, we find our previous conclusion, $\overline{\Delta} = 0$ is strengthened.

We therefore concentrate on the case where Δ is given by $\tilde{\Delta}$. Using (4.10) and (4.8) in (4.4),

we find

$$\eta^i_\mu = (\partial_\mu \delta_3)\hat{B}(3)^i + f'\hat{\rho}_\mu \delta_3 \hat{B}(2)^i - \frac{\sin f}{\rho}\hat{\phi}_\mu \delta_3 \hat{B}(1)^i \ , \tag{4.11}$$

where now

$$\hat{B}(1)^i = \hat{\rho}^i, \quad \hat{B}(2)^i = (\hat{z}^i \sin f + \hat{\phi}^i \cos f), \quad \hat{B}(3)^i = (\hat{z}^i \cos f - \hat{\phi}^i \sin f) \ . \tag{4.11a}$$

Substituting (4.8,4.10) and (4.11) into (4.5) gives only one non trivial equation for δ_3

$$\Big([1 + \frac{b}{a}\frac{\sin^2 f}{\rho^2}](\frac{\partial^2}{\partial \rho^2}) + [1 + \frac{b}{a}\frac{\sin f}{\rho}(2f' \cos f - \frac{\sin f}{\rho})](\frac{1}{\rho}\frac{\partial}{\partial \rho}) + [1 + \frac{b}{a}f'^2](\frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2})$$
$$+ [1 + \frac{b}{a}(f'^2 + \frac{\sin^2 f}{\rho^2})](-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2})$$
$$+ \frac{\sin^2 f}{\rho^2}[1 + \frac{b}{a}f'^2] + f'^2[1 + \frac{b}{a}\frac{\sin^2 f}{\rho^2}] - m_\pi^2 \cos f \Big)\delta_3 = 0 \ . \tag{4.12}$$

When $f = 0$, this reduces to the Klein-Gordon equation. Since f is a function only of ρ, equation (4.12) separates completely, and can be solved with the functions

$$\delta_3 = \mathrm{T}(t)\ Z(z)\ \Phi(\phi)\ R(\rho) \ ,$$

$$\mathrm{T}(t) = e^{\Omega t}, \quad \Phi(\phi) = e^{\pm i m_\phi \phi}, \ m_\phi = \text{integer},$$
$$Z(z) = \cos(k_z z), \ k_z = \frac{(2n+1)\pi}{L}, \quad \text{or} \quad Z(z) = \sin(k_z z), \ k_z = \frac{2n\pi}{L}, \quad n = \text{integer} \ , \tag{4.13}$$

where $Z(z)$ obeys the boundary conditions at $z = \pm L/2$. Putting (4.13) into (4.12) gives:

$$\Big([1 + \frac{b}{a}\frac{\sin^2 f}{\rho^2}](\frac{\partial^2}{\partial \rho^2}) + [1 + \frac{b}{a}\frac{\sin f}{\rho}(2f' \cos f - \frac{\sin f}{\rho})](\frac{1}{\rho}\frac{\partial}{\partial \rho})$$
$$- [1 + \frac{b}{a}(f'^2 + \frac{\sin^2 f}{\rho^2})](\Omega'^2) - [1 + \frac{b}{a}f'^2](\frac{m_\phi^2}{\rho^2})$$
$$+ \frac{\sin^2 f}{\rho^2}[1 + \frac{b}{a}f'^2] + f'^2[1 + \frac{b}{a}\frac{\sin^2 f}{\rho^2}] - m_\pi^2 \cos f \Big) R(\rho) = 0 \ , \tag{4.14}$$

where we define the reduced eigenvalue,

$$\Omega'^2 = \Omega^2 + k_z^2 \ . \tag{4.15}$$

Since we are interested in the most unstable mode, we will seek the solution where $k_z = \pi/L$, $m_\phi = 0$, and Ω is real, positive, and as large as possible. So we set $m_\phi = 0$, and examine the limits of (4.14) for large and small ρ. Since $f(\rho) \to 0$ for $\rho \to \infty$, and $f \to \pi - \alpha\rho$ for $\rho \to 0$, we find that we can solve (4.14) with the Bessel functions, $K_0(\kappa_1\rho)$ and $J_0(\kappa_2\rho)$ for large and small ρ respectively, where

$$\kappa_1 = \sqrt{\Omega'^2 + m_\pi^2}\ , \quad \text{and} \quad \kappa_2 = \left(\frac{m_\pi^2 + 2\alpha^2(1 + b\alpha^2/a) - \Omega'^2(1 + 2b\alpha^2/a)}{(1 + b\alpha^2/a)}\right)^{1/2} . \tag{4.16}$$

Using these limits, we have solved equation (4.14) for $R(\rho)$ and for the largest possible value of Ω'^2. Using the solution discussed in section 2, obtained using the values of Jackson and Rho [3] for a, b and m_π, we find $\Omega'^2 = 3.78\ \text{fm}^{-2}$. The solution is shown in Fig. 8. Notice that the magnitude of $R(\rho)$ decreases monotonically, and dies away exponentially for large ρ, the r.m.s. radius is about ~ 2 fm.

The minimum length for which decay can occur can be calculated from the reduced eigenvalue Ω'^2, using (4.15) and $k_z = \pi/L$, we find

$$\Omega(L) = \sqrt{\frac{\Omega'^2 L^2 - \pi^2}{L^2}} . \tag{4.17}$$

For $\Omega'^2 = 3.78\ \text{fm}^{-2}$, the numerator in the square root goes negative at a critical length of $L_c = 1.62$ fm (very close to the length of string containing the mass of one skyrmion) and below this length, the string is stable. The characteristic time for the instability to grow is given by $\tau(L) = 1/\Omega(L)$, which diverges at $L = L_c$, but decreases rapidly above L_c so that for $L = 2$ fm, $\tau = 0.87$ fm. In the limit $L \to \infty$ we find $\tau \to 0.51$ fm.

Since the possible values of k_z are all integer multiples of this fundamental value π/L, the higher modes in z all have critical lengths which are likewise integer multiples of this fundamental one. Note that for higher values of k_z, the function $Z(z)$ goes negative, implying that the decay direction reverses and the baryon current flows in opposite directions in different parts of the string.

5. Discussion and Conclusions

Our purpose in this paper has been to demonstrate that string-like configurations in the Skyrme

model have intriguing properties in common with the Q.C.D. string. We have shown that these configurations arise naturally when objects with complimentary baryon number fractions are separated. We have examined the decay of these configurations in some detail, and found that this process is *inevitably* accompanied by the production of baryon-antibaryon number density. This production occurs wherever and whenever the string snaps, a feature startlingly reminiscent of the production of quark anti-quark pairs in Q.C.D. string models. We can restate these results in a particularly clear way using the arguements of section 3. Topological charge is confined into integer lumps. When fractions of a baryon (say f and $(1-f)$) are seperated from each other *without production of charge anti-charge density* then the energy of such a configuration will increase *at least* linearly with the seperation. To circumvent this result, it is necessary to produce topological charge-anticharge density between the seperated fractions. This charge anti-charge density *screens* the original topological charge density. This fits very nicely with the conjecture advanced by Witten [13].

We have tried to keep the discussion as general as possible, and have considered the properties of the decay of tube-like configurations that are topological in origin. Our reasons for doing so are simple. While one may not believe the precise dynamical properties of the Skyrme model alone, we believe that since the topological content of the model is, to a large part, independent of these details, they should hold when modifications to the model are made. Thus, whilst we have the concrete example of the string solution of section 2 in mind, many of our arguments are specifically designed to be applicable to quite general configurations covering surfaces in internal space. It is therefore worth stressing that most of the arguments in section 3 are based on continuity and topology alone.

The analysis of section 4 is an exception to that approach, and we expect the time and length scales deduced there to be subject to modification when additional dynamical factors are taken into account. Since the energy of the solution is lowered when it decays, this process must be accompanied by the emission of pions. The dynamical details of the decay will also be effected if the couplings of other mesons to the pion field are included. To see how these details could be modified, consider, very roughly, the effect of coupling the ω meson to the baryon current in the

manner of Adkins and Nappi [16]. Since the baryon density in the static string solution vanishes everywhere, the ω meson does not couple to it - for this reason we expect that the solution itself will survive unscathed in such a theory. However, when the string begins to decay a baryon current develops in the z direction, and this acts as a source for the z component of the ω field. We must then take into account the possibility of the emission of ω mesons. Given these uncertainties, the most probable decay time for a string of a given length is likely to be longer than the time-scale calculated here.

Several problems remain before one can hope to firmly associate the string-like configurations discussed here with Q.C.D. strings. One of the most serious of these is that the string tension and decay time for the string (1.1) are $O(N)$ and $O(1)$, respectively. The predicted size of these quantities for the fundamental string are [17] $O(1)$ and $O(1/N)$, respectively. However, when $N = 3$, the difference between removing 1 quark from a baryon, and removing 1/2 the quarks (*i.e.* $N/2$), is small. If, instead of separating a baryon into two halves, we had removed 1/3 of the baryonic charge, we would certainly have lowered the string tension. However, in practice, not by too much. For example use the form (3.1), and put $\sin(\mathcal{T}\pi) = 1/3$, then the energy density (3.3) is very little changed from its value at $\mathcal{T}\pi = 0$, since $\cos^2(\mathcal{T}\pi) = 8/9$. Of course, this change will destroy the symmetry between the two decay modes of the string and alter the probabilities for baryon production as noted in section 3. Moreover, there is no time independent solution to the equations of motion with this form.

However, if the connection of the Skyrme string to the Q.C.D. string can be made stronger [18], many applications suggest themselves. For example, in modeling the relativistic heavy ion collisions performed at CERN, frequent use is made of string models [19]. The density of strings arising from individual nucleon nucleon collisions in these models is on the order of $2/\text{fm}^2$. It is therefore of urgent practical interest to understand the interactions that are possible between these strings. The interaction between Skyrme strings can be studied in the same way as the interactions between skyrmions and two-dimensional skyrmions, using the product ansatz [5] or lattice methods [7,19]. The latter suggests interesting possibilities. When the density of two-dimensional skyrmions gets large, a phase transition takes place restoring chiral symmetry and the skyrmions lose their

individual identity.

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Figure Captions

Fig. 1. The geometrical form of the string solution. The outline represents a section of the cylindrical surface for which $U = i\tau^i\hat{\rho}^i$, the arrows represent the direction of the pion field vector, and the line through the center of the cylinder represents the line where $U = -1$.

Fig. 2. The chiral angle, f, for the solution to equation (2.8), using $m_\pi = 137$ MeV, and the parameters of Jackson and Rho [3], $\epsilon^2 = 0.00552$, $f_\pi = 93$ MeV. The solid curve is the numerical solution, the dashed curve is the asymptotic form $f(\rho) = 1.1972\ K_1(m_\pi\rho)$.

Fig. 3. The internal space organized as a ball. All points on the surface of the ball are identified as the single point $U = 1$. The circle with arrows represents the sphere on which $U = i\tau^i\hat{\pi}^i$.

Fig. 4. The internal space organized as a ball, showing the surface Σ' separating the complimentary regions A' and B'.

Fig. 5. The surfaces Σ' and Σ'_c in internal space, surrounding a volume covered between them.

Fig. 6. The surfaces Σ' in internal space, as the homotopy (3.1) proceeds. $\mathcal{T} = 0$ is top left, and $\mathcal{T}$ increases clockwise to $\mathcal{T} = 1/2$ at bottom left. When $\mathcal{T} = 1/2$, the surface has swept out 1/2 of the volume of the internal space.

Fig. 7. A sequence of illustrative snapshots of the chiral field of the string seen in cross section through the z-axis as it decays according to the homotopy (3.10-12,3.18). The field has rotational symmetry about the z-axis at all times and is fixed at the left and right edges of the boxes. Open circles represent the field $U = -1$, crosses $U = +1$, and the vectors represent the pion field vector. The decay proceeds clockwise from top left. a) The initial state of the string with translational symmetry in the z direction. b) The decay starts, and the field vectors around $z = 0$ rotate towards the $\hat{z}^i$ direction. A baryon current develops in the $-z$ direction. c) The decay proceeds and the baryon density begins to accumulate. d) Very near the end of the decay, a half skyrmion and a half anti-skyrmion have accumulated on the left and right hand edges of the box, respectively. Near the center of the box, the field has nearly reached the vacuum value.

Fig. 8. $R(\rho)$, the radial form of the decay mode against radius is shown in the solid curve. Also shown for comparison in the dashed curves are the asymptotic forms $J_0(\kappa_2\rho)$, and $K_0(\kappa_1\rho)$.

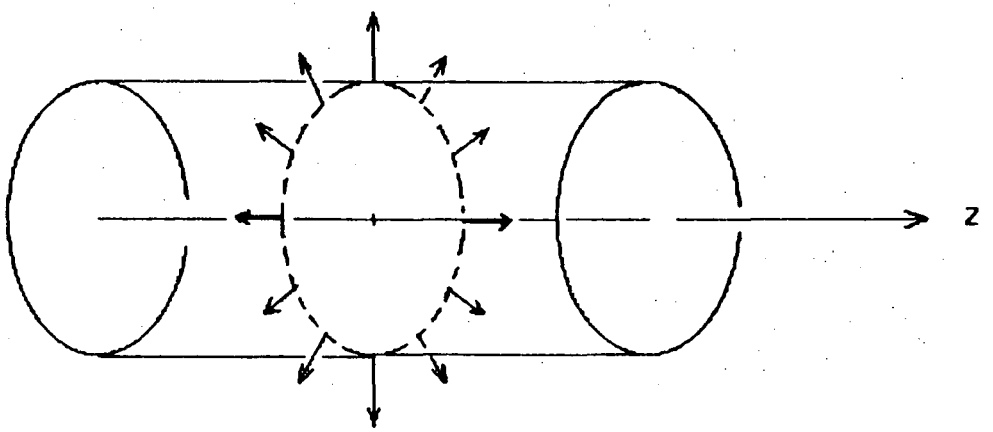

Fig. 1.

Chiral angle, F, against ρ.

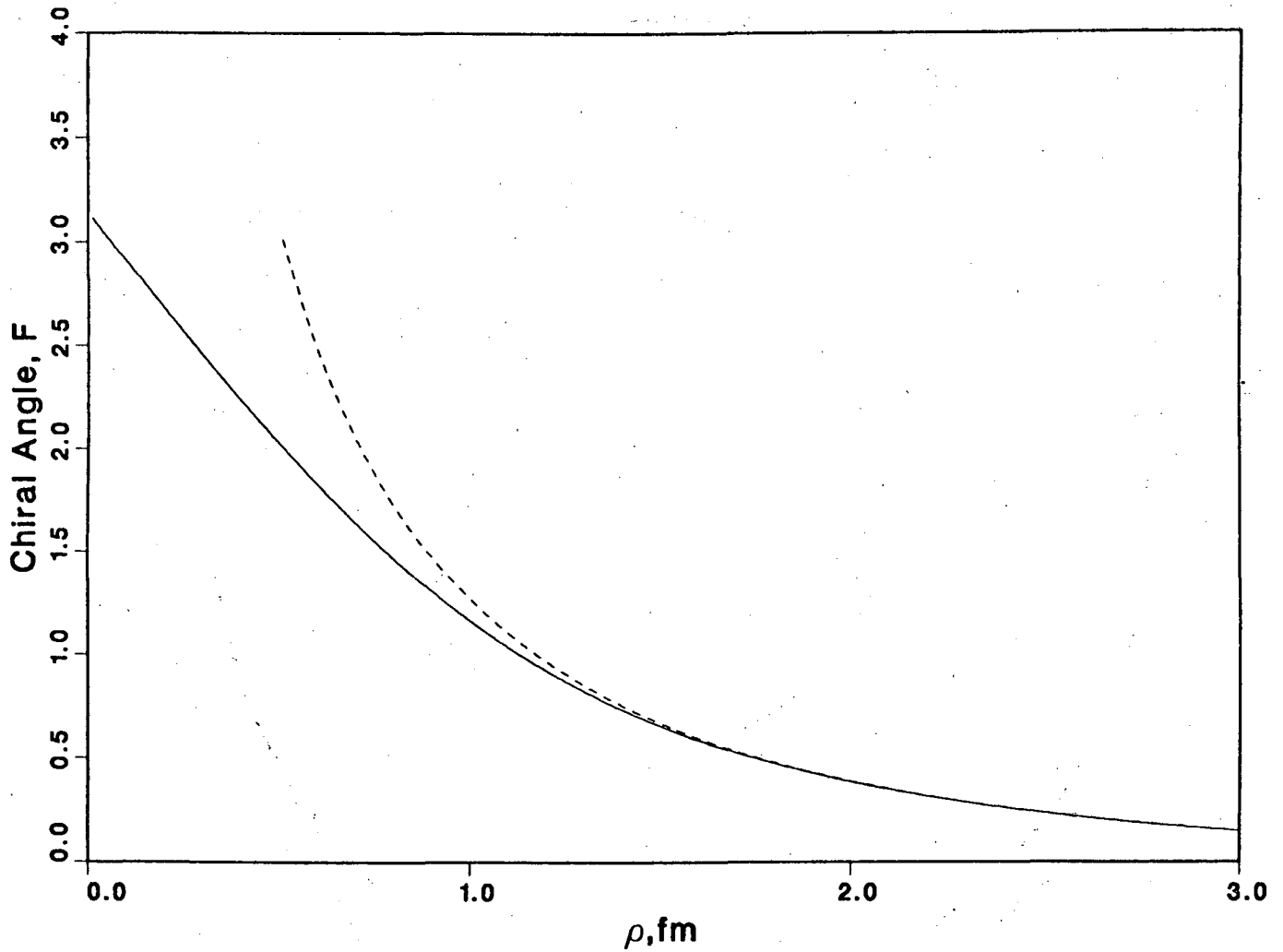

Fig. 2.

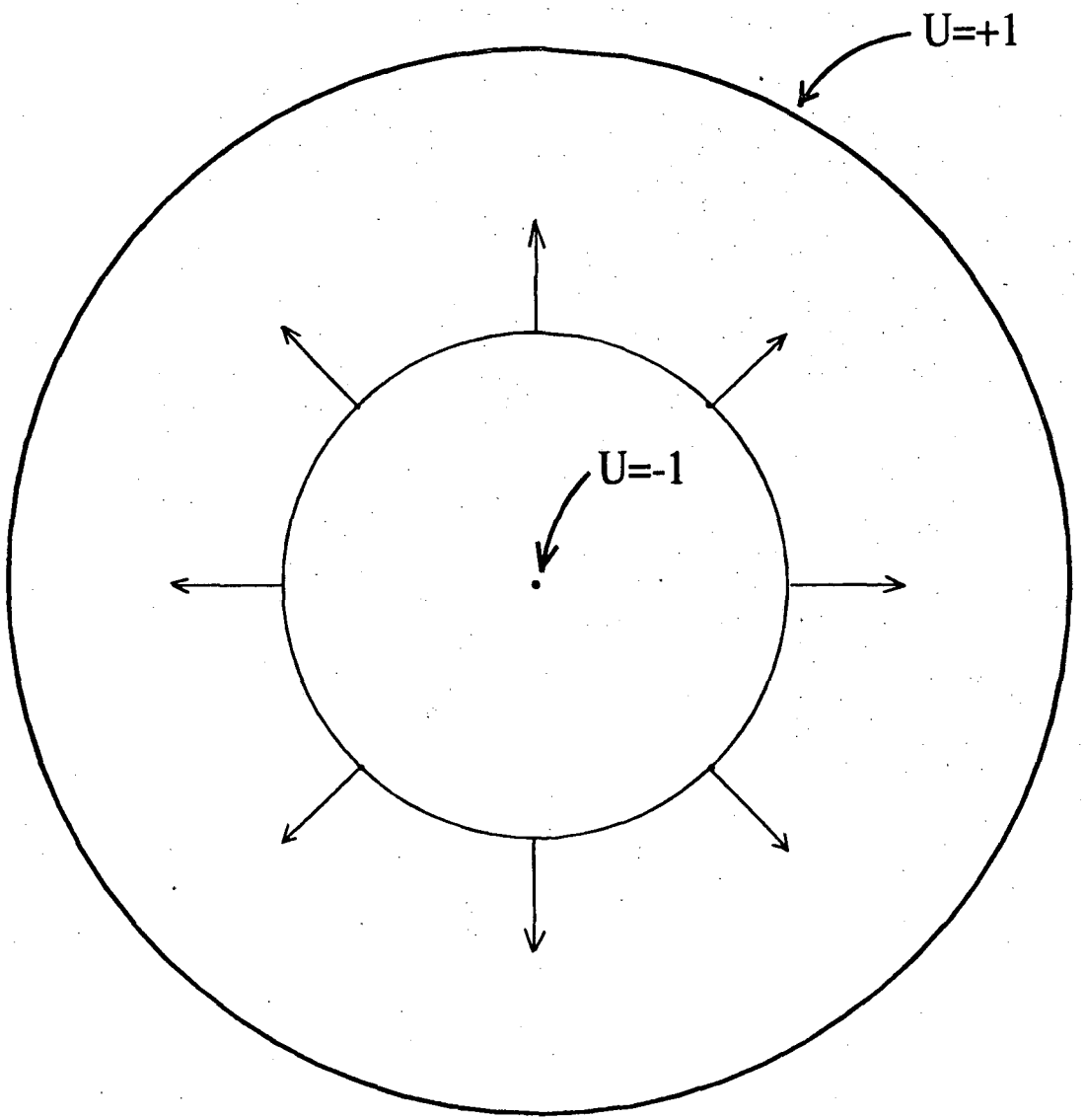

Fig. 3.

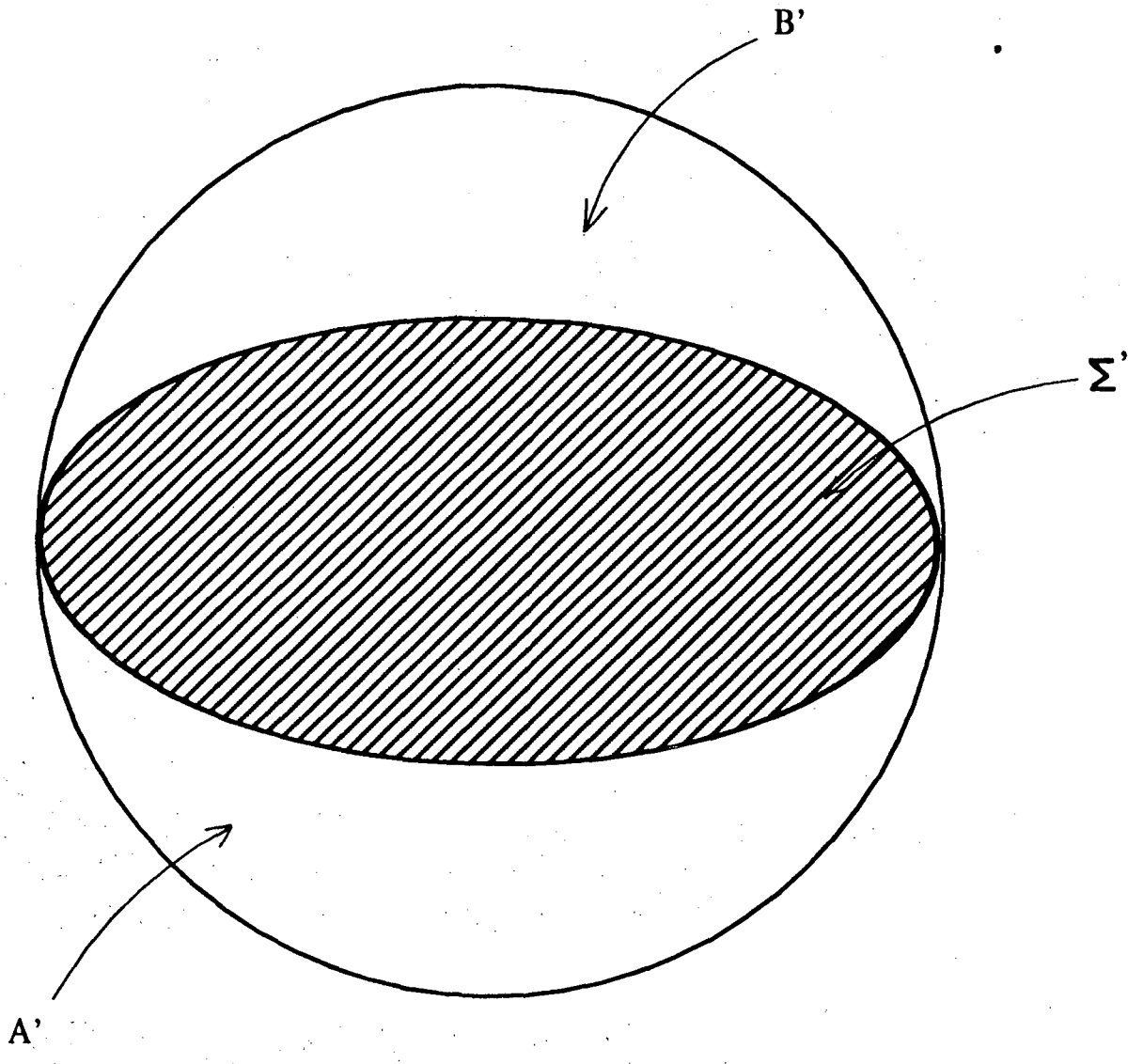

Fig. 4.

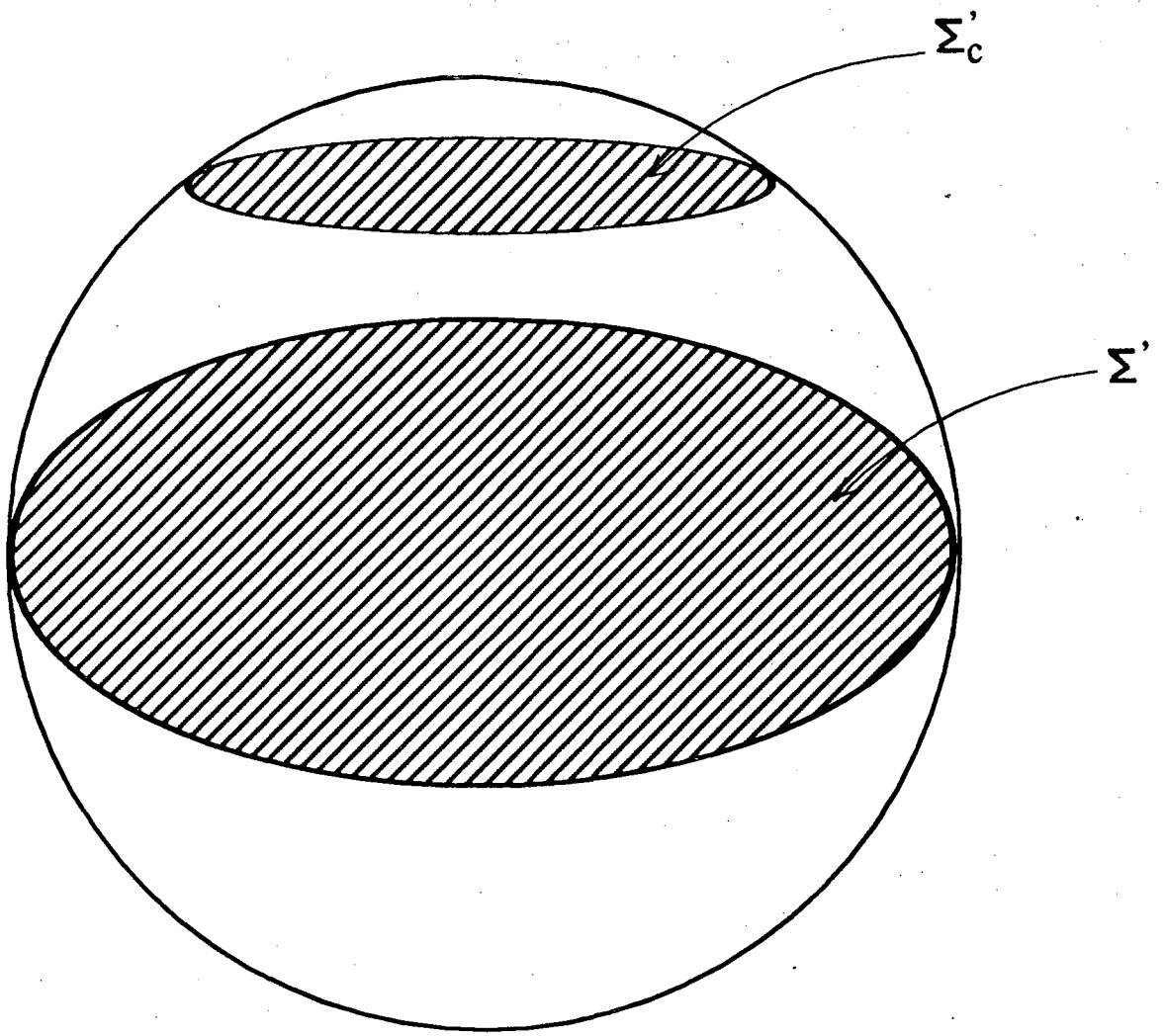

Fig. 5.

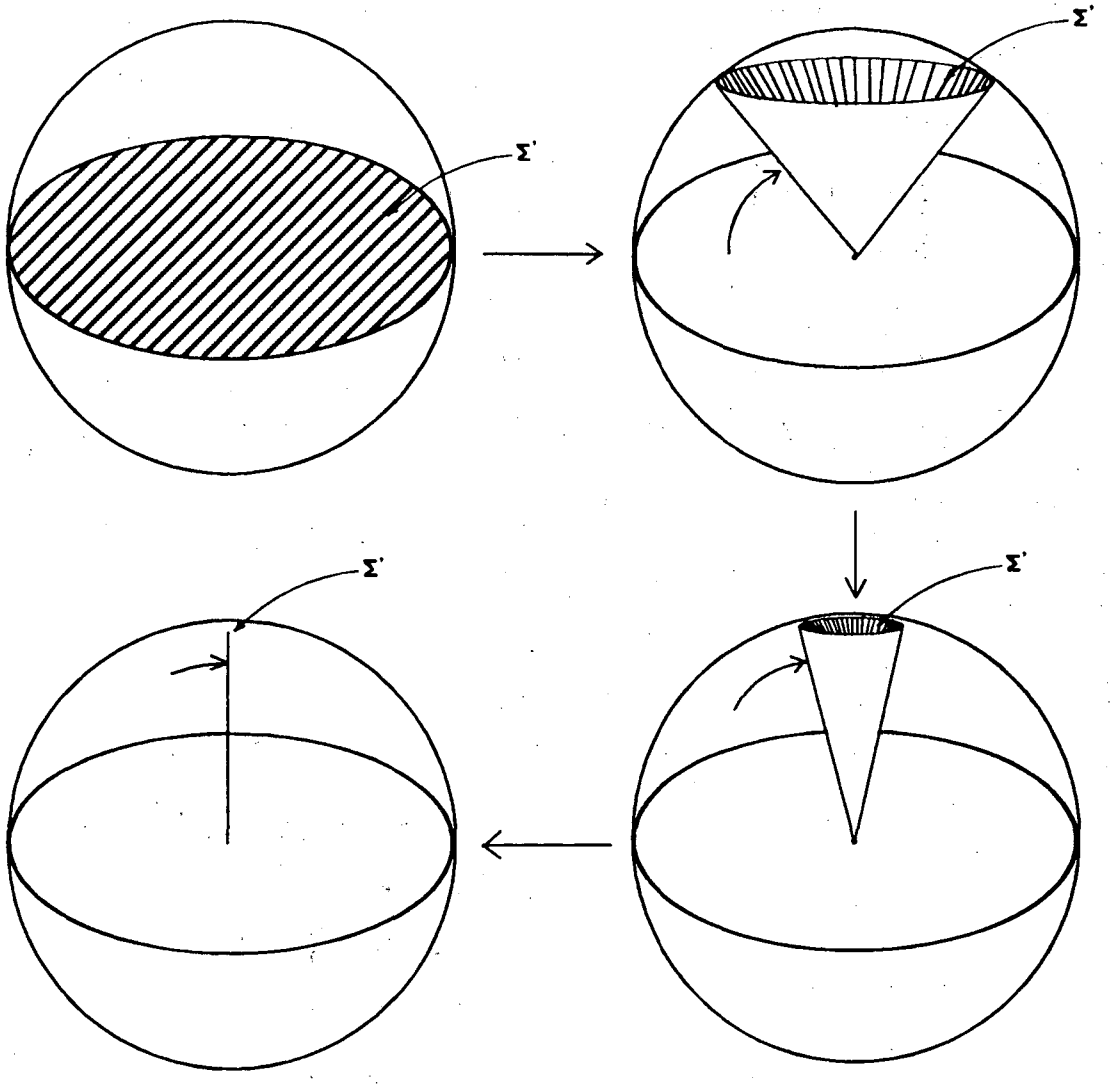

Fig. 6.

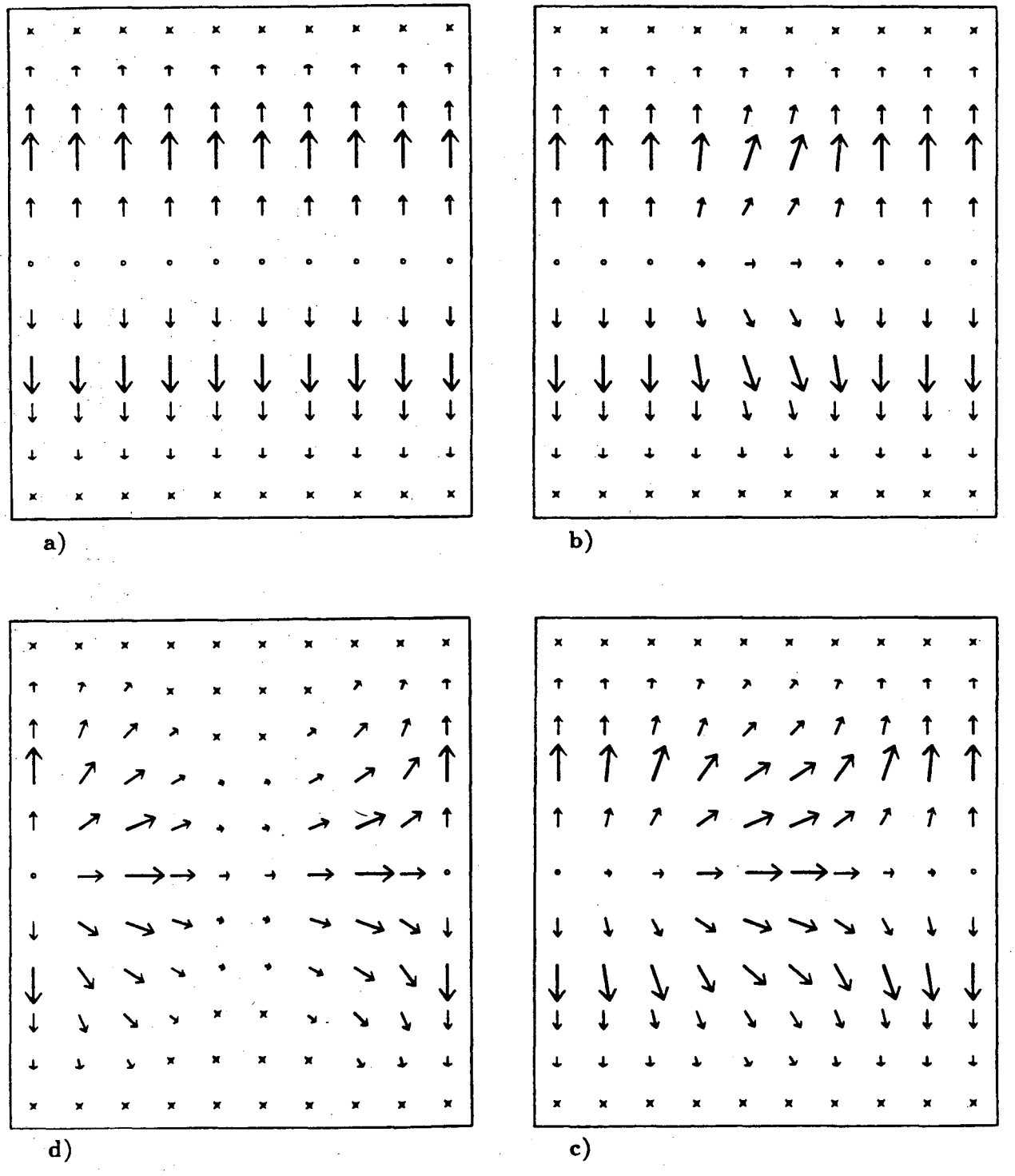

Fig. 7.

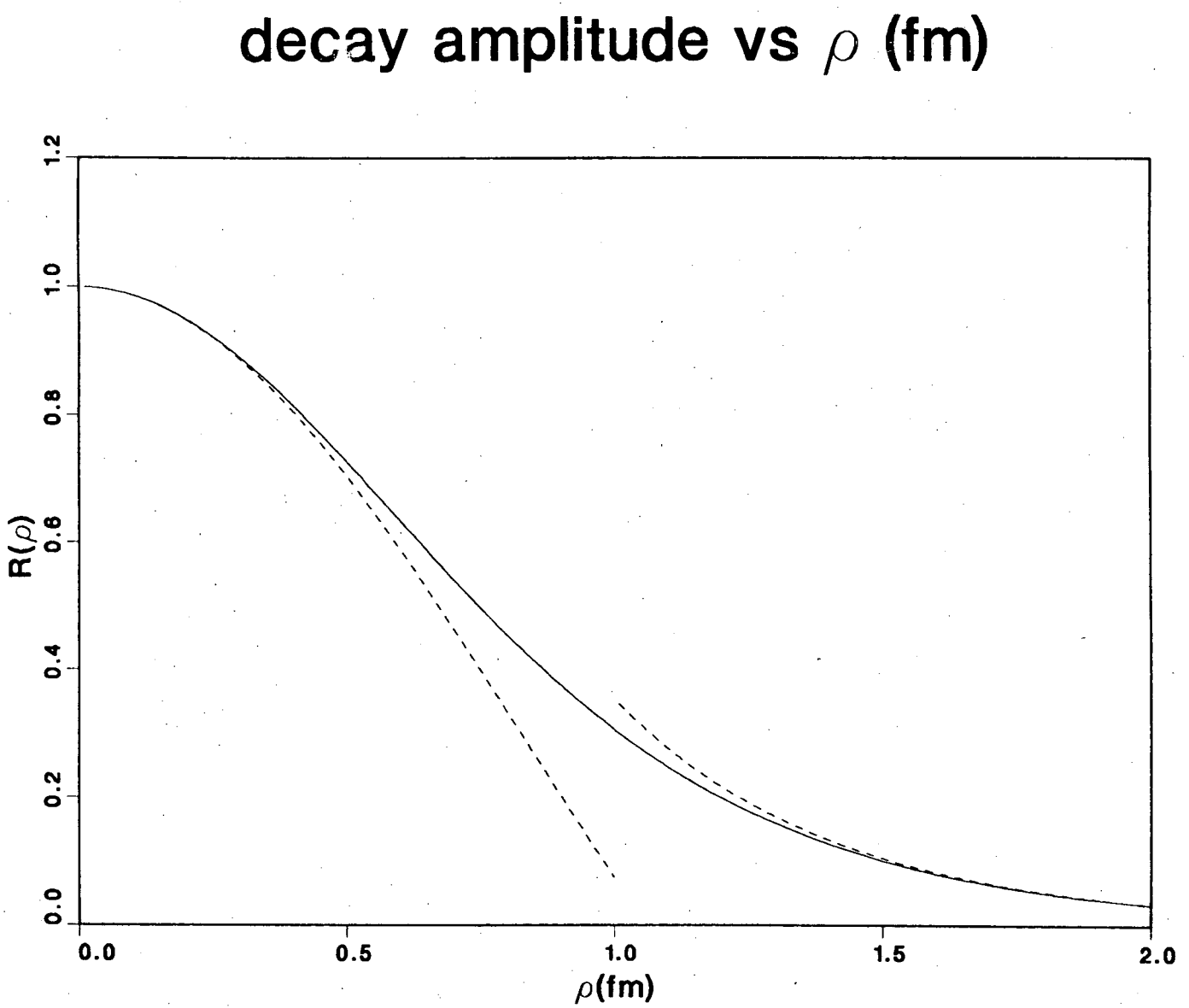

Fig. 8.

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