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## Nuclear Multifragmentation: Models and Observables

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October 1988

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# Nuclear Multifragmentation: Models and Observables

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# NUCLEAR MULTIFRAGMENTATION: MODELS AND OBSERVABLES*

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First two major classes of models for nuclear dynamics are considered, namely one-body (mean-field) models and $A$-body (molecular-dynamics) models, and some areas in need of further progress are identified; a brief description is also given of a recently developed quasi-classical molecular-dynamics model that incorporates a Pauli potential and mimics nuclear systems well. Then the attention is turned towards statistical models of multifragmentation, and recent work on contrasting sequential binary breakup with prompt multifragmentation is described.

## 1. INTRODUCTION

Nuclear collisions at intermediate energies typically lead to final states containing many complex fragments. As the beam energy is raised from a few tens to several hundreds of MeV/N, the multiplicity of final fragments increases steadily and the experimental characterization of the events becomes progressively more difficult. Yet, the field has enjoyed a remarkable growth through the past decade, as signified by the commissioning of ever more powerful accelerator facilities and associated instrumentation, culminating with the impressive SIS-18/ESR complex presently being completed at GSI. The study of nuclear collisions at medium energies is primarily motivated by the unique possibilities for probing the physical properties of hot and dense nuclear matter. However, the extraction of unambiguous information depends on our ability to discern the reaction mechanisms. The complexity of the collision dynamics makes it difficult to interpret the experimental data on multifragmentation processes. The difficulty is compounded by the fact that a plethora of models exist for nuclear collisions at intermediate energies, and typically the (limited) data available can be accounted for by models based on mutually conflicting assumptions.

A meeting like the present one may be likened to an *Autobahn*, with the drivers being the theorists and the cars representing their various models. With this picture in mind, it appears very appropriate that the present meeting is denoted a *workshop*, since that's where you go when your car needs being repaired. In this spirit, I shall comment on some of the currently employed models, with the aim of identifying aspects in need of further development. I shall also briefly report some recent results that may be useful in our endeavor to understand the intriguing phenomena associated with nuclear collision dynamics.

As indicated at the appropriate places, part of the work reported here was carried out in collaboration with Claudio Dorso, Sergio Duarte, and Jorge López.

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## 2. DYNAMICAL MODELS

In the present section, two main classes of dynamical models will be briefly discussed, namely one-body models and $A$-body models. In the simplest form, the former type of model is often referred to as a mean-field model, while the latter has been dubbed molecul· dynamics.

### 2.1 One-body models

The starting point for mean-field models is the well-established fact that, at sufficiently low "temperatures", the nuclear properties, static as well as dynamical, can be understood in terms of individual nucleons moving in a one-body mean field and subject to residual two-body interaction. In these models the dynamical information retained corresponds to the reduced one-body density matrix and, accordingly, the emphasis is on studying one-body observables.

The principal one-body model has long been the Time-Dependent Hartree-Fock model, which has been used extensively to study and interpret nuclear dynamical phenomena at relatively low energies. Its classical analog is the Vlasov model, in which the (appropriately prepared) one-body phase-space density (the classical analog of the Wigner distribution) is propagated by Liouville's equation, rather than Schrödinger's equation. As the energy is raised, the effect of the residual interactions grows increasingly significant as the exclusion principle becomes less effective in blocking the final states for direct two-body collisions between the constituent nucleons.

A general method for incorporating such direct two-body collisions was first formulated by Nordheim[1] and it has been adapted for nuclear studies under the name of the Vlasov-Uehling-Uhlenbeck (VUU) or Boltzmann-Uehling-Uhlenbeck (BUU) model. In these models, the average effect of the residual two-body scattering is incorporated self-consistently, for example by performing, at each time step, an average over a large number of parallel evolutions, each of which represents one particular manifestation of the $A$-body system. [It is important to note that even though the system is thus represented by $A$-body information during each time step, the only dynamical information retained after the averaging is the one-body information. Thus, the model remains a one-body model, but modified to incorporate the average effect of the two-body collisions.] The resulting type of model represents a significant step forward in our description of nuclear dynamics at medium energies.

Even so, such a model still has the significant drawback that the same initial condition will always produce the same final one-body density: it is fully deterministic. Since the many possible evolutions are averaged over after each time step, any particular fluctuation is not given a chance to develop over finite times. This lack of dynamical branching is particularly serious when instabilities are present, since the system is forced to compromise between future histories that differ substantially, rather than being allowed to choose between them. In particular, the phenomenon of multifragmentation (where the final channels differ significantly) can not be adequately addressed. Thus there is an urgent need for developing the Nordheim-type models further, so that the spontaneous fluctuations generated by the two-body collisions may develop independently of any of the other possible histories of the particular system. I consider this the presently most important challenge in the area of nuclear dynamics.

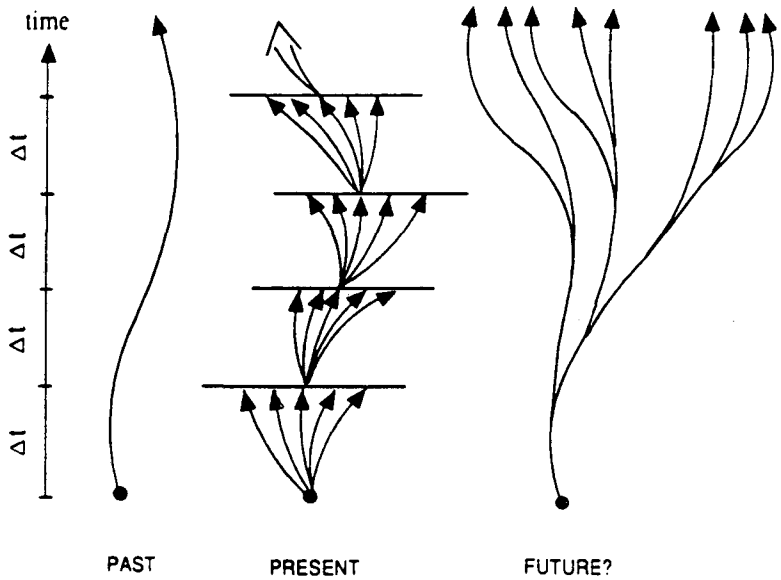



FIGURE 1.
Schematic illustration of the gradual refinement of one-body models for nuclear dynamics.

The gradual refinement of one-body models for nuclear dynamics is illustrated in fig. 1. On the left is depicted the trajectory of a pure one-body model, such as the TDHF or the Vlasov model. In these models a single dynamical trajectory results. [For example, in TDHF the wave function remains a single Slater determinant throughout.]. These models have been very useful in the past, when the dynamical processes considered exhibited relatively little fluctuation. The drawing in the middle shows the extension of the one-body models to incorporate the average effect of spontaneous fluctuations, generated by direct collisions between the individual nucleons: starting from a given dynamical state, many parallel histories are followed for a short time $\Delta t$, after which the average change is calculated and the procedure repeated. Thus, this treatment incorporates the average effect of the two-body collisions, while still leading to a single dynamical trajectory (which is in general different from the one resulting when the collisions are neglected, as on the left). This type of model (commonly referred to as BUU or VUU) is the present state of the art. The drawing on the right illustrates what kind of model is needed for a more satisfactory description of processes where the dynamical fluctuations propagate to such a degree that histories that are significantly different may evolve (*e.g.* breakup into channels of different fragment multiplicity). For such processes some form of dynamical simulation is probably required.

The situation is similar to that of Brownian motion, though far more complex, of course. The pure one-body treatment corresponds to ignoring entirely the interaction of the particle with the heat bath in which it is embedded. Taking into account the average effect of the collisions with the gas molecules leads to a mean-trajectory description, effected by adding a friction term to the free equation of motion. Although an auxiliary equation of motion can be derived for the second moments, the actual diffusion of the trajectories can probably only be incorporated by performing a direct numerical simulation of the associated transport equation.

2.2 Molecular dynamics

Molecular-dynamics models for nuclear reactions are adapted from chemical problems. These models describe individual classical nucleons interacting directly, ordinarily via specified two-body forces. Thus the entire $A$-body phase-space density is followed and many-body quantities, such as clusterization, can readily be studied. Although the dynamical evolution is strictly deterministic, molecular dynamics nevertheless contains some degree of fluctuation, due to the fluctuations present in the initial state.

A few years ago, a classical molecular-dynamics model was developed by Pandharipande *et al.*[2]. This model was designed to simulate a system a Ar *atoms*, rather than nuclei. Having the advantage of being relatively well defined, this approach has yielded instructive insight into the collision dynamics of of clusters of Ar atoms. However, since the characteristic properties of Ar matter are significantly different from those of nuclei, the model is not directly applicable to nuclear systems and the modifications required for addressing nuclear problems are not straightforward.

A major problem in adapting molecular dynamics to nuclear problems is the absence of the Pauli exclusion principle in a classical approach. This deficiency causes such models to be particularly unrealistic at low excitations where the absence of nucleonic motion makes it hard to reproduce the most basic features characteristic of nuclear matter. In particular, the ground states implied are usually quite unrealistic.

In an attempt to circumvent this inherent shortcoming, Aichelin and Stöcker[3] have devised the so-called *Quantum Molecular Dynamics* model. The "Quantum" indicates that the initial nuclei are prepared "by hand" so as to comply with the phase-space distribution of the corresponding quantal system. Although the thus constructed "nuclei" prove to remain reasonably stable when left in isolation, it is important to realize that they are not inherently stable, so that the uncontrollable disturbances generated by the collision process may destroy their resemblance with real nuclei and produce fragments closer to the collapsed ground-state clusters characteristic of such models. It is clear that this feature raises serious problems with regard to the interpretation of the outcomes of nuclear collision simulations.

We have recently sought to remedy this problem. Our goal has been to establish a molecular-dynamics model that simulates the Pauli exclusion principle and approximates nuclear systems as well as possible. Our success in this regard is briefly summarized below. It should be noted that the developed model is not unique and parallel efforts towards the same goal are valuable so that it can be ensured that any conclusions reached are not dependent on the specific model employed. We are aware of two concurrent efforts, one by David Boal *et al.* and another by Georg Peilert *et al.*.

2.3 Quasi-classical molecular dynamics

The work reported here was carried out with Claudio Dorso and Sergio Duarte[4,5].

A molecular-dynamics model is specified by its $A$-body Hamiltonian, which we take to be of the simple form

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m_i} + \sum_{i<j} V_{ij} \ , \ V_{ij} = V_P(r_{ij}, p_{ij}) + V_N(r_{ij}) + V_C(r_{ij}) \ . \tag{1}$$

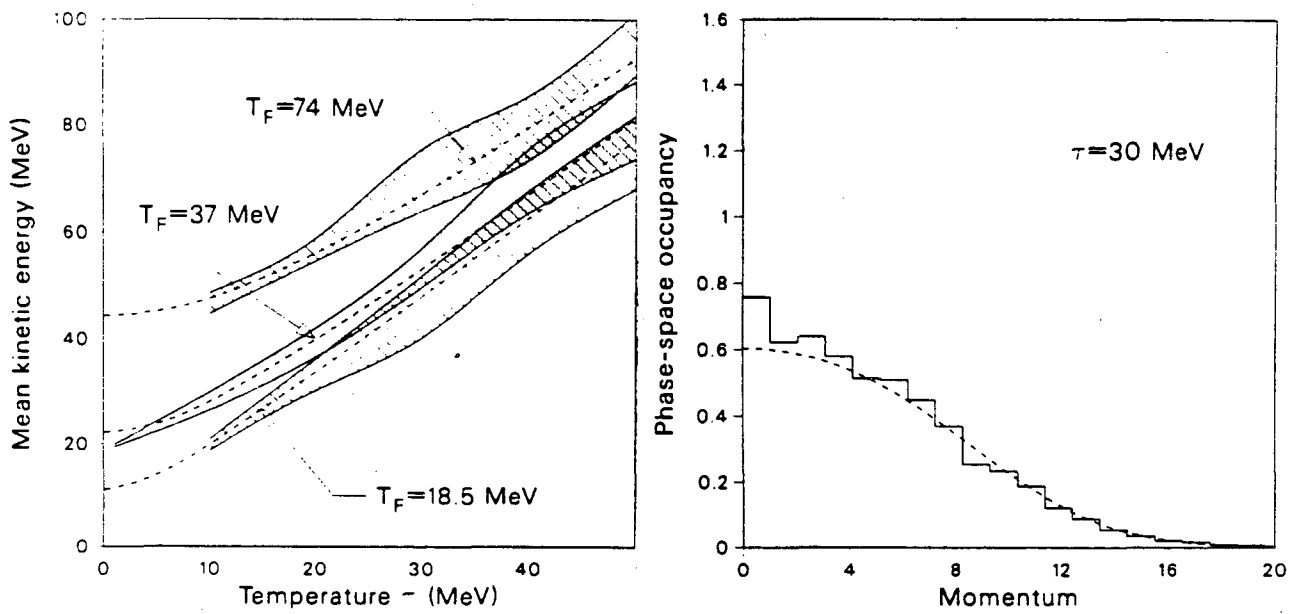



FIGURE 2.

On the left is shown the mean kinetic energy per nucleon in an infinite gas of specified density end temperature, as calculated with the Pauli potential $V_P$ of eq. (2); the dashed curves are the exact result for a free Fermi gas under the same conditions. On the right is shown the momentum distribution in such a system when the conditions corresponds to standard nuclear matter at a temperature of $\tau$=10 MeV; the smooth curve is the the corresponding Fermi-Dirac distribution. These results are from ref. 4.

The first term in the interaction energy is the *Pauli potential.* This momentum-dependent repulsion is designed to simulate the effect of the Pauli exclusion principle on the phase-space distribution of the nucleons and it acts between nucleons with the same spin-isospin components. Such an approach was first tried by Wilets *et al.*[6]. We have recently reconsidered the problem and demonstrated that the potential

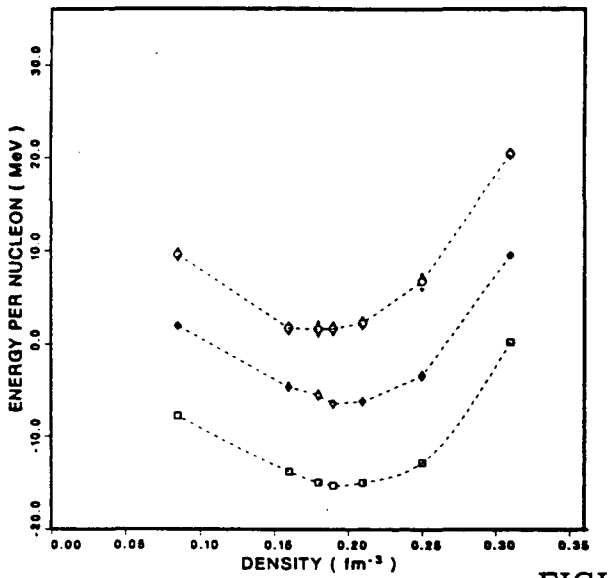
$$V_P = V_P^0 \left(\frac{\hbar}{p_0 q_0}\right)^3 e^{-p_{ij}^2/2p_0^2 - r_{ij}^2/2q_0^2} , \tag{2}$$

where $V_p^0$=34.32 MeV, $p_0$=2.067 MeV$\cdot 10^{-22}$ s/fm, $q_0$=6.00 fm, leads to a rather good reproduction of the momentum distribution in a free Fermi gas, over an interesting range of temperatures and densities.[4] This is illustrated in fig. 2.

Subsequently, the following modified Lennart-Jones nuclear potential has been determined,

$$V_N = V_N^0 \left[\left(\frac{r_1}{r_{ij}}\right)^{p_1} - \left(\frac{r_2}{r_{ij}}\right)^{p_2}\right] \frac{1}{1 + e^{(r_{ij}-d)/a}} , \tag{3}$$

where $V_N^0$=25.93 MeV, $p_1$=6.2, $r_1$=1.757 fm, $p_2$=3.0, $r_2$=1.771 fm, $d$=3.350 fm, $a$=5/6.[5] This potential, in conjunction with the above Pauli potential, gives a reasonable behavior




| Nucleus | Experiment | Calculation |
|---|---|---|
| $^{4}$He | 28.30 | 18.8 |
| $^{6}$Li | 31.993 | 34.5 |
| $^{8}$Be | 56.50 | 48.4 |
| $^{12}$C | 92.16 | 88.92 |
| $^{16}$O | 127.62 | 124.48 |
| $^{20}$Ne | 160.65 | 163.8 |
| $^{28}$Si | 236.63 | 247.8 |
| $^{40}$Ca | 342.06 | 375.2 |
| $^{50}$Ti | 437.79 | 473.5 |
| $^{64}$Zn | 559.01 | 611.2 |
| $^{82}$Kr | 714.28 | 738.4 |

FIGURE 3.

On the left is shown the calculated energy per nucleon in nuclear matter over a broad range of densities and temperatures. On the right are listed the measured and calculated binding energies for light and medium-heavy nuclei. These results are from ref. 5.

of the energy of nuclear matter as a function of density and temperature. In particular, the model yields reasonable values for both the saturation energy and the saturation density (although the latter value is around 10% too high). The effective compressibility, as indicated by the energy of cold matter at double density, is also close to that produced by standard nuclear models. The addition of a point-charge Coulomb repulsion between protons makes it possible to consider finite nuclei as well and a satisfactory global behavior is achieved for both binding energies and radii, with these quantities being typically 10% and 20% too large, respectively. The degree to which the model mimics nuclear systems is illustrated in fig. 3.

Notwithstanding this good overall reproduction of the most important static nuclear properties, certain general deficiencies remain. One concerns the specific heat at low temperatures, which is linear in a classical model, whereas it is quadratic in a quantal system. In our model, this difference is insignificant for temperatures above a few MeV and thus of minor concern for the anticipated applications to medium-energy heavy-ion collisions. Another deficiency is that there is no zero-point motion for the nucleons. This is particularly apparent in the $\alpha$-particle whose four nucleons are at relative rest (since their different spin-isospin components render them insensitive to the Pauli exclusion). Even so, the binding energies of light nuclei are quite acceptable. For heavy nuclei, the zero-point motion is small in comparison with the Fermi motion, which is well reproduced by the Pauli potential.

The work summarized above was motivated by the need for models for nuclear dynamical processes that both incorporate fluctuations and give a reasonable reproduction of general thermostatic nuclear properties. This latter requirement serves to ensure that the results for dynamical processes be interpretable and informative, and it ought to constitute a minimum requirement for any model employed for the study of energetic nuclear collisions.

Having established that our quasi-classical model accounts reasonably well for the general thermostatic nuclear properties, we are in a well-founded position to address dynamical problems. Once the Hamiltonian has been specified, the molecular-dynamics equations of motion immediately follow, $\dot{\mathbf{p}}_i = \partial H/\partial \mathbf{p}_i$ , $\dot{\mathbf{r}}_i = \partial H/\partial \mathbf{r}_i$. Using the nuclei resulting from the determination of the ground-state properties, nuclear reactions can readily be simulated numerically and such studies are presently in progress.

## 3. STATISTICAL MODELS OF MULTIFRAGMENTATION

Because of the complexity of the nuclear disassembly process, it is natural to apply statistical considerations to the problem – statistical models are often remarkably economical and powerful. Accordingly many statistical models have been developed and it is beyond the scope of this discussion to review those efforts.

The presently most refined statistical models of nuclear disassembly consider assemblies of excitable and interacting fragments. As an illustration, in the microcanonical approximation the *density of states* is given by the sum over all accessible multifragment states $F$ of the system and can be expressed as a sum over *configurations*,

$$\rho(\Omega, A, E) \equiv \sum_F \delta(A_F - A)\delta(E_F - E) = \sum_N \prod_{n=1}^{N} \left[ \sum_{A_n} \int \frac{d\mathbf{r}_n}{\Omega} \int d\epsilon_n \right] W(C) \ . \tag{4}$$

Here the *statistical weight* of a given configuration $C = \{A_n, \epsilon_n, \mathbf{r}_n,\ n = 1, ..., N\}$ is given by

$$W(C) = \frac{1}{N!} \frac{1}{\Gamma(\frac{3}{2}N)} \prod_{n=1}^{N} \left[ \Omega (\frac{mA_n}{2\pi\hbar^2})^{\frac{3}{2}} \rho(\epsilon_n) \right] \delta(\sum_n A_n - A)\ K^{\frac{3}{2}N-1} \ , \tag{5}$$

when label permutations are considered to be significant. Here $K(C)$ is the total kinetic energy of the $N$ fragments. Similar treatments can be given for the canonical and grand-canonical approximations, and the formulation readily lends itself to numerical implementation in terms of a Metropolis sampling of the configuration space.[7] [For the above result (5) only energy conservation has been imposed. Conservation of the overall momentum is readily incorporated as well; that refinement will merely reduce the power of $K$ by $\frac{3}{2}$. The conservation of overall angular momentum and overall center-of-mass position would be more complicated to include analytically, but would still not present an essential calculational obstacle.]

A statistical model of the type described above lends itself most easily to studies of infinite (*i.e.* periodic) matter, prepared at a given mean density and at a given energy or temperature. Such studies are of direct astrophysical relevance and it would be interesting to explore the phase diagram of hot, dilute matter with such a model. However, reliable results must await more realistic implementations.

For nuclear-matter studies, the confining volume $\Omega$ is simply the test volume considered. It is far from obvious how to properly apply the model to predict the outcome of a nuclear collision, which is a dynamical process in which equilibrium may not be established. Ordinarily, one relies one some sort of transition-state approximation and assumes

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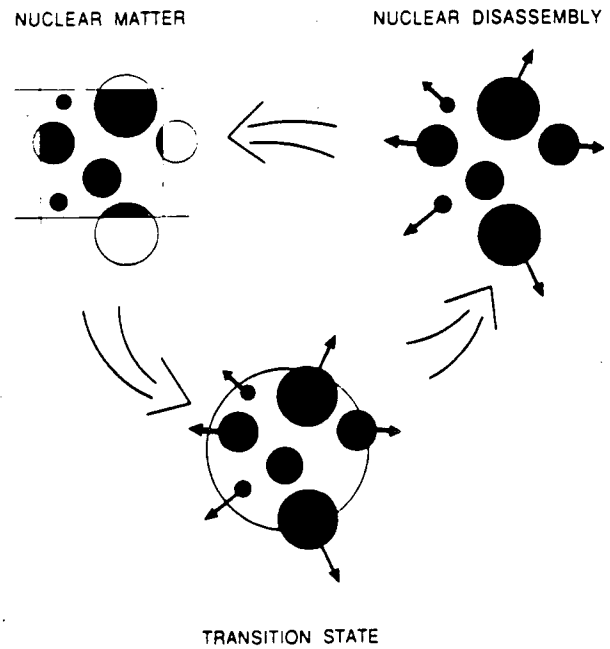


FIGURE 4.
The relation between the statistical properties of hot and dilute nuclear matter and fragment production in nuclear collisions. See the text below for more explanation.

that the produced fragments reflect the statistical properties within a certain freeze-out volume, identified with the quantity $\Omega$ entering in the statistical weights in (5).

Figure 4 illustrates the relationship between the properties of hot and dilute nuclear matter and fragment production in nuclear collisions. A major motivation for studying nuclear multifragmentation processes is to learn about the statistical properties of nuclear matter far from ordinary conditions. Conversely, knowledge of those properties can be used to predict fragment production in nuclear collisions. To do that one calculates the fragment distribution within a specified freeze-out volume $\Omega$ and assume that these fragments cease to interact and freely propagate away from the source zone (possibly to deexcite later on in a more standard sequential manner). This procedure amounts effectively to invoking a transition-state approximation.

Such statistical models have been remarkably successful in reproducing a variety of experimental data. [See, for example, the preceding contribution by Gross and references therein.] Nevertheless, there are still several important aspects that are not treated satisfactorily in the current statistical multifragmentation models. Some of the major ones concern:

1) *Nucleon vapor.* At the high temperatures of interest, the nuclear fragments are unable to confine the nucleons within their interior and it is necessary to allow the nucleons to roam the entire volume. Although some progress has been made with regard to the incorporation of such a nucleon vapor[7] (or, equivalently, the proper description of highly excited nuclear fragments), further development is necessary for achieving a sufficiently realistic description.

The above mechanism of sequential binary decay is rather well defined. It is interesting to contrast it with the mechanism of simultaneous multifragment breakup, which may be referred to as *true* multifragmentation. In order to facilitate the comparison, it is instructive to consider events that differ only kinematically, *i.e.* contain the same fragment species. For this purpose, we have devised a scheme that produces a multifragmentation event for each event generated by the sequential-binary mechanism described above. The partner event consists of the exact same fragments, and has the same total energy, but the individual fragment velocities are different. In this manner the inherent kinematical differences between the two mechanisms can be brought out most clearly.

A "true" multifragmentation event is generated as follows. The excited fragments are placed in a random non-overlapping configuration with their centers confined within a sphere whose volume is twice that of the nuclear volume for the original compound nucleus ${}^{A_0}Z_0$. For the resulting configuration, the potential energy $V$ associated with the mutual Coulomb repulsion between the fragments is then calculated and the excess energy is distributed among the translational degrees of freedom of the fragments. This "freeze-out" state is then propagated dynamically, under the action of the Coulomb repulsion, until the fragments have attained their asymptotic velocities.

The event samples produced in this simple statistical multifragmentation model match, event by event, those produced by the sequential mechanism, with respect to total kinetic energy, fragment composition, and fragment excitation. As an illustration of the results, consider a source with $A_0$=150 and $Z_0$=62 at excitation energy of 5 MeV/N. The two different disassembly mechanisms produce notable differences in the velocity distributions of the fragments. A visual impression of this can be obtained by overlaying the velocity distributions of many events, after suitable reorientations have been made. The emerging cloud of representative points in velocity space exhibit a rather elongated shape for the sequential decay whereas the simultaneous breakup leads to a fairly spherical shape. These structures can be quantified in a variety of ways:

*Sphericity analysis.* It is instructive to consider the flow tensor $T_{ij} = \sum_n p_n^i p_n^j / 2m_n$, where $p_n^i$ denotes the $i^{\text{th}}$ Cartesian component of the momentum of the $n^{\text{th}}$ fragment and $m_n$ is its mass. One can use the ordered eigenvectors of $T$, $t_1 < t_2 < t_3$, to define the reduced quantities $q_i \equiv t_i^2 / \sum_{j=1}^3 t_j^2$, in terms of which the sphericity and coplanarity shape parameters are given by $S = \frac{3}{2}(1 - q_3)$ and $C = \frac{1}{2}\sqrt{3}(q_2 - q_1)$, respectively. In the $S - C$ plane the origin would correspond to a rod-like object, the point $(1, 0)$ to a spherical shape, and $(3/4, \sqrt{3}/4)$ to a disk. The left side of fig. 5 presents the result of this analysis. The differences in shape are clearly visible, with the sequential-binary decays having a rod-like shape and the multifragmentation events being more spherical.

*Folding angle.* A direct consequence of these different shapes of the velocity distributions is the angular correlation of the two heaviest fragments of each event. This angle is a natural generalization of the folding angle commonly employed in ordinary fission studies. [A drawback of the sphericity analysis is that it requires kinematic information about many (preferably all) of the fragments. The folding-angle analysis is considerably simpler, since it involves only the heaviest fragments.] For the sequential binary breakup we expect that the two largest fragments most often arise as residues of two early fission partners and therefore tend to appear back to back, whereas these heavy fragments are expected to have a broader angular distribution for the prompt breakup mechanism.

2) *The volume.* In order to carry out the sum over states (4), it is necessary to somehow confine the integration over the fragment positions. This is ordinarily done by demanding that they be restricted to some volume $\Omega$, whose size must be specified by the user of the model. This arbitrariness is clearly unsatisfactory and there is a need for developing some better-founded means of constraining the position integrations within the model itself.

3) *Fragment interactions.* Although generalization has now been made to interacting fragments[7] as illustrated in (5), there remains a significant practical problem with regard to actually calculating the fragment interaction energy. Schematic treatments in terms of point-charge Coulomb repulsion, possibly augmented by some proximity potential, are inadequate, since the fragments are typically quite densely packed so that major distortions are expected. [In order to appreciate the importance of this problem, it is instructive to recall that in ordinary binary fission (the simplest "multifragmentation" process), the transition state can usually *not* be described in terms of two interacting spherical fragments. This problem is clearly compounded when breakup into several fragments occur.]

In addition to such formal problems, a major practical problem is posed by the fact that insufficient data is available on multifragmentation processes. The fact that models based on different physical pictures can account comparably for the data indicates that more discriminating observables should be considered, both by theory and experiment.

3.1 Prompt versus sequential disassembly

The work reported in the following has been carried out with Jorge López[8]. With the aim of identifying observables that may help to elucidate the underlying reaction dynamics, we have examined the kinematical differences between two opposite extremes, namely sequential binary fission and *true* multifragmentation, *i.e.* simultaneous multifragment breakup.

In the first extreme, the source is considered as a compound nucleus and is assumed to disassemble by sequential binary decay. We shall refer to any such split as a "fission" process, although possible binary channels are considered, including light-particle emission. In general, the binary split produces two new compound nuclei which may then undergo further binary decay, provided that their excitation energy is high enough. This tree of fission processes results in a final multifragment state consisting of individual nucleons and bound complex nuclear fragments.

The partial decay width for a specific binary decay, $A_0Z_0E_0^\star \rightarrow A_1Z_1E_1^\star + A_2Z_2E_2^\star$ is given by the canonical transition-state method as[9]

$$\frac{d\Gamma(A_0 \rightarrow A_1A_2)}{dK_{12}^\infty} = \frac{\rho_{12}(A_0, E_0^\star - B_{12} - K_{12})}{\rho_0(A_0, E_0^\star)} , \tag{6}$$

where $K_{12}^\infty$ is the asymptotic kinetic energy of the relative fragment motion. The level density of the "activated complex" at the conditional saddle point for the particular mass-asymmetry considered is denoted by $\rho_{12}(E_{12}^\star)$. Here the intrinsic excitation energy of the nucleus at the saddle point is given by $E_{12}^\star = E_0^\star - K_{12} - B_{12}$, where $B_{12}$ is the barrier height and $K_{12}$ denotes the kinetic energy of relative motion of the two (pre)fragments at the saddle point. By repeating the decay procedure for each (sufficiently excited) daughter fragment, an ensemble of final multifragment events is conveniently generated.

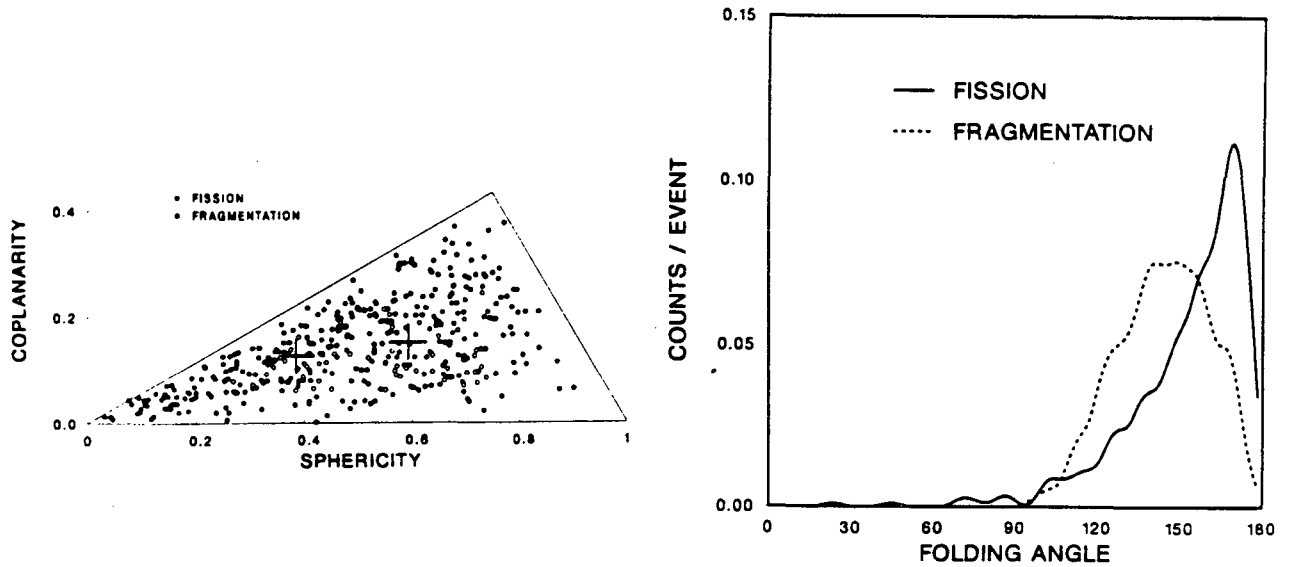


FIGURE 5.

Analysis of 200 final states following the disassembly of $^{100}Sm^*$ at $E_0^*$=5 MeV/N, in the two opposite disassembly scenarios considered. On the left is shown the distribution with respect to sphericity and coplanarity, and on the right is shown the distribution of the folding angle between the two heaviest fragments.

On the right side of fig. 5 we present the distribution of the angles between the two heaviest fragments of each event, as obtained for the two breakup mechanisms considered. As expected for a fission process, the two largest fragments in the sequential mechanism are highly correlated at large angles and appear to have been emitted almost back to back. The simultaneous process, on the contrary, peaks around 140° with a wider distribution. This brings out the different geometry characterizing the two mechanisms of fragment production.

In summary, we have studied the kinematical differences between the two opposite breakup mechanisms considered. In addition to certain differences in Coulomb peaks for the proton distribution, we have found that the mechanisms differ markedly in velocity correlations, sphericity-coplanarity coordinates, and heavy-fragment angular correlations. Unfortunately, we have also found these differences to be energy dependent and difficult to pinpoint experimentally, in addition to being somewhat model dependent.

A tentative (and as yet preliminary) application of the suggested methods of analysis has been made to the disassembly of excited $^{16}O$ nuclei, produced by projectile excitation in peripheral reactions at 620 MeV at the LBL 88-Inch Cyclotron. [See the contribution by Stokstad in these proceedings.] So far, the most complete data analysis has been made for the $4\alpha$ final state. On the basis of this preliminary (and incomplete) analysis, it appears that the disassembly of the produced excited $^{16}O$ nuclei proceeds by way of sequential binary breakup.

3.2 Transition-state treatment of multiple breakup

We are presently seeking to develop a model for true multifragmentation (*i.e.* the simultaneous breakup into several fragments) by appropriate generalization of the treatment of binary fission described above.[10] The rate for disassembly into a given channel is then given by

$$\frac{d\Gamma(A_0 \rightarrow A_1 \cdots A_N)}{dK^{\infty}_{1\ldots N}} = \frac{< \rho_{1\ldots N}(A_0, E^*_0 - B_{1\ldots N} - K_{1\ldots N}) >}{\rho_0(A_0, E^*_0)} \tag{7}$$

Here the final channel is specified by the mass numbers $A_1$, ..., $A_N$ and the asymptotic kinetic energy of all the fragments, $K^{\infty}_{1\ldots N}$. The indicated average is over the (many) different spatial geometries of the transition configuration. The corresponding barrier is denoted by $B_{1\ldots N}$ and the kinetic energy associated with the expansion, at the transition point, is $K_{1\ldots N}$.

The total width for prompt disassembly into $N$ fragments can be obtained by summing the corresponding partial widths over the various possible mass partitions,

$$\Gamma_N(A_0, E^*_0) = \sum_{A_1 \leq \cdots \leq A_N} \delta(\sum_{n=1}^{N} A_n - A_0)\, \Gamma(A_0 \rightarrow A_1 \cdots A_N)\ . \tag{8}$$

In fig. 6 are shown preliminary results for $\Gamma_2$, $\Gamma_3$, and $\Gamma_4$. It is seen how the dominant binary breakup is meeting with increasingly strong competition from multiple breakup as the excitation energy of the source is raised. These calculations also indicate that there is a rapid increase of the dominant multiplicity at a certain excitation energy. For the system considered, this onset of multifragmentation occurs at around 6-8 MeV/N.

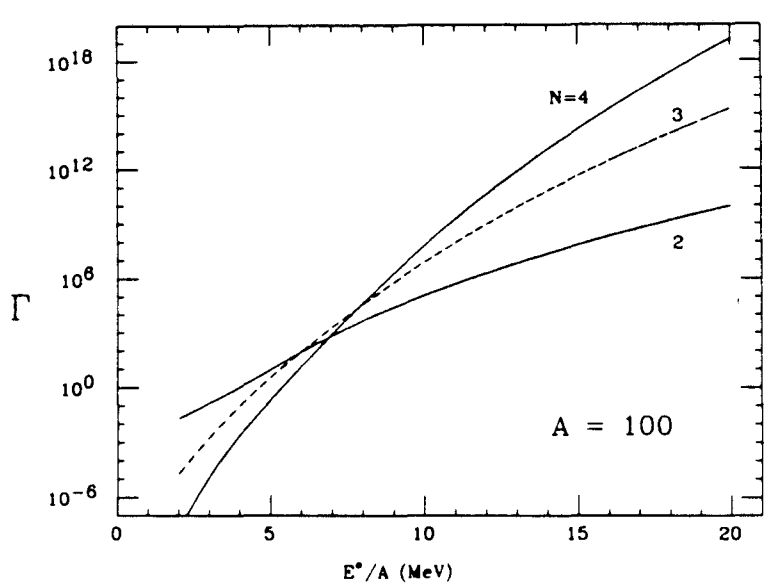


FIGURE 6.
Competition between binary, ternary, and quaterny breakup. See text.

## 4. CONCLUDING REMARKS

Nuclear multifragmentation presents challenging theoretical problems. In general, the possibility of breakup into competing channels presents the system with a choice between alternatives leading to qualitatively different final states. The current one-body models for nuclear dynamics do not incorporate such dynamical branching and are therefore of limited utility in addressing the experimental observables (in which channel specificity plays a central role). Until such further development has been accomplished, instructive insight may be gained from suitably constructed molecular-dynamics models. In order to make the results interpretable, it is important that such models mimic nuclear systems as closely as possible. Recent progress with regard to simulating the Pauli exclusion principle in terms of a momentum-dependent repulsion presents an important advance in this endeavor.

The unique physical scenarios created (albeit transiently) in nuclear collisions at medium energy also call for further developments of statistical models, particularly with regard to incorporating the nucleon vapor, the fragment interactions, and the proper definition of the freeze-out volume. The two latter problems may be solved by developing a suitable generalization of the transition-state method to multiple breakup.

It is also important to ascertain to which extent various model "predictions" are specific to a given model and, conversely, there is a need for identifying observables that are well suited for discrimination between different physical assumptions. With regard to this latter aspect, recent work on contrasting sequential binary breakup with prompt multifragmentation has suggested some novel ways of analyzing multifragment data.

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