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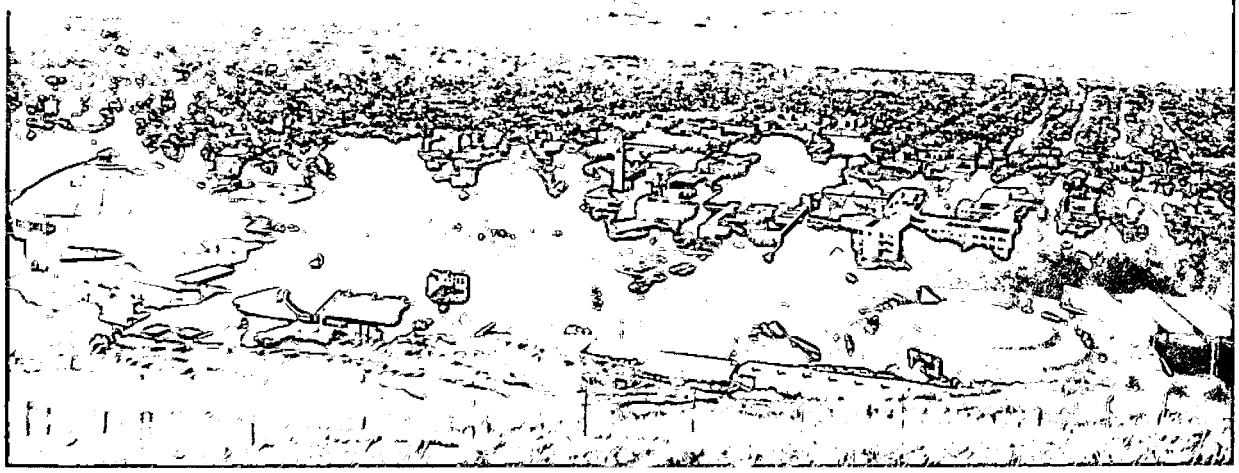

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Abstract

The permeability of a fracture that consists of smooth, parallel faces and which has randomly-located, uniform-sized cylindrical asperities is investigated. The viscous resistance due to the asperities is accounted for by an in-plane permeability coefficient, and a Brinkman-type equation is used to find the velocity distribution across the thickness of the fracture. The resulting expression for the permeability of the fracture reduces to the known result for parallel plates as the concentration of asperities approaches zero, and reduces to the known result for long, parallel cylinders as the distance between the plates goes to infinity.

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I. INTRODUCTION

The flow of a Newtonian fluid through a rock fracture is of importance in many geophysical and geotechnical processes. For hydrological purposes, rock fractures have traditionally been modeled as two smooth parallel walls separated by a distance h, which leads to a permeability of $h^2/12$.[1] More recent work[2,3] has attempted to account for the in-plane tortuosity caused by fluid flowing around the regions where the opposing rock faces are in contact (the asperities). In this paper we estimate the effect that viscous drag along the sides of the asperities has in reducing the permeability.

II. THEORETICAL DEVELOPMENT

Consider a fracture that consists of two smooth parallel walls of infinite extent, separated by a distance h. Let the x–y coordinate axes be parallel to the plane of the fracture, and the z axis be perpendicular to the fracture walls, which are located at $z = \pm h/2$. The x–y coordinates can be oriented so that the macroscopic pressure gradient, and the mean flow, are in the x direction (see Fig. 1). For very low Reynolds' numbers, such as are typically found in geological systems, the problem can be analyzed as follows. At each position z within the fracture, fluid "particles" follow a tortuous path around the cylindrical obstacles, but have zero velocity in the z direction. The viscous resistance offered by these asperities can be accounted for by a "two-dimensional" permeability coefficient K_2, defined so that the average velocity $\bar{u}$ (averaged locally over the transverse x–y plane) for uniform, steady-state flow equals $-(K_2/\mu)\nabla P$. Methods of estimating this permeability, for random or periodic distributions of circular obstacles, have been discussed by Hasimoto,[4] Howells,[5] Rubinstein and Keller,[6] and Sangani and Yao.[7] The average velocities on the various planes parallel with the fracture walls will then vary with z, due to the viscous drag between adjacent fluid "sheets". This effect can be treated within the framework of the Navier-

Stokes equations, in the sense that the viscous drag between the various sheets is proportional to $\partial\bar{u}/\partial z$.

The time-dependent equations that govern flow within the fracture are obtained by combining the two-dimensional Navier-Stokes equations with the effective permeability concept, yielding (*cf*. Brinkman[8], Vafai and Tien[9])

$$\frac{\rho}{\phi^2}\frac{\partial\bar{u}(x,z,t)}{\partial t} = -\frac{d(p+\rho g\chi)}{dx} - \frac{\mu}{K_2}\bar{u}(x,z,t) + \frac{\mu}{\phi^2}\frac{\partial^2\bar{u}(x,z,t)}{\partial z^2}, \tag{1}$$

where ρ is the density of the fluid, μ is the viscosity, g is the gravitational acceleration, χ is a coordinate measured in the direction of the gravitational field, and ϕ is the fraction of space in the $x-y$ plane that is not occupied by obstacles. The velocity $\bar{u}$ is averaged locally over the $x-y$ plane, and hence is a function only of x and z. This velocity is a "Darcy velocity" or "filter velocity"[10], and can be related to the actual average velocity of the fluid particles by $\bar{u}=\phi^2 u$ (actual). The second term on the right side of equation (1) represents the potential drop due to viscous drag along the sides of the obstacles. One factor of ϕ was introduced by Vafai and Tien[9] to account for the reduced area available for flow, while the second is introduced here to account for "tortuosity", in the sense that the actual travel path of a fluid particle between two points $x=x_1$ and $x=x_2$ must exceed the distance $|x_2-x_1|$. Although the identification of the tortuosity with $1/\phi$ is not exact, this idea has had some success in predicting the electrical conductivity of porous rocks[11]. The potential gradient $d(p+\rho g\chi)/dx$ is averaged over the $y-z$ plane, and hence depends only on x.

For fully-developed flow, the average velocity and the potential gradient will not vary in the x direction, and the flow field will not vary with time, so the equation of motion takes the form

$$\frac{dH}{dx} = \frac{\mu}{\phi^2}\frac{\partial^2 \bar{u}(z)}{\partial z^2} - \frac{\mu}{K_2}\bar{u}(z) , \tag{2}$$

where H represents the potential $p + \rho g \chi$. The no-slip boundary condition on the fracture walls requires that

$$\bar{u}(z = \pm h/2) = 0 . \tag{3}$$

The governing equation (2) can be integrated, using the boundary condition (3), to yield the following velocity profile:

$$\bar{u}(z) = \frac{-K_2}{\mu}\frac{dH}{dx}\left[1 - \frac{\cosh\left[\phi z/\sqrt{K_2}\right]}{\cosh\left[\phi h/2\sqrt{K_2}\right]}\right] . \tag{4}$$

In the limit as the concentration of obstacles goes to zero, $\phi \rightarrow 1$ and $K_2 \rightarrow \infty$. Since $\cosh\zeta \approx 1 + \zeta^2/2$ as $\zeta \rightarrow 0$, the velocity profile reduces to

$$\bar{u}(z) = \frac{-h^2}{8\mu}\frac{dH}{dx}\left[1 - \left[\frac{2z}{h}\right]^2\right] , \tag{5}$$

which is the well-known result for flow between two parallel walls.

III. FRACTURE PERMEABILITY

The total volumetric flux can be found by averaging the velocity again, this time over the z direction, yielding

$$\bar{\bar{u}} = \frac{1}{h}\int_{-h/2}^{+h/2} \bar{u}(z)\,dz = \frac{-K_2}{\mu}\frac{dH}{dx}\left[1 - \frac{\tanh\left[\phi h/\sqrt{K_2}\right]}{\left[\phi h/2\sqrt{K_2}\right]}\right] . \tag{6}$$

The equivalent "three-dimensional" fracture permeability can now be found by comparing (6) with Darcy's law

$$\bar{\bar{u}} = \frac{-K_3}{\mu}\frac{dH}{dx} , \tag{7}$$

to yield

$$K_3 = K_2\left[1 - \frac{\tanh\left[\phi h/2\sqrt{K_2}\right]}{\left[\phi h/2\sqrt{K_2}\right]}\right] . \tag{8}$$

The above expression reduces to the expected results in the two limiting cases of $\phi \to 1$ (no obstacles), and $h \to \infty$ (no side walls). In the former case, note that $K_2 \to \infty$ when $\phi \to 1$ with h held constant, so that the argument of the tanh function goes to zero. Use of the series $\tanh\zeta = \zeta - \zeta^3/3 + \cdots$ in (8) leads to

$$K_3 \to \frac{h^2}{12} \quad \text{as} \quad \phi \to 1 , \tag{9}$$

which is the known[1] permeability for flow between parallel flat plates. In the other limit of $h \to \infty$ (with ϕ held fixed), $\tanh(\infty) = 1$ in (8), so that

$$K_3 \to K_2 \quad \text{as} \quad h \to \infty \,. \tag{10}$$

In this case the permeability reduces to that of flow across an array of infinitely long, parallel circular cylinders.

In order to be more specific about the results, and to quantify what is meant by "large h", we need to assume an expression for K_2. In general, this is a difficult problem which is not yet completely solved. When the obstacle concentration is small, however, asymptotic expressions for K_2 are available. For cylindrical obstacles of radius a located on a square lattice, Hasimoto[4] showed that the permeability K_2 is given by

$$K_2 = \frac{a^2}{8c}(-\ln c - 1.476 + 2c + \cdots) \,, \tag{11}$$

where $c = 1 - \phi$ is the areal concentration of the obstacles (see Figure 2). The permeability in (11) scales with a^2; this is also true for any extension of such an equation to higher asperity concentrations. Although (11) was derived for a cubic array of cylinders, the influence of cylinder location (as opposed to concentration) only assumes importance for values of c greater than about 0.4.[4,7]

It is clear that h/a is the important dimensionless parameter, so that the limit $h \to \infty$ really corresponds to $h \gg a$. There is another important limit in which $h/a \to 0$ for fixed c, in which case (8) reduces to

$$K_3 = \frac{\phi^2 h^2}{12} = \frac{h^2}{12}(1-c)^2 . \tag{12}$$

If the concentration of obstacles c is small but finite, (12) reduces to $(1-2c)h^2/12$, which is the result found by Walsh[2] for a thin fracture containing a small concentration of randomly distributed circular asperities. Walsh used the Hele-Shaw approximation to reduce the flow equations to a Laplace equation for the pressure, and then used known results from the analogous field of heat conduction. The present analysis clarifies the fact that Walsh's result requires $h \ll a$.

For higher concentrations of randomly-located asperities, Sangani and Yao[7] used a lubrication-type approximation for the flow between nearby cylinders to derive (see Figure 2)

$$K_2 = \frac{a^2}{3.34c}\left[1 - 1.10\sqrt{c}\right]^{5/2} . \tag{13}$$

Whereas (11) is expected to be accurate for small values of c, (13) is asymptotically accurate for large values of c. If one of the K_2 expressions is used in conjunction with (8), the three-dimensional permeability K_3 can be predicted (Figure 3). K_3 is normalized with respect to the "unobstructed" parallel plate value $h^2/12$, in order that the plotted permeabilities be finite for all values of h and a. The permeability curve labelled $h/a = 0$ represents the limiting case where the viscous drag along the parallel faces of the fracture greatly exceeds the drag along the sides of the asperities. An increase in the h/a parameter increases the viscous drag along the asperites, and hence decreases the permeability below the "thin fracture" value.

IV. CONCLUSIONS

An expression has been derived for the permeability of a fracture that consists of two smooth parallel walls separated by an aperture h, and which contains cylindrical asperities of radius a. The predicted permeability reduces to that of an assemblage of infinitely long parallel cylinders when $h/a \rightarrow \infty$, and to the parallel plate permeability when the asperity concentration goes to zero. The results show that viscous drag along the faces of asperities appreciably reduces the permeability of a fracture.

ACKNOWLEDGMENTS

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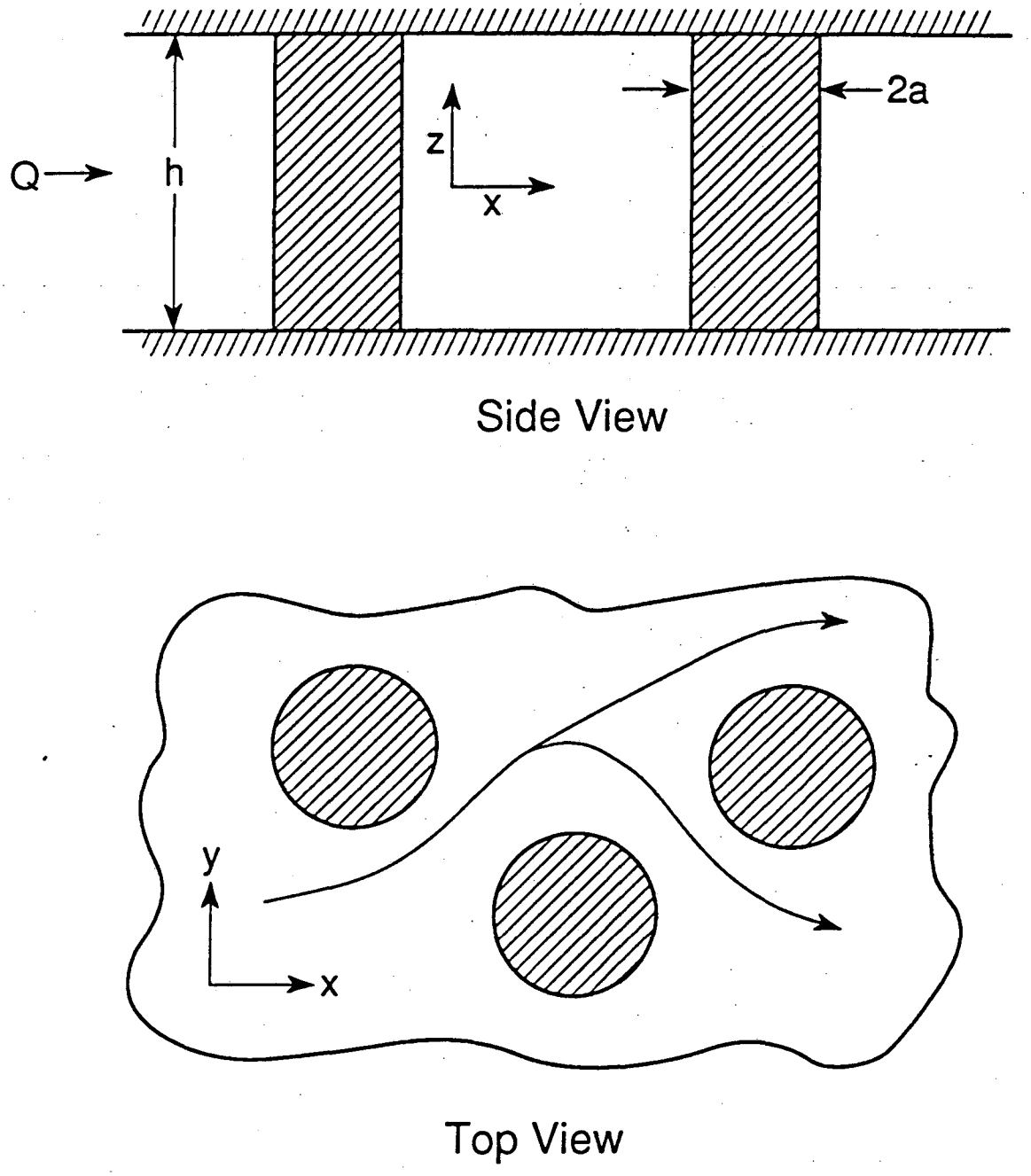
FIGURE CAPTIONS

FIG. 1. Schematic diagram of a parallel-wall fracture propped open by cylindrical asperities.

FIG. 2. Two-dimensional permeability for uniform flow across an array of randomly-located parallel cylinders of radius a.

FIG. 3. Three-dimensional fracture permeability, using two different expressions for K_2 in Equation (8).



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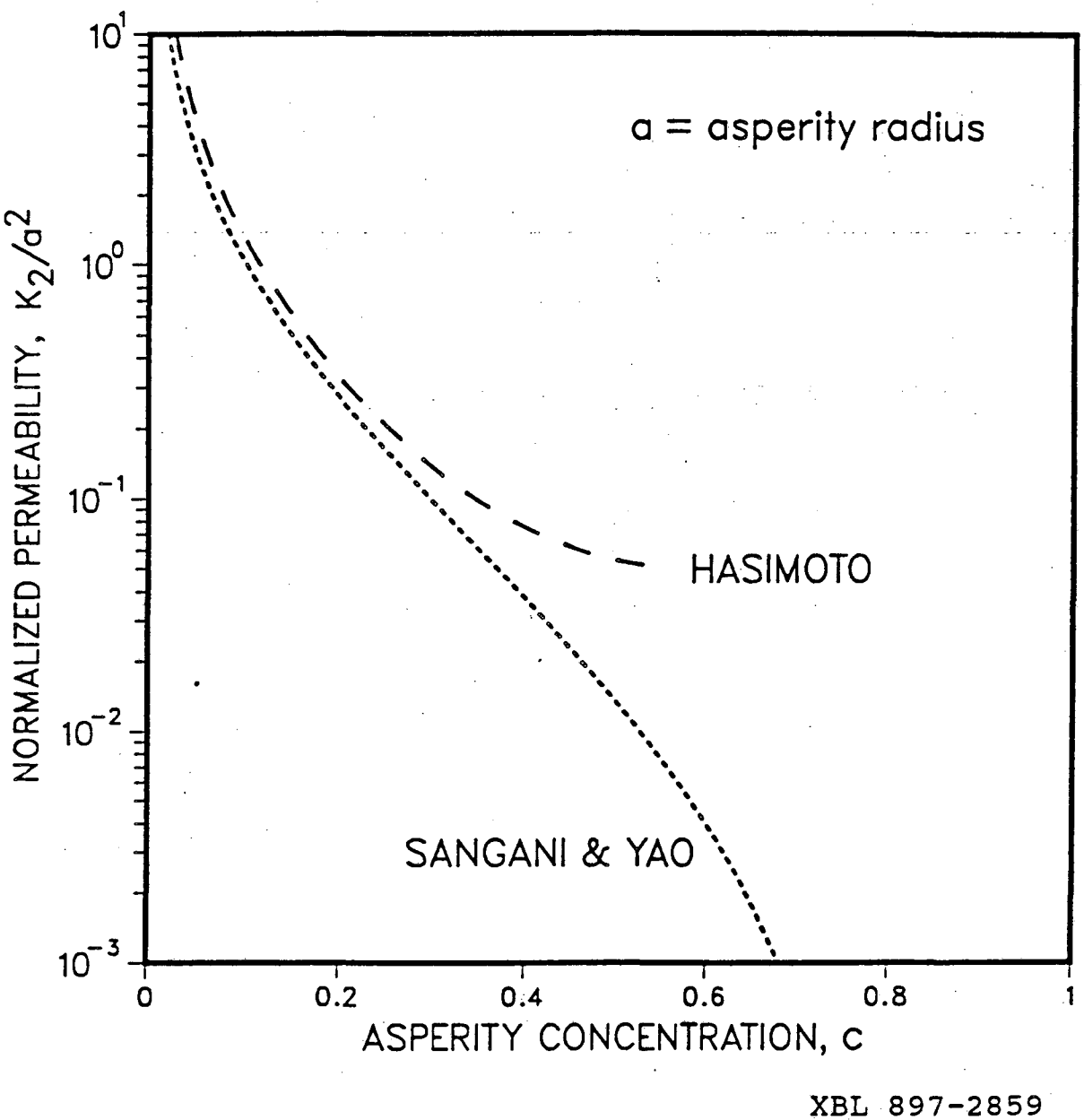

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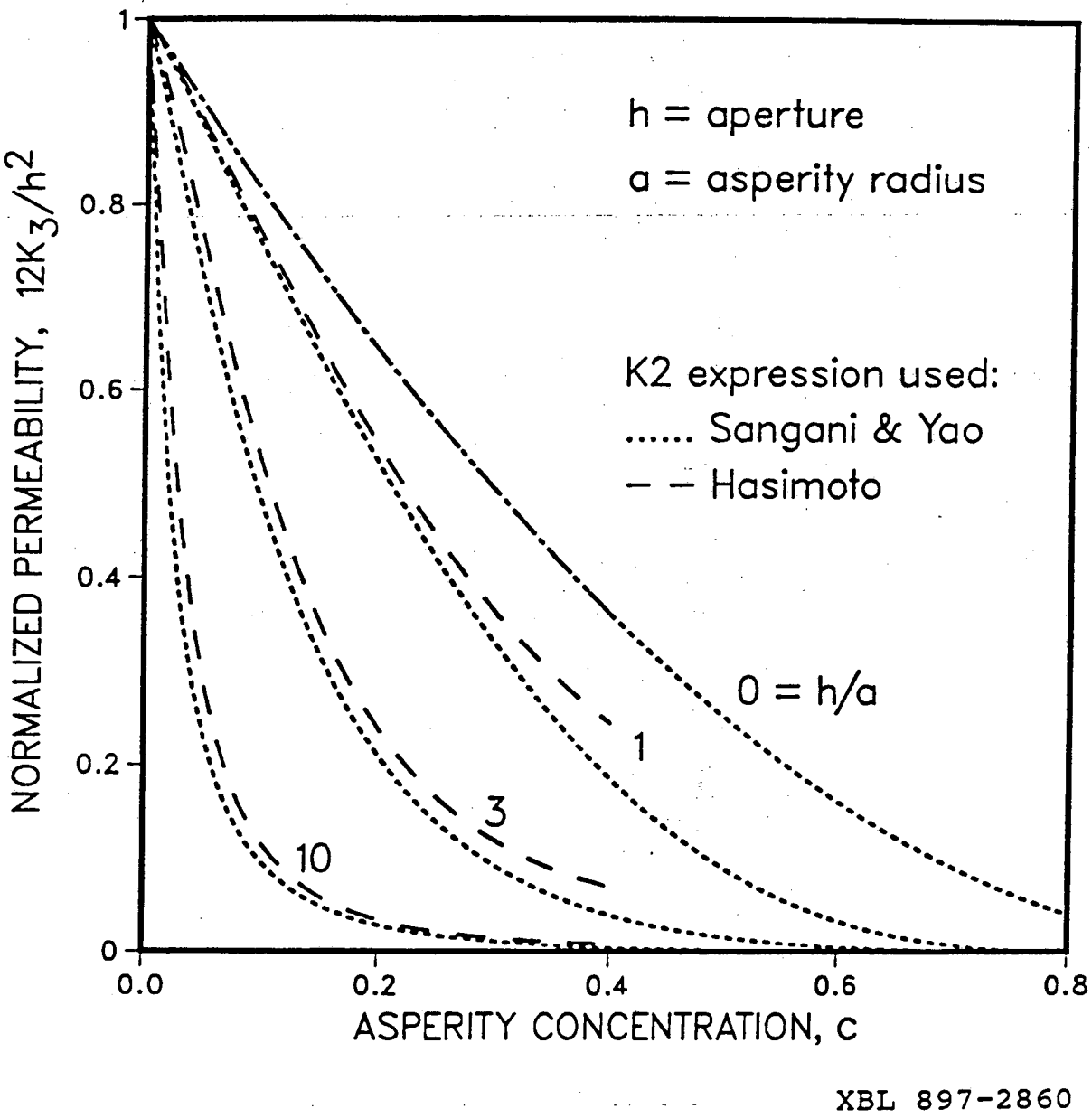

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