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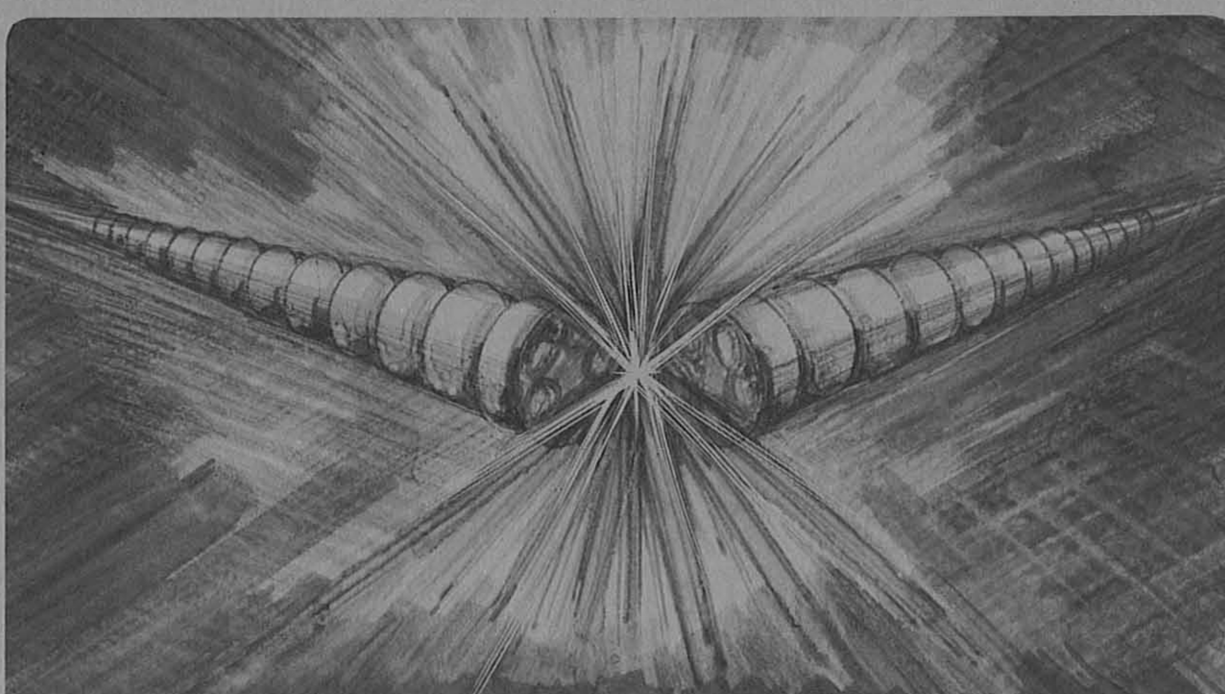

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# A SYMPLECTIC COHERENT BEAM-BEAM MODEL*

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A SYMPLECTIC COHERENT BEAM-BEAM MODEL†

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We consider a simple one-dimensional model to study the effects of the beam-beam force on the coherent dynamics of colliding beams. The key ingredient is a linearized beam-beam kick. We study only the quadrupole modes, with the dynamical variables being the 2nd-order moments of the canonical variables $q$, $p$. Our model is self-consistent in the sense that no higher-order moments are generated by the linearized beam-beam kicks, and that the only source of violation of symplecticity is the radiation. We discuss the round beam case only, in which vertical and horizontal quantities are assumed to be equal (though they may be different in the two beams). Depending on the values of the tune and beam intensity, we observe steady states in which otherwise identical bunches have sizes that are equal, or unequal, or periodic, or behave chaotically from turn to turn. Possible implications of luminosity saturation with increasing beam intensity are discussed. Finally, we present some preliminary applications to an asymmetric collider.

## Introduction

The study of the coherent modes of oscillation of colliding beams has a long history, with many contributions to this important and difficult problem; space limitations prevent us from giving here a full set of references [1]. A while ago Hirata [2] proposed a simplified model to study the problem including the essential coupled-beam features, but is in principle inconsistent with Vlasov's equation because it assumes a bunch distribution that remains Gaussian at all times. However, it explains qualitatively the "flip-flop" effect and the saturation of the luminosity and beam-beam parameter at high intensity. Furthermore, a Gaussian bunch shape is generally accepted as being a good approximation to a self-consistent solution, so the numerical results from this model may be reasonable despite the theoretical inconsistency; in fact, a more recent [3] approximation which takes into account higher-order moments improves the agreement with multiparticle simulations. We summarize here the results of a simpler model [4], defined along similar lines, that has the virtue of being fully self-consistent (*i.e.*, symplectic in the absence of radiation, with Gaussian beams remaining Gaussian) since it involves the essential ingredient of a *linearized* beam-beam force. Several of the features are derived analytically. The consistency with Vlasov's equation is achieved at the price of ignoring Maxwell's equations altogether, since the force is assumed to be linear at all distances while the bunch size is finite. This is clearly not a good approximation for any reasonable distribution. However, since we study only the quadrupole modes of beams that collide head on, the linear part of the force has the most important effect, and in this sense it is reasonable to make such an approximation. By comparing our results with Hirata's we hope to determine those features that are generic to this type of model.

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## Model (symmetric case)

### Ingredients

We consider a collider composed of two rings with identical linear lattices and common tune $\nu_0$, a single interaction point, and one bunch per beam with the same number of particles $N$ and same energy. We label them the $e^+$ and $e^-$ bunches, although our discussion allows for like-charged beams ($e^\pm e^\pm$) just as well. The bunches collide head-on. We consider only the dynamics in one dimension, say the vertical, described by $y$, $y'$, and define the normalized coordinates $q$, $p$ for a particle in each beam as $q_\pm \equiv y_\pm/\sqrt{\beta_y}$ and $p_\pm \equiv (\beta_y y'_\pm + \alpha_y y_\pm)/\sqrt{\beta_y}$. Note that $\beta_y$ is the "bare" lattice function, *i.e.*, it does not include any corrections due to the beam-beam interaction. We represent the beam-beam interaction by the linearized kick

$$q'_\pm = q_\pm, \qquad p'_\pm = p_\pm - k_\mp q_\pm \tag{1}$$

where $k_\pm$ is the dimensionless strength of the kick,

$$k_\pm = \frac{2 r_0 N \beta_y}{\gamma \left\{(\sigma_x + \sigma_y)\sigma_y\right\}_\pm} \equiv \frac{\beta_y}{f_\pm} \tag{2}$$

This is the only source of coupling and of nonlinearity since the strength of the kick on the + beam, $k_-$, depends inversely on the size of the opposing bunch, $\sigma_-$, which is a *dynamical variable*, as is $\sigma_+$. In the above $r_0$ is the classical radius of the particle, $\gamma$ the usual relativistic factor and $f_\pm$ the kick's focal length. The $-$ sign in front of $k_\mp$ in Eq. (1) implies the convention that $k_\mp > 0$ for attractive kicks (opposite-charged beams). We consider here only the extreme case of round-beam shape, $\sigma_x = \sigma_y \equiv \sigma$. In terms of the nominal beam-beam parameter $\xi_0$, we have

$$\begin{aligned} k_\pm &= 4\pi\xi_0 \left(\frac{\sigma_0}{\sigma_\pm}\right)^2 \equiv 4\pi\xi_\pm \\ \sigma_0^2 &= \beta\epsilon_0, \qquad \xi_0 = \frac{r_0 N \beta}{4\pi\gamma\sigma_0^2} \end{aligned} \tag{3}$$

where $\epsilon_0$ and $\sigma_0$ are the nominal, equilibrium emittance and beam size (we assume that $\beta_x = \beta_y \equiv \beta$ and $\epsilon_{x0} = \epsilon_{y0} \equiv \epsilon_0$ for this round-beam case) and $\xi_\pm$ are the actual, dynamically determined, beam-beam parameters. Note that in the weak-beam limit $\xi_+$ is the tune shift of the $-$ beam and viceversa. The analysis of the flat-beam case, in which $k \propto 1/\sigma$ rather than $1/\sigma^2$, can be carried out analogously [4].

Following Hirata [2], we represent the effect of the synchrotron radiation loss and its compensation by the RF cavities by the stochastic localized kick

$$\begin{pmatrix} q'_\pm \\ p'_\pm \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} q_\pm \\ p_\pm \end{pmatrix} + \hat{r}_\pm \sqrt{\epsilon_0(1-\lambda^2)} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

where the $\hat{r}_\pm$ are independent random numbers with $\langle \hat{r}_\pm \rangle = \langle \hat{r}_+ \hat{r}_- \rangle = 0$ and $\langle \hat{r}_\pm^2 \rangle = 1$, and $\lambda$ is related to the damping decrement of the ring $\delta$ by $\lambda = \exp(-2\delta)$ (we assume the rings to have equal damping decrements). The first term in the above equation describes the damping due to the energy loss by radiation, and the second term the noise induced by the RF cavity that restores the energy to the particle.

The third and final ingredient is a linear transport through a phase advance $2\pi\nu_0$, given by

$$\begin{pmatrix} q'_\pm \\ p'_\pm \end{pmatrix} = \begin{pmatrix} C & S \\ -S & C \end{pmatrix} \begin{pmatrix} q_\pm \\ p_\pm \end{pmatrix} \tag{5}$$

where $C = \cos(2\pi\nu_0)$, $S = \sin(2\pi\nu_0)$.

The bunches undergo collision, transport, radiation, collision, etc. The one-turn map for a given particle has a stochastic inhomogeneous part arising from the last term in Eq. (4). A deterministic (but still inhomogeneous) map is obtained for the bilinear combinations of $q$ and $p$ and averaging these over all radiation events in each bunch. In this way we convert the problem into the study of a 6-dimensional map for the second-order moments (3 for each beam) of the bunch distributions. Since the beam-beam kick is linear, the second order moments characterize the problem completely (assuming head-on collisions) because no higher-order moments can be generated. With a surface of section just before the beam-beam kick we find

$$\begin{bmatrix} \langle q_+^2 \rangle \\ \langle p_+ q_+ \rangle \\ \langle p_+^2 \rangle \end{bmatrix}_{n+1} = \widetilde{M}(k_{-,n}) \begin{bmatrix} \langle q_+^2 \rangle \\ \langle p_+ q_+ \rangle \\ \langle p_+^2 \rangle \end{bmatrix}_n + \epsilon_0(1-\lambda^2) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{6}$$

where the $3\times3$ matrix $\widetilde{M}(k_{-,n})$ depends on $\langle q_-^2 \rangle_n$, $\nu_0$, $\lambda$ and $\xi_0$ (there is a simultaneous companion map with $+ \leftrightarrow -$). In practical calculations we use the dimensionless moment vectors $\mathbf{X} \equiv (\langle q_+^2 \rangle, \langle q_+ p_+ \rangle, \langle p_+^2 \rangle)/\epsilon_0$ and $\mathbf{Y} \equiv (\langle q_-^2 \rangle, \langle q_- p_- \rangle, \langle p_-^2 \rangle)/\epsilon_0$ so that, with $\mathbf{e} \equiv (0,0,1)$, the full 6-dimensional one-turn map reads

$$\begin{aligned} \mathbf{X}_{n+1} &= \widetilde{M}(Y_{1,n})\mathbf{X}_n + (1-\lambda^2)\mathbf{e} \\ \mathbf{Y}_{n+1} &= \widetilde{M}(X_{1,n})\mathbf{Y}_n + (1-\lambda^2)\mathbf{e} \end{aligned} \tag{7}$$

The basic dynamical physical quantities that we are interested in studying from the map are the beam sizes $\sigma_\pm$ and emittances $\epsilon_\pm$ given by

$$\begin{aligned} \sigma_\pm^2 &= \beta \langle q_\pm^2 \rangle, \\ \epsilon_\pm^2 &= \langle y_\pm^2 \rangle \langle y'^2_\pm \rangle - \langle y_\pm y'_\pm \rangle^2 \\ &= \langle q_\pm^2 \rangle \langle p_\pm^2 \rangle - \langle q_\pm p_\pm \rangle^2 \end{aligned} \tag{8}$$

from which we can extract "observables" such as the luminosity and the beam-beam parameters $\xi_\pm$, relative to their nominal values.

### Digression on the Radiation Kick

The radiation kick, Eq. (4), is constructed so that, if the beam-beam force is turned off, the beam decays exponentially to an equilibrium configuration $\mathbf{X}_{eq} = \mathbf{Y}_{eq} = (1,0,1)$ which corresponds to a beam ellipse that is matched to the bare lattice, with emittance equal to the nominal value, $\epsilon = \epsilon_0$. It can be shown that this "fixed point" is absolutely stable, *i.e.*, it is reached from any initial state provided only the tune $\nu_0$ is not too close to 0 (how close depends on the damping decrement $\delta$, which we assume small). In the decay process the beam emittance changes with every passage through the radiation kick according to $\epsilon'^2 = \lambda^2\epsilon^2 + (1-\lambda^2)\epsilon_0 \langle q^2 \rangle$ until it reaches the nominal equilibrium value. We regard this process as a simple physical model of the damping that occurs in an actual electron storage ring.

Actually Eq. (4) has been questioned and alternatives have been proposed. Hirata and Ruggiero [5] propose a kick symmetric in $q$ and $p$,

$$\begin{pmatrix} q' \\ p' \end{pmatrix} = \sqrt{\lambda} \begin{pmatrix} q \\ p \end{pmatrix} + \sqrt{\epsilon_0(1-\lambda)} \begin{pmatrix} \hat{r}_1 \\ \hat{r}_2 \end{pmatrix} \tag{9}$$

where $\langle \hat{r}_1^2 \rangle = \langle \hat{r}_2^2 \rangle = 1$, $\langle \hat{r}_1 \rangle = \langle \hat{r}_2 \rangle = \langle \hat{r}_1 \hat{r}_2 \rangle = 0$. In the absence of the beam-beam interaction this form for the radiation kick also has an absolutely stable equilibrium configuration $\mathbf{X}_{eq} = (1,0,1)$ whose emittance has the nominal value. An arbitrary initial state decays exponentially to it with the emittance changing at each step according to $\epsilon'^2 = \lambda^2\epsilon^2 + (1-\lambda)^2\epsilon_0^2 + \lambda(1-\lambda)\epsilon_0(\langle q^2 \rangle + \langle p^2 \rangle)$.

Krishnagopal and Siemann [6] propose a similar expression,

$$\begin{pmatrix} q' \\ p' \end{pmatrix} = \sqrt{\lambda} \begin{pmatrix} q \\ p \end{pmatrix} + \sqrt{\epsilon_0(1-\lambda)} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \tag{10}$$

where $\langle r_1 \rangle = \langle r_2 \rangle = \langle r_1 r_2 \rangle = 0$, $\langle r_1^2 \rangle = \sqrt{\langle q^2 \rangle / \langle p^2 \rangle}$ and $\langle r_2^2 \rangle = \sqrt{\langle p^2 \rangle / \langle q^2 \rangle}$. This kick is supposed to be used only at a symmetry point of the lattice, where $\langle qp \rangle = 0$ by symmetry. It differs from the previous expressions in that the noise term has a *dynamical dependence on the beam.* In the absence of beam-beam interaction it, too, has an absolutely stable equilibrium state $\mathbf{X}_{eq} = (1,0,1)$ with nominal emittance. An initial state with arbitrary emittance decays to it exponentially with a turn-by-turn change in the emittance given by $\epsilon' = \lambda\epsilon + (1-\lambda)\epsilon_0$. This equation shows that, if the initial state is not completely arbitrary but has an emittance $\epsilon = \epsilon_0$, then *the emittance is preserved at every turn at its nominal value.* Since the beam-beam force is symplectic, this property remains true when the beam-beam interaction is operating. The authors of (10) claim this property to be essential, and they use the above expression for the kick in multiparticle tracking simulations in which the beams have nominal emittance only, which is therefore guaranteed to be preserved.

Our (limited) experience shows that the above three forms of the radiation kick yield qualitatively similar results except possibly for very intense beams, although the algebraic expressions in analytical calculations are quite different. We adopt the point of view that the simpler the model for the radiation kick the better, provided only that in the absence of the beam-beam interaction the beam should decay from any initial state to the equilibrium state with that is matched to the bare lattice, which has nominal emittance. We regard the emittance nonconservation of (4) as a virtue, since this property emulates the behavior of beams out of equilibrium. When the beam-beam force is included our model yields a variety of steady-state solutions, which may or may not be constant in time. The emittances and beam sizes of these solutions are not, in general, equal to their nominal values, and are obtained as output quantities. Expression (4) has the additional virtue of yielding simple analytic expressions for the period-one fixed point.

## Results

### Period-One Fixed Points

We first seek steady solutions by setting $\langle\cdots\rangle_{n+1} = \langle\cdots\rangle_n$ in (6) for all six moments, which yields a set of two equations for $\langle q_+^2\rangle$ and $\langle q_-^2\rangle$. By defining $k_+ \equiv (\lambda+1)x$, $k_- \equiv (\lambda+1)y$, $\rho \equiv 4\pi\xi_0/(\lambda+1)$ and $\chi \equiv \cot(2\pi\nu_0)$, we obtain

$$x/\rho = 1 + 2\chi y - y^2, \quad y/\rho = 1 + 2\chi x - x^2 \tag{11}$$

These equations admit solutions with $x = y$ (equal-size beams, or "normal solutions"), and with $x \neq y$ (unequal-size beams, or "flip-flop solutions") which can be found analytically in a straightforward way. Note that they depend on $\nu_0$ and $\rho$, but do not depend separately on $\lambda$; also note that $\rho \propto$ beam intensity $N$. Once $x$ and $y$ are found, the specification of the fixed point is completed by finding the moments $\langle pq\rangle$ and $\langle p^2\rangle$. These are also obtained from Eq. (6), which yields very simple algebraic expressions in terms of $\langle q_\pm^2\rangle$. In order to be physical, the solutions must be real and have the same sign. The normal solutions are always real: the positive solutions are physical for the $e^+e^-$case, the negative solutions for the $e^\pm e^\pm$case. The flip-flop solutions, on the other hand, are physical only in certain regions of the $\nu_0 - \rho$ plane, which is shown shaded in Fig. 1. Round beams do not admit $e^\pm e^\pm$ flip-flop solutions, but this not true in the flat-beam model, which does admit them [4]. In addition to being real, the solutions must be stable. This is determined from the eigenvalues of the $6\times 6$ stability matrix, which is obtained by expanding the map infinitesimally close to the fixed point. Results for the size and stability of the $e^+e^-$ case are shown in Fig. 2, for the specific case of $\nu_0 = 0.15$ and $\delta = 0.07$ ($\lambda = 0.8694$). The normal solutions are stable only in $0 \leq \rho \lesssim 0.3$; since $\xi_0 \simeq \rho/2\pi$, this corresponds to $0 \leq \xi_0 \lesssim 0.048$. Flip-flop solutions are real only in the interval $1 \lesssim \rho \lesssim 2$ ($0.16 \lesssim \xi_0 \lesssim 0.32$) with one beam growing in size rapidly as $\rho$ increases. However, they are stable only in the regions $1.1 \lesssim \rho \lesssim 1.3$ and $1.6 \lesssim \rho \lesssim 2$. In those regions of $\rho$ where neither the normal or the flip-flop solutions are stable other type of solutions appear, such as higher-order fixed points, as we now describe.

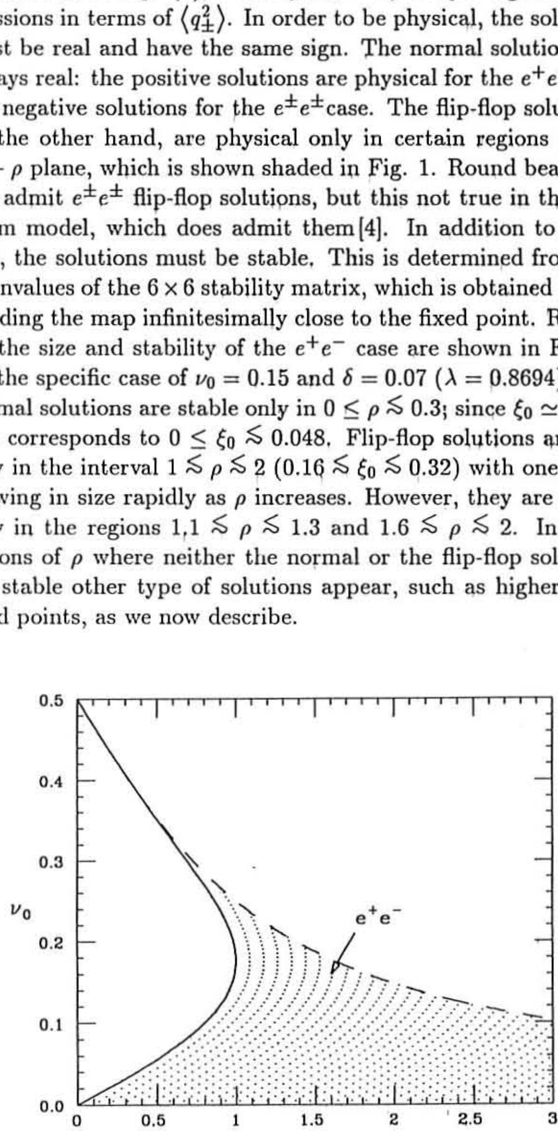


Fig. 1. Region where the round–beam flip–flop solutions are real (though not necessarily stable). The $e^\pm e^\pm$ solutions are always complex.

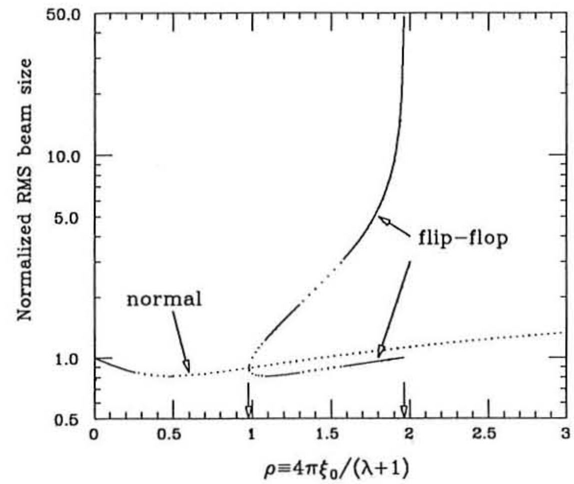


Fig. 2. RMS beam sizes and stability for the period–1 fixed point solutions (solid=stable, dots=unstable). Round–beam case, $\nu$=0.15, $\lambda$=0.8694.

### Iteration of the Map

By starting with a given set of values for the 6 moments we iterate the map (7) until it converges or diverges. All results presented here are for the $e^+e^-$case, for $\nu_0 = 0.15$ and $\delta = 0.07$ ($\lambda = 0.8694$). This is an unrealistically large value of $\delta$; however, because our model is symplectic in the absence of radiation, all our results have a smooth $\lambda \to 1$ limit, and are *quantitatively similar* for any $\lambda$ sufficiently close to 1 (a large $\delta$ has the practical advantage of fast convergence of the map iteration). Results are shown in Fig. 3. Dots represent chaotic behavior, in which the two beams are preferentially of different size; + represents period–1 fixed points, in which the beams are of equal or unequal size, depending on the value of $\rho$ (they correspond to the beam sizes shown in Fig. 2); ×, ◇ and □ represent period–2, –3, and –4 fixed points with beams of equal size. Other types of solutions may well exist, but are hard to find. If more than one solution is possible, the one to which the map converges depends on the initial conditions. For $\rho \simeq 0.3$ the chaotic solutions are the most stable. For other values of $\rho$, generally speaking, the period–1 fixed point is the most stable unless it coexists with higher-order fixed points. In this case the the period–3 fixed point is the most stable. By "most stable" we mean that this solution is the most likely one to be reached when varying the initial conditions.

The effects of the map can be evaluated by looking at "observables" such as the luminosity or the effective beam-beam parameter, which depend on the actual (*i.e.*, dynamically determined) beam sizes. Thus a quantity that measures the physical effects of our model is the "enhancement factor" $E$ defined by $E \equiv \mathcal{L}/\mathcal{L}_0 = \xi/\xi_0$,

$$E = \frac{2\epsilon_0}{\langle q_+^2\rangle + \langle q_-^2\rangle} \tag{12}$$

which we plot in Fig. 4 (for the higher-order fixed points, for which $E$ varies from turn to turn, we compute its average over the period of the most stable fixed point). Note the saturation effect due to chaotic behavior and higher-order fixed points.

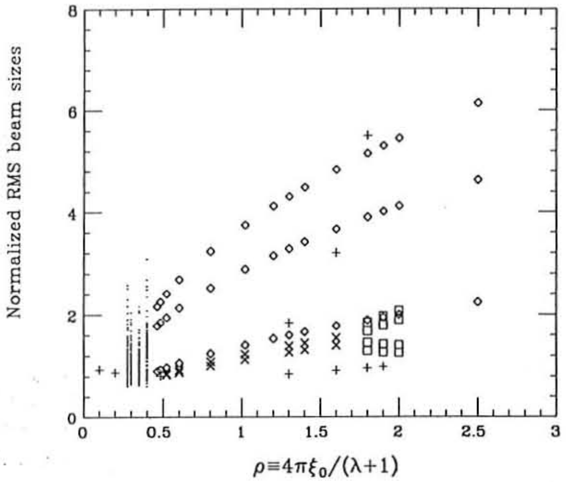


Fig. 3. Round beam sizes from map iteration; dots=chaotic; +=period-1 (equal or unequal sizes); ×=period-2, ◇=period-3, □=period-4 (equal sizes). $\nu$=0.15, $\lambda$=0.8694.

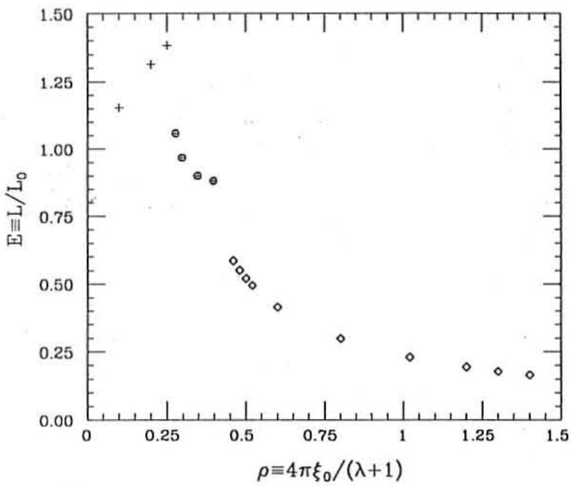


Fig. 4. Round-beam enhancement factor from period-1 (+), period-3 (◇), or chaotic (⊚) fixed points. $\nu$=0.15, $\lambda$=0.8694.

## Asymmetric Case

Recently asymmetric colliders have been proposed as high-luminosity "factories" for the study of specific particles such as beauty mesons [7]. We present in the following the results of a brief, ongoing and preliminary study of applying our model to asymmetric colliders. There are two asymmetries: the lattices can be different, and the beams can be different, although we still assume that they are round. The rings have tunes $\nu_{0\pm}$, damping decrements $\delta_\pm$ and beta-functions $\beta_\pm$. The bunches have $N_\pm$ particles, energy/mass $\gamma_\pm$ and nominal emittances $\epsilon_{0\pm}$. The dynamical and nominal beam quantities are

$$\begin{aligned} k_\pm &= 4\pi\xi_{0\pm}\left(\frac{\sigma_{0\pm}}{\sigma_\pm}\right)^2 \equiv 4\pi\xi_\pm \\ \sigma_{0\pm}^2 &= \beta_\pm\epsilon_{0\pm}, \qquad \xi_{0\pm} = \frac{r_0 N_\pm \beta_\mp}{4\pi\gamma_\mp\sigma_{0\pm}^2} \end{aligned} \tag{13}$$

with $\sigma_\pm^2 = \beta_\pm\langle q_\pm^2\rangle$. In addition, we simulate the effect of the longitudinal dynamics by modulating the tunes according to

$$\begin{aligned} \nu_{0\pm} \to \nu_\pm &= \nu_{0\pm} + \Delta_\pm \sin(\nu_{s\pm} n), \\ \Delta_\pm &= \frac{\sigma_{s\pm}\nu_{s\pm}}{2\beta_\pm} \end{aligned} \tag{14}$$

where $\nu_{s\pm}$ and $\sigma_{s\pm}$ are the synchrotron tunes and bunch lengths, and $n$ = turn number.

In order to describe the beam asymmetry we use in what follows four quantities: average nominal ($\xi_0$) and dynamical ($\xi$) beam-beam parameters, and nominal ($A_0$) and dynamical ($A$) beam asymmetry parameters, defined by

$$\begin{aligned} \xi_0 &= (\xi_{0+} + \xi_{0-})/2, \qquad A_0 = \frac{\xi_{0+} - \xi_{0-}}{\xi_{0+} + \xi_{0-}} \\ \xi &= (\xi_+ + \xi_-)/2, \qquad A = \frac{\xi_+ - \xi_-}{\xi_+ + \xi_-} \end{aligned} \tag{15}$$

Typically we use as inputs $\xi_0$ and $A_0$; thus the nominal beam-beam parameters are given by $\xi_{0\pm} = \xi_0(1 \pm A_0)$. Obviously a corresponding expression exists for the dynamical parameters $\xi_\pm$.

Fig. 5 shows $\xi$ *vs.* $\xi_0$ for various values of $A_0$ for a collider with symmetrical rings with tune $\nu_0 = 0.765$, damping decrement $\delta = 0.005$ and no tune modulation, showing a saturation at $\xi \simeq 0.07$. Although we do not show $\xi_+$ and $\xi_-$ individually, typically $\xi_+$ grows linearly for all values of $\xi_0$, while $\xi_-$ grows linearly at small $\xi_0$, reaches a maximum and goes back down to 0 at large $\xi_0$. There is beam instability beyond $\xi_0 \simeq 0.14$.

Fig. 6 shows the dynamical asymmetry parameter $A$ *vs.* $\xi_0$ for various values of $A_0$, for the same symmetric collider parameters as in Fig. 5. Note that the dynamics causes an increase in asymmetry ($A$ grows from $A = A_0$ at $\xi_0 = 0$ to $A = 1$ at large $\xi_0$). When $A = 1$ it means that one beam is strong and the other one is infinitesimally weak. Note also the spontaneous breaking of the symmetry for the symmetric case, $A_0 = 0$, at $\xi_0 \gtrsim 0.12$; this breaking is as likely to yield $A > 0$ as it is to yield $A < 0$, but for convenience we show only the first case.

Fig. 7 is similar to Fig. 6, except that we have added tune modulation, which causes seemingly unpredictable changes in the dynamical asymmetry $A$ above $\xi_0 \simeq 0.03$. Actually these effects occur after many turns, on the order of $10^5$; for the first $\sim 10^4$ turns the results are similar to those of Fig. 6 (the synchrotron period is $\sim 19.2$ turns).

Fig. 8 again shows $A$ *vs.* $\xi_0$, this time for an asymmetric collider, as the parameters at the right indicate (these correspond to APIARY [7]). Note the competition between the beam asymmetry and the lattice asymmetry: at low $\xi_0$ the dynamical asymmetry $A$ *decreases* from its nominal positive value $A_0$. This means that it is the $-$ beam that "wants" to be the strong one, even though it is nominally the weak one. However, at $\xi_0 \simeq 0.03$ the competition becomes unstable and the dynamics forces the beams to a strong-weak state ($A = \pm 1$). In fact it is so unstable that which beams becomes strong and which weak depends more on the initial beam configurations than the asymmetry in the dynamics.

## Conclusions

For the symmetric case:

(1) For low beam intensity (small $\rho$), only normal (constant, equal beam size) solutions exist and are stable, as it should be expected.

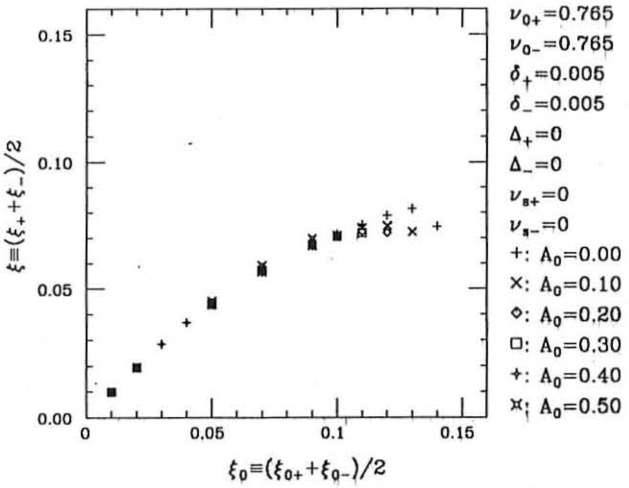


Fig. 5. Dynamical vs. nominal beam–beam parameter for various values of the nominal beam asymmetry parameter.

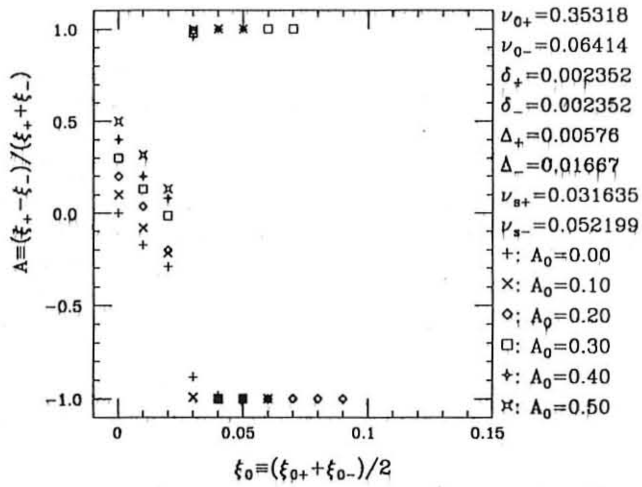


Fig. 8. Same as Fig. 7, except that the rings as well as the beams are asymmetrical.

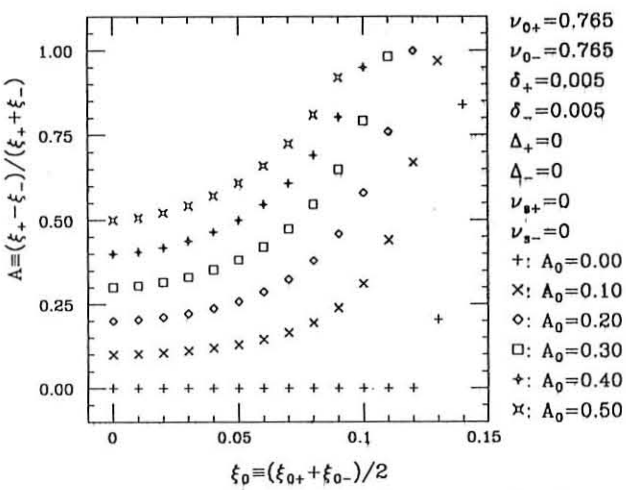


Fig. 6. Dynamical asymmetry parameter vs. nominal beam–beam parameter for various values of the nominal asymmetry. The rings are identical, with no tune modulation.

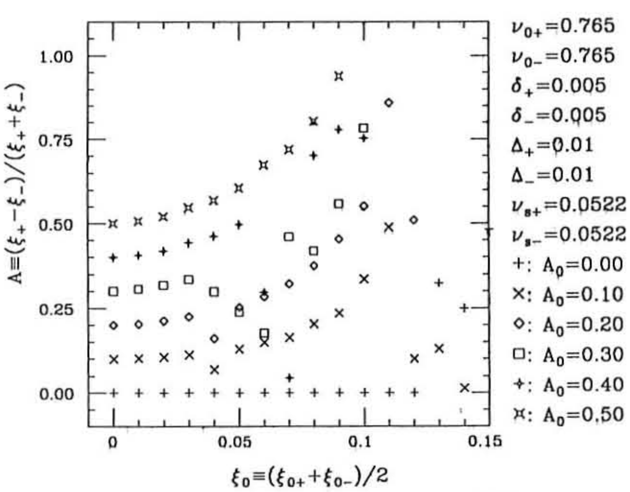


Fig. 7. Same as Fig. 6 except with tune modulation.

(2) As the intensity is increased, other solutions appear which cause the change in behavior of the luminosity from $\propto I^2$ to $\propto I$, and the saturation of beam-beam parameter at $\xi_0 \simeq \rho/2\pi \simeq 0.043$. The saturation mechanism is due to the appearence of a chaotic region followed by a higher-order fixed point rather than to a bifurcation. This seems to be a generic difference with Hirata's result [2].

(3) For a given tune, flip-flop solutions always exist and are real in a range of values of $\rho$. However, they are not always stable, and are unnatural for small $\rho$. By this we mean that they require a delicate relationship between $\nu_0$ and $\rho$, as can be seen from Fig. 1. Therefore the flip-flop effect may have a natural explanation in our model only for unrealistically high beam intensities. This seems to be in qualitative agreement with Hirata's model. It should be interesting to decide whether higher-order fixed points occur in other models; otherwise they might be an artifact of the linearization of the beam-beam force. These higher-order fixed points are confirmed by multiparticle tracking simulations for the linear-force model [8].

For the asymmetric case:

(1) In the absence of tune modulation the beam asymmetry grows with intensity from its nominal value.

(2) In the presence of tune modulation the dynamical asymmetry seems unpredictable after many turns; we do not presently understand even the systematics of this behavior.

(3) It is possible to compensate beam asymmetries with lattice asymmetries, but the equilibrium reached seems fairly unstable, except at low intensities. It should be interesting to see whether this is also true of more realistic models for the beam-beam force.

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