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C.S. Wang and W.D. Myers

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C.S. Wang
Center of Theoretical Physics, CCAST (World Laboratory)
Department of Technical Physics, Peking University,
Beijing 100871, China

W.D. Myers
Nuclear Science Division
Lawrence Berkeley Laboratory
1 Cyclotron Road
Berkeley, CA 94720 USA

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ABSTRACT

A model is proposed for cold dense matter at subsaturation densities which is composed of neutron rich nuclei immersed in a neutron gas as well as an electron gas. The equation of state is calculated by means of a Wigner-Seitz approximation. The result shows a discontinuity at a critical matter density ~ 5×10^{12}g/cm^3. This discontinuity, which corresponds to the point where one valley on the energy surface empties into another from above, is related to a sharp peak in the dependence of the nuclear surface tension on the nuclear asymmetry.

This paper concerns the equation of state of cold dense matter at densities that are below the saturation density of nuclear matter, which (for symmetric, N=Z nuclear matter) corresponds to nuclear particle number densities $\rho \leq 0.15/\text{fm}^3$. Uniform nuclear matter is unstable at a density somewhat below saturation, which (when defined as the density where the nuclear incompressibility becomes negative) is at a particle number density of about $0.1/\text{fm}^3$ for symmetric nuclear matter and somewhat lower for asymmetric nuclear matter [1]. The thermodynamically favored configuration is then a periodic structure of bulk matter immersed in a neutron gas and an electron gas which is required for overall electrical

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neutrality. At each density the equilibrium configuration can be determined by a minimization of the total energy E_t at fixed volume V_t and fixed number of nucleons A_t.

Since this subsaturation density regime exists in a collapsing stellar core during the formation of a supernova [2,3], it has been studied by many authors. In an extensively studied case, the bulk matter is assumed to consist of spherical neutron rich nuclei. At higher densities the role of the nuclear matter and the neutron gas are reversed and the dense matter is assumed to consist of neutron gas bubbles surrounded by neutron rich nuclear matter. In both of the above cases which have been studied by means of macroscopic [4-7] as well as microscopic [8-12] models, the periodic structure is 3-dimensional. It has also been shown [13,14] that one can expect 2-dimensional and even 1-dimensional periodic structures consisting of rods, tubes or slabs, to be thermodynamically stable configurations. All of these studies show that the formation of stable configurations is very sensitive to the surface tension, as the configuration is determined by competition between the surface energy and the Coulomb energy. Therefore, a consistent treatment of the bulk matter, the neutron gas and the nuclear surface energy is essential for a macroscopic model of this dense matter system, as has often been emphasized by Ravenhall et al. [15].

It is interesting to note that the behavior of the nuclear surface tension γ as a function of nuclear asymmetry δ is qualitatively quite similar but quantitatively different depending on whether the phenomenological Skyrme interaction or the Seyler-Blanchard interaction is chosen for use in a macroscopic model [16,17]. (The quantity $\delta = (\rho_n - \rho_p)/\rho$, where ρ_n and ρ_p are the neutron and proton density respectively and $\rho = \rho_n + \rho_p$ is the total nucleon density.) The surface tension $\gamma(\delta)$ given by the Seyler-Blanchard interaction [16] has a much sharper dependence on δ than that given by the Skyrme interaction [17]. Consequently, we felt it was worthwhile to determine if there is also a difference in the stable configurations predicted for dense matter at nuclear subsaturation densities. The purpose of this paper is to present some preliminary results on the configuration of a neutron rich nucleus immersed in a mixture of neutron gas and electron gas as a function of increasing density.

For simplicity our model calculation assumes there is a neutron rich nucleus, surrounded by a neutron gas in the center of the elementary cell which is a sphere of radius r_c in the Wigner-Seitz approximation. An electron gas is assumed to be distributed

uniformly in this cell, and the number of electrons equal to the number of protons Z in the nucleus for overall charge neutrality. The total energy of a unit cell can be written as

$$E = E_N + E_G + E_{el} + E_{lat}, \tag{1}$$

where E_N is the energy of nucleus, E_G is the energy of the neutron gas, E_{el} is the energy of the electron gas, and E_{lat} is the lattice Coulomb energy of a unit cell.

An extended nuclear droplet model [18] is used for the neutron rich nucleus where the nucleus is assumed to be composed of a nuclear bulk core of radius r_p and a neutron skin of thickness t, so we have

$$E_N = E_b + E_s + E_C + E_{ex}, \tag{2}$$

where E_b is the bulk energy, E_s is the surface energy, E_C is the Coulomb direct energy, and E_{ex} is the Coulomb exchange energy. The bulk energy E_b can be calculated as

$$E_b = \mu_p(\rho,\delta)Z + \mu_n(\rho,\delta)N - P(\rho,\delta)V, \tag{3}$$

where μ_p and μ_n are the bulk proton and neutron chemical potential respectively, $P(\rho,\delta)$ is the bulk pressure, ρ is the number density of bulk nucleons and δ is the bulk asymmetry. Three of the quantities μ_p, μ_n and P can be calculated from the equation of state of asymmetric nuclear matter based on the Thomas-Fermi model with Seyler-Blanchard interaction [1,16,18]. The number of protons Z and the number of neutrons N can be calculated respectively as

$$Z = \frac{1}{2}(1-\delta)\rho\frac{4\pi}{3}r_p^3, \tag{4}$$

$$N = \frac{1}{2}(1+\delta)\rho\frac{4\pi}{3}(r_p + t)^3, \tag{5}$$

where the neutron skin thickness t, as a function of the bulk asymmetry δ, is given by a Thomas-Fermi calculation [16]. The nuclear volume V depends on r_p, t, Z and N according to the expression

$$V = \frac{4\pi}{3}\left[\frac{Z\, r_p + N\,(r_p + t\,)}{Z + N}\right]^3. \tag{6}$$

The surface energy can be calculated as $E_s = \gamma(\delta)\, S$, where the nuclear surface tension γ is given by the Thomas-Fermi calculation [16], and the surface area S is

$$S = (\,4\pi\,)^{1/3}\,(\,3V\,)^{2/3}. \tag{7}$$

The Coulomb direct energy E_C and the exchange energy E_{ex} can be calculated respectively as [19,20]

$$E_C = \frac{3}{5}\frac{e^2Z^2}{r_p}\,(1 - 2.5\,\beta^2 + 3.0216\,\beta^3 + \beta^4) \tag{8}$$

$$E_{ex} = -\frac{3}{4}\left(\frac{3}{2\pi}\right)^{2/3}\frac{e^2Z^{4/3}}{r_p}\,(1 - 0.736\,\beta), \tag{9}$$

where $\beta = b(\delta)/r_p$, and b is the asymmetry dependent nuclear charge surface diffuseness given by the Thomas-Fermi calculation [16].

If the Seyler-Blanchard expression for the energy per particle of bulk matter as a function of the density ρ and asymmetry δ is written as $e(\rho,\delta)$, then the energy of the neutron gas E_G is given by

$$E_G = e(\rho_G, 1)\, N_G, \tag{10}$$

where

$$N_G = \rho_G\, V_G = \rho_G\,\frac{4\pi}{3}\,[r_c^3 - (r_p + t)^3] \tag{11}$$

is the number of neutrons in the gas, V_G is the volume of the neutron gas, and r_c is the radius of the unit cell.

The relativistic Fermi gas model is used to calculate the energy of the electron gas,

$$E_{el} = \frac{m_e^4 c^5}{8\pi^2\hbar^3}\left[(2t^2+1)\, t\sqrt{t^2+1} - \ln\left(t+\sqrt{t^2+1}\right)\right] V_c\,, \tag{12}$$

where $t = \hbar(3\pi^2\rho_e)^{1/3}/m_e c$ and $\rho_e = Z/V_c$ is the number density of electrons and the volume of the unit cell is $V_c = 4\pi r_c^3/3$.

The Wigner-Seitz approximation is used to calculate the lattice energy, so we have

$$E_{lat} = -\frac{9}{10}\frac{e^2Z^2}{r_c}\left[1 - \frac{1}{3}\left(r_p/r_c\right)^2\right]. \tag{13}$$

From the equations above the energy per nucleon for a unit cell $E/(Z+N+N_G)$ can be calculated as a function of bulk density ρ, bulk asymmetry δ, charge radius r_p, gas density ρ_g and cell radius r_c, or equivalently as a function of number of protons Z, number of neutrons N, bulk asymmetry δ, number of neutrons in the gas N_G and the nucleon density of a cell $\rho_c = (Z+N+N_G)/V_c$. For a given density ρ_c, the minimization of $E/(Z+N+N_G)$ with respect to the other four variables Z, N, δ, N_G will give the thermodynamic equilibrium configuration and thus the equation of state of this dense matter.

Figure 1 shows the number of protons Z as a function of the matter density $m_N\rho_c$. The dotted curve is an early result of Baym, Bethe and Pethick using a liquid drop model [4], the thin curve is the result of Mackie and Baym using a compressible liquid drop model [5], the dashed curve is a result of Ravenhall, Bennett and Pethick using a Hartree-Fock model [8], and the dot-dashed curve is from Buchler and Barkat using a Thomas-Fermi model [9]. The crosses are from Negele and Vautherin [10] using a Hartree-Fock model for the complete unit cell. (The structure in their result can probably be attributed to shell effects.) The heavy curves are our result.

It is worthwhile to note that there are two very interesting points in our result. Firstly, there is a discontinuity in the charge number at the matter density ~ 5×10^{12}g/cm^3, while all of the other results (except Negele and Vautherin's, which is only discrete points) are continuous. Investigation shows that this discontinuity corresponds to a sudden drop in

the energy per particle as one valley on the energy surface empties into another from above. The lower thick dashed curve indicates the equilibrium Z value corresponding to the other valley before the jump takes place and the upper thick dashed curve corresponds to the locus of Z values along the ridge separating the two valleys. The jump happens at a point where the ridge and the first valley come together. At the present stage of our investigation we are not sure whether this discontinuity implies a real phase transition of dense matter, or is an artifact of our macroscopic model. A study similar to the work of Buchler and Barkat [9]) but using the Seyler-Blanchard force could probably resolve this question.

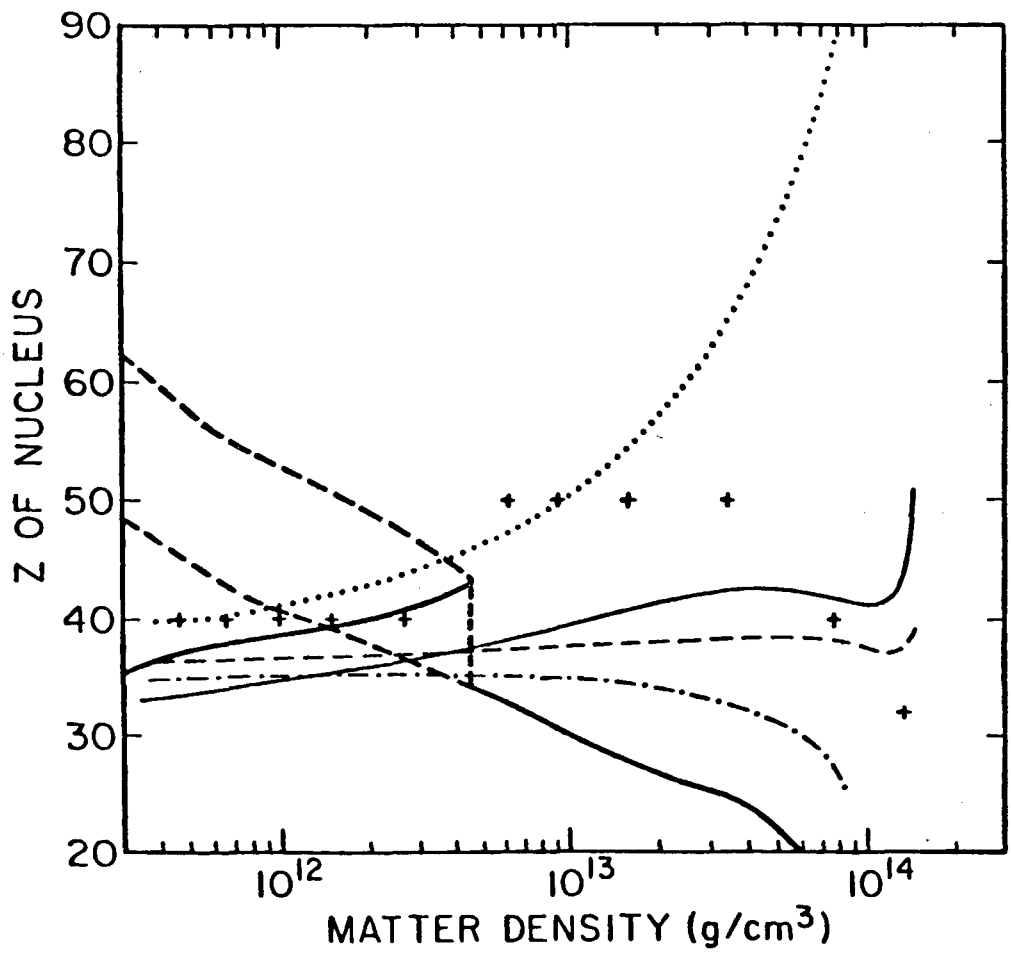

Fig. 1. The charge number Z of a nucleus centered in a unit cell, as a function of the matter density. The dotted curve comes from ref. [4]), the thin curve comes from ref. [5]), the dashed curve comes from ref. [8]), the dot-dashed curve comes from ref. [9]), the crosses come from ref. [10]), and the thick curves are the present result.

Secondly, as the matter density approaches the value of normal nuclear matter $2.5 \times 10^{14} g/cm^3$, the charge number of the stable lattice nucleus Z decreases rapidly in our work, and thus the radius r_p, as well as the radius of the unit cell r_c, decreases rapidly. Although Buchler and Barkat's results show the same trend as ours, this trend is different from most of the other calculations. Does the size of the periodic unit increase or decrease with the density of matter? This question is more serious than the first question, and a more fundamental study is required to answer it. From our work it is clear that the nature of the equilibrium configuration is very sensitive to how the surface energy is treated.

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LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
INFORMATION RESOURCES DEPARTMENT
1 CYCLOTRON ROAD
BERKELEY, CALIFORNIA 94720