LBL-27453
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Submitted to Physical Review Letters

Tests of Time Reversal Invariance via Transmission Experiments

H.E. Conzett

October 1989

LOAN COPY
Circulates
for 2 weeks
Bldg. 50 Library.
LBL-27453
Copy 2

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

TESTS OF TIME REVERSAL INVARIANCE VIA TRANSMISSION EXPERIMENTS

H.E. Conzett

Nuclear Science Division, Lawrence Berkeley Laboratory

One Cyclotron Road, Berkeley, CA 94720

ABSTRACT

The existing formalism used to describe spin observables in neutron transmission experiments is found to be inadequate. A suitable formalism is developed, through which time-reversal violating (and parity non-conserving) forward scattering amplitudes are identified, along with their corresponding spin observables. It is noted that new and more precise tests of T-symmetry are provided in transmission experiments and that such investigations are applicable more generally in nuclear and particle physics.

Recent transmission experiments with slow neutrons have shown remarkable enhancements in two parity non-conserving (PNC) observables, the neutron spin rotation[1] and the neutron longitudinal analyzing power, A_z[2]. In the latter case, a value of A_z close to 10% has been measured in the transmission of neutrons through ^{139}La at the 0.734–eV resonance, and this nuclear enhancement of 10^5 to 10^6 is explained in terms of parity-mixed levels of ^{140}La and p-wave barrier hindrance of the parity conserving transition.[3]

The suggestion that nuclear effects could also provide enhancements in time-reversal violating (TRV) neutron transmission observables, which become accessible with polarized targets[4], has generated considerable interest and acitivity in the investigation of that possibility[5]. Stodolsky and Kabir have developed a formalism to describe the spin aspects of neutron transmission, and they have suggested TRV observables to be measured.[4] I have found some difficulties with their treatment, which I discuss in this paper. I then consider, again, the prospect for measureable TRV observables in transmission experiments.

Stodolsky and Kabir use the plane-wave neutron coherent amplitude in the target material,

$$M = e^{i(n-1)kd}\text{, with } n-1 = \frac{2\pi}{k^2}\rho f(o),$$

$$f(o) = f_o + f_x\,\hat{\sigma}\cdot\hat{s} + f_y\,\hat{\sigma}\cdot(\hat{k}\times\hat{s}) + f_z\,\hat{\sigma}\cdot\hat{k}\,, \tag{1}$$

where $n, k, d, \rho, f(o)$ are, respectively, the index of refraction, neutron wave number, distance of transmission in the target, density of target nuclei, and the neutron-nucleus forward elastic scattering matrix. It should be emphasized that only coherent non-spin-flip amplitudes contribute to the index of refraction, since scattering in which the target state is changed is incoherent.[6] In (1) $\hat{\sigma}$ is the neutron spin operator and $\hat{s}$ the target polarization, so f_z is a PNC

term and f_y is both PNC and TRV[4]. Choosing the projectile helicity frame, so that unit vectors along the coordinate axes are given by

$$\hat{z} = \hat{k}, \hat{x} = \hat{s}, \hat{y} = \hat{z} \times \hat{x} \ , \tag{2}$$

the forward scattering matrix becomes

$$f(o) = f_o + f_x \sigma_x + f_y \sigma_y + f_z \sigma_z \ . \tag{3}$$

Then, taking $$C_j = \frac{4\pi}{k} f_j \rho d \ , \ j = o, x, y, z.$$

$$M = \exp\left[\frac{i}{2} \sum_j C_j \sigma_j\right] . \tag{4}$$

Since M is a two by two matrix in the neutron spin space, Stodolsky and Kabir take its most general form,

$$M = \sum_j B_j \sigma_j \ , \ j = o, x, y, z \ , \ \sigma_0 = 1 \ , \tag{5}$$

and use (5) in the calculation of spin observables. For example, an analyzing-power component

$$A_j = \frac{Tr\, M\, \sigma_j M^\dagger}{Tr\, M\, M^\dagger} \ . \tag{6}$$

The transformation from the C_j of (4) to the B_j of (5) is not simple, and one loses the direct association and interpretation of the spin observables with the spin-dependent terms of the scattering matrix.

There are some clear difficulties with this procedure. In (3), f_y is an off-diagonal neutron spin-flip amplitude. However, since helicity is conserved in forward scattering, the equivalent condition is that $f(o)$ be invariant with respect to rotation about the z-axis, R_z, so

$$f(o) = f_o + f_z \sigma_z \tag{7}$$

is the most general form. In this treatment, then, a TRV observable is excluded by conservation of angular momentum (z-component), whether the scattering is either coherent or incoherent.

Since (7) includes the PNC term f_z, it is of interest to calculate the PNC neutron *coherent* transmission observables in terms of C_o and C_z (thus f_o and f_z) of eq. (4). Expressing the complex amplitude

$$C_j = \alpha_j + i\beta_j \quad , \qquad j = o, z \ , \tag{8}$$

since σ_o and σ_z commute, (4) can be written as

$$M = R_o D_o R_z D_z \tag{9}$$

with

$$R_j = \exp\left[\frac{i}{2}\alpha_j \sigma_j\right] \ , \ D_j = \exp\left[-\frac{1}{2}\beta_j \sigma_j\right], \tag{10}$$

so R_j (D_j) is a rotation (attenuation) matrix. Defining the transmission factor, through a target distance d,

$$\frac{I(d)}{I(o)} \equiv T(d) = \frac{1}{2} Tr M M^{\dagger} \ , \tag{11}$$

the analyzing power A_z and the polarization transfer (rotation) coefficient K_x^y are given by

$$T A_z = \frac{1}{2} Tr M \sigma_z M^\dagger$$

$$T K_x^y = \frac{1}{2} Tr M \sigma_x M^\dagger \sigma_y \quad . \tag{12}$$

Using (9) – (12),

$$T = e^{-\beta_0} \cosh \beta_z \Rightarrow e^{-\beta_0}$$

$$C_z \ll 1$$

$$A_z = -\tanh \beta_z \Rightarrow -\beta_z \tag{13}$$

$$K_x^y = -K_y^x = \frac{-\sin \alpha_z}{\cosh \beta_z} \Rightarrow R_z(-\alpha_z).$$

Thus, one sees the known PNC differential attenuation of positive and negative helicity neutrons, i.e., A_z, due to β_z, the imaginary part of the amplitude C_z, and the PNC neutron transverse spin rotation around the direction of propagation, due to α_z, the real part of C_z. Since β_z is large ($A_z \simeq 0.1$) at the 0.734–eV neutron—^{139}La resonance, it would be most interesting to determine α_z there via a measurement of the transverse spin rotation coefficient K_x^y, assuming, of course, that the forward scattering is coherent at this energy (at 1–eV, $\lambda \simeq 3 \times 10^4$ fm).

Since target polarization is required in order to provide a TRV term in the forward scattering matrix, it is necessary for that matrix to encompass *both* the projectile and target spin-matrices. That is, an observable that involves only the projectile (target) polarization can be expressed in terms of the projectile (target) spin-matrix amplitudes alone, as in (13), but the

combined spin-space amplitudes are required for an observable that involves both projectile and target polarizations. To investigate, then, the possibility of finding a TRV observable in transmission experiments, I consider in detail, as a prototype, the simplest case with spin-$\frac{1}{2}$ projectile and target. There the 4x4 scattering matrix, including the same factor as in (4),

$$F(o) = \frac{4\pi}{k} \rho d\, f(o) \quad , \tag{14}$$

is the direct product of the 2x2 projectile and target spin matrices:

$$\begin{aligned} F &= F_1 \otimes F_2 \quad , \text{ with} \\ F_n &= \sum_j C_{nj}\, \sigma_{nj} \quad , \quad n = 1,2 \end{aligned} \tag{15}$$

and n = 1 (2) labeling the projectile (target) matrix. Then, a typical term in F is $C_{1x}\ C_{2y}\ \sigma_{1x} \otimes \sigma_{2y}$, which I will abbreviate to $C_{xy}\ \sigma_x\ \sigma_y$, so that the ordering of the subscripts provides the n = 1,2 labeling. Then, with $\sigma_{1x} \equiv \hat{\sigma}_1 \cdot \hat{x}$, etc., we have the following transformations under the P, T symmetry operations:

$$\begin{array}{c|ccc} & \sigma_x & \sigma_y & \sigma_z \\ \hline P & \sigma_x & -\sigma_y & -\sigma_z \\ T & \sigma_x & -\sigma_y & \sigma_z \end{array} \tag{16}$$

The 16 F-matrix amplitudes,

$$C_{oo},\ C_{ox},\ \underline{C_{oy}},\ \underline{C_{oz}},\ C_{xo},\ C_{xx},\ \underline{C_{xy}},\ \underline{C_{xz}},\ \underline{C_{yo}},\ \underline{C_{yx}},\ C_{yy},\ C_{yz},\ \underline{C_{zo}},\ \underline{C_{zx}},\ C_{zy},\ C_{zz}, \tag{17}$$

can then be classified according to their P and/or T symmetry. For example, the underlined in (17) are PNC amplitudes. The requirements of R_z, P, and T-symmetries then yield, from (15) and (16), the $F(o)$-matrix

$$F(o) = C_{oo} + C_{xx}(\hat{\sigma}_1 \cdot \hat{\sigma}_2) + C'_{zz}\, \sigma_z \sigma_z \quad ,$$

so (18)

$$F(o) = C_{oo} + C_{xx}(\sigma_x \sigma_x + \sigma_y \sigma_y) + C_{zz}\, \sigma_z \sigma_z \quad ,$$

Thus, only 3 independent amplitudes survive, and, since, the C_{xx} term conserves helicity via a double spin-flip process, it cannot contribute to the coherent neutron amplitude.

Now add the PNC terms from (17) that are R_z-invariant,

$$C_{oz}\, \sigma_o \sigma_z + C_{zo}\, \sigma_z \sigma_o \ , \tag{19}$$

and the sixth R_z-invariant term,

$$C_{xy}\, (\hat{\sigma}_1 \times \hat{\sigma}_2)_z = C_{xy}\, (\sigma_x \sigma_y - \sigma_y \sigma_x) \ , \tag{20}$$

which, from (16), is seen to be both PNC and TRV. Thus,

$$F(o) = C_{oo} + C_{oz}\, \sigma_o \sigma_z + C_{zo} \sigma_z \sigma_o + C_{xx}\, (\sigma_x \sigma_x + \sigma_y \sigma_y) + C_{zz} \sigma_z \sigma_z + C_{xy}\, (\sigma_x \sigma_y - \sigma_y \sigma_x) \tag{21}$$

is the complete R_z-invariant PNC and TRV forward-scattering matrix. Again, the C_{xy} term is a double spin-flip amplitude, so I conclude that in the $\frac{1}{2} + \frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2}$ system, coherent forward scattering does not provide a TRV observable.

However, and perhaps more importantly, the term C_{xy} in the forward scattering matrix suggests that a corresponding PNC, TRV observable is available in the more ordinary and

widespread possibilities for *incoherent* transmission experiments in nuclear and particle physics. There one uses a treatment that features transmitted *intensities* rather than amplitudes; and the spin-dependent observables, i.e. the total cross-sections, are then related to the spin-dependent forward scattering amplitudes by the optical theorem.

The transmission factor (11) is now

$$T_{(d)} = \exp\,[-\sigma_T \rho d] \equiv \exp\,[-\sigma] \ , \tag{22}$$

where σ_T is the unpolarized total cross-section, and thus σ is a dimensionless "total cross-section" which includes the areal density factor ρd. The corresponding spin-dependent cross sections are

$$\sigma_{jk} = \sigma(1 + p_j\, p_k\, A_{jk})\ ;\ j, k = x, y, z \ , \tag{23}$$

where $p_j(p_k)$ is the projectile (target) polarization along the $j(k)$ direction, and A_{jk} is the corresponding (total cross-section) spin-correlation coefficient, which is essentially defined by this equation. Then, with $\sigma_{jk}(++)$ ($\sigma_{jk}(+-)$) defined as the cross section for the pure spin states $p_j = p_k = 1$ ($p_j = -p_k = 1$), we have

$$\begin{aligned} \sigma_{jk}(++) &= \sigma_{jk}(--) = \sigma(1 + A_{jk}) \ , \\ \sigma_{jk}(+-) &= \sigma_{jk}(-+) = \sigma(1 - A_{jk}) \ . \end{aligned} \tag{24}$$

Using these spin dependent cross-sections, the corresponding transmission factors are defined as

$$T_{jk} = \frac{1}{2}\{\exp\,[-\sigma_{jk}(++)\,] + \exp\,[-\sigma_{jk}(+-)\,]\} \ , \tag{25a}$$

and

$$\Delta T_{jk} = \frac{\exp\,[-\sigma_{jk}(++)\,] - \exp\,[-\sigma_{jk}(+-)\,]}{\exp\,[-\sigma_{jk}(++)\,] + \exp\,[-\sigma_{jk}(+-)\,]} \ . \tag{25b}$$

Thus, T_{jk} is the transmission factor for a completely polarized beam transmitted through an unpolarized target (and vice versa), while ΔT_{jk} is the *transmission asymmetry* of the polarized beam for opposite states of the target polarization. Using (24),

$$T_{jk} = e^{-\sigma} \cosh \sigma A_{jk} , \tag{26a}$$

$$\Delta T_{jk} = - \tanh \sigma A_{jk} . \tag{26b}$$

We now use the spin-dependent form of the optical theorem[7] to express σA_{jk} in terms of the imaginary part of the corresponding forward scattering amplitude. That is

$$\sigma_T(\rho) = \frac{4\pi}{k} Im\ Tr\ [\rho\, f(o)], \tag{27}$$

where ρ_{jk} is the density matrix representing the inital polarizations and $\sigma_T(p_j, p_k)$ is the corresponding total cross-section. The normalization $Tr\rho = 1$ has been chosen. Then, in the established notation,

$$\rho_{jk} = \frac{1}{4} (\sigma_o + p_j\, \sigma_j) \otimes (\sigma_o + p_k \sigma_k) \tag{28}$$

so

$$\rho_{jk}\ (+\pm) = \frac{1}{4} (1 + \sigma_j\, \sigma_o \pm \sigma_o\, \sigma_k \pm \sigma_j\, \sigma_k) . \tag{29}$$

With (14), (27) becomes

$$\sigma_{jk} = Im\ Tr\ [\rho_{jk}\, F(o)] , \tag{30}$$

and with

$$\sigma A_{jk} = \frac{1}{2}\,[\sigma_{jk}(++) - \sigma_{jk}(+-)\ ]\,, \tag{31}$$

we have

$$\sigma A_{jk} = \frac{1}{4}\,Im\,Tr\,[\,(\sigma_o\,\sigma_k\ + \sigma_j\,\sigma_k)\,F(o)\,]. \tag{32}$$

Then noting that

$$T_r\,(\sigma_j\,\sigma_k)\,(\sigma_{j'}\,\sigma_{k'})\ =\ 4\ \ \delta_{jj'}\,\delta_{kk'} \tag{33}$$

the only terms in $F(o)$, (21), that survive in (32) are the terms C_{ok} and C_{jk}, once the polarizations p_j, p_k have been selected. For our purpose here, to identify a TRV observable corresponding to the amplitude C_{xy}, the appropriate choice is $p_j = p_x, p_k = p_y$ (or vice versa), for which

$$\sigma A_{xy} = Im\ C_{xy} = \beta_{xy}\,. \tag{34}$$

Finally, then, the corresponding transmission asymmetry,

$$\Delta T_{xy} = \text{-tanh}\ \beta_{xy} \overset{C_{xy}<<1}{\rightarrow} -\beta_{xy}\ \ , \tag{35}$$

is the observable to measure, with a vector (rank 1) polarized target, in the search for a combined PNC, TRV effect. This is also the simplest experimentally, for which high accuracy can be achieved.

Since it is clear from the foregoing development that there is no purely TRV forward amplitude in the $\frac{1}{2} + \frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2}$ spin system, it is important to examine the suggestion[8] for an additional T-odd, P-even term [presumably in $f(o)$, (1)] of the form $(\hat{\sigma} \cdot \hat{k} \times \hat{s})\ (\hat{k} \cdot \hat{s})$, with target spin $S \geq 1$ since S^2 represents an alignment. It is clear that this term would not contribute to f_z in

(3), so it is excluded by R_z-invariance. It can, however, be provided by a term in the F–matrix (15) for a system with the spin structure $\frac{1}{2}+1 \rightarrow \frac{1}{2}+1$. Since space limitations preclude a discussion here in the detail of that for the $\frac{1}{2}+\frac{1}{2} \rightarrow \frac{1}{2}+\frac{1}{2}$ system, I will simply indicate the result, and the details will be available elsewhere.[9] In (15), F_2 now becomes a 3x3 spin-1 matrix, with

$$F_2 = \sum_j C_j P_j + \sum_{k\,l} C_{kl}\ P_{kl}\,,\ j = o, x, y, z\,;\ k, l = x, y, z\ \ , \tag{36}$$

with the second sum limited to five independent terms. The P_j (P_{kl}), $j \neq 0$, are the vector, rank 1 (tensor, rank 2) components of the spin-1 matrix-operator.[10] Thus, the 9-term F_2 combined with the 4-term F_1 provides the required 36 terms of the F-matrix. The forward scattering matrix, imposing R_z-invariance, is reduced to 10 terms. P-symmetry reduces it to 5 independent amplitudes and T-symmetry to 4, so that there is one TRV amplitude. It is provided by the term

$$C_{y,zx}\,(\sigma_x P_{yz} - \sigma_y P_{zx})\ . \tag{37}$$

in a straightforward extension of the previous notation. Thus, $C_{y,zx}$ is the TRV amplitude and the corresponding TRV observable is the transmission asymmetry

$$\Delta T_{y,zx} = \tanh \beta_{y,zx} \overset{C_{y,zx} \ll 1}{\rightarrow} \beta_{y,zx}\ . \tag{38}$$

As the notation indicates, this corresponds to neutron polarization p_y in combination with the target tensor polarization p_{zx}, i.e. alignment along the direction z = x.

In summary, it has been established that the complete spin-space matrix must be used in the description of TRV effects in transmission experiments, and it is seen that transmission observables can provide new and sensitive tests of T-symmetry. An especially important result is that the TRV observable is given directly by the imaginary part of the corresponding TRV

forward scattering amplitude, in contrast to the situation that exists for the standard (non-forward) scattering experiment. There, T-symmetry tests can be accomplished only via comparison of two *separate* observables (e.g. $P_y = A_y$), and, as has been established,[11] no single null test is possible. Now, $\Delta T_{y,zx}$ (38) provides just such a null test of T-symmetry, and such an experiment is capable of attaining an unprecedented precision, comparable to that reached in P-symmetry tests, where, for example, A_z values of the order 10^{-7} have been measured.[12]

Finally, although the motivation for the development of this spin formalism for transmission experiments was derived from the exciting results achieved with slow neutrons, this development clearly has application at higher energies in nuclear and particle physics with respect to attaining considerably higher precision in tests of T-symmetry.

I am grateful to M. Simonius and E. M. Henley for clarifying discussions. This work was partially supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contract DE-AC03-76SF00098. Partial support was also provided by the Institut National de Physique Nucléaire et de Physique des Particules during a sabbatical year spent at the University of Marseille, when some initial ideas were generated concerning spin observables in transmission experiments. I am grateful to J.-P. Longequeue and E. Perret of the IN2P3 and to my hosts at Marseille, J. Soffer and J.J. Aubert, for their generous assistance. I also thank K. Kilian for raising a provocative question some time ago.

REFERENCES

1. B. Heckel et al., Phys. Rev. C 29, 2389 (1984) and references therein.

2. V.P. Alfimenkov et al., Nucl. Phys. A398, 93 (1983).

   Y. Masuda et al., *Tests of Time Reversal Invariance in Neutron Physics*, edited by N.R. Roberson, C.R. Gould, and J.D. Bowman (World Scientific, Singapore, 1987), p. 191; to be published, Nucl. Phys. (1989).

3. V.E. Bunakov and V.P. Gudkof, Nucl. Phys. A401, 93 (1983).

4. L. Stodolsky, Phys. Lett. B 172, 5 (1986).

   P.K. Kabir, Phys Rev. Lett. 60, 686 (1988), Phys. Rev. D37, 1856 (1988).

5. *Tests of Time Reversal Invariance in Neutron Physics* (Ref. 2).

6. W. Marschall and S.W. Lovesey, *Theory of Thermal Neutron Scattering* (Oxford University Press, London, 1971), for example.

7. R.J.N. Phillips, Nucl. Phys. 43, 413 (1963).

8. P.K. Kabir, *The Investigation of Fundamental Interactions with Cold Neutrons*, NBS Special Publ. No. 711 (U.S. GPO, Washington, D.C., 1986), p. 81.

9. H.E. Conzett, in preparation.

10. For example, G.G. Ohlsen, Rep. Prog. Phys. 35, 717 (1972).

11. F. Arash et al., Phys. Rev. Lett. 54, 2649 (1985).

12. S. Kystryn et al., Phys. Rev. Lett. 58, 1616 (1987).

LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
1 CYCLOTRON ROAD
BERKELEY, CALIFORNIA 94720