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# Pion-nucleus Complex Refractive Index*

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ABSTRACT

We find it possible to obtain a formal complex refractive index that depends on r, $k_o$, and $\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}$ for the pion-nucleus interaction in terms of a full pion-nucleus optical potential operator including the Lorentz-Lorenz effect and $\rho^2(r)$ terms. The proton and neutron densities are taken as Woods-Saxon form. Numerical calculations are made for a radial propagation of a pion approaching a lanthanum (A=139, Z=57) with different energies. The real and imaginary refractive index (which are related to the complex pion wave number), the optical potential, and the absorption probability density are displayed as a function of r.

KEYWORD ABSTRACT

NUCLEAR REACTIONS: Pion-nucleus complex refractive index. Pion-nucleus elastic and inelastic interaction. Pion absorption.

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## I. Introduction

Considerable attention has been paid in recent years to pion production in heavy ion collisions[1-7] and pion-nucleus reactions,[8-14] hoping that one can learn about nuclear structure and reaction mechanisms. This is because some of the useful properties of the pion as probe are: its lack of spin, its isospin triplet state (which is a powerful probe to the isospin character of the nucleus compared to the nucleon, which is a doublet isospin state), its striking property of scattering through the $\Delta_{33}$ resonance ($T_\pi \simeq 100$-$300$ MeV) with large ratio of elementary cross section: $d\sigma(\pi^- p)/d\sigma(\pi^- n) = d\sigma(\pi^+ n)/d\sigma(\pi^+ p) = 9$, and the ease with which it can be detected and distinguished from other products of nuclear reactions.

In heavy ion reaction models of pion behavior, the pions have usually been treated classically, despite their deBroglie wave length being greater than nucleon dimensions. In this paper we begin to assess quantal effects for pions in nuclei.

Once the pion has collided with the nucleus, its scattering via the potential that represents the pion-nucleus interaction must be considered. So, further experimental and theoretical work gave more information about the pion-nucleus optical potential.[15-28] The optical potential developed by Stricker et al[19] and Carr et al[19] has the interesting feature that it can describe both the pionic atoms and low energy pion-nucleus scattering with a parameter set not strongly changing with energy. However, they extended their analyses to higher energy regions[17] by properly taking into account the quasielastic scattering process. In their work the $\pi$-nucleus optical potential of Ericson and Ericson[21] that fits pionic atom data is shown to reproduce low-energy scattering data by including the energy dependence of the coefficients, the nuclear density distribution $\rho(r)$, and the effect due to

angular transformation terms (ATT),[22] which has a kinematic origin. The Lorentz-Lorenz Ericson-Ericson term (LLEE) seems to include most of the significant physics and to be the most convenient as a description of the low-energy phenomena. The LLEE term is complicated since it acts on the product of the density distribution ($\rho(r)$, $\rho^2(r)$, $\rho(r)\rho_p(r)$, where $\rho$ and $\rho_p$ are the nucleon and proton density distributions, respectively) and the pion wave function. In Ref. 13 an approximation was made by considering a sharp boundary representing the nuclear surface, setting $\rho(r)$ to equal the nuclear density at the center of the nucleus, and letting the pion wave function be transparent with respect to the LLEE term, and hence this term acts only on the nuclear density. This approximation is justified, since the motivation is to cover a wide range of energies. Of course, if one is interested in propagation of pions through nuclei, low-energy pion-nucleus scattering, and pionic atoms, a special attention to the LLEE term must be paid because of its dynamical origin.

## II. Propagation of Pions Through Nuclei

The problem of interaction of pions with nuclei (many-particle systems) at medium energies can be treated nonrelativistically based on Schroedinger mechanics. The treatment proved itself to be reliable lowest-order approximation to the physics of the problem. Moreover, it meshes naturally with the nonrelativistic description of the nuclear structure.

For a massive nucleus (of mass number A and charge Z) relative to a pion of mass $m_\pi$($\mu=m_\pi c^2\simeq 140$ MeV), the center of mass of the system, approximately, coincides with the center of mass of the nucleus (the ACM). Then the problem of propagating a pion with initial coordinate $\mathbf{r}_o$, relative to the center of the nucleus, and momentum $\mathbf{P}_o=\hbar\mathbf{k}_o$, through a nucleus which is at rest in the

lab frame, can be described in the ACM frame by the following Schroedinger equation:

$$\{\nabla^2 + \frac{2\mu^*}{(\hbar c)^2}[E_\pi - \tilde{V}_{opt}(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}})]\}\Psi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k}) = 0 \ , \tag{1}$$

where $\mu^*$ is the pion reduced rest mass energy

$$\mu^* = \{1/\mu + 1/(AM_n c^2)\} \text{ with } M_n c^2 \simeq 931 \text{ MeV} \ ,$$

and $E_\pi$ is the initial pion total c.m. energy identified by the initial variables $\mathbf{r}_o$ and $\mathbf{p}_o$ and given by

$$E_\pi = (\hbar c)^2/(2\mu^*)k_o^2 + V_{opt}(r_o,k_o,\hat{\mathbf{r}}_o\cdot\hat{\mathbf{k}}_o) \ . \tag{2}$$

In Eq. (1), and hereafter, unit vectors are identified by a caret (^) above the vectors and operators by a tilde (~) above the variables. Also, $\tilde{V}_{opt}$ is given by

$$\tilde{V}_{opt}(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}}) = \tilde{U}_{opt}(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}}) + V_c(r) \ , \tag{3}$$

where $\tilde{U}_{opt}$ is the optical potential operator that describes the interaction of a fictitious particle of rest mass energy $\mu^*$, momentum $\mathbf{p}(=\hbar\mathbf{k})$, and coordinate $\mathbf{r}$ with respect to the center of the nucleus and $V_c$ is the coulomb potential. Acting with $\tilde{V}_{opt}$ on $\Psi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k})$ we have:

$$\tilde{V}_{opt}\Psi = V_{opt}\Psi \ , \tag{4}$$

and then one can write $V_{opt}$ as:

$$\begin{aligned} V_{opt}(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}}) &= \Psi^{-1}_{\mathbf{k}_o}(\mathbf{r},\mathbf{k})\tilde{V}_{opt}(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}})\Psi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k}) \\ &= \Psi^{-1}_{\mathbf{k}_o}(\mathbf{r},\mathbf{k})\tilde{U}_{opt}(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}})\Psi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k}) + V_c(r) \\ &= U_{o\ pt}(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}}) + V_c(r) \ , \end{aligned} \tag{5}$$

where

$$U_{opt}(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}}) = \Psi_{\mathbf{k}_o}^{-1}(\mathbf{r},\mathbf{k})\ \tilde{U}_{opt}(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}})\ \Psi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k})\ .$$

With the help of Eq. (4) we can write Eq. (1) as:

$$(\nabla^2 + K^2)\ \Psi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k}) = 0\ , \tag{6}$$

where

$$K^2 = \frac{2\mu^*}{(\hbar c)^2}\ [E_\pi - V_{opt}(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}})]\ . \tag{7}$$

The wave number K is a complex variable if $V_{opt}$ is complex and it can be written as:

$$K = k + i\kappa\ . \tag{8}$$

In Eq. (8) k is the real part of the wave number and $\kappa$ is the complex one.

A solution of the form

$$\Psi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k}) = e^{iKr(\hat{\mathbf{k}}\cdot\hat{\mathbf{r}})}\ ,\ \text{(for constant K only)}, \tag{9}$$

will satisfy Eq. (6) if K is constant, with $\hat{\mathbf{k}}$ as a unit vector specifying the direction of propagation of the pion wave function ($\mathbf{k} = k\hat{\mathbf{k}}$) inside a slab of uniform nuclear material. In our case $V_{opt}$, and hence K, is not constant but varies smoothly with r. In this case it is reasonable to use the first order WKB approximation for $E_\pi > \mathrm{Re}V_{opt}$, and write the solution of Eq. (6) as:

$$\Psi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k}) = \exp\ \{i\ \xi_{\mathbf{k}_o}(r,\hat{\mathbf{k}}\cdot\hat{\mathbf{r}})(\hat{\mathbf{k}}\cdot\hat{\mathbf{r}})\}\ , \tag{9a}$$

where

$$\xi = \begin{cases} \int_{r}^{r_o} K_{k_o}(r',\hat{k}'\cdot\hat{r}')dr' & \text{incoming } (r\le r_o) \\ \int_{r_t}^{r_o} K_{k_o}(r',\hat{k}'\cdot\hat{r}')dr' + \int_{r_t}^{r} K_{k_o}(r',\hat{k}'\cdot\hat{r}')dr' & \text{outgoing } (r\ge r_t) \end{cases} \tag{9b}$$

This approximation is valid only if $\nabla^2\xi$ remains relatively small, which is true if the potential does not vary too violently. In Eq. (9b) $\mathbf{k}'$ is the pion wave vector at a point specified by $\mathbf{r}'$ and $r_t$ is the distance of closest approach ($r_t$ = o in the case of radial pion propagation). Also, the integration is done on the trajectory at the center of the pion wave packet.

This result expresses the fact that inside the nucleus the original external wave, of propagation vector $\mathbf{k}_o$, has been replaced by a new wave directed along $\hat{\mathbf{k}}$, and $\hat{K}$ replaces $k_o$. In discussing this modification it is essential to realize that the complex wave number $K = k + i\kappa$ does not merely represent a change in the numerical value of the wave propagation vector, but in fact embodies a new physical feature in the propagation among the scatterers, namely, absorption; and the presence of $\kappa$ represents a loss in the propagated pion beam due to encounters with the scatterers. One should notice that, in Eqs. (7-8), the dependence of K on variables like r,... etc. is omitted for simplicity since that dependence is clear. This omission will be used hereafter for functions like refractive index, wave function etc. when it is convenient.

## III. Pion-nucleus Interaction and the Optics Approach

In optics there exists a completely analogous equation to Eq. (6), which is the equation describing the propagation of electromagnetic waves[23] in a conducting medium with permittivity $\varepsilon$, permeability $\mu$, and conductivity $\sigma$ (see pg. 612, Eq. (11, XII) of Born and Wolf, Ref. 23). In a problem of this type one has to consider the attenuation of electromagnetic waves through the conducting medium.

Indeed it emerges that much of the essential physics of our pion-nucleus interaction resembles optics, and one can learn a great deal about medium-energy interactions from the optics analogy. We could attempt to explain, briefly, the analogy between the quantum mechanical problem of pion-nucleus interaction in medium energies and the electromagnetic propagation in a conducting medium by defining the complex index of refraction[24] of the nucleus with respect to the pion as:

$$\begin{aligned} n(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}}) &= n_R(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}}) + i\, n_I(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}}) \\ &= K/k_o \\ &= \left\{\frac{2\mu^*}{(\hbar c)^2}\,[E_\pi - V_{opt}(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}})]\right\}^{\frac{1}{2}}/k_o \end{aligned} \tag{10a}$$

Accordingly, one can define:

$$\begin{aligned} \xi/k_o &= L \\ &= L_R + iL_I\ , \end{aligned} \tag{10b}$$

where

$$L_R = \begin{cases} \int_r^{r_o} n_R(r',k_o,\hat{\mathbf{k}}'\cdot\hat{\mathbf{r}}')dr' & \text{incoming } (r\le r_o) \\ \int_{r_t}^{r_o} n_R(r',k_o,\hat{\mathbf{k}}'\cdot\hat{\mathbf{r}}')dr' + \int_{r_t}^{r} n_R(r',k_o,\hat{\mathbf{k}}'\cdot\hat{\mathbf{r}}')dr' & \text{outgoing } (r\ge r_t) \end{cases}\ , \tag{10c}$$

and similar relations hold for $L_I$ by replacing $n_R$ by $n_I$ in the last equation. Squaring Eq. (10a) we obtain, upon equating the real and imaginary parts, the following relations:

$$n_R^2 - n_I^2 = \frac{2\mu^*}{(\hbar c k_o)^2}\,\mathrm{Re}[E_\pi - V_{opt}]\ , \tag{11}$$

$$2n_R n_I = \frac{2\mu^*}{(\hbar c k_o)^2}\,\mathrm{Im}[E_\pi - V_{opt}]\ , \tag{12}$$

There is difficulty in obtaining an expression for $n_R$ and $n_I$ from Eqs. (11) and (12) if full inclusion of the LLEE term in $V_{opt}$ is to be considered. This difficulty arises from the fact that $V_{opt}$ depends on the wave solution itself. However, this difficulty can be circumvented by adopting the eikonal approximation, as will be shown later, and hence an expression for $n_R$ and $n_I$ can be obtained. This means that with this approximation, Eq. (10) enables us to write the pion wave function in the pion-nucleus interaction region in terms of the real and imaginary parts of the refractive index of the nucleus as:

$$\Psi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k}) = \chi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k})\psi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k}) \tag{13}$$

where

$$\chi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k}) = e^{-k_o L_I(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}})(\hat{\mathbf{k}}\cdot\hat{\mathbf{r}})} , \tag{14}$$

is the attenuation amplitude, which is in the form of exponential function, for the pion local plane wave:

$$\psi_{\mathbf{k}_o}(\mathbf{r},\mathbf{k}) = e^{ik_o L_R(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}})(\hat{\mathbf{k}}\cdot\hat{\mathbf{r}})} . \tag{15}$$

The probability, dP, of finding the pion per unit volume at point $\mathbf{r}$, which is $|\Psi|^2$, becomes

$$\begin{aligned} dP_{\mathbf{k}_o}(r,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}}) &= e^{-2k_o L_I(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}})(\hat{\mathbf{k}}\cdot\hat{\mathbf{r}})} \\ &= e^{-s/\bar{\lambda}} , \end{aligned} \tag{16}$$

where $\bar{\lambda}$ resembles the mean free path, for a pion wave traveling through a medium of constant density, since in this case dP falls to 1/e of its value after the wave has gone a distance $\bar{\lambda}$ (see Fig. 1). Thus the probability of finding the pion diminishes exponentially with s and it is characterized by the local pion mean free path which is defined as:

$$\bar{\lambda}(\text{local mean free path}) = 1/[2k_o|\langle n_I(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}})\rangle|] \ . \tag{17}$$

The fraction of absorbed pions is $1 - \chi^2$.

In reality we do not have a local pion plane wave, as given by Eqs. (13-17), but a pion wave packet of width $\Delta r$ in r around a particular pion coordinate (with $\Delta r \ll$ the dimension of the nucleus) and a spread $\Delta k$ in k around $k_o$ when $r \simeq \infty$ (with $\Delta k \ll k_o$). However, the measurable physical quantities do not depend on the shape of the wave packet, and we may carry out our investigation to the pion-nucleus interaction in the time-independent framework of Eq. (1).

Now, as the pion wave with wave vector $\mathbf{k}_o$ approaches the nucleus from a large distance, the imaginary parts of the index of refraction $n_I$ are initially zero; and hence the mean free path $\bar{\lambda}$ is infinite. This situation changes as the pion wave reaches the nucleus. The pion absorption process will start to take place, and hence $n_I$ acquires values greater than zero, resulting in a finite smaller value of $\bar{\lambda}$ as compared to the case where $r \simeq \infty$. On the other hand, because of the dependence of $n_I$ on the coordinate r of the pion with respect to the center of the nucleus and on the angle between the two vectors $\mathbf{r}$ and $\mathbf{k}$, $\bar{\lambda}$ will also depend on direction and energy of the pion wave. The same concept applies to the pion probability density, since it depends on $\bar{\lambda}$ or simply on $n_I$. In summary, calculations of the pion probability density are not straightforward, since the refractive index depends on the wave function. Thus, one needs to solve Eqs. (11-12) for $n_R$ and $n_I$ and use them for calculating $L_I$. This implies that one should get detailed information about the propagation of the pion wave packet through the nucleus. That is, one must calculate $\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}$ along the propagation of the pion wave packet. In radial propagation $\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}$ is known for both the incoming and outgoing waves.

## IV. The Eikonal Approximation

The fact that $V_{opt}$ in Eqs. (11-12) depends on the wave function itself, which has an attenuation amplitude $\chi$, forced us to seek a reasonable approximation for the case where $\chi$ varies very slowly. The approximation is appropriate where the essential physics of the pion-nucleus interaction resembles optics. One sensible and physically acceptable approximation, especially after introducing the nuclear refractive index, is the eikonal approximation,[25] which is used successfully in optics.[23] As in optics, it is found that it is convenient to define a real scalar function of position known as "the optical path," where constant optical path may be called the geometrical wavefront (plane of constant phase). This idea is adopted here in Eq. (15) by defining:

$$\phi(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}}) = L_R(r,k_o,\hat{\mathbf{r}}\cdot\hat{\mathbf{k}})[\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}] \ . \tag{18}$$

Based on Eq. (18), Eqs. (14-15) can be written as

$$\chi_{\mathbf{k}_o} = \exp(-k_o L_I L_R^{-1}\phi) \ , \tag{19}$$

$$\psi_{\mathbf{k}_o} = \exp(ik_o\phi) \ , \tag{20}$$

and

$$\Psi_{\mathbf{k}_o} = \exp(-k_o L_I L_R^{-1}\phi)\ \exp(ik_o\phi) \ . \tag{21}$$

We now employ a phenomenological approximation method to find the relation that $\phi$ must satisfy. Substituting Eq. (21) into Eq. (6) to obtain, upon equating the real parts on both sides of the resulting equation and neglecting the term which contains $k_o^{-2}\chi^{-1}\nabla^2\chi$ , an equation which $\phi$ must satisfy:

$$\nabla\phi\cdot\nabla\phi = n_R^2 - n_I^2 \ . \tag{22}$$

Eq. (22) is known as the eikonal equation.[23] Also, we obtain, upon equating the imaginary parts of the same resulting equation and neglecting the term which contains $\chi^{-1}\nabla\chi$ , the following relation:

$$\nabla^2\phi = 2k_o n_R n_I \tag{23}$$

In obtaining Eq. (22), the slow variation of $\chi$ allows us to neglect terms containing $\nabla^2\chi$ since it is essentially zero even if $k_o$ is not large. Of course, this approximation is more justified for small pion wave length (large $k_o$). Also, in obtaining Eq. (23), the term containing $\chi^{-1}\nabla\chi$ is neglected since $\nabla\chi$ is essentially zero on the tail of nuclear density and very small compared to the rest of Eq. (23) inside the nucleus, as we will see later. We will see how useful the two Eqs. (22-23) are in calculating $n_R$ and $n_I$ if the LLEE term of the optical potential is to be included.

The eikonal equation allows us to define a unit vector which is directed along the trajectory of the center of the pion wave packet. This trajectory can be defined as the orthogonal trajectories to the pion geometrical wave front $\phi$ = constant. If **r** is the pion position vector from the center of the nucleus at point, P, on a trajectory and s the length of the trajectory measured from a fixed point, $p_o$, on it (fig. 1), then

$$\hat{\mathbf{k}} = \frac{d\mathbf{r}}{ds} = -[n_R^2 - n_I^2]^{-\frac{1}{2}}\nabla\phi \ . \tag{24}$$

Eq. (24) will allow $n_R$ and $n_I$ to depend on the angle between **r** and **k** vectors (as we will see later) if the LLEE term is involved. Also, from the definition of gradient we get:

$$\frac{d\phi}{ds} = [n_R^2 - n_I^2]^{\frac{1}{2}} \ . \tag{25}$$

Eq. (25) can be used to find the phase $\phi$ along the trajectory of the center of the pion wave packet. Although $\phi$ depends on $n_R$ and $n_I$ along the path length,

s, for the case of zero impact parameter it can be calculated with the help of the phase equation, Eq. (25), since $s \equiv r$ in this case. The phase for this special case will be given by

$$\phi(r,k_o,\mp 1) = \begin{cases} \int_r^{r_o} [n_R^2 - n_I^2]^{\frac{1}{2}} dr & \text{incoming } (r \le r_o) \quad (25a) \\ \int_o^{r_o} [n_R^2 - n_I^2]^{\frac{1}{2}} dr + \int_o^{r} [n_R^2 - n_I^2]^{\frac{1}{2}} dr & \text{outgoing } (r>0) \quad (25b) \end{cases}$$

where we set $\theta(r_o,k_o,-1)$ to be zero. Note that, when we start the pion at large $r_o$ where $U_{opt} \approx o$, $n_R$ must be 1 if $V_c = 0$ and $n_I$ should be zero.

## V. Nuclear Refractive Index

The aim of this section is to find an expression for $n_R$ and $n_I$ based on the optical potential developed by, first, Stricker, Carr and McManus[19] (SCM) and, second, by Carr, McManus and Stricker[18] (CMS). The CMS potential is of interest to us, since $\pi$-atom data, pion-nucleus scattering data and pion-nucleus absorption data are all described by one set of nearly energy independent parameters. The treatment will be general for any nucleus of mass number A, Z protons, and N neutrons, and also all terms, including the nonlocal LLEE term, will be considered. It is convenient to write the CMS optical potential as a sum of local and nonlocal terms:

$$\tilde{U}_{opt} = - \frac{2\pi(\hbar c^2)}{\bar{\omega}} \{\tilde{\mathcal{L}}(r) + \tilde{\mathcal{L}}_{LLEE}(r,k_o,\hat{r}\cdot\hat{k})\} , \qquad (26)$$

where

$$\tilde{\mathcal{L}}(r) = \mathcal{L}_{\pi N}^{SC}(r) + \mathcal{L}_{\pi}^{ab}(r) + \tilde{\mathcal{L}}_{AT}(r),$$

$$\mathcal{L}_{\pi N}^{SC}(r) = f_1 b(r) \quad \text{(single pion-nucleon scattering term)},$$

$$\mathcal{L}_{\pi}^{ab}(r) = f_2 B(r) \quad \text{(pion absorption term)},$$

$\tilde{\mathcal{L}}_{AT}(r) = \frac{1}{2}(1-f_1^{-1})\nabla^2 c(r) + \frac{1}{2}(1-f_2^{-1})\nabla^2 C(r)$ (angle transformation term),

$\tilde{\mathcal{L}}_{LLEE} = -\nabla\cdot[L(r)/\{1+(\frac{4\pi}{3})\lambda L(r)\}]\nabla$ (Lorentz-Lorenz Ericson-Ericson nonlocal term),

$b(r) = \bar{b}_o\rho(r) - \varepsilon_\pi b_1 \delta\rho(r)$ ,

$B(r) = B_o\rho^2(r) - \varepsilon_\pi B_1 \rho(r)\delta\rho(r)$,

$c(r) = c_o\rho(r) - \varepsilon_\pi c_1 \delta\rho(r)$,

$C(r) = C_o\rho^2(r) - \varepsilon_\pi C_1 \rho(r)\delta\rho(r)$,

$\varepsilon_\pi = o, +1, -1$ for $\pi^o$, $\pi^+$ and $\pi^-$, respectively,

$\delta\rho(r) = \rho_n(r) - \rho_p(r) = \rho(r) - 2\rho_p(r)$,

Here $\bar{\omega}$ is the pion reduced energy,

$\bar{\omega} = 1/[1/\omega + 1/(AM_n c^2)]$;

$f_1$ and $f_2$ are kinematic factors,

$f_1 = [1 + \varepsilon]/[1 + \varepsilon/A]$,

$f_2 = [1 + \frac{1}{2}\varepsilon]/[1 + \frac{1}{2}\varepsilon/A]$,

$\varepsilon = \omega/(M_n c^2)$,

$\omega^2 = (\hbar c)^2 k_o^2 + \mu^2$;

$\bar{b}_o$ is the effective s-wave scattering length in the nucleus,

$\bar{b}_o = b_o - 3k_F(b_o^2 + 2b_1^2)/(2\pi)$ with $k_F$ the Fermi momentum, $k_F = 1.4\ fm^{-1}$,

$L(r) = f_1^{-1}\ c(r) + f_2^{-1}C(r)$;

the densities $\rho(r)$, $\rho_p(r)$ and $\rho_n(r)$ are normalized to A, Z and N respectively; the parameters $b_o$, $b_1$, $c_o$, $c_1$, $B_o$, $B_1$, $C_o$, $C_1$ and $\lambda$ are taken from Carr's[17,13] fit to the pion-nucleus scattering cross-sections.

It is interesting at this point to start the pion at some large distance $r_o$ where $U_{opt} \simeq 0$ in order to find $n_R$ and $n_I$ for all values of $0 \le r \le r_o$ outside and inside the nucleus. By taking this condition into consideration, then substitution of Eq. (26) into Eqs. (11-13) will yield:

$$n_R^2 - n_I^2 = 1 - \frac{2\mu^*}{(\hbar c k_o)^2}[V_C(r) - V_C(r_o)] + \frac{4\pi\mu^*}{\bar{\omega} k_o^2}\mathrm{Re}\{\mathcal{L}(r) + \mathcal{L}_{LLEE}(r, k_o, \hat{\mathbf{r}}\cdot\hat{\mathbf{k}})\} \tag{27}$$

and

$$2 n_R n_I = \frac{4\pi u^*}{\bar{\omega} k_o^2}\,\mathrm{Im}\{\mathcal{L}(r) + \mathcal{L}_{LLEE}(r, k_o, \hat{\mathbf{r}}\cdot\hat{\mathbf{k}})\} \tag{28}$$

In Appendix A the term $\mathcal{L}_{LLEE} = \Psi^{-1}\,\tilde{\mathcal{L}}_{LLEE}\Psi$ will be expressed in terms of $n_R$ and $n_I$, and its results can be used in Eqs. (26-28) to find (after some algebraic manipulation):

$$n_R = W^{\frac{1}{2}}\{\eta + [\eta^2 + 1]^{\frac{1}{2}}\}^{\frac{1}{2}}, \text{ (then } k = k_o n_R), \tag{29}$$

$$n_I = W/n_R, \quad \text{(then } \kappa = k_o n_I) \tag{30}$$

where

$\eta = Q/W$,

$W = \frac{1}{2}[H + \beta\ell_1(2Q)^{\frac{1}{2}} + 2h_2 Q]/[1 - h_1]$,

$Q = \frac{1}{2}\{[-B \pm [B^2 - 4AC]^{\frac{1}{2}}]/(2A)\}^2$ , (the positive sign is used for our radial propagation)

$A = 1 + h_1^2 + h_2^2 - 2h_1$ ,

$$B = -\beta[\ell_2(h_1-1) - \ell_1 h_2] ,$$

$$C = (h_1 - 1)D + h_2 H ,$$

$$h_1 = \frac{4\pi\mu^*}{\bar{\omega}} \text{ Re } f,$$

$$h_2 = \frac{4\pi\mu^*}{\bar{\omega}} \text{ Im } f,$$

$$\ell_1 = \frac{4\pi\mu^*}{\bar{\omega}} \text{ Re } \frac{df}{dr} ,$$

$$\ell_2 = \frac{4\pi\mu^*}{\bar{\omega}} \text{ Im } \frac{df}{dr} ,$$

$$D = 1 - \frac{2u^*}{(\hbar c k_o^2)} [V_C(r) - V_C(r_o)] + \frac{4\pi\mu^*}{\bar{\omega} k_o^2} \text{ Re } \mathcal{f}(r),$$

$$H = \frac{4\pi\mu^*}{\bar{\omega} k_o^2} \text{ Im } \mathcal{f}(r),$$

$$\beta = \hat{\mathbf{r}}\cdot\hat{\mathbf{k}}/k_o,$$

$$f = L(r)/[1 + (4\pi/3)\lambda L(r)] .$$

Of course, all the variables listed above have r dependence and some of them depends on $k_o$ and $\hat{\mathbf{r}}\cdot\hat{\mathbf{k}}$, but this dependence is omitted for simplicity. Also, $\mathcal{f}(r) = \Psi^{-1}\tilde{\mathcal{f}}(r)\Psi$ and the Lorentz-Lorenz Ericson-Ericson function L(r) are defined by the CMS optical potential given by Eq. (26).

## VI. Application to Nuclei Having Woods-Saxon Form

The nucleon and proton distributions, $\rho(r)$ and $\rho_p(r)$, for a particular nucleus may be represented quite accurately by a Woods-Saxon (or Fermi) distribution

$$\rho(r) = \rho_o/\left[1 + \exp\left(\frac{r - R_o}{a}\right)\right], \text{ with } a=0.54 \text{ fm for all nuclei,}$$

$$R_o = [1.12\ A^{1/3} - 0.33\ A^{-1/3}]\text{fm},$$

$$\rho_p(r) = \rho_{op}/[1 + \exp(\frac{r-R_{op}}{a_p})], \text{ with } a_p=0.454 \text{ fm for all nuclei},$$

$$R_{op} = [1.106 + 1.05 \times 10^{-4}A]A^{1/3} \text{ fm}. \tag{31}$$

The matter density $\rho(r)$ is normalized such that $\int\rho(r)d\tau = A$ and $\int\rho_p(r)d\tau = Z$. For the Lanthanum nucleus $^{139}$La these normalizations give $\rho_o$=0.162 nucleons/fm$^3$, $R_o$ = 5.74 fm, $\rho_{op}$=0.066 protons/fm$^3$ and $R_{op}$=5.81 fm. Figure 2 shows how the densities for all nucleons, for neutrons, and for protons vary with r in the case of $^{139}$La.

In calculating $n_R$ and $n_I$ from Eqs. (29-30) one needs to evaluate $V_c(r)$, $\frac{df}{dr}$, and $\pounds(r)$ at any value of r using Eq. (31) for $\rho(r)$ and $\rho_p(r)$ at any value of r. First, for the total coulomb potential energy of a charged pion at any value $0\le r\le\infty$ one can use:

$$V_c(r) = \begin{cases} Ze^2\varepsilon_\pi r^{-1} & \text{if } r>R_{zero} \\ 4\pi e^2\varepsilon_\pi[r^{-1}\int_o^r \rho_p(r')r'^2dr' + \int_r^{R_{zero}} \rho_p(r')r'dr'] & \text{if } r<R_{zero}, \end{cases} \tag{32}$$

where $R_{zero}$ is the radius at which $\rho_p$ is essentially zero, i.e., $\rho_p(r\ge R_{zero}) = 0$.

Second, for $\frac{df}{dr}$ and $\pounds(r)$ one can relate them to $\frac{d\rho}{dr}(r)$ and $\nabla^2\rho(r)$, and similar forms for $\rho_p(r)$. Hence, for a Woods-Saxon distribution these derivatives are simply given by:

$$\frac{d\rho}{dr} = -a^{-1}\rho(r)(1-\rho(r)/\rho_o),$$

and

$$\nabla^2\rho = \{2r^{-1} - a^{-1}[1 - 2\rho(r)/\rho_o]\}\frac{d\rho}{dr}\ . \tag{33}$$

## VII. Numerical Results and Discussion

We have carried out numerical calculations mainly for neutral pions on lanthanum-139 with some negative pion calculations also. Lanthanum has served as both beam and target in a number of Berkeley BEVALAC experiments, so it has special interest. In this paper we will not try to relate results to particular experimental data sets. Indeed, neutral pion data are rare in heavy ion work due to experimental difficulties, but it is natural to begin theoretical studies of quantal effects with neutral pions so as to avoid complications of the Coulomb interaction.

In Fig. 2 we show the Woods-Saxon density distributions used in the calculations. These curves give an orientation as to the radial distance scale for comparison with later figures.

We have solved for wave functions and potentials in the eikonal approximation described. The initial conditions are an inward moving wave at zero impact parameter (leftward-moving from the right margin on the displayed figures). The solutions are equivalent to a left-moving plane wave on a slab of nuclear matter with diffuse boundaries and of thickness equal to the lanthanum nuclear diameter.

Figure 3 stacks vertically the plots of various calculated quantities for 36 MeV neutral pions. Note that the nuclear center is in the middle, so the right-hand entrance surface as well as the left-hand exit surface are displayed. This figure shows the real and imaginary parts of the CMS optical potential, the corresponding refractive index and the damped oscillating wave function with its envelope. The fraction of the pion wave absorbed is shown in the lowest plot (Fig. 4e). Note that the wave amplitude envelope falls to about 10% of its incident value, corresponding to 99% absorption traversing

the La nucleus. It is of interest to check this result with our earlier calculations in constant density nuclear matter. (Mehrem et al.) Fig. 1 of Ref. 13 gives a mean free path in nuclear matter of about 2.5 fm at 36 MeV. Taking the diameter for mass 139 from Eq. (32) we get 11.47 fm between the half-density points. That is, the pion path traverses 4.59 mean free paths for an attenuation to 1.02%, in good agreement with the newer calculations displayed in Fig. 3. That the present more sophisticated treatment of the diffuse nuclear surface and the gradient terms in the LLEE potential gives the same answer for over all attenuation would not have been obvious. In particular we note that the imaginary part of the optical potential (Fig. 3a) and consequently the imaginary part of the refractive index (Fig. 3c) strongly peak in the nuclear surface region. The real parts show an oscillation at the surface resulting from the combination of density-dependent and density-squared-dependent terms and the radial gradients. The net real potential at 36 MeV is about 8 MeV attractive in the interior. Fig. 2 of Ref. 13 shows the real potential for neutral pions on Pb-207 nearly zero, going repulsive at lower energies and attractive at higher energies. The small difference between the older and newer calculations may arise from slightly different density distributions assumed for Pb-207 and La-139. Unlike ordinary optics the refractive index can be less than 1, since the wave number can be smaller than in vacuum for regions of net repulsive potential.

The behavior of the complex optical potential for increasing incident pion energies is shown in Fig. 4. The interior real potential becomes increasingly attractive as a consequence of the p-wave pion-nucleon attraction. The oscillations due to gradient terms in the surface become less pronounced, though there is a persistence of the surface-peaking in the imaginary component.

Our simple eikonal approximation (equivalently lowest-order WKB) runs into difficulty at lower energies, where the interior repulsive interaction gives rise to classically forbidden regions and turning points. Without a wave function the LLEE term in the optical potential cannot be evaluated. We show in Fig. 5 the optical potential and refractive index for 10 MeV, excluding the nuclear interior. Potential parameters for the pionic atom are used. It is evident that a surface absorption and a slight surface attraction persists in the outer surface before the interior repulsion begins to dominate.

Finally, in Fig. 6 we show the optical potential and refractive index plots for negative pions of 10 MeV incident energy. The extra Coulomb energy for pions in the interior avoids the difficulties mentioned in connection with neutral pions in Fig. 5. The overall potential shows a repulsion in the outer half of the nucleus, and the absorptive imaginary potential is, as always, surface-peaked.

We do not presume to improve upon the earlier optical potential calculations of pion nuclear scattering, except to afford some insight into the results by examination of the potentials and some wave propagation examples. There are some minor troublesome problems and cautions with respect to use of these complex optical potentials at kinetic energies so low that classically forbidden regions appear. In such regions the DEL-SQUARED operator in the LLEE term changes sign, and imaginary terms change from absorptive to source terms, and as discussed in Ref. 13 the negative mean free paths encountered by Hecking (Ref. 26) could come from this problem. It may be that this problem has no practical effect on the pionic atom and scattering calculations of Carr et al., since surface absorption is always so strong. However, positive pion scattering theory at low kinetic energies bears re-

examination. To apply quantal corrections to pion production on heavy nuclei, whether by nuclear or antiprotonic interactions, will force us to deal with all energies of pions, including the lowest energy pions. In this connection there may be situations of pion propagation through hot nuclear matter where imaginary potential source terms are appropriate to treat the stimulated emission or pion-laser action.

There have been earlier speculations that "bumps" in heavy-ion produced pion spectra might arise from quantal interference effects or partial pion orbiting in the nuclear surface. We hope to go on in a later paper to explore trajectories with non-zero angular momentum, but the prospects for "orbiting" on cold nuclear fragments seem minimal in view of the weak or repulsive potentials at low pion velocities and the strength of the absorption. An interesting extension of pion optical potential methods would be the treatment of pion propagation in hot nuclear matter at different densities and composition. That absorption will decrease and change sign as temperature increases seems obvious, but the temperature dependence of this and other optical potential parameters needs more fundamental theoretical work.

## ACKNOWLEDGMENTS

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## Appendix A

### Evaluation of the Lorentz-Lorenz Ericson-Ericson Term

The Lorentz-Lorenz Ericson-Ericson term defined by

$$\mathcal{L}_{LLEE} = \Psi^{-1}\,\tilde{\mathcal{L}}_{LLEE}\Psi$$

can take the form[27]

$$\begin{aligned}\mathcal{L}_{LLEE} &= -\Psi^{-1}\nabla\cdot[f\nabla\Psi] \\ &= -\Psi^{-1}[f\nabla^2\Psi + \nabla f\cdot\nabla\Psi],\end{aligned} \tag{A1}$$

where f is defined in the text. From the definition of $\Psi$ which is given by Eq. (21) one can find that

$$\nabla\Psi \simeq ik_o\Psi\nabla\phi \quad (\text{for } L_I < L_R \text{ and } \nabla n_I \approx 0), \tag{A2}$$

and hence

$$\nabla^2\Psi \simeq k_o\Psi[i\nabla^2\phi - k_o\,\nabla\phi\cdot\nabla\phi].$$

with the use of the eikonal approximation given by Eqs. (22-23), we can write,

$$\nabla^2\Psi \simeq -k_o^2[2in_Rn_I + (n_R^2 - n_I^2)]\Psi\ . \tag{A3}$$

The second term of Eq. (A1) can be simply given by:

$$\nabla f\cdot\nabla\Psi \simeq ik_o\Psi\ f'\ \hat{\mathbf{r}}\cdot\nabla\phi.$$

Now, with the help of Eq. (24) we can write $\nabla f\cdot\nabla\Psi$ as

$$-ik_o\Psi\ f'[n_R^2 - n_I^2]^{\frac{1}{2}}\ \hat{\mathbf{r}}\cdot\hat{\mathbf{k}}. \tag{A4}$$

Finally, the LLEE term will take the following form:

$$\mathcal{L}_{LLEE} \simeq k_o^2 f[(n_R^2 - n_I^2) + 2in_In_R] + ik_o\ f'[n_R^2 - n_I^2]^{\frac{1}{2}}\ \hat{\mathbf{r}}\cdot\hat{\mathbf{k}}\ . \tag{A5}$$

## References


1. J.Miller, Ph.D. thesis, Lawrence Berkeley Laboratory, Report No. LBL-24275, 1988 (unpublished).
2. J. Miller, J. Bercovitz, G. Claesson, G. Krebs, G. Roche, L.S. Schroeder, W. Benenson, J. van der Plicht, J.S. Winfield, G. Landaud and J.-F. Gilot, Phy. Rev. Lett, 58, 2408 (1987).
3. A.F. Barghouty and G. Fai, Phys. Rev. C, 35, 950 (1987).
4. R.B. Clare and D. Strottman, Phys. Rev. C, 33, 1288 (1986).
5. J.W. Harris, R. Bock, R. Brockmann, A. Sandoval, R. Stock, H. Stroebele, G. Odyniec, H.G. Pugh, D. Schall, D. Bangert, W. Pauch and K.L. Wolf, Phys. Lett. 153B, 377 (1985).
6. K.A. Frankel, J.A. Bistirlich, R. Bossingham, H.R. Bowman, K.M. Crowe, C.J. Martoff, D. Murphy, J.O. Rasmussen, J.P. Sullivan, W.A. Zajc, J.P. Miller, O. Hashimoto, M. Koike, J. Peter, W. Benenson, G.M. Crawley, E. Kashy, O.A. Nolan, Jr., and J. Quebert, Phys. Rev. C, 25, 1102 (1982).
7. J.P. Sullivan, J.A. Bistirlich, H.R. Bowman, R. Bossingham, T. Buttke, K.M. Crowe, K.A. Frankel, C.J. Martoff, J. Miller, D.L. Murphy, J.O. Rasmussen, W.A. Zajc, O. Hashimoto, M. Koike, J. Peter, W. Benenson, G.M. Crawley, E. Kashy, and J.A. Nolen, Jr., Phys. Rev. C, 25, 1499 (1982).
8. D.E. Nagle, M.B. Johnson and D.f. Measday, Physics Today, 40, No. 4, 56 (1987); P.G. Roos, AIP Conf. Proc. 163, 210 (1987).
9. C.J. Harvey, H.W. Baer, J.A. Johnstone, C.L. Morris, S.J. Seestrom-Morris, D. Dehnhard, D.B. Holtkamp and S.J. Greene, Phys. Rev. C, 33, 1454 (1986).
10. H.-C. Chiang and M.B. Johnson, Phys. Rev. C, 31, 2140 (1985).
11. D. Marlow, P.D. Barnes, N.J. Colella, S.A. Dytman, R.A. Eisenstein, R. Grace, P. Pile, F. Takeutchi, W.R. Wharton, S. Bart, D. Hancock, R.

Hackenberg, E. Hungerford, W. Mayes, L. Pinsky, T. Williams, R. Chrien, H. Palevsky, and R. Sutter, Phys. Rev. C, 30, 1662 (1984).

12. H.M.A. Radi, R. A. Mehrem and J.O. Rasmussen, to be published.

13. R.A. Mehrem, H.M.A. Radi and J.O. Rasmussen, Phys. Rev. C, 30, 301 (1984).

14. H.M.A. Radi, J.O. Rasmussen, K.A. Frankel, J.P. Sullivan and H.C. Song, Phys. Rev. C, 27, 606 (1983).

15. D.R. Giebink and D.J. Ernst, Comput. Phys. Commun., 48, 407 (1988).

16. M.B. Johnson and E.R. Siciliano, Phys. Rev. C, 27, 730 (1983). Cf. also M.B. Johnson and H.A. Bethe, Nucl. Phys. A305, 418 (1978).

17. J.A. Carr, Ph.D. Thesis, Michigan State University, 1982 (unpublished).

18. J.A. Carr, H. McManus, and K. Stricker, Phys. Rev. C, 25, 952 (1982).

19. K. Stricker, J.A. Carr and H. McManus, Phys. Rev. C, 22 2043 (1980).

20. K. Stricker, H. McManus and J.A. Carr, Phys. Rev. C, 19, 929 (1979).

21. M. Ericson and T.E.O. Ericson, Ann. Phys. (NY) 36, 323 (1966).

22. M. Thies, Phys. Lett. 63B, 43 (1976), and references therein.

23. M. Born and E. Wolf, Principles of Optics (Pergamon Press, Oxford, 1975).

24. J.M. Eisenberg and D.S. Koltun, theory of meson interaction with nuclei (A Wiley-Interscience Publication Co., 1980).

25. A.S. Iljinov, V.I. Nazaruk and S.E. Chigrinov, Nucl. Phys. A382, 378(1982).

26. P. Hecking, Phys. Lett. 103B, 401 (1981).

27. R.A. Mehrem (Indiana University), Private Communication (1988).

## Figure Legends

Fig. 1. Schematic drawing of pion trajectory in nuclear field.

Fig. 2. Nucleon density distributions used in calculations.

Fig. 3. For incident neutral pions of 36 MeV kinetic energy and normal incidence on $^{139}$La (a) Real and imaginary components of the effective optical potential, (b) Real part of the refractive index, (c) Imaginary part of the refractive index, (d) real part of the left-going wave, with envelope, and (e) probability of the pion wave being absorbed. Note that the pion ray goes through the center of the nucleus and that the nuclear center is in the middle of the abscissa.

Fig. 4. The pion optical potential for normal incidence neutral pions of higher kinetic energies of (a) 50 MeV, (b) 67 MeV, and (c) 80 MeV.

Fig. 5. The pion optical potential and refractive index for normal incidence neutral pions of 10 MeV kinetic energy (Carr parameters for pionic atom used). The plot covers only the exterior of the nucleus, since a classically forbidden region due to S-wave repulsion is encountered at smaller radii.

Fig. 6. Same as Fig. 5, except for negative pion of 10 MeV kinetic energy. There is no classically forbidden region, so the entire radial distance range is displayed.

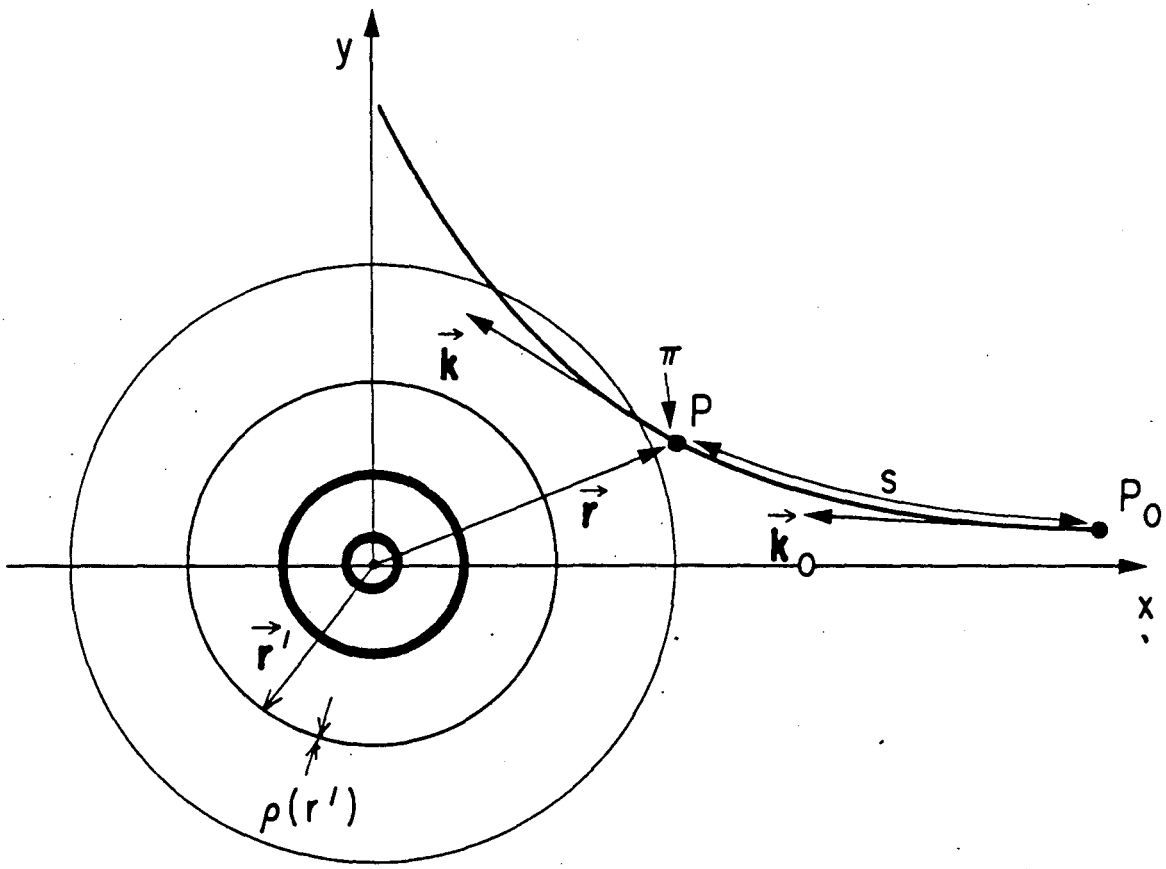


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Fig. 1

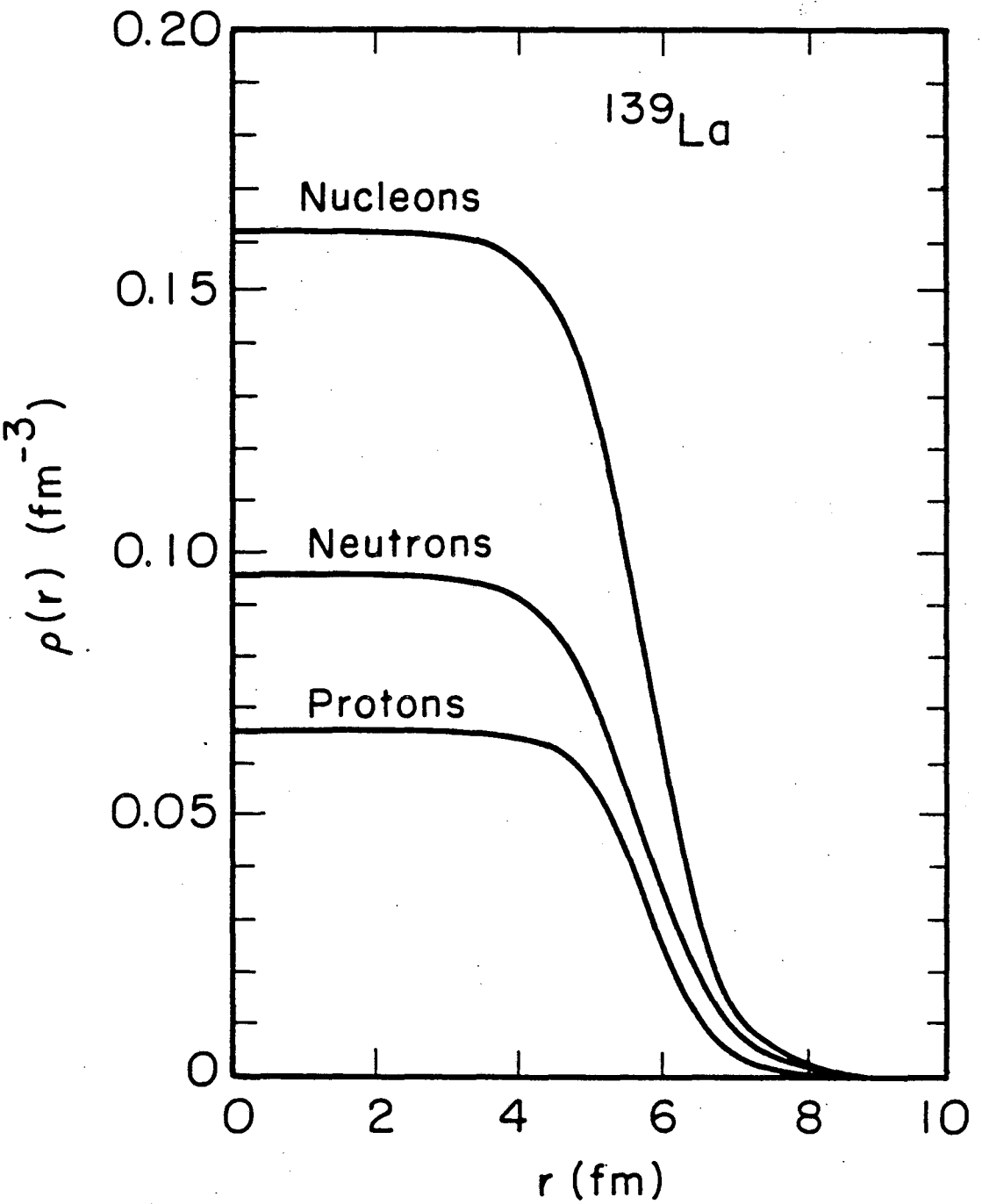


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Fig. 2

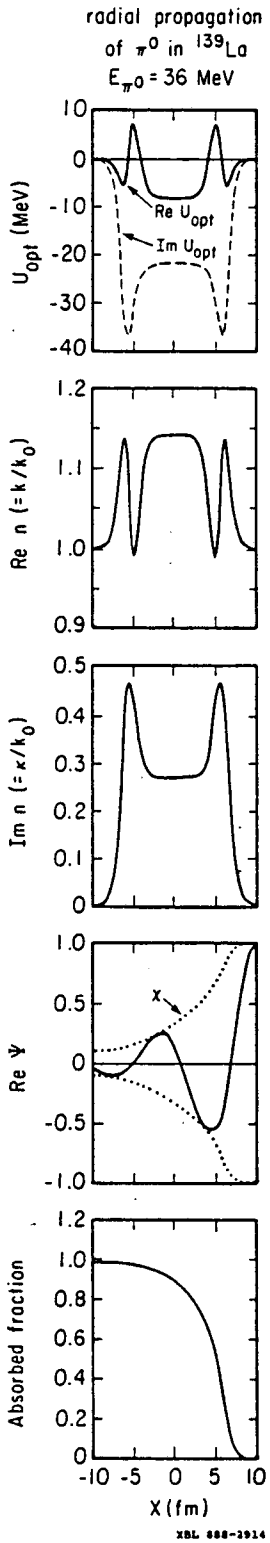
radial propagation
of $\pi^0$ in $^{139}$La
$E_{\pi^0}$ = 36 MeV
$U_{opt}$ (MeV)
Re $U_{opt}$
Im $U_{opt}$
Re n (= k/$k_0$)
Im n (= $\kappa$/$k_0$)
Re $\Psi$
x
Absorbed fraction
X (fm)
XBL 888-2914

Fig. 3

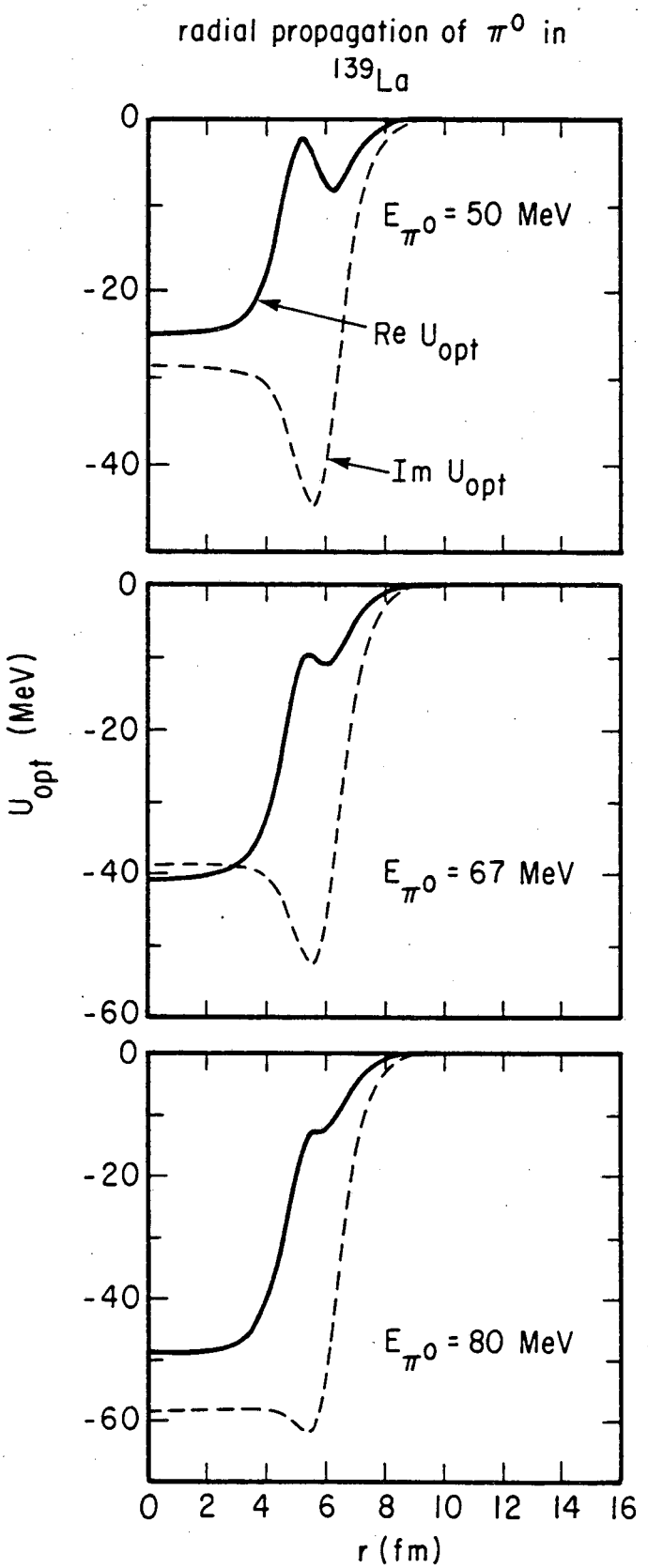
radial propagation of $\pi^0$ in $^{139}La$
$E_{\pi^0}$ = 50 MeV
Re $U_{opt}$
Im $U_{opt}$
$E_{\pi^0}$ = 67 MeV
$E_{\pi^0}$ = 80 MeV
$U_{opt}$ (MeV)
0
-20
-40
-60
0 2 4 6 8 10 12 14 16
r (fm)
XBL 888-2919

Fig. 4

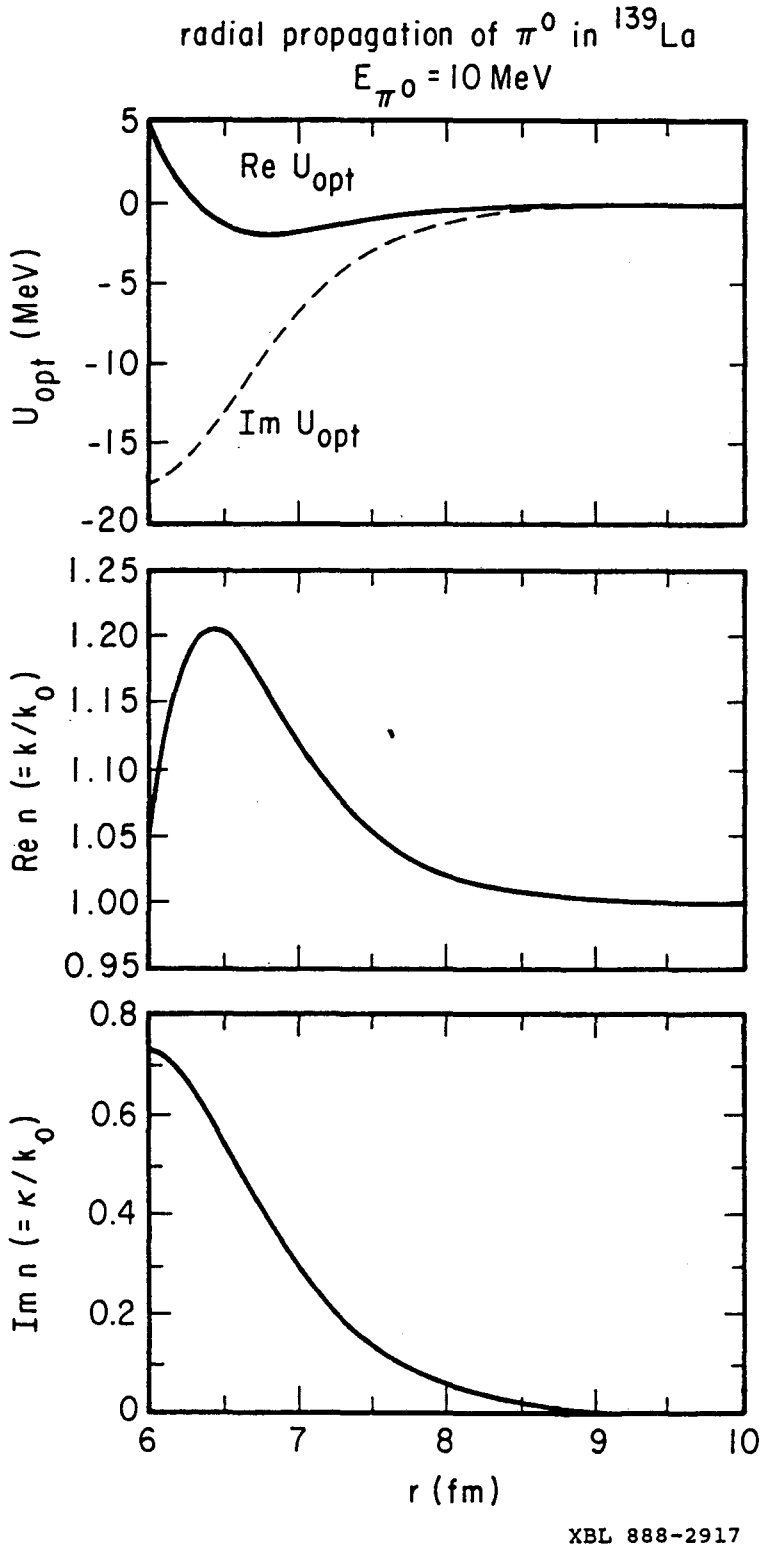
radial propagation of $\pi^0$ in $^{139}$La
$E_{\pi^0}$ = 10 MeV
Re $U_{opt}$
Im $U_{opt}$
$U_{opt}$ (MeV)
5
0
-5
-10
-15
-20
Re n (= $k/k_0$)
1.25
1.20
1.15
1.10
1.05
1.00
0.95
Im n (= $\kappa/k_0$)
0.8
0.6
0.4
0.2
0
6
7
8
9
10
r (fm)
XBL 888-2917

Fig. 5

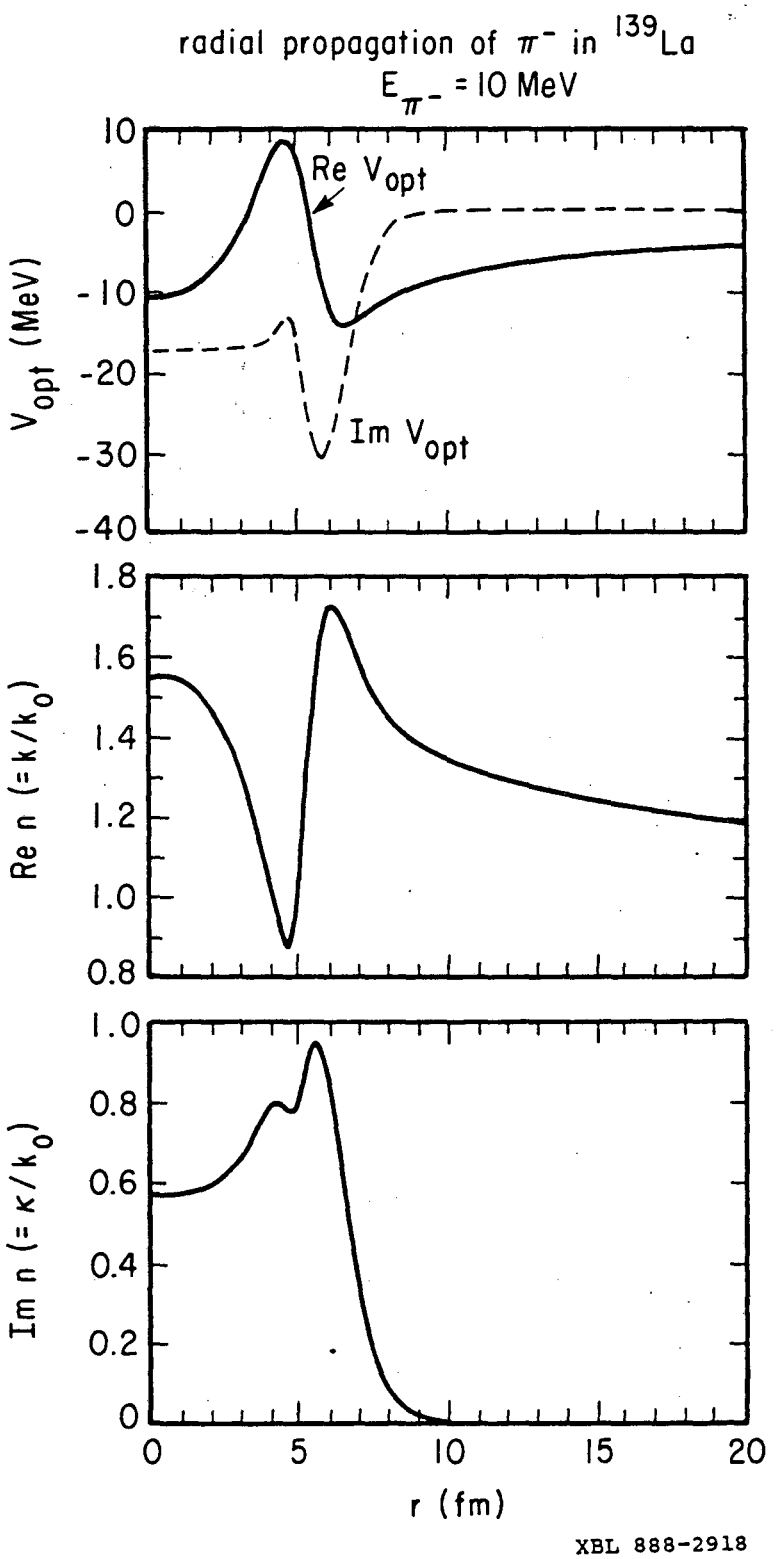
radial propagation of $\pi^-$ in $^{139}$La
$E_{\pi^-}$ = 10 MeV
Re $V_{opt}$
Im $V_{opt}$
$V_{opt}$ (MeV)
10
0
-10
-20
-30
-40
Re n (= k/k$_0$)
1.8
1.6
1.4
1.2
1.0
0.8
Im n (= κ/k$_0$)
1.0
0.8
0.6
0.4
0.2
0
0
5
10
15
20
r (fm)
XBL 888-2918

Fig. 6

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