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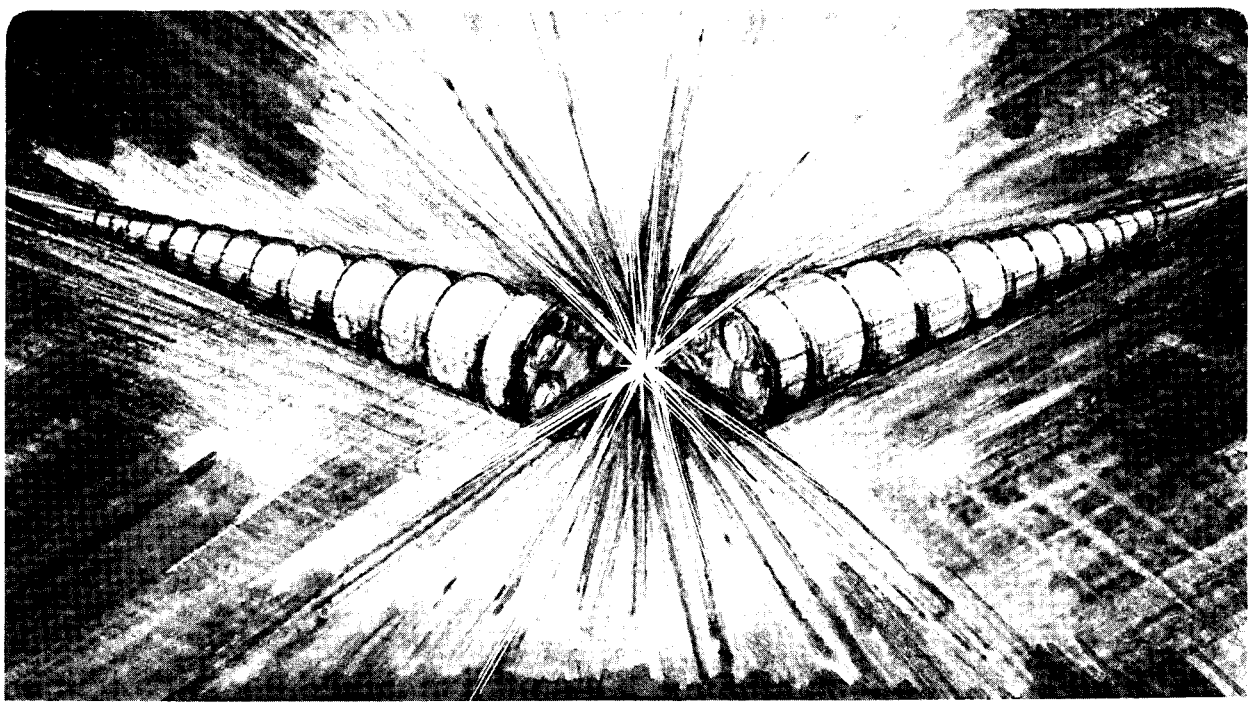

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EMITTANCE CHANGE DUE TO A WIRE GRID*

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Emittance Change due to a Wire Grid

William M. Fawley and Edward P. Lee

Recently there has been interest [**1**] in degrading the emittance of the MBE-4 beam by passing it through a grid of conducting wires just in front of the ground plane that serves as the effective anode. The strong transverse electric fields, due to image charge induced on the wires by the anode-cathode voltage difference, lead to a change in $v_\perp$ that is highly position dependent. A calculation of this process was made by M. Tiefenbach in his thesis[**2**], but his (correct) results are are obscured by his assumption that the longitudinal electric field goes to zero far above the grid as opposed to the more probable boundary condition that the grid is held at a fixed potential. In this note, we reconsider this calculation and determine $\Delta(\varepsilon^2)$ as a function of grid position and spacing by several different methods.

I. "Handwaving" Analysis

Let us assume that the wire grid lies a distance d downstream of the anode which itself is spaced a distance D from the effective cathode plane with $d \leq D$ (see Fig. 1). Let us further assume that the grid wire spacing w is quite small compared with either d or $d - D$. This assumption allows us the treat the electric field near either the anode or cathode plane as being independent of transverse position and in the longitudinal direction $\hat{y}$ only. We adopt V_o as the accelerating potential between the anode and cathode. The wire grid, anode, and cathode are all assumed to extend infinitely in the transverse plane.

We first do a "hand-waving" analysis of the emittance growth based on suggestions by A. Faltens [**3**]. For convenience, the anode plane is taken to have $\phi = 0$. In the absence of the wire grid, the potential at $y = d$ is simply $\phi = V_o \times d/D$. Thus, *if* the grid were held at this potential, there will be no net induced image charge on the grid (apart from that due to the beam's own space charge) and, ignoring collisions between the beam ions and the wire atoms, essentially no emittance growth as the beam goes past. We then exploit the principle of superposition in potential theory and let $\phi(y) = V_o \times y/D \; + \; \Delta V(y)$. For the standard case of the grid being held at the cathode potential (*i.e.* $\phi(d = y) \equiv \phi_g = V_o$), we have on the wires themselves

$$\Delta V(d) \equiv V_* = V_o(1 - d/D) \tag{1}$$

and $\Delta V = 0$ at $y = 0,\ D$. The actual potential is thus the sum of the following:

1. An A-K gap field with a total voltage drop of V_o and the intervening grid at $y = d$ held at potential $V_o \times d/D$ which produces no emittance growth.

2. A field due to the grid held at potential $V_* = \phi_g - V_o d/D$ between two ground planes. For $V_* \neq 0$ there will be emittance growth. It is this second problem that we will now "solve".

At positions y where $|y - d| \gg w$, the electric field produced by the wires is almost purely in the y direction with

$$\begin{aligned} E_y &= -V_*/d \qquad & y < d \\ &= V_*/(D-d) \qquad & y > d \end{aligned} \tag{2}$$

At all positions not on the grid or electrodes, $\nabla \cdot E = 0$ holds and we have

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z} = -\frac{\partial E_y}{\partial y} \tag{3}$$

If we now examine the case of a grid with wires extending in only one direction ($\hat{z}$), one sees from (3) that

$$\frac{\partial}{\partial x}\int_0^D dy\, E_x = -\int_0^D dy\, \frac{\partial E_y}{\partial y} = E_y(0) - E_y(D) = -\frac{V_* D}{d(D-d)} \tag{4}$$

The above result is identical to M. Tiefenbach's and the remainder of our derivation in this section is parallel to his. Adopting a coordinate system in x such that the wires are at positions $x = 0, \pm w, \pm 2w, \ldots$, we have

$$\int dy\, E_x = \left(\frac{w}{2} - \Delta x\right) \frac{V_* D}{d(D-d)} \quad \text{where} \quad \Delta x \equiv x - w \cdot \text{integer} \tag{5}$$

Since $\Delta v_x / v_y = \int dy\, eE_x / m v_y^2$ (presuming $v_y \approx$ constant and $v_x \ll v_y$ wherever E_x is large), averaging in x gives

$$< \Delta x'^2 >= \left(\frac{eV_* D}{2T\, d(D-d)}\right)^2 \frac{2}{w}\int_0^{w/2} dx \left(x - \frac{w}{2}\right)^2 = \left(\frac{eV_* D}{Td(D-d)}\right)^2 \cdot \frac{w^2}{48} \tag{6}$$

where T is the kinetic energy of an ion (presumed constant during its interaction with the grid). For the case with the grid at the anode potential,

$$V_* = V_o(1 - d/D) \quad \text{and}$$

$$< \Delta x'^2 >= \left(\frac{eV_o}{T}\right)^2 \left(\frac{w}{d}\right)^2 \times \frac{1}{48} \tag{7}$$

Expression (4) is equivalent to saying the total impulse in x felt by a given ion is proportional to the number of longitudinal field lines "cut" by the ion over its trajectory

from the anode to cathode. Similar arguments can be given for the case of a bi-directional wire grid (*i.e.* like a window screen rather than an egg slicer) to show

$$< \Delta\theta^2 > \equiv < \Delta x'^2 > + < \Delta y'^2 > \approx \left(\frac{eV_o}{T}\right)^2 \left(\frac{w}{d}\right)^2 \times \frac{1}{96} \tag{8}$$

The reduction in average scattering angle in each transverse plane by a factor of two is due to the reduction in the average line image charge density held on each wire (or equivalently, only half as many field lines are terminated on each wire as before).

II. A more detailed solution based upon Fourier series expansion

It is possible to obtain a more exact solution to the local electric fields by exploiting their periodic nature (in x). Let us again assume that the wires are spaced uniformly in x with a separation w and that the wires extend an infinite distance in z. The potential responsible for the emittance degradation may then be expressed in general as

$$\begin{aligned} \Delta V(y) &= V_w \cdot \frac{y}{d} + \sum_{m=1}^{\infty} a_m \cos k_m x \frac{\sinh k_m y}{\sinh k_m d} && y \le d \\ &= V_w \cdot \frac{D-y}{D-d} + \sum_{m=1}^{\infty} b_m \cos k_m x \frac{\sinh k_m (D-y)}{\sinh k_m (D-d)} && y \ge d \end{aligned} \tag{9}$$

where V_w, the a_m and b_m must be determined. Here $k_m \equiv 2\pi m/w$ and we have chosen a cosine series due to the eveness of $\Delta V(y)$ around $x = nw$. The potential $\Delta V(y)$ must satisfy three conditions:

1. It be continuous at $y = d$ for all x.

2. Its $y-$derivative must be continuous at $y = d$ *except* on the actual wires for which $\nabla^2(\Delta V) \neq 0$.

3. Its value on the actual wires must be equal to V_*. The first condition forces $a_m = b_m$. The second condition is a little bit trickier to implement in order to determine the a_m's uniquely.

Evaluating the $y-$derivative at $y = d$, we find

$$\begin{aligned} \Delta V'(y=d) &= \frac{V_w}{d} + \sum a_m k_m \cos k_m x \coth k_m d \\ &= -\frac{V_w}{D-d} - \sum a_m k_m \cos k_m x \coth k_m (D-d) \end{aligned} \tag{10}$$

Let us presume that the wire spacing w is much less than both d and $D-d$ in order to approximate the argument of the hyperbolic cotangent as 1. If $\partial\phi/\partial y$ were strictly continuous, we could combine the two expressions from (10) and obtain

$$-\frac{V_w D}{d(D-d)} = \sum_1^{\infty} 2 a_m k_m \cos k_m x \quad . \tag{11}$$

However, note that we are attempting to represent a constant function by a discrete Fourier series starting at $m = 1$. This is clearly impossible and leads to a more careful examination of the jump condition on E_y. If we let $\lambda/\delta t$ be the surface charge density on ribbon wires of width $\delta t (\ll w)$, then

$$\left(-\frac{\partial}{\partial y}\Delta V\right)\Bigg|_{d-}^{d+} = \left\{\begin{array}{c} +\dfrac{\lambda}{\varepsilon_o \delta t} \\ 0 \end{array}\right\} \quad \text{for } |x - w \cdot \text{integer}| \left\{\begin{array}{l} < \delta t/2 \\ > \delta t/2 \end{array}\right. \tag{12}$$

Inserting expression (10) gives

$$\frac{V_w}{D-d} + \sum a_m k_m \cos k_m x + \frac{V_w}{d} + \sum a_m k_m \cos k_m x$$

$$= \left\{\begin{array}{c} +\dfrac{\lambda}{\varepsilon_o \delta t} \\ 0 \end{array}\right\} \quad \text{for } |x - w \cdot \text{integer}| \left\{\begin{array}{l} < \delta t/2 \\ > \delta t/2 \end{array}\right. \tag{13}$$

Averaging over x, we see

$$V_w \frac{D}{d(D-d)} = \frac{\lambda}{w\varepsilon_o} \tag{14}$$

and find

$$\sum 2a_m k_m \cos k_m x = \left\{\begin{array}{c} \dfrac{V_w D}{d(D-d)\delta t}\left(\dfrac{w}{\delta t} - 1\right) \\ -\dfrac{V_w D}{d(D-d)} \end{array}\right\} \quad \text{for } |x - w \cdot \text{integer}| \left\{\begin{array}{l} < \delta t/2 \\ > \delta t/2 \end{array}\right. \tag{15}$$

Integrating (15) with $\cos k_m x$ results in

$$a_m = V_w \frac{D}{(D-d)} \frac{1}{k_m d} \frac{2}{k_m \delta t} \sin \frac{k_m \delta t}{2} \tag{16}$$

In the limit $\delta t \to 0$,

$$a_m \approx V_w \frac{D}{(D-d)} \frac{1}{k_m d}$$

and

$$V_* = V_w \times \left(1 + \sum \frac{1}{k_m d} \frac{D}{(D-d)}\right) \tag{17}$$

The summation term diverges logarithmically which is a consequence of placing a finite potential on an infinitely thin wire. If, instead, we take $0 < \delta t \ll w$, the summation can approximated by taking $(2/k_m \delta t)(\sin(k_m \delta t/2)) \approx 1$ for $m \leq w/2\delta t$ and zero elsewhere. One then has

$$V_* \approx V_w \times \left(1 + \frac{1}{2\pi} \frac{w}{d} \frac{D}{(D-d)} \left(0.577 + \ln \frac{w}{2\delta t}\right)\right) \tag{18}$$

Reasonable choices of δt, w/d, and d/D make the logarithmic term small and we will take $V_* \approx V_w$.

We then evaluate the y–integrated transverse field and find for $w \ll d$, $(D-d)$ that

$$\int E_x \, dy \approx \frac{V_* D}{d(D-d)} \sum \frac{2}{k_m} \sin k_m x \tag{19}$$

The expression within the sum is identical to the Fourier sine series for $(\frac{w}{2} - x)$ on the interval $[0, w]$ and thus expressions (5) and (19) are equivalent.

III. Solution by Complex Potential Method

In this approach we slightly modify the geometry of the last two sections. Once again that the wires are at a distance d above a ground plane but, rather than a cathode plane, we presume there is a specified asymptotic potential $\phi(\infty) = V_o$ as $y \to \infty$. There is also a potential (undetermined) on the wires that depends on their line charge density λ, their spacing w and radius a, and their separation d from the anode. An unscaled plot of field lines in shown in Fig. 2. To solve for the potential for $y \geq 0$ note that the real and imaginary parts of an analytical function satisfy $\nabla^2 \phi = 0$. We need a function of $z = x + iy$ periodic in x, with logarithmic singularities at the wires (and their images below the anode plane). Try

$$f(z) = \ln\left[\sin\left(\frac{\pi(z-id)}{w}\right)\right] - \ln\left[\sin\left(\frac{\pi(z+id)}{w}\right)\right] \tag{20}$$

and

$$\phi = -\frac{2\lambda}{4\pi\epsilon_o} \quad \mathrm{Re}\, f(z) \tag{21}$$

The branch cuts may be taken to run off to $\pm i\infty$, but they only appear in the imaginary part of $f(z)$ and are thus no problem. Note that

$$\begin{aligned} \left|\sin\left(\frac{\pi(z-id)}{w}\right)\right|^2 &= \sinh^2\left(\frac{\pi(d-y)}{w}\right)\cos^2\left(\frac{\pi x}{w}\right) + \cosh^2\left(\frac{\pi(d-y)}{w}\right)\sin^2\left(\frac{\pi x}{w}\right) \\ &= \frac{1}{2}\left[\cosh\frac{2\pi(d-y)}{w} - \cos\frac{2\pi x}{w}\right] \end{aligned} \tag{22}$$

We then have

$$\phi = -\frac{\lambda}{4\pi\epsilon_o}\left(\ln\left[\cosh\frac{2\pi(d-y)}{w} - \cos\frac{2\pi x}{w}\right] - \ln\left[\cosh\frac{2\pi(d+y)}{w} - \cos\frac{2\pi x}{w}\right]\right)$$

and

$$\phi(\infty) = +\frac{\lambda}{4\pi\epsilon_o}\,\frac{4\pi d}{w} \tag{23}$$

which relates the line charge λ to the asymptotic potential. To determine the potential on the wire let $\Delta x \equiv x - w \cdot$ integer, $\Delta y \equiv y - d$ and $r^2 \equiv (\Delta x)^2 + (\Delta y)^2$. Then, assuming r/w, $w/d \ll 1$, we find

$$\begin{aligned}\phi &\approx -\frac{\lambda}{4\pi\epsilon_o}\left(\ln\left[1+\frac{1}{2}\left(\frac{2\pi\Delta y}{w}\right)^2 - 1 + \frac{1}{2}\left(\frac{2\pi\Delta x}{w}\right)^2\right] - \ln\left[e^{\frac{4\pi d}{w}}\right]\right)\\ &\approx -\frac{\lambda}{4\pi\epsilon_o}\left(\ln\left[\frac{2\pi^2 r^2}{w^2}\right] - \frac{4\pi d}{w}\right)\end{aligned} \tag{24}$$

Consequently,

$$\left(\frac{\partial\phi}{\partial r}\right) \approx -\frac{2\lambda}{4\pi\epsilon_o r} \tag{25}$$

as expected.

Inasmuch as the wires are assumed to be close together relative to their separation d from the anode, the transverse deflections only occur close to the wires (within $r \leq w/2$). We take v_y constant and integrate aong a vertical line of given coordinate x to find

$$\begin{aligned}\frac{\Delta v_x}{v_y} &\approx -\frac{qe}{mv_y^2}\int_0^\infty dy\,\frac{\partial\phi}{\partial x}\\ &= \left(\frac{-qe}{mv_y^2}\right)\left(\frac{-\lambda}{4\pi\epsilon_o}\right)\frac{2\pi}{w}\sin\left(\frac{2\pi x}{w}\right)\int_0^\infty dy\left(\frac{1}{\cosh\frac{2\pi(d-y)}{w} - \cos\frac{2\pi x}{w}}\right.\\ &\qquad\left. - \frac{1}{\cosh\frac{2\pi(d+y)}{w} - \cos\frac{2\pi x}{w}}\right)\end{aligned} \tag{26}$$

Replacing v_y^2 by its approximate value

$$v_y^2 \approx -\frac{2qe}{m}\frac{\lambda}{4\pi\varepsilon_o}\frac{4\pi d}{w} \tag{27}$$

we find the last expression is equivalent to

$$\frac{\Delta v_x}{v_y} = -\frac{w}{4\pi d}\left(\tan^{-1}\left[\frac{e^{2\pi d/w} - \cos 2\pi x/w}{\sin 2\pi x/w}\right] - \tan^{-1}\left[\frac{e^{-2\pi d/w} - \cos 2\pi x/w}{\sin 2\pi x/w}\right]\right) \tag{28}$$

In the limit $w/d \ll 1$, we obtain in the interval $0 < x < w$,

$$\frac{\Delta v_x}{v_y} = -\frac{w}{4d}\left(1 - \frac{2x}{w}\right) \tag{29}$$

The RMS average over a wire spacing is then

$$\begin{aligned}\langle(\Delta x')^2\rangle &= \left\langle\left(\frac{\Delta v_x}{v_y}\right)^2\right\rangle = \frac{w^2}{16d^2}\int_0^{w/2}\frac{2\,dx}{w}\left(1 - \frac{2x}{w}\right)^2\\ &= \frac{w^2}{48\,d^2}\end{aligned} \tag{30}$$

as was obtained from the other methods (eq. (7)).

IV. Application to MBE-4

Proposed experiments with MBE-4 [1] require an emittance increase by a factor of about 2 from its present "natural" value. Equivalently,

$$\Delta(\varepsilon^2) \;=\; 3\varepsilon_o^2 \tag{31}$$

If we presume an initial K-V distribution, then

$$\left\langle (x_o')^2 \right\rangle \;=\; \frac{\varepsilon_o^2}{16 < x^2 >} \;=\; \frac{\varepsilon_o^2}{4a^2} \tag{32}$$

where a is the beam edge radius ($\approx$ 6mm) and ε_o is the unnormalized, RMS emittance ($\approx$ 12mm-mrad for V_o=200 kV.) Thus, for a bi-directional wire mesh, expression (8) applies and

$$\left\langle (\Delta x')^2 \right\rangle \;=\; 3\left\langle (x_o')^2 \right\rangle \;=\; \frac{3\times(12\times10^{-6})^2}{4\times(6\times10^{-3})^2} \;=3\times10^{-6}\ \mathrm{rad}^2 \;=\; \frac{1}{2}\left(\frac{w}{d}\right)^2\frac{1}{96} \tag{33}$$

giving

$$\frac{w}{d} \;=\; 24\times10^{-3} \tag{34}$$

for a wire mesh. For a cathode-anode spacing of 4 inches, the mesh wire spacing should be about 96 mils (=2.44mm).

It may be important to note that this form of scattering, in the limit $w \ll d$ and $w \leq O(3a)$, produces a nearly uniform spread of scattered angles over the entire beam with the consequence that the beam's phase space no longer is remotely described by a K-V distribution. Whether this effect will present difficulties in comparing post-mesh behavior of the MBE-4 beam with that of the unscattered beam remains to be seen.

References

[1] T. Fessenden, private communication.

[2] M. G. Tiefenbach, Ph.D. Thesis, Univ. of Calif. at Berkeley, issued as *LBL-22465*, 1986.

[3] A. Faltens, private communication.

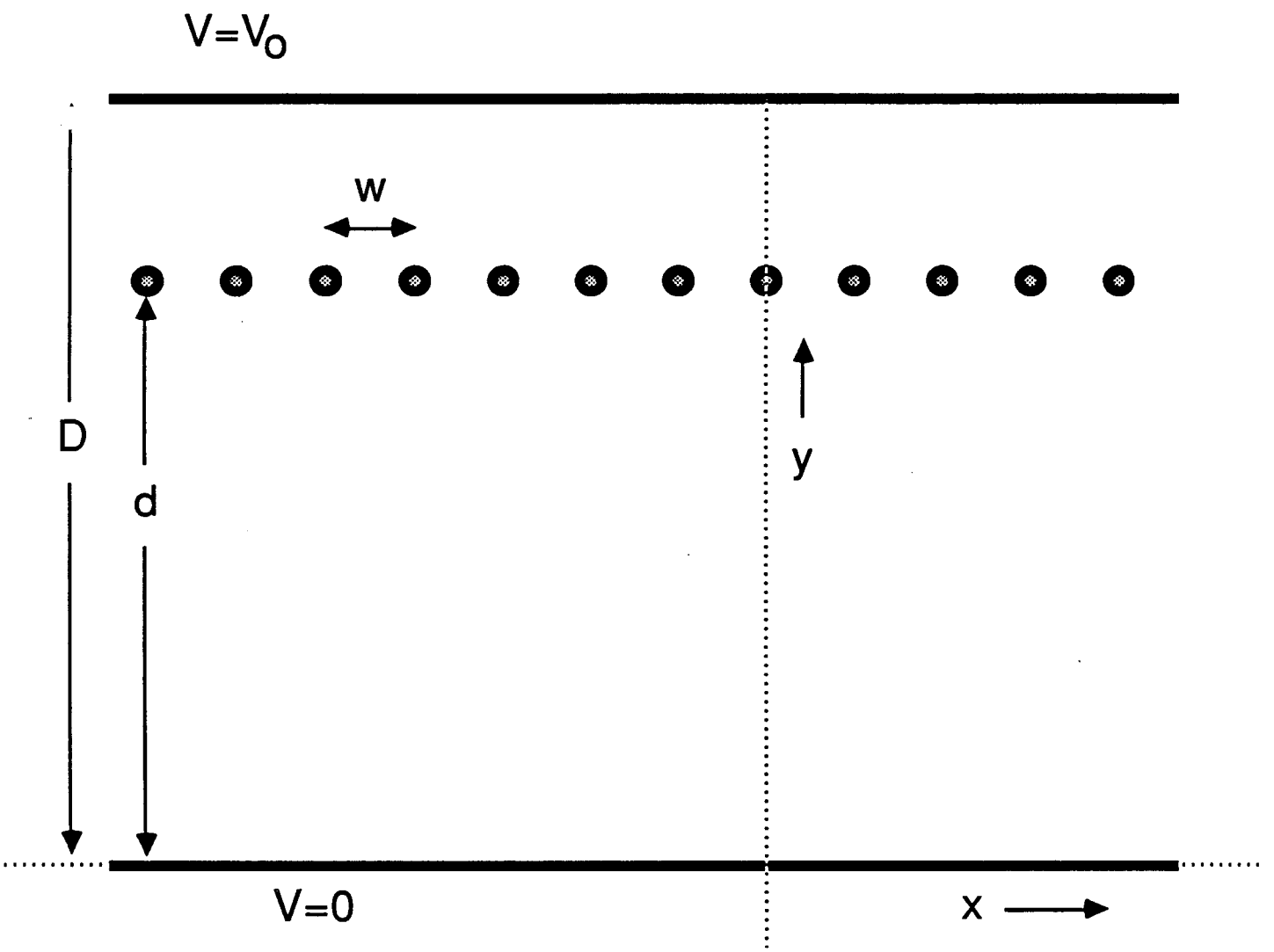

Figure 1. The geometry of §I and §II showing the anode plane ($y = 0$), the cathode plane ($y = D$), and the wires located at $y = d$ spaced with a separation of w in x. The anode, cathode, and wires extend infinitely in x and z.

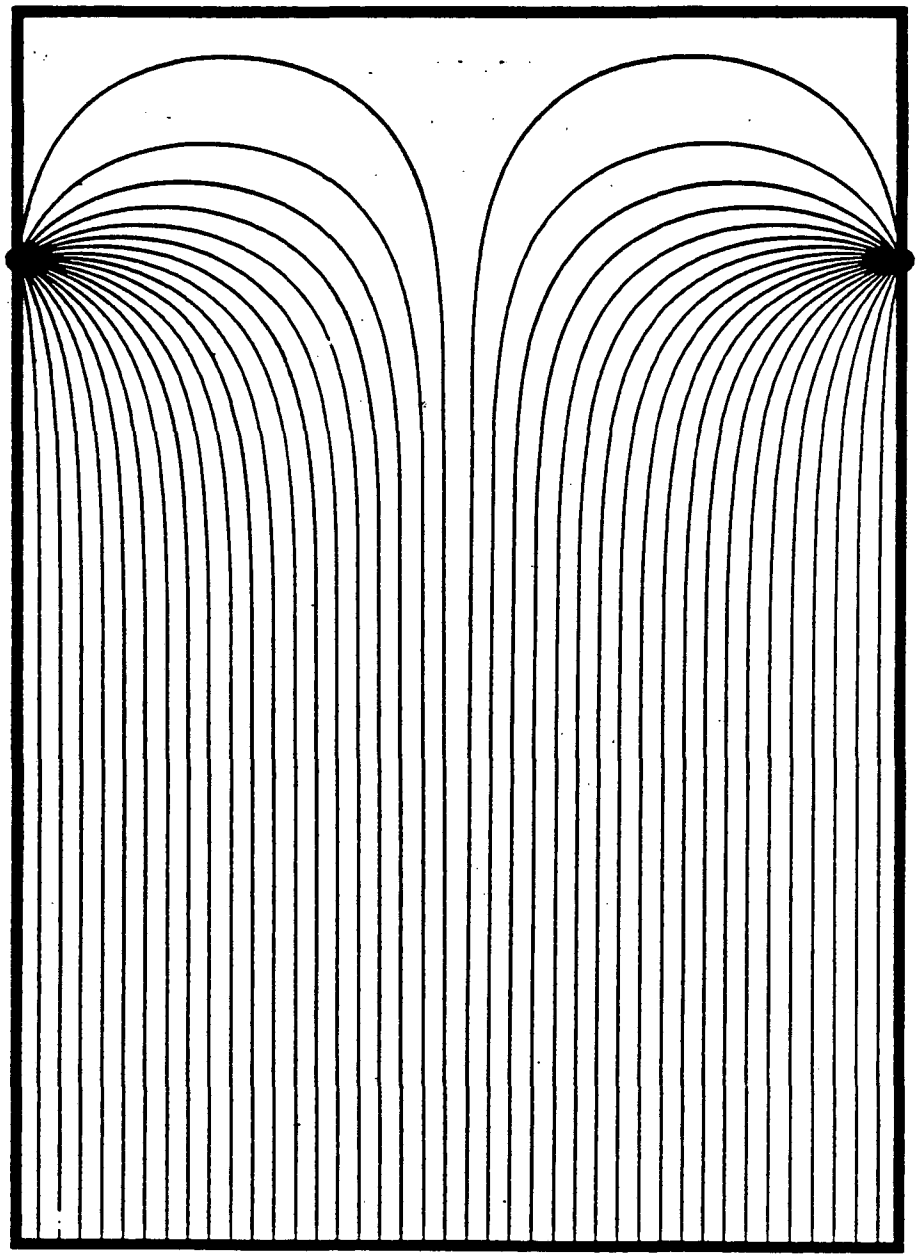

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Figure 2. An artist's representation of the electric field lines near the wire grid reproduced from [**2**]. In this case, it was assumed that $E_y \to 0$ for large y (as opposed to the condition that $V = V_o$ at the cathode plane adopted in our analysis).

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