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Space-Time Aspects of Hadronic Cascading in Lepton Nucleus Scattering*

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Abstract:
A Monte Carlo model of hadronic cascading in inelastic lepton nucleus scattering is constructed to investigate space-time scenarios consistent with the momentum space description of string models of multiparticle production. The prospects for resolving the ambiguity inherent in the definition of a formation length for composite hadrons are emphasized.

1 Introduction

It was realised [1] a long time ago that, as a result of time dilation, the formation length ℓ of a high energy particle should grow linearly with its energy,

$$\ell(x) \sim \frac{E}{\kappa} \sim \frac{x\nu}{\kappa} \tag{1}$$

where $x \equiv (E/\nu)$, and ν is the total energy available. The constant κ has dimensions of an effective string tension parameter, and should be on the order of a GeV/fm. More generally, the fluctuations and correlations of x and ℓ are described in terms of a distribution $\mathcal{D}(x,\ell)$, the shape of which depends sensitively on the specific particle production mechanism considered (e.g., string fragmentation).

Ideally, what we would like to learn from eA data may be expressed in terms of questions such as: What is the size of κ? What information can be obtained concerning the phase space distribution $\mathcal{D}(x,\ell)$, and how does this help do decide between competing dynamical models? Furthermore, as hadrons are composite particles, an ambiguity arises as far as the definition of ℓ is concerned: should it refer to the formation of the final state hadron, or of its constituents (partons)? Finally, what impact do the answers to the above questions have on our understanding of the evolution of the energy density in high energy nuclear collision experiments at BNL and CERN?

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In the following, we investigate secondary hadronic cascading in inelastic lepton nucleus scattering experiments. Renewed interest in this topic stems from new experimental proposals (PEGASYS [2]) that may provide answers to the above questions. As will be discussed in detail below, previous studies [3,4,5] of the experimentally observed [6,7] nuclear attenuation effect in the forward region either neglected the contribution of rescattered secondaries, or else are unable to describe the recently observed energy dependence [7].

In section 2, we briefly review some basic properties of the Lund string model [8] related to the formation length; concerning the ambiguity in defining ℓ for a composite hadron, we introduce the concepts of the yoyo formation length and the constituent formation length [9] as two extreme alternatives for assigning a formation point to a hadron. The consequences of adopting either one of these two scenarios for the study of inelastic eA interactions, as well as the differences between them, are discussed in terms of a schematic analytic model. Section 3 contains a description of the Monte Carlo model PERSEUS [10], and numerical results obtained for a 14.5GeV/c electron beam scattering on noble gas targets. Finally, conclusions as well an outlook for further theoretical studies are presented in section 4.

2 The Formation Length in String Models

A successful description of a wide range of observables and phase space distributions for hadronic multiparticle production in e^+e^-, pp , pA and (to an accuracy of about 20%) also in AA collisions, has been provided by the string models developed by the Lund group [8]. Below, we shall apply the basic concepts of these models (concerning string excitation, scattering and fragmentation) as well as the available Monte Carlo routines [11,12] as an input for our analysis of nuclear effects on the fragmentation function.

A Lund string is specified by its light cone momenta $E^{\pm} = E \pm P_z$, its transverse momentum $\mathbf{P}_{\perp}$ and the quark flavors at the ends; excitation and fragmentation of strings into secondary hadrons are represented graphically in momentum space diagrams such as Fig.1. While the Lund model by itself does not contain any configuration space description of either string motion or string fragmentation, it is suggestive to think of a string as a system of two relativistic point particles in 1+1 dimensions that are bound by a linear (confining) potential $V = \kappa r$, where κ is the string tension ($\kappa \approx 1$ GeV/fm). The corresponding equations of motion describe "relativistic yoyos", and a momentum space diagram such as Fig.1 can be mapped onto coordinate space by rescaling all distances with a factor $1/\kappa$; in particular, the length scale characteristic of the yoyo motion of a string of energy E is then given by $L = E/\kappa$.

In order to test experimentally such a space-time scenario, it is important to consider both pA and eA scattering data, as these two types of processes yield quite different phase space constraints. Specifically, apart from any cascading effects, for

reasonable values of the projectile energy, the initial pN interaction is always non-perturbative, with momentum transfers $Q^2 \sim \Lambda^2_{QCD}$, whereas the initial eN vertex is perturbative for $Q^2 > 1$ GeV^2. Furthermore, a single pN collision already creates at least two strings with variable masses, while an eN collision produces exactly one string, the 4-momentum of which can be determined from the kinematic variables of the scattered lepton. Finally, in the absence of rescattering of secondaries, eA collisions can be expected to behave roughly like the corresponding eN processes; by contrast, pA produces on the average about $1 + A^{1/3}$ multistring excitations, which implies characteristic differences in the rapidity distributions of secondaries between eA and pA (cf.Fig.2). As the purpose of our investigation is to use rescattering effects as a probe of the underlying space time-picture, we see that eA data is much cleaner because of the absence of multistring effects and can serve to fix the theoretical "input" for a study of the more complex pA and AA reactions.

One of the relevant questions one hopes to answer by analysing the A-dependence of the data concerns the ambiguity inherent in the definition of a formation length for a composite hadron. In the context of a Lund type string model, the problem is illustrated in Fig.3. Consider a hadron (H_2 in Fig.3) which is formed out of a quark q and an antiquark $\bar{q}$ and carries a fraction x of the string energy. One possibility is to assume that it can rescatter beyond its "yoyo point", $z = \ell_y$, where the trajectories of q and $\bar{q}$ intersect for the first time. Alternatively, one could hypothesize that its constituent antiquark $\bar{q}$ may interact with the nuclear medium immediately after it is created at $z = \ell_c$, which would correspond to a smaller effective formation length for the hadron. Subsequently, ℓ_y and ℓ_c will be referred to as yoyo length and constituent length, respectively. In terms of x, one finds $\ell_y - \ell_c = xL$; clearly the differences between ℓ_y and ℓ_c are largest for $x \to 1$ (cf.Fig.4, where we plot the mean values of $\ell_c(x)$ and $\ell_y(x)$, and of the naive inside-outside cascade formula eq(1), for the Lund model). Ultimately, the question whether attenuation due to rescattering in the nuclear medium starts at ℓ_y or at ℓ_c, or at some different point, can only be answered by the experiment. The answer is of considerable importance for the ongoing discussion about the possibility of creating a quark-gluon-plasma in the relativistic nucleus-nucleus collision experiments at CERN and at BNL. For a 200 AGeV oxygen beam interacting with Au target nuclei, Fig.5 shows that the evolution of the energy density (as calculated with the ATTILA code [13] extending the Lund model to coordinate space along the lines discussed here) is extremely sensitive to the formation length used as an input for cascade calculations.

To get a qualitative understanding of the expected A-dependence of particle production in the forward region in eA collisions, let us briefly consider the following schematic model illustrated in Fig.6, where the z-axis is chosen in the direction of the 3-momentum of the virtual photon, R denotes the dimension of the nucleus in this direction, and z_0 ($0 \leq z_0 \leq R$) denotes the position of the hadronic vertex. A hadron that initially carries an energy fraction x_0 will be formed at the point $z_0 + \ell(x_0)$ and thus can be scattered on its way out of the nucleus if $z' \equiv R - z_0 - \ell(x_0) \geq 0$. The details depend (a) on the nuclear geometry, and (b) on the dynamics of the

rescattering process. Concerning (a), if the nuclear density ρ_0 is taken to be uniform, the probability for a hadron to rescatter n times may be parametrized as a Poisson distribution,

$$\begin{aligned} P_n(x_0, z_0) &= \Theta(z_0 + \ell(x_0) - R)\ \delta_{n0} \\ &+ \Theta(R - z_0 - \ell(x_0))\ \frac{1}{n!}\ \left(\frac{R - z_0 - \ell(x_0)}{\lambda}\right)^n \exp(-\frac{R - z_0 - \ell(x_0)}{\lambda}) \end{aligned} \tag{2}$$

where $\lambda \equiv (\rho_0 \sigma)^{-1}$ is the inelastic hadronic mean free path in the target nucleus.

As for point (b), the dynamics are described in terms of a scattering kernel $K_n(x, x_0)$, which is defined as the probability for a hadron of an initial energy fraction x_0 to be left with a fraction x after being scattered n times on target nucleons.

The fragmentation function $D^{eA}(x)$ may then be written in a multiple collision expansion,

$$D^{eA}(x) = \sum_{n=0}^{\infty} D_n^{eA}(x) \tag{3}$$

where

$$D_n^{eA}(x) = \int_0^R \frac{dz_0}{R} \int_x^1 dx_0\ D^{eN}(x_0)\ K_n(x, x_0)\ \ P_n(x_0, z_0) \tag{4}$$

Note that the $n = 0$ contribution depends only on nuclear geometry,

$$D_0^{eA}(x) = D^{eN}(x)\ [\frac{\ell(x)}{R} + \frac{\lambda}{R}\{1 - \exp(-\frac{R - \ell(x)}{\lambda})\}\ ] \tag{5}$$

The higher order terms depend on K_n. For the sake of illustration, consider the case of a fixed inelasticity $(1 - \alpha)$ associated with every single collision, i.e. $K_n(x, x_0) = \delta(x - \alpha^n)$, which implies, for $n \geq 1$,

$$D_n^{eA}(x) = \frac{1}{\alpha^n}\ D^{eN}(\frac{x}{\alpha^n})\ \tilde{P}_n(\frac{x}{\alpha^n}) \tag{6}$$

where

$$\tilde{P}_n(x) = \frac{\lambda}{R}\ [1 - \sum_{j=0}^{n} \frac{1}{j!}\ (\frac{R - \ell(x)}{\lambda})^j\ \exp(-\frac{R - \ell(x)}{\lambda})] \qquad (n \geq 1) \tag{7}$$

The resulting ratio of fragmentation functions, $\mathcal{R}_A(x) \equiv D^{eA}(x)/D^{eN}(x)$, is sketched in Fig.7, where the simple parametrizations $\ell_y \sim xL$ and $\ell_c \sim x(1-x)L$ have been used for the yoyo and for the constituent formation length, respectively. In the kinematic region of interest for the PEGASYS experiment [2], $E_{lab} \leq 15$GeV, in the high and intermediate x region $\mathcal{R}_A(x)$ will be dominated by the survival probability $\tilde{P}_0(x)$, while hadrons that suffer $n \geq 1$ collisions will typically contribute in the region below $x \sim \alpha^n$. See the Appendix for an analytic model employing the

ansatz $D(x) = \frac{1}{x}(1 - x^2)$ for the fragmentation function as well as more reasonable parametrizations for the scattering kernel. That model yields results consistent with numerical results from the PERSEUS Monte Carlo routine that will be discussed in the following section. Here, we would like to emphasize that

- $D_0^{eA}(x)$ depends on the dimensionless ratios (λ/R), $(R/L) \equiv (\kappa R/\nu)$ which can be studied experimentally by varying A and E_{lab}, and on $g(x) \equiv \ell(x)/L$ which is a model dependent input function.
- For $n \geq 1$, $D_n^{eA}(x)$ depends in addition on the inelasticity distribution (on the parameter α in our schematic model calculation).

3 Hadronic Cascading in eA Collisions

In this section, we introduce a Monte Carlo model (PERSEUS) of hadronic cascading in eA collisions that includes effects of multistring excitation, rescattering and fragmentation in a way consistent with the momentum space description of the Lund/Fritiof models [8,14]. We investigate the consequences of three different assumptions concerning which formation length is relevant for the onset of hadronic cascading: (a) the yoyo formation length, (b) the constituent formation length and (c) zero formation length ($\ell(x) \equiv 0$). Our model contains the following basic steps:

1. The initial (q-qq) string configuration formed in an inelastic lepton nucleon scattering process is set up by calling the appropriate Lund routine, LEPTO4.3 [12], and the space coordinate of the target nucleon involved in the initial eN collision is randomly selected according to a three parameter Wood Saxon distribution.

2. The first generation (=primary) hadrons produced in the fragmentation of the initial string are allowed to rescatter within the nucleus: for each primary hadron

a) the formation point $\mathbf{z_p}$ is determined depending on the model for $\ell(x)$

b) the mean number of inelastic collisions $< n_c >$ is calculated assuming straight line propagation,

$$< n_c > = \sigma \int_0^\infty ds \; \rho(\mathbf{z_p} + s \; \hat{\mathbf{p}}) \tag{8}$$

where σ is the inelastic cross section (taken to be 20mb for mesons, and 30mb for baryons) and $\hat{\mathbf{p}}$ is the unit vector in the direction of the hadron's three momentum. The actual number of inelastic collisions that hadron suffered is then chosen from a Poisson distribution,

$$P(n_c) = \frac{< n_c >^{n_c}}{n_c!} \exp(- < n_c >) \tag{9}$$

c) n_c subsequent collisions are processed according to the Fritiof string-string scattering algorithm [14], where each of the colliding hadrons (=strings) can be excited to a higher mass M according to a M^{-1} law.

3) Finally the fragmentation of all excited strings in the final state is executed according to the Lund JETSET6.3 routine [11].

Below, we show results obtained for 14.5 GeV electrons scattering on neon and on xenon targets. All distributions and observables relate to the rest frame of the target nucleus. Unless explicitly stated otherwise, calculations were done for the canonical parameter value of the string tension, $\kappa = 1$ GeV/fm. The limit of large string tension corresponds to the case of vanishing formation length. Thus, the curves labeled $\kappa = 10$ GeV/fm refer to that limit, $\ell \approx 0$.

Figs.8 to 10 illustrate the effects of nuclear geometry, in conjunction with a string length $L = (\nu/\kappa) \sim 10$fm (a value to be compared with nuclear rms radii of 5 to 6 fm). In Fig.8, the distribution of the position z of the hadronic formation point relative to the center of the nucleus (z=0) is plotted for hadrons of inverse ranks $i = 0, ..., 4$ (where inverse rank 0 corresponds to the hadron produced from the diquark of the initial qq-q jet, rank 1 corresponds to the next hadron produced, etc.). Clearly, there are considerable differences between the yoyo and the constituent formation cases, while the effects of going from Ne to Xe targets are small by comparison. Fig.9 shows the average number of collisions, $< n_c >$, as a function of inverse rank for a Xe target. Since the hadronic mean free path, $\lambda \approx 3$fm for a meson, is close to the radius of the target nucleus, R_A, the average number of collisions varies only between one and two. This puts severe limits on the information to be gained from such events. By contrast, a factor 2-3 in $< n_c >$ may be gained if one triggers on high multiplicity events, by selecting events where the initial eN interaction occurred in the backward hemisphere of the nucleus ($z \approx -R_A$ in Fig.8). The corresponding distributions in the number of recoil nucleons, for the "no bias" events as well as for our "high multiplicity trigger" events, are presented in Fig.10.

Having seen the limits imposed by nuclear geometry, we now come to the distributions $D^{eA}(x)$, i.e. to the dependence of the fragmentation functions on the nuclear medium. In Fig.11, the function $D_{\pi^+}(x)$ is plotted for different assumptions concerning the formation length relevant for the onset of hadronic cascading. We have also included data [15] obtained in neutrino-proton scattering experiments. Clearly, on an absolute scale the difference between the constituent case and the yoyo case (the latter is approximately the same as the case of eN) is very hard to determine; in fact, in the region $x \geq 0.5$ it is comparable to the size of the error bars in the νp-data. Again, the situation greatly improves if one triggers on high multiplicities.

In Fig.12, the ratio of fragmentation functions, $\mathcal{R}_A(x) = D^{eA}(x)/D^{eN}(x)$, is plotted rather than absolute values (which emphasizes the effect of nuclear attenuation in the forward region). As noted above, for the yoyo formation length with $\kappa = 1$GeV/fm no effect is observed. In the constituent case, the value at $x = 1$ is determined by nuclear geometry alone ($\ell_c(x = 1) = 0$, cf. Fig.4). As can be seen explicitly in Fig.13, in the region of low and intermediate x the rescattered hadrons ($n_c \geq 1$) contribute significantly and actually give rise to an "anti-attenuation" effect ($\mathcal{R}_A(x) > 1$) for $x \geq 0.2$. The data, taken from [7] (experimental errors are of the size of the symbols), are from an earlier SLAC experiment [6] performed with an electron beam of 20 GeV/c and a Sn target. In the region of large and intermediate

x, the data are not well described by any of the curves that correspond to a finite formation length, but seem to be consistent with the assumption $\ell \approx 0$. In the low x region dominated by the contribution from secondary cascading, they disagree with all the model calculations; in fact, as can be seen from Fig.13, the data points would seem to agree with the zero rescattering component for the constituent formation case and $\kappa = 1\text{GeV/fm}$. At this point one might be tempted to speculate that the assumption of zero formation length together with very high inelasticities (which would shift the $n_c \geq 1$ contribution to x-values below 0.05) could indeed offer a consistent explanation of the experimental data. However, apart from problems with understanding such large inelasticities, a model with $\ell(x) = 0$ disagrees with the recent experimental observation [7] that the effect of nuclear attenuation disappears at higher energies (i.e., at higher ν).

Previous studies that managed to fit the data did so at the cost of either neglecting the secondary cascading ($n_c \geq 1$) contribution [4,7] or of adopting scenarios [3,5] that do not yield the recently observed energy-dependence. Thus, the model of ref.[5] describes a simple inside-outside cascade scenario (cf.eq.(1)), with an extreme $\kappa_{eff} \approx 7$ GeV/fm, and ref.[3] corresponds to a Glauber model ($\ell = 0$) for intranuclear scattering of the leading quark of the intial (q-qq) string configuration, with an energy-independent ansatz for the inelasticity distribution.

Unfortunately, the results of the old SLAC experiment represent the only data presently available in the regime of $\nu \sim 10\text{GeV}$. If those observations were confirmed they could point to a significant lack in our understanding of the underlying space-time picture. Clearly, at this point it is highly desirable to get new and more sensitive experimental data.

4 Conclusions and Outlook

We have investigated the sensitivity of hadronic cascading effects in eA scattering on the underlying space-time scenario of hadronization. The necessary theoretical input consists of

- the momentum space dynamics, specified by the inelastic hadron nucleon cross sections, $\sigma_{in}(hN)$, as well as a scattering kernel $K_n(x, x_0)$ to describe nuclear stopping power, and
- additional model assumptions concerning the mapping of momentum space information onto coordinate space, as specified by the string tension κ (to fix the length scale) and by the x-dependence of the formation length (the unknown function $g(x) \equiv \ell(x)/L$)

Here, we used the parametrization of phase space distributions provided by the Fritiof/Lund string models [8,14,11,12] as an input, and we considered the two extreme cases where either the yoyo or the constituent formation length are taken to determine the onset of intranuclear cascading. For the kinematic range of interest,

$< \nu > \sim$ 10GeV, it turns out that, unless one chooses to trigger on special classes of events, very precise measurements are required in order to decide between the two different scenarios. The reason for this is that in the average event hadrons formed within the nucleus traverse a distance $d \sim A^{1/3}$fm on their way out, which means that on the average they will suffer not more than 1-2 collisions with target nucleons. In addition to the constraints that can be obtained by varying the energy transfer ν and the nuclear targets used, one may get considerably more sensitive information by triggering on high multiplicities, thus effectively increasing the range of intranuclear distances d probed (up to $2A^{1/3}$fm, which would correspond to an $A_{eff} \sim 8A$!).

For the canonical string tension parameter value κ =1 GeV/fm, our calculations with the yoyo formation model show virtually no nuclear attenuation (as measured by the ratio $\mathcal{R}_A(x) \equiv D^{eA}(x)/D^{eN}(x)$). For the constituent formation model, a clear effect is found: for large and intermediate values of x, $\mathcal{R}_A(x)$ is determined by the survival (no scattering) probability $P_0(x)$, while the low x region ($x \leq 0.3$) is dominated by the rescattered ($n_c \geq 1$) component. It is precisely in that low x region where we find the strongest discrepancy between our results and data [7] from a SLAC experiment [6]. These data seem to be well described by the $n_c = 0$ component alone, which would suggest, as pointed out in [7], that virtually all the rescattered hadrons would have to be shifted to values $x \leq 0.05$! However, the Fritiof/Lund description, as well as competing models for inelastic hN scattering, cannot account for such extreme inelasticities.

At this point, we stress again that previous studies that succeeded in fitting the SLAC-data either have done so by ignoring the secondary cascade component (and thus their fits only test if the zero rescattering component can account for the data), or else fail to describe the experimental observation [7] that the attenuation effect goes away in the limit of large ν. The latter fact clearly rules out the case of zero formation length.

At present, no experimental information other than the old SLAC data is available for the kinematic region of $\nu \sim 10$ GeV. In this situation, the need for new much higher precision data is obvious. If these should indeed confirm the surprising behavior of $\mathcal{R}_A$ in the low x regime, one may be forced to abandon hadronic rescattering scenarios altogether and consider alternatives, such as scattering of the initial (q-qq) string, or of the leading quark, within the nuclear medium. Work along these lines is in progress; the results will be presented elsewhere.

Appendix

Consider the case $D(x) = \frac{1}{x}(1-x^2)$, for which it has been shown [9] that the constituent length $\ell_c(x)$ and the yoyo length $\ell_y(x)$ are given by

$$\ell_c(x) = xL \frac{\ln(1/x^2) - 1 + x^2}{1-x^2} \tag{10}$$

$$\ell_y(x) = xL \frac{\ln(1/x^2)}{1-x^2} \tag{11}$$

where $L \approx \nu/\kappa$ is the length of the string.

We can rewrite the scattering kernel as

$$\begin{aligned} K_n(x,x_0) &= \int_0^1 du\, f_n(u)\, \delta(x - [x_{min} + u(x_0 - x_{min}]) \\ &= \frac{1}{x_0 - x_{min}} f_n(\frac{x - x_{min}}{x_0 - x_{min}}) \end{aligned} \tag{12}$$

where $x_{min} \equiv (m_T/W)^2$ is a lower cutoff for x, x_0 due to the finiteness of the mass of the string, W, and of the transverse mass m_T of the scattered hadrons.

We shall parametrize the distributions $f_n(u)$ in the canonical form

$$f_n(u) = a_n \; u^{a_n - 1} \tag{13}$$

where $< u >_n = a_n/(a_n+1)$; we choose $a_1 = 3$ and $a_n = 1$ for $n \geq 2$ to compare with the Monte Carlo results obtained with the Fritiof string scattering algorithm.

As in section 2, the integration over z_0 may be carried out immediately and replaces the $P_n(x, z_0)$ by the $\tilde{P}_n(x)$ in eq(4). For the case of the constituent formation length with $\kappa \geq 1$ GeV/fm (i.e., also for the limiting case $\ell \equiv 0$), in the kinematic regime under investigation ($\nu \approx 10$ GeV) we have $\tilde{P}_0(x') < 1$ for all $0 \leq x' \leq 1$, and the $\tilde{P}_n(x')$ vary slowly with x', such that the integrands are dominated by the $(x_0 - x_{min})^{-a_n}$ factors. One may then take the $\tilde{P}_n$ out of the integral and evaluate them at the lower limit of integration, x. For the relevant cases of parameter values a_n, the remaining integrals can be evaluated analytically, and one finds

$$\mathcal{R}_A(x) = P_0(x) + \frac{x}{1-x^2} \sum_{n \geq 1} a_n (x - x_{min})^{a_n - 1}\, \tilde{P}_n(x)\, [s_{a_n}(1) - s_{a_n}(x)] \tag{14}$$

with

$$s_1(x) = \frac{1}{x_{min}}[(1 - x_{min}^2)\, \ln x - x_{min} - \ln(x) - x_{min} x] \tag{15}$$

$$s_2(x) = \frac{1}{x_{min}^2}\, [\ln(x) - (1 + x_{min}^2) \ln(x - x_{min}) - x_{min}(1 - x_{min}^2)\, \frac{1}{x - x_{min}}]$$

$$s_3(x) = \frac{1}{x_{min}^3}\, [\ln(\frac{x - x_{min}}{x}) + (1 + x_{min}^2)\, \frac{x_{min}}{x - x_{min}} - (1 - x_{min}^2)\, \frac{x_{min}^2}{2(x - x_{min})^2}]$$

Figure Captions

Fig. 1 String excitation and fragmentation in the Fritiof/Lund model

Fig. 2 Rapidity distributions in eA and in pA

Fig. 3 Constituent formation length and yoyo formation length of the hadron H_2

Fig. 4 The x-dependence of the mean values for the constituent formation length, the yoyo formation length and the formation length of a naive inside-outside cascade scenario as in eq(1)

Fig. 5 Time evolution of the energy density in high energy nuclear collisions, for various formation zone models. The calculations were done with the ATTILA/Fritiof code [13].

Fig. 6 Secondary cascading in eA

Fig. 7 x-dependence of the ratio of the eA and the eN fragmentation function in the schematic model of eqs(3-7), for the constituent and for the yoyo formation model. For the constituent case, the separate contributions due to hadrons that suffered n inelastic collisions are shown as well.

Fig. 8 Formation point distributions for hadrons of inverse rank 0,...,4

Fig. 9 Average number of inelastic collisions plotted against the inverse rank of the hadron

Fig.10 Distribution of the multiplicity of recoil nucleons for various formation length models and a Xe target; included are curves for the "high multiplicity trigger" (top). For the constituent formation model, the effects of varying the nuclear targets are shown (bottom).

Fig.11 π^+- yields per event, as a function of the energy fraction carried by the π^+. Data points are from [15].

Fig.12 Ratio of the π^+ yields per event of Xe and D, for various formation models. The data are from [7].

Fig.13 As Fig.12, but with the contributions due to hadrons that suffered a specific number N_c of inelastic collisions made explicit (for the constituent model, with κ =1GeV/fm).

References

[1] O.V. Kancheli, JETP Lett.18 (1973) 274; N.N. Nikolaev, Sov.J.Part.Nucl.12 (1981) 63; J.D. Bjorken, in "Current Induced Reactions", Lecture Notes in Physics vol.56 (Springer, Berlin, 1976), p.73; R. Annishetty, P. Koehler and L. McLerran, Phys.Rev.D22 (1980) 2793

[2] PEGASYS Letter of Intent, SLAC, December 1987

[3] G. Nilsson, B. Andersson and G. Gustafson, Phys.Lett.83B (1979) 379

[4] A. Dar and F. Takagi, Phys.Rev.Lett.44 (1980) 768

[5] N.N. Nikolaev, Z.Physik C5 (1980) 291

[6] L.S. Osborne et al., Phys.Rev.Lett.40 (1978) 1624

[7] P.B. Renton, "New Results from the European Muon Collaboration on Structure Function Ratios and Hadron Production in Nuclear Matter", Invited talk given at the 3rd Conference on the Intersections between Particle and Nuclear Physics, Rockport, Maine (May 1988)

[8] B. Andersson et al, Phys.Rep.97 (1983) 31

[9] A. Bialas and M. Gyulassy, Nucl.Phys.B291 (1987) 793

[10] PERSonal Electron Uranium Scattering code

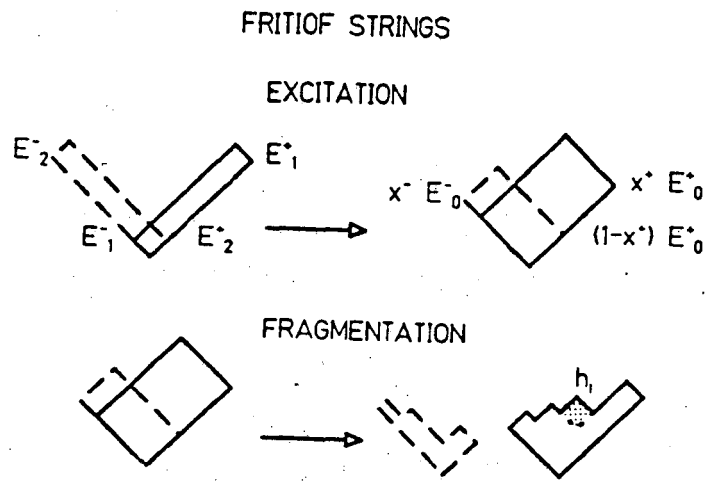
[11] T. Sjöstrand, JETSET manual, Lund preprints LU-TP-85-10, LU-TP-86-22

[12] G. Ingelman, "LEPTO version 4.3", CERN library long writeup THE LUND MONTE CARLO PROGRAMS (April 1987)

[13] M. Gyulassy, CERN preprint CERN-TH-4794 (1987)

[14] B. Andersson, G. Gustafson and B. Nilsson-Almqvist, Nucl.Phys. B281 (1987) 289

[15] P.B. Renton and W.S.C. Williams, Ann.Rev.Nuc.Part.Sci.31 (1981) 193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EXCITATION
E^-_2
E^+_1
E^-_1
E^+_2
$x^- E^-_0$
$x^+ E^+_0$
$(1-x^+) E^+_0$
FRAGMENTATION
h_1

Fig. 1

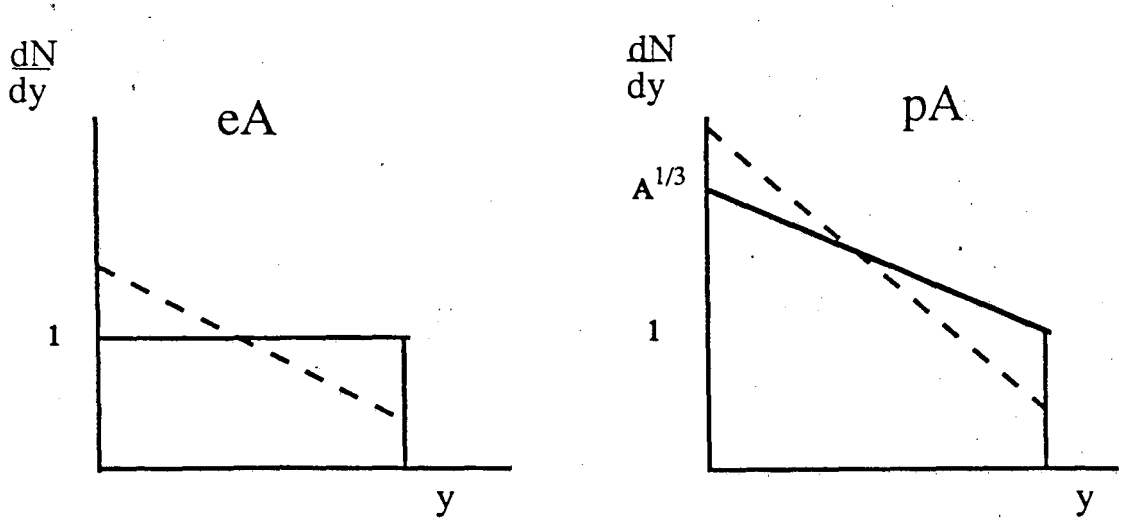
dN/dy
eA
1
y
dN/dy
pA
$A^{1/3}$
1
y

Fig. 2

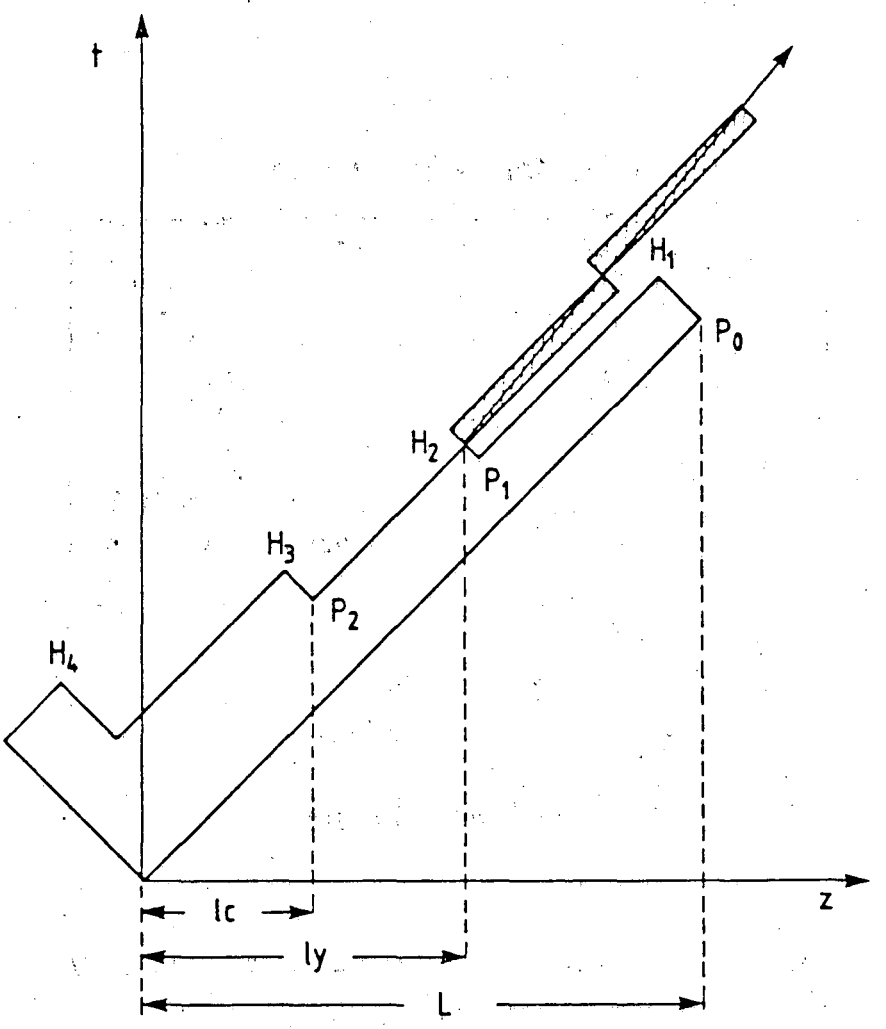

Fig. 3

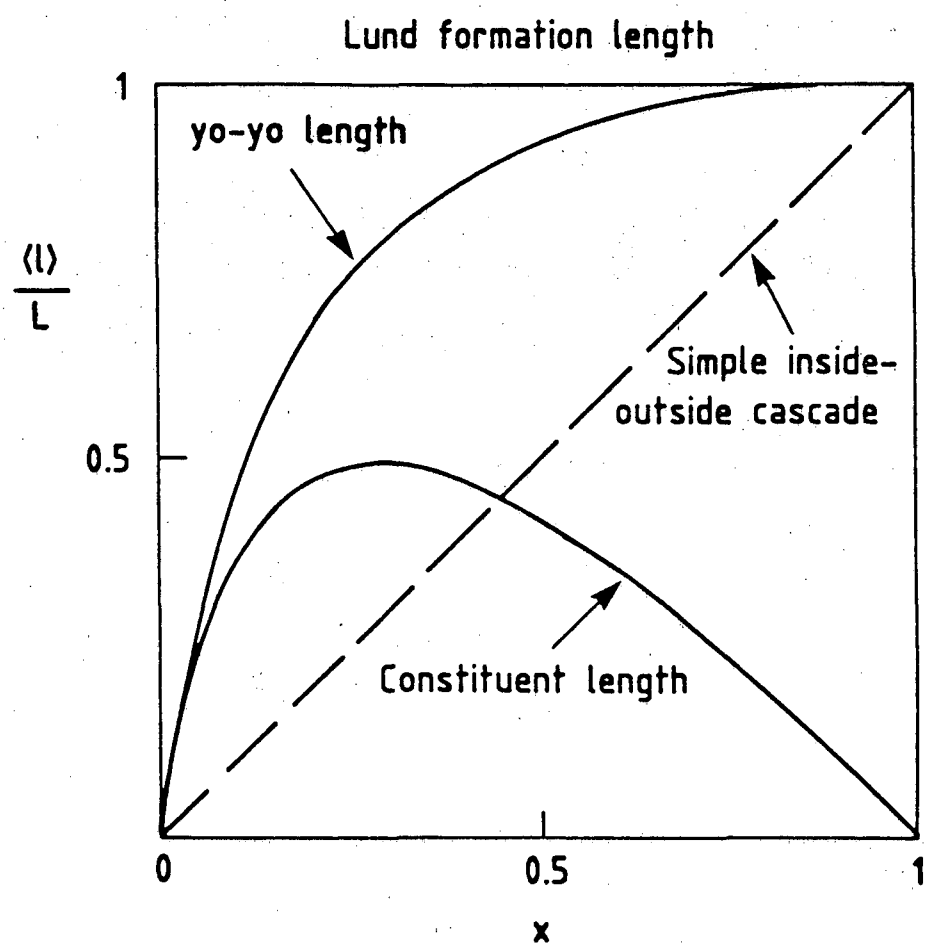
Lund formation length
1
yo-yo length
⟨l⟩
L
Simple inside-
outside cascade
0.5
Constituent length
0
0.5
1
x

Fig. 4

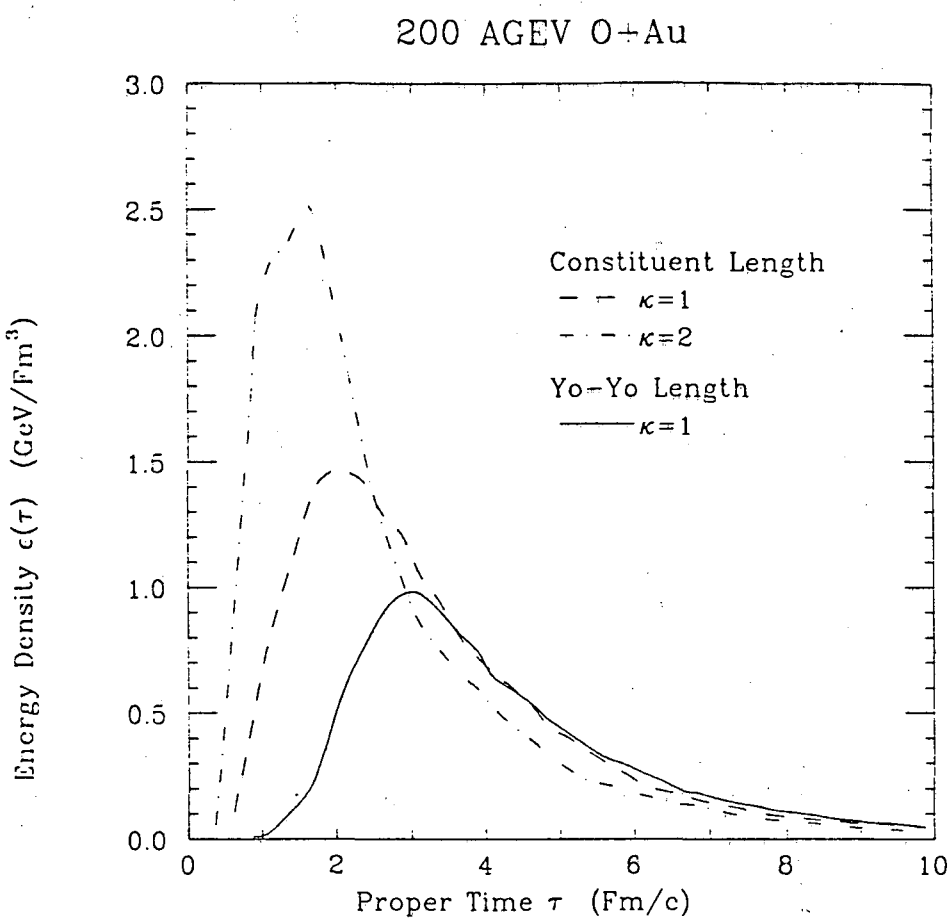
200 AGEV O+Au
Constituent Length
κ=1
κ=2
Yo-Yo Length
κ=1
Energy Density ε(τ) (GeV/Fm³)
Proper Time τ (Fm/c)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10

Fig. 5

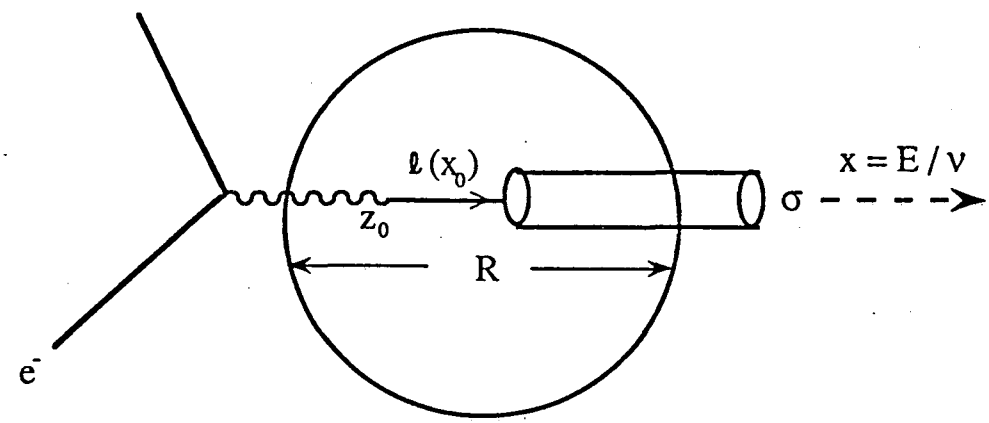
ℓ(x0)
x = E / ν
σ
z0
R
e-

Fig. 6

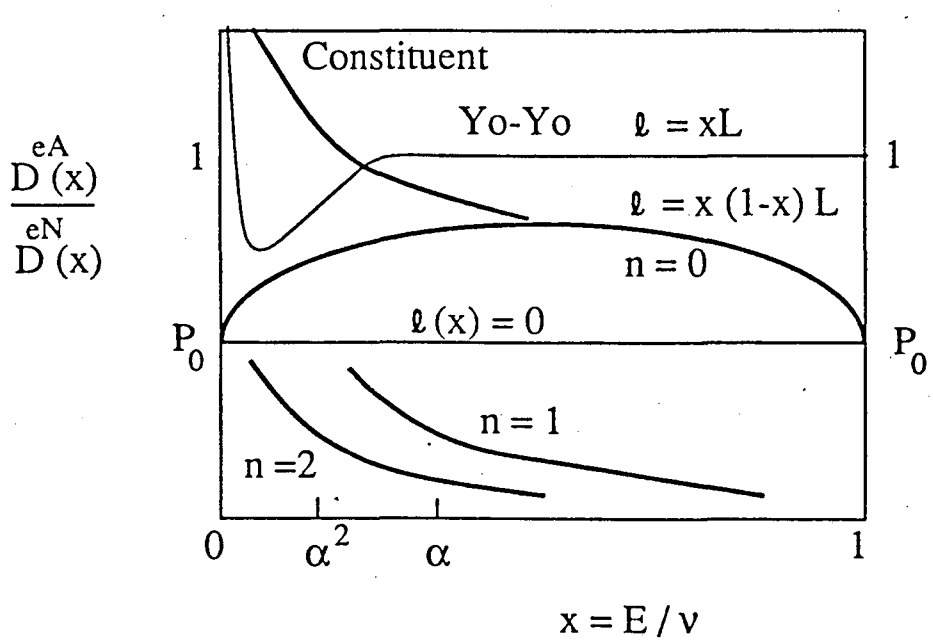
Constituent
Yo-Yo
ℓ = xL
ℓ = x (1-x) L
n = 0
ℓ (x) = 0
n = 1
n =2
1
1
P0
P0
0
α2
α
1
x = E / ν
eA D (x) / eN D (x)

Fig. 7

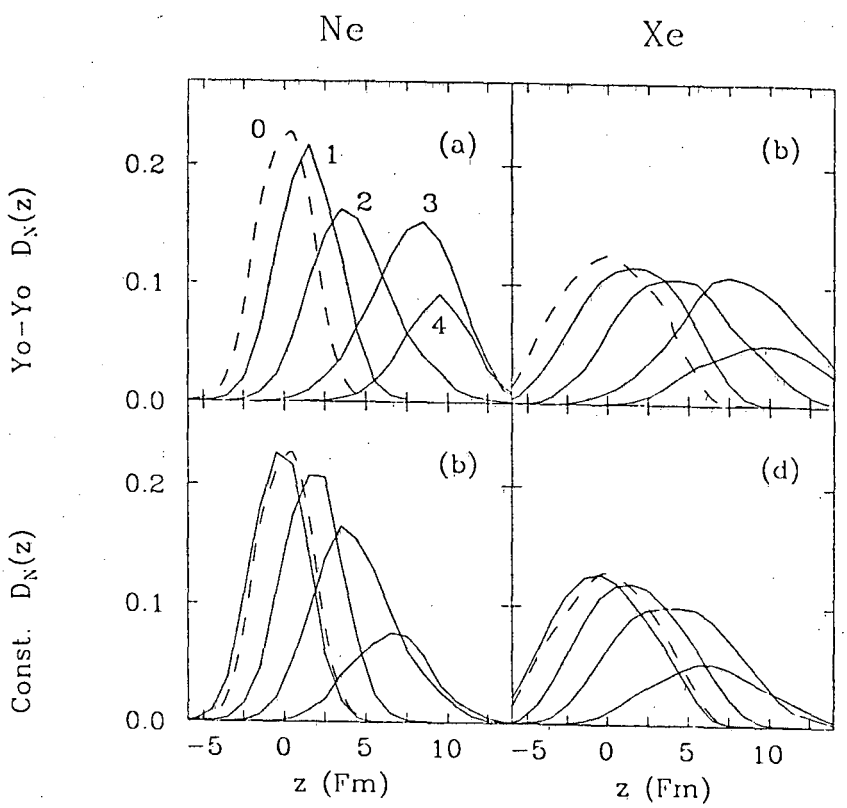

Fig. 8

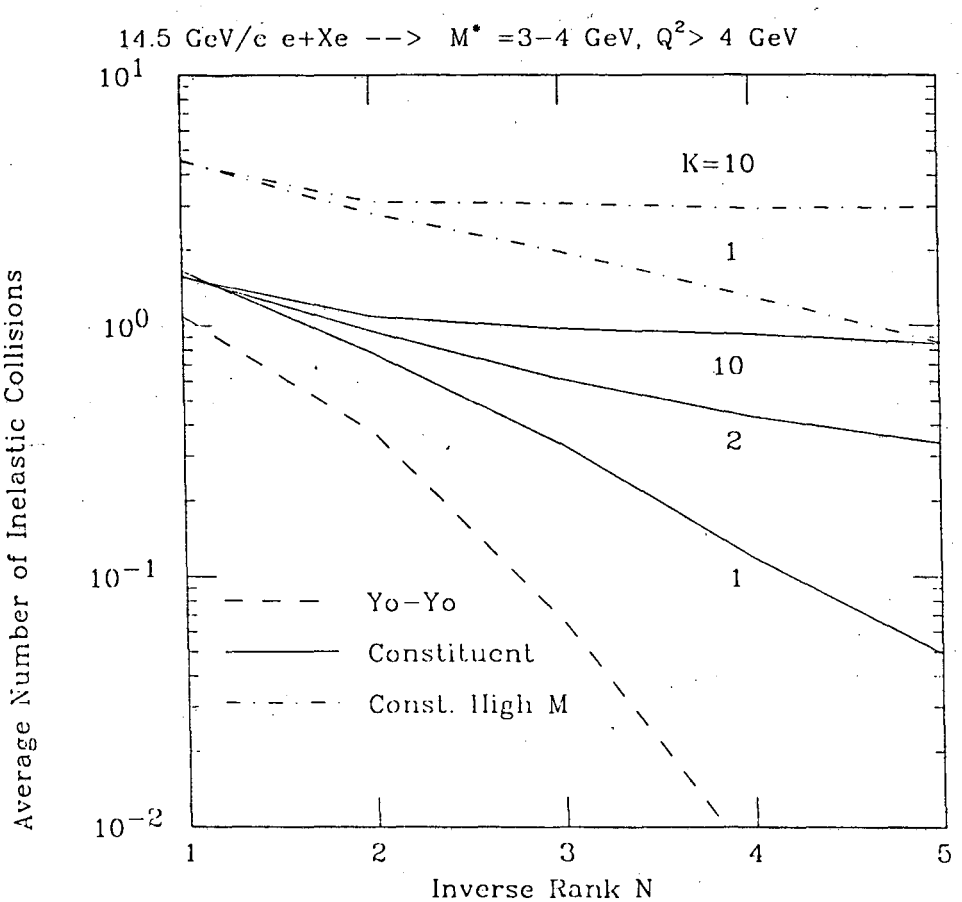
14.5 GeV/c e+Xe --> M* =3-4 GeV, Q²> 4 GeV
Average Number of Inelastic Collisions
Inverse Rank N
K=10
1
10
2
1
Yo-Yo
Constituent
Const. High M

Fig. 9

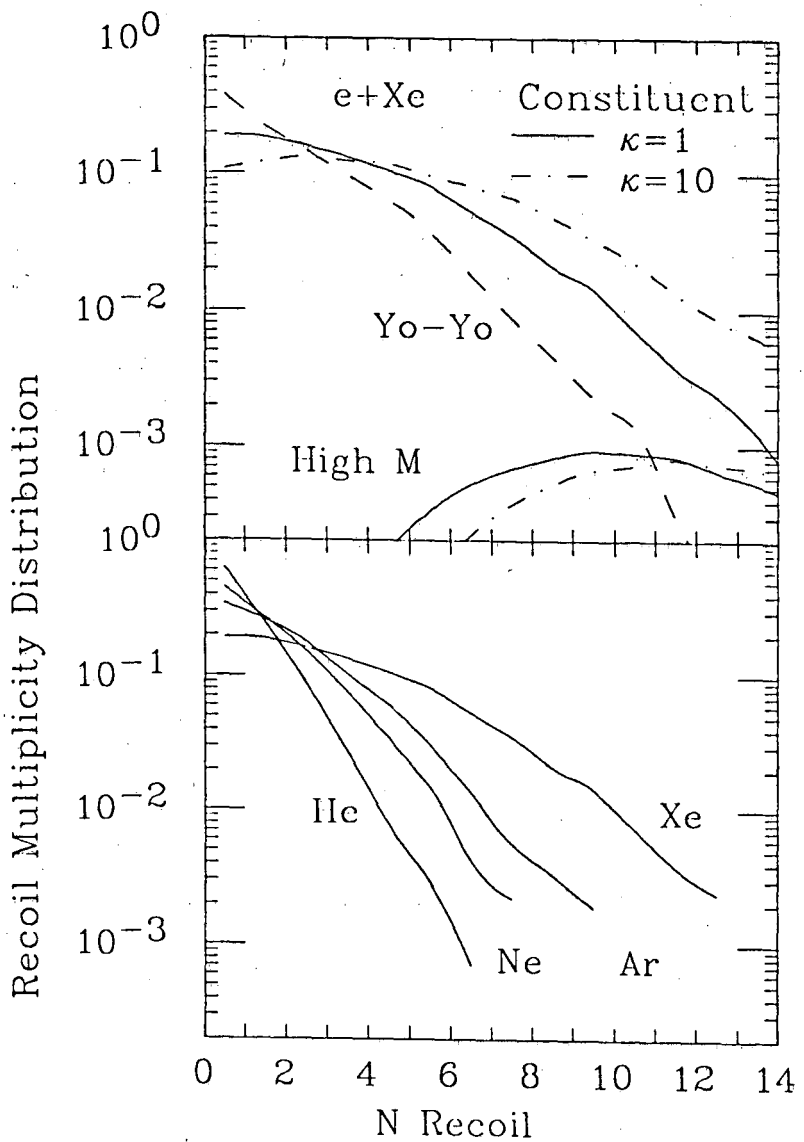
e+Xe
Constituent
κ=1
κ=10
Yo−Yo
High M
He
Ne
Ar
Xe
Recoil Multiplicity Distribution
N Recoil
10^0
10^−1
10^−2
10^−3
0
2
4
6
8
10
12
14

Fig. 10

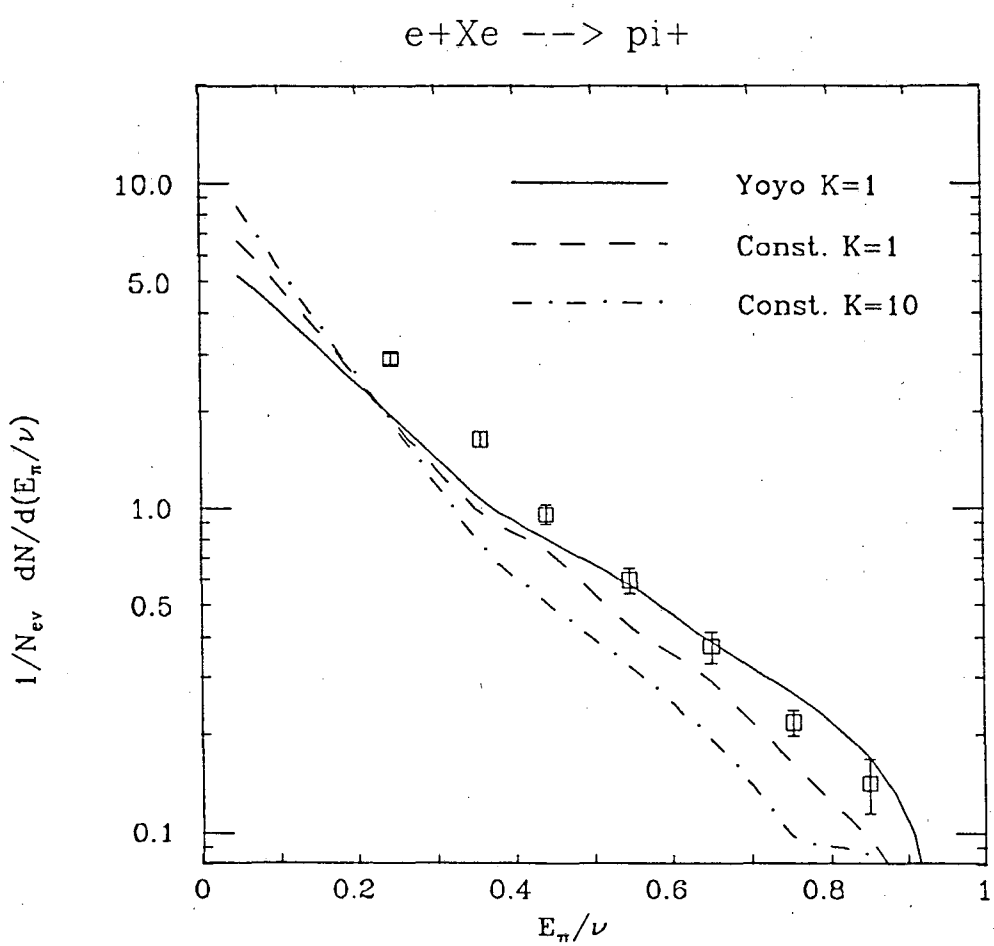
e+Xe --> pi+
Yoyo K=1
Const. K=1
Const. K=10
10.0
5.0
1.0
0.5
0.1
0
0.2
0.4
0.6
0.8
1
$1/N_{ev}$ $dN/d(E_\pi/\nu)$
E_π/ν

Fig. 11

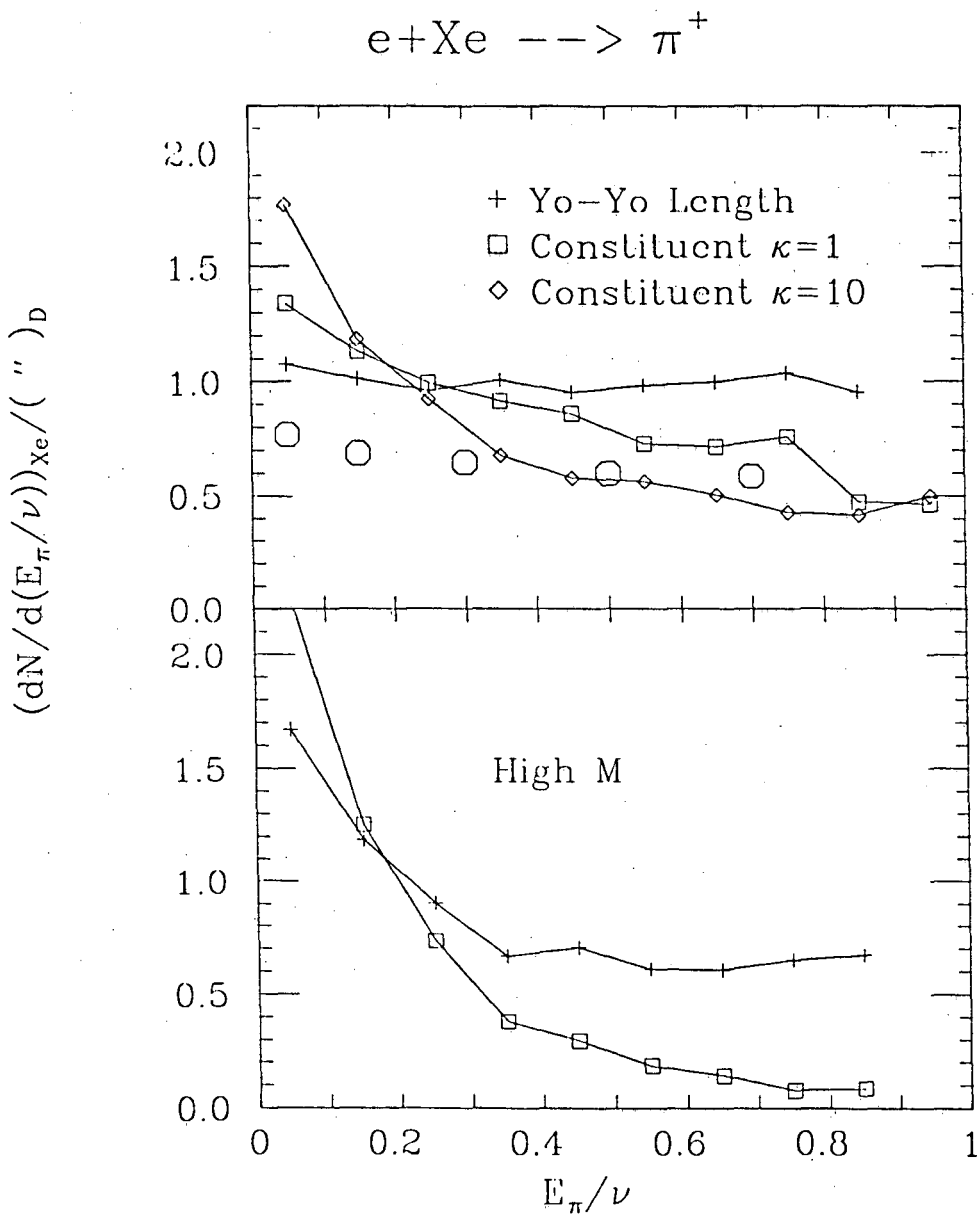
e+Xe --> π+
+ Yo-Yo Length
□ Constituent κ=1
◇ Constituent κ=10
High M
(dN/d(Eπ/ν))Xe/(")D
Eπ/ν
0.0
0.5
1.0
1.5
2.0
0
0.2
0.4
0.6
0.8
1

Fig. 12

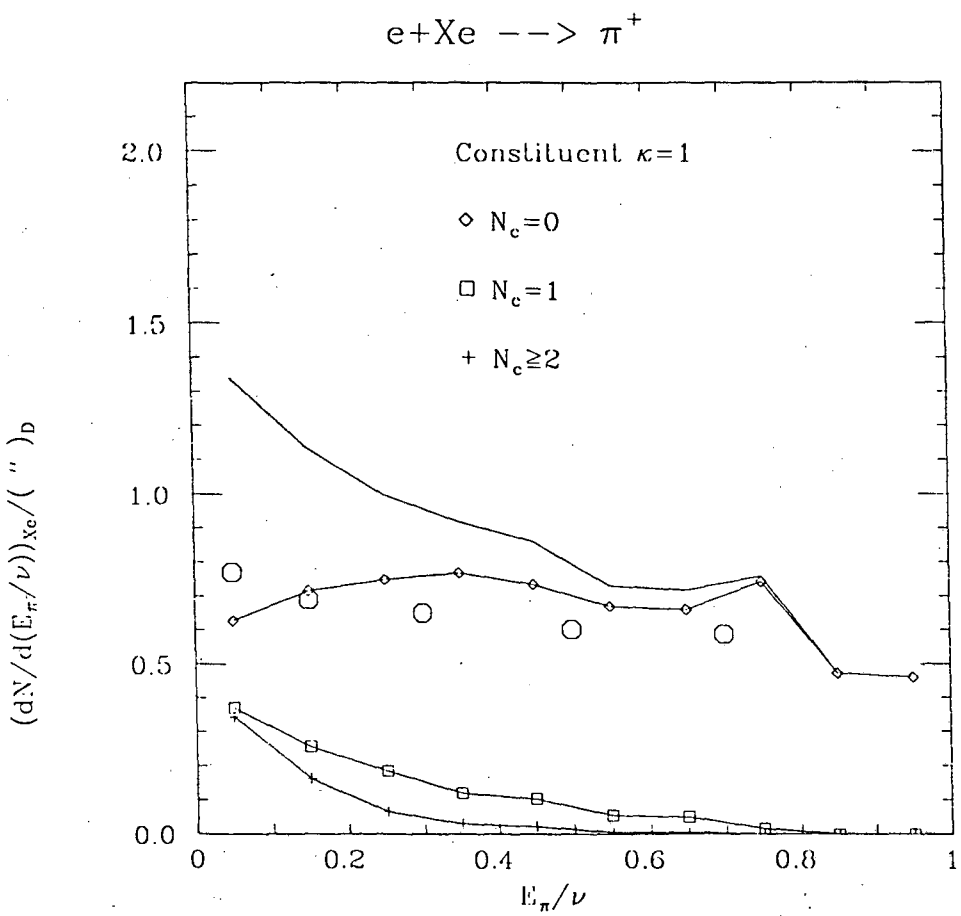
e+Xe --> π^+
Constituent $\kappa=1$
$N_c=0$
$N_c=1$
$N_c \geq 2$
2.0
1.5
1.0
0.5
0.0
0
0.2
0.4
0.6
0.8
1
$(dN/d(E_\pi/\nu))_{Xe}/(\ ''\ )_D$
E_π/ν

Fig. 13

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