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Fermion Loops in the Effective Potential of N = 1 Supergravity, with Application to No-Scale Models

J.W. Burton
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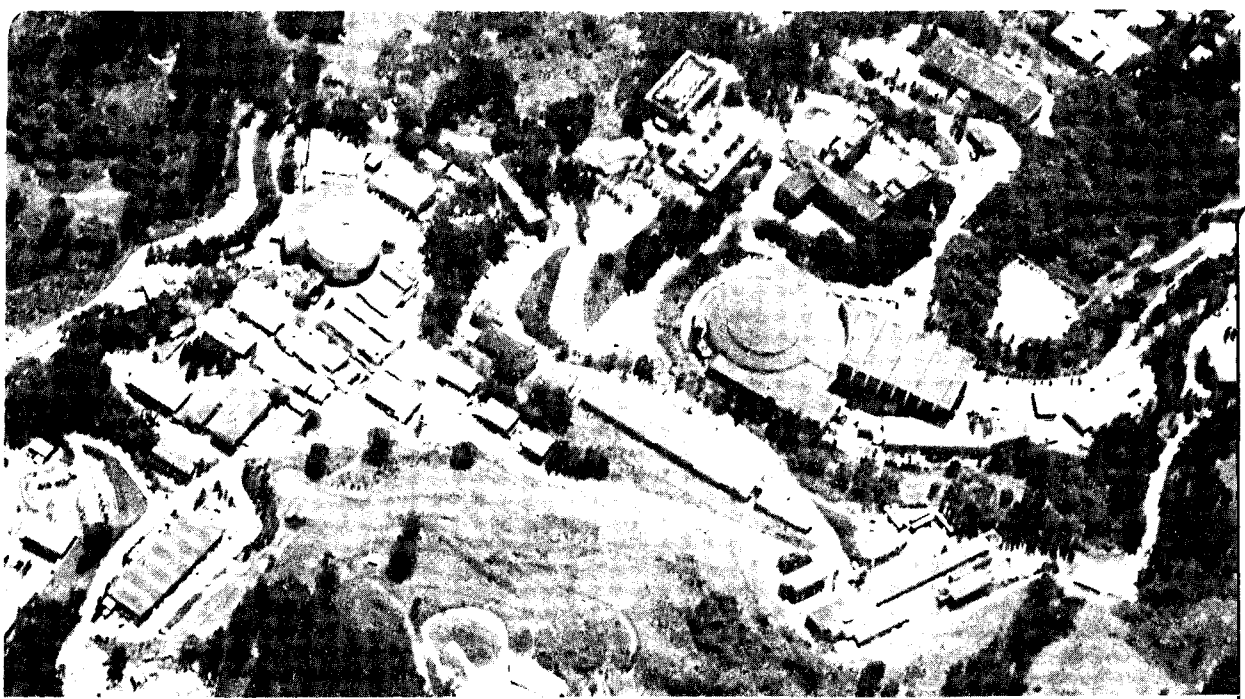

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Fermion Loops in the Effective Potential of $N = 1$ Supergravity, with Application to No-Scale Models*

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Ph. D. Dissertation

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Fermion Loops in the Effective Potential of $N=1$ Supergravity, with Application to No-Scale Models

By

Joshua Wingate Burton

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Abstract

Powerful and quite general arguments suggest that $N = 1$ supergravity, and in particular the superstring-inspired no-scale models, may describe the physics of the four-dimensional vacuum at energy densities below the Planck scale. These models are not renormalizable, since they arise as effective theories after the large masses have been integrated out of the fundamental theory; thus, they have divergences in their loop amplitudes that must be regulated by imposing a cutoff.

Before physics at experimental energies can be extracted from these models, the true vacuum state or states must be identified: at tree level, the ground states of the effective theories are highly degenerate. Radiative corrections at the one-loop level have been shown to break the degeneracy sufficiently to identify the states of vanishing vacuum energy.

As the concluding step in a program to calculate these corrections within a self-consistent cutoff prescription, all fermionic one-loop divergent corrections to the scalar effective potential are evaluated. (The corresponding bosonic contributions have been found elsewhere.) The total effective scalar Lagrange density for $N = 1$ supergravity is written down, and comments are made about cancellations between the fermionic and bosonic loops. Finally, the result is specialized to a toy no-scale model with a single generation of matter fields, and prospects for eventual phenomenological constraints on theories of this type are briefly discussed.

Approved:

. . . . . .

Chair **Date**

For my parents of blessed memory,

Mabel Guerra y Morandeira de Burton

and

Cyrus Matthew Burton

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CHAPTER 1

Background and Motivation

One of the major themes of twentieth-century physics has been the discovery of surprising logical and causal connections between the behavior of systems at radically different scales of length, mass and energy [1]. No philosopher of an earlier age would have guessed, for example, that the fate of a dying star could depend critically on the precise mass of an iron atom, or that the rotation curves of galaxies could hold clues to the results of future collider experiments. Yet one of the largest disparities of scale in all of science, that between the energy at which we understand elementary particles and the energy at which nature (or at any rate the geometry of spacetime) appears to understand them, remains to this day not merely puzzling but inexplicable within the context of the standard model.

In particular, the gauge bosons that carry the electroweak forces we observe, and the hypothetical scalar fields that give them masses by the Higgs mechanism, have masses that are approximately seventeen orders of magnitude lower than the natural scale (the so-called Planck mass) that arises from attempts to quantize Einstein's theory of gravity. Worse still, the quantum field theories we have, though spectacularly successful in their quantitative predictions, are understood only perturbatively. Even if a mechanism were found to make the $W^{\pm}$ and Z^0 boson masses small at zeroth order in perturbation theory, quantum corrections would mix their masses with any available large mass scales in the theory; since the leading corrections depend quadratically, and not logarithmically, on the masses of the heavy modes, the fields we see at accelerator energies could be kept light only by very unnatural fine-tuning.

It is easy, in contrast, to keep the masses of fermionic, or matter, fields small: because these fields have a chiral symmetry that is not preserved by mass terms,

they cannot acquire masses until that symmetry is broken. The strongest motivation for studying supersymmetric models, in which a very peculiar symmetry connects each boson with a fermion (through rotation by an "angle" described by an anti-commuting number), is that the bosons in such a theory can stay strictly massless, to all orders in perturbation theory, until supersymmetry, or SUSY, is broken.

A second reason to take supersymmetry seriously, in spite of the present paucity of experimental evidence for it, can be found in empty space itself. The largest cosmological constant, or vacuum energy density, that is consistent with the observed Hubble parameter is more than 120 orders of magnitude smaller than the Planck density; the universe is far flatter, and far older, than quantum mechanics and general relativity would lead us to expect. It is a general property of supersymmetric models, however, that their ground state has vanishing vacuum energy, so long as the supersymmetry is unbroken. Since we do not in fact observe mass degeneracies between the known fields and their superpartners (another necessary consequence of an unbroken SUSY generator), we must break SUSY either explicitly or spontaneously, presumably in such a way that the Higgs field can acquire a mass near the electroweak scale, while the induced cosmological constant is highly suppressed. While few would maintain that the blackness of the nighttime sky is experimental evidence for supersymmetry, it is at least encouraging that we now have a mechanism capable of explaining, in principle, the smallness of the present vacuum energy density.

This is not the place to provide a comprehensive review of supersymmetric theory and phenomenology; the reader is instead referred to the already extensive review literature [2]. The point of principal interest at present is that models in which there is only one generator of supersymmetric rotations (hence $N = 1$), and in which that generator is local, or gauged (hence super*gravity*) have the best

prospect of successfully describing the low-energy physics we observe. Theories in which more than one SUSY generator survives at low energies have the very undesirable property that all low-mass fermions must be in real representations of the underlying gauge group, unlike the known quarks and leptons, which occur in complex, or chiral representations. A global or ungauged supersymmetry would be sufficient to protect the Higgs mass from large corrections; however, workable models with such a symmetry seem to be difficult to construct [3]. Also, since supergravity, or SUGRA, is a natural feature of all well-behaved string theories (purely bosonic theories of quantized strings have tachyons and other diseases), and indeed is likely to be needed in any finite theory of gravity at the Planck energy, parsimony dictates that we attempt to make use of it, rather than invoking some new mechanism by which a global supersymmetry could arise.

A class of $N = 1$ supergravity models, the no-scale models [4–7], have features that make them good candidates for explaining physics below the Planck scale. At tree level in these models, the cosmological constant vanishes without fine-tuning, and there is a degeneracy in some of the parameters of the theory. In particular, the gravitino mass is undetermined at tree level. This is important since it is a nonzero gravitino mass that is the signal for supersymmetry breaking, which must occur in order to explain why we do not observe light scalars with the same mass as the light fermions.

Also important is the requirement that the supersymmetry breaking should affect the fields and parameters of the observable world only weakly; otherwise, without fine-tuning, radiative corrections would make them too large. Typically, the supersymmetry is spontaneously broken by the vacuum expectation values of "hidden-sector" fields; that is, fields that interact only with gravitational strength with observed particles. The gravitino mass, the scale of supersymmetry breaking

in the observed sector, and the scale of weak interaction physics are all determined dynamically.

The model that we shall consider in most detail is a toy model "inspired" by superstrings [8,9]. It has one generation of matter fields, and is derived by a simple Calabi-Yau compactification from the zero-slope limit of the heterotic superstring; *i.e.*, from ten-dimensional $N = 1$ supergravity with an $E_8 \otimes E_8'$ gauge group. Although this is not a realistic model, it is at least believed to provide a prototype that is worth studying. General features of this model are expected to appear in more realistic four- or ten-dimensional string-derived (rather than string-inspired) models, with several generations. In addition, in a supersymmetric world, no-scale models are good effective models valid below the Planck scale, in a sense that is largely independent of the underlying physics. Even if some other, as yet undiscovered, theory replaces string theory as a model of physics valid above the Planck scale, we may still expect its low-energy limit, from the point of view of field theory, to be some type of no-scale model.

This situation can be compared with the large Higgs-mass limit of the renormalizable standard model of electroweak interactions [10,11]. In this limit the physical Higgs is removed from the theory; one is left with an effective nonrenormalizable model valid below the Higgs mass. However, it is precisely the details of the symmetry-breaking sector that are not completely known. By removing the physical Higgs we have gained a model that, although nonrenormalizable, is in a sense more general than the standard model. Even if the standard model is superseded by a better model, we expect the low-energy limit to be very similar to the the large Higgs-mass limit of the standard model. In the same way, even though we do not yet have a good understanding of strings, or even know if strings are good models for Planck-scale physics, we might expect that some type of nonrenormalizable no-scale

supergravity model will prove to be the low-energy limit of the real, perhaps finite, underlying physics.

There is, to be sure, one important difference between the large Higgs-mass limit of the standard model and an effective low-energy supergravity model: whereas the weak scale is presently accessible to direct experimental probes, the Planck scale is not. As a consequence, although we have a definite model for low-energy electroweak physics, we do not have a definite model for low-energy physics below the Planck scale. What we have instead is a belief that such a model will be of the no-scale type. How can we choose from all the possible no-scale models that we could write down? As we have already mentioned, the tree-level cosmological constant vanishes. An obvious first step is to check if radiative corrections nonetheless generate too large a cosmological constant. In addition, we should study how radiative corrections affect—and perhaps even determine—low-energy parameters that we can measure. By studying radiative corrections in these models we can hope to gain more concrete knowledge about what constitutes a good no-scale model.

Of course, since supergravity is nonrenormalizable, radiative corrections will generate divergent terms that do not appear in the tree-level Lagrangian. It is clear how to interpret these divergences if we appeal once again to the analogy of electroweak physics. The large Higgs-mass limit of the standard model is also a nonrenormalizable theory that yields divergent loop corrections not in the tree-level Lagrangian. If the corresponding corrections are computed in the (renormalizable) standard model they must be finite. In this case it is the Higgs boson that enters to make the otherwise divergent corrections finite. We expect that by cutting off the momentum integrals in our large Higgs-mass model at the Higgs mass we should reproduce, up to threshold uncertainties, the $O(M^2_{\rm Higgs})$ and $O(\ln M_{\rm Higgs})$ results of standard-model calculations. In fact, this is just what is found [11]. In the same

way, the divergent momentum integrals of our supergravity calculations should be cut off at the scale at which the underlying physics comes into play. We expect this procedure should reproduce, again up to threshold effects, the leading corrections computed in the underlying theory.

Unfortunately, calculating the radiative corrections for supergravity models is a technically complex undertaking. The large number of particles and interactions conspire to make determining even the leading one-loop corrections a nontrivial task. (In Chapter 2, a simpler fermion loop calculation, due to Gaillard [12], will serve to outline the basic program, and the main calculation of Chapters 3–5 will follow this outline fairly closely.) In addition, whereas in the electroweak-physics example there is only one particle, the Higgs, that we take to be heavy, here strings dictate an infinite tower of massive modes. Thus one has to be very careful when considering how these heavy modes affect the low-energy results.

The toy model that we consider in this work is the most extensively studied and the best understood string-inspired no-scale model. Some of the complications associated with the radiative corrections and the relevance of the ultraheavy modes to the effective low-energy theory are understood. As has already been stressed, this model is expected to have much in common with more realistic models; all the tools and techniques used to study it should be just as applicable to such models.

A description of what is known about this toy model can be found in the literature [8,9,3,13]. The toy model has a hidden sector that is pure Yang-Mills and is asymptotically free. All the gauge couplings are unified at, or very near, the compactification scale, and the hidden-sector gauge coupling runs so that it quickly becomes strong. When this happens a fermion condensate forms, just as in technicolor models. In particular, a gaugino condensate breaks supersymmetry. Supersymmetry is also broken by the vacuum expectation value of the compact

part of the field strength of the antisymmetric two-form that appears in the ten-dimensional supergravity multiplet. In general, both these supersymmetry-breaking terms are present below the condensation scale, but only the latter is present between the condensation scale and the compactification scale.

In more realistic models we might also expect other sources of supersymmetry breaking. For example, supersymmetry may be spontaneously broken by coordinate-dependent compactifications [14]. In the string context, in all such models that have been studied to date, one also finds effective four-dimensional models with a structure of the no-scale type [15]. There may also be nonperturbative string effects that are surprising from the point of view of point field theory, but as yet no one knows what these effects may be. In any case, our toy model will have only the two nonperturbative (point field theory) supersymmetry-breaking effects mentioned above.

The tree-level toy model is described by its Kähler potential, given by [8,9]:

$$\begin{aligned} \mathcal{G} &= -\ln(s+\bar{s}) - 3\ln(t+\bar{t}-k|\phi|^2) + \ln|W|^2 \\ &\equiv G + \ln|W|^2, \end{aligned} \tag{1.1}$$

where $\mathrm{Re}\, s$ and $\mathrm{Re}\, t$ are gauge singlets related to the ten-dimensional dilaton ϕ^0 and the breathing mode σ of the compact manifold by

$$\mathrm{Re}\, s = e^{3\sigma}(\phi^0)^{-3/4} \quad \text{and} \quad \mathrm{Re}\, t = e^{\sigma}(\phi^0)^{3/4} + \frac{k}{2}|\phi|^2, \tag{1.2}$$

and the fields $\phi^1, \ldots, \phi^N$ are N light gauge nonsinglet scalars. We will set the parameter k to one for the remainder of this work. The superpotential W at the compactification (GUT) scale is at least trilinear in the gauge nonsinglets. Below the condensation scale, the superpotential is modified by an s-field dependent term that arises from integrating out the heavy gauge and gaugino fields of the hidden sector [9]. The s-field plays a special role, since in the toy model the matrix-valued

function f_{ab} of the scalar fields (which enters, for example, into the gauge kinetic energy term) is given by

$$f_{ab} = s\delta_{ab}. \tag{1.3}$$

In particular, the superpotential is given by

$$W = c + he^{-3s/2b_0} + W(\phi^i), \tag{1.4}$$

where c parametrizes the vev of the antisymmetric three-form:

$$\langle H_{LMN} \rangle \propto \langle c m_{\mathrm{Pl}}^3 \epsilon_{LMN} \rangle; \tag{1.5}$$

and h parametrizes the vev of the gaugino bilinear term:

$$\langle \bar{\lambda}\lambda \rangle \propto \langle h \Lambda_{\mathrm{GUT}}^3 e^{-3s/2b_0} \rangle. \tag{1.6}$$

Here m_{Pl} is the Planck mass, $\Lambda_{\mathrm{GUT}} = \langle 1/\sqrt{\mathrm{Re}\, s\, \mathrm{Re}\, t} \rangle$ is the compactification scale, and b_0 determines the β-function of the hidden sector. Then the gauge coupling constants at the compactification scale are all equal to

$$g_{\mathrm{GUT}} = \langle 1/\sqrt{\mathrm{Re}\, s} \rangle. \tag{1.7}$$

The final result in Chapter 6 will therefore be presented in a form in which the s-field dependence is explicit.

It is straightforward to check that this model is of the no-scale form; that is, that at tree level it has vanishing vacuum energy and that the gravitino mass is undetermined due to flatness of the potential in certain directions in field space [9,3]. The one-loop effective potential is just given by the supersymmetric Coleman-Weinberg result and has been studied in various papers [16,17,3]. In reference [3] the effective potential has been computed by using the Kähler potential given above and taking $h \neq 0$ for momenta within the integration range $0 < p^2 < \Lambda_C^2$, and

$h = 0$ for $\Lambda_C^2 < p^2 < \Lambda_{GUT}^2$. In this case it has been shown at tree plus one loop that either the potential is unbounded or *else* the cosmological constant vanishes, with either unbroken supersymmetry or broken supersymmetry and undetermined gravitino mass. It has also been shown that the gauge-nonsinglet scalars remain massless at one loop and no other soft symmetry-breaking terms (so-called A-terms) are generated in the observed scalar sector. (At two loops, however, gauge and gravitational interactions can both contribute to the scalar potential in the same diagram, and it is no longer possible for the global symmetries of the Kähler potential to protect the scalar masses.) The first result is due to the invariance of G under global nonlinear Heisenberg transformations among the t and ϕ^i fields, while the latter is closely related to the vanishing of the cosmological constant. One-loop corrections to the gaugino masses have also been studied. It is found that corrections from momentum integrals below the condensation scale will vanish if the ground state vacuum energy vanishes. Of course, corrections from momentum integrals intermediate between the condensation scale and the compactification scale must also be included. In this case, the evaluation is plagued with technical difficulties associated with the artificial step-like behavior of h at the condensation scale.

Reference [3] draws no definite conclusion about the one-loop gaugino masses, and in fact makes the observation that radiative corrections computed with the effective tree model defined by the Kähler potential (1.1) and the superpotential (1.4) do not include all the necessary loops. Loops containing hidden-sector gauge and gaugino fields are missing, since these have been integrated out at tree level. The correct procedure is to compute the radiative corrections from all the fields and then integrate out the heavy hidden-sector fields to define an effective theory below the condensation scale. In this case, there is reason to believe [3] that the gaugino masses may vanish at one loop.

The first steps in such a program were made in reference [13], which presents the leading-N one-loop corrections quadratic in the compactification scale, where N is the number of chiral supermultiplets. Corrections from just the light modes (*i.e.*, excluding the Kaluza-Klein and string modes), cut off at the field dependent compactification scale, are not themselves supersymmetric. In fact, to obtain a supersymmetric answer one has to carefully consider the heavy modes whose mass is near the compactification scale. In general, computing the $O(\Lambda_{GUT}^2)$ corrections from these heavy modes would require a knowledge of their spectrum and couplings.

However, symmetries of the effective low energy theory may in some cases be sufficient to find these additional corrections. Reference [13] considers a general class of no-scale models with partial nonlinear symmetries among the scalar fields, of which the toy model is a particular example. These symmetries, which are remnants of the ten-dimensional theory, along with the constraints of low-energy local supersymmetry, are enough to determine all the leading $O(\Lambda_{GUT}^2)$ corrections for this class of no-scale models. In particular, it was found that the net result of all the corrections was to redefine the Kähler potential (after wavefunction renormalizations were performed). These results were then used to define an effective theory below the condensate scale. It was found that the ground-state degeneracy is not lifted and there are no observable soft supersymmetry-breaking effects.

The aim of the present work is to carry the program further by computing, up to $O(\Lambda_{GUT}^2)$ and $O(\ln \Lambda_{GUT}^2)$, all the one-loop corrections to the effective scalar Lagrangian that arise from light fields. Thus, we will set $h = 0$ in (1.4) in explicit calculations. However, our results will first be presented in a completely general form that can be applied to any supergravity model. We also determine similar one-loop corrections to the gauge field terms, excluding corrections that arise because the gauge fields have a noncanonical kinetic energy. The purely scalar loop

corrections [16,17,12], the scalar and gauge corrections [18], the scalar and graviton corrections [19], and many of the gaugino and chiral fermion corrections [12,3,13] have already been computed. Also, Gaillard and Jain have calculated the additional corrections arising from mixed gauge and graviton loops [20,21]. We will combine these results with all the remaining fermion-loop corrections, particularly those arising from gravitino and mixed gravitino-spinor loops.

Loop corrections to theories with derivative scalar self-couplings can be evaluated with covariant-derivative expansion techniques [22,12,23,24]. These methods involve expanding the fields about a background and then functionally evaluating the path integral over the quantum fields. These methods are powerful since they allow one to compute the radiative corrections in a manner that manifestly respects the symmetries, both linear and nonlinear, of the theory. For example, since a nonlinear σ-model is invariant under reparametrizations of the scalar fields, their expansion about a background involves the use of normal coordinates [22,25,26]. A manifestly scalar-field reparametrization-invariant evaluation of the one-loop corrections to a scalar nonlinear σ-model can be found in reference [12]. Generalization of these techniques for gauged nonlinear σ-models can be found in references [10] and [18], and for nonlinear σ-models in curved spacetime in reference [27]. An application of these techniques to our Lagrangian will yield manifestly scalar-field reparametrization- and gauge-invariant corrections, but not manifestly supersymmetric corrections, which will require further generalization. However, the fact that we are working in a supersymmetric theory will reveal itself in certain cancellations between the bosonic and fermionic loops.

Our starting point is the supergravity Lagrangian of Cremmer *et al.* [28]. In reduced Planck-mass units ($m_{\rm Pl} = \sqrt{1/8\pi G_{\rm N}} \equiv 1$), the relevant purely bosonic Lagrangian for an $N = 1$ SUGRA theory with Yang-Mills covariant couplings is

given by

$$\begin{aligned}\frac{1}{\sqrt{g}}\mathcal{L}_B = \mathcal{G}_{ij} D_\mu z^i g^{\mu\nu} D_\nu z^j - e^{\mathcal{G}}(\mathcal{G}_i(\mathcal{G}^{-1})^{ij}\mathcal{G}_j - 3) \\ - \frac{1}{4}\,\mathrm{Re}\, f_{ab} F^{a\mu\nu} F^b_{\mu\nu} - \frac{1}{4}\,\mathrm{Im}\, f_{ab} F^{a\mu\nu} F^b_{\mu\nu} + \frac{1}{2} R.\end{aligned} \tag{1.8}$$

For the scalars we use the notation $z^i \equiv \bar{z}^{\bar{i}}$ and as usual the Kähler metric $\mathcal{G}_{ij}$ is defined in terms of the Kähler potential $\mathcal{G}(z, z^\dagger)$ by

$$\mathcal{G}_{ij} = \frac{\partial^2 \mathcal{G}(z, z^\dagger)}{\partial z^i \partial z^j}. \tag{1.9}$$

The gauge covariant derivative is just

$$D_\mu z = (\partial_\mu - ieA_\mu) z. \tag{1.10}$$

For the toy model of interest, the Kähler potential is the real function of the scalar fields $z = s$, t, and $\phi^1, \ldots, \phi^N$ defined by equations (1.1) and (1.4), and $f_{ab}(z)$ is given by equation (1.3). In addition, our gauge group is assumed to be unified, with a single coupling constant defined at the GUT scale by equation (1.7).

Since we are interested here in only the scalar and gauge corrections, we need only retain backgrounds for these fields. (The graviton field can be expanded about the Minkowski metric.) Then, since one-loop corrections come from terms quadratic in the quantum fields, we can drop all terms of quartic or higher order in the fermion fields. Neglecting spacetime curvature, and using the sign conventions of Itzykson and Zuber [29] for the metric and Dirac matrices, the relevant fermionic Lagrangian, including Yukawa and gauge interactions with the bosonic sector, is [28]

$$\begin{aligned}\mathcal{L}_F = {} & \frac{i}{4}\bar{\lambda}^a \gamma^\mu D_\mu \lambda^b\, \mathrm{Re}\, f_{ab} - \frac{1}{8} D_\mu(\bar{\lambda}^a \gamma^\mu \gamma_5 \lambda^b)\, \mathrm{Im}\, f_{ab} - \frac{i}{4}\bar{\lambda}^a_R \gamma^\mu \lambda^b_R\, \mathrm{Re}\, f_{ab} \mathcal{G}_i D_\mu z^i \\ & + \frac{1}{4}\bar{\lambda}^a_L \lambda^b_R \frac{\partial f^*_{ab}}{\partial z^k} \mathcal{G}_l (\mathcal{G}^{-1})^{lk} + i\bar{\chi}^i_R \gamma^\mu D_\mu \chi^j_R \mathcal{G}_{ij} \\ & - i\bar{\chi}^i_R \gamma^\mu \chi^k_R (\mathcal{G}_{ki,j} - \frac{1}{2}\mathcal{G}_{ki}\mathcal{G}_j) D_\mu z^j - \bar{\chi}^i_R \chi^j_L e^{\mathcal{G}/2}(\mathcal{G}_{ij} + \mathcal{G}_i\mathcal{G}_j - \mathcal{G}_l(\mathcal{G}^{-1})^{lk}\mathcal{G}_{kij}) \\ & - \frac{i}{4}\bar{\chi}^i_R \sigma^{\mu\nu} \lambda^b_L F^a_{\mu\nu} \frac{\partial f_{ab}}{\partial z^i} - \frac{i}{2}\bar{\chi}^k_R \lambda^c_L (1/\mathrm{Re}\, f)^{ab} \frac{\partial f_{bc}}{\partial z^k} \mathcal{G}_i T^i_{aj} z^j - 2i\bar{\lambda}^a_L \chi^l_R \mathcal{G}_{lj} T^j_{ak} z^k\end{aligned}$$

$$
\begin{aligned}
&+ \frac{1}{4}\bar{\psi}_\mu \epsilon^{\mu\nu\rho\sigma}\gamma_\nu\gamma_5 D_\rho\psi_\sigma + \frac{1}{8}\bar{\psi}_\mu \epsilon^{\mu\nu\rho\sigma}\gamma_\nu\psi_\rho \mathcal{G}_i D_\sigma z^i + \frac{i}{4}\bar{\psi}_\mu \sigma^{\mu\nu}(1+\gamma_5)\psi_\nu e^{\mathcal{G}/2} \\
&- \frac{1}{2}\bar{\psi}_\mu \gamma^\mu \lambda^a_n \mathcal{G}_i T^i_{aj} z^j + \frac{1}{8}\bar{\lambda}^a \gamma^\mu \sigma^{\nu\rho}\psi_\mu F^+_{\nu\rho}\,\mathrm{Re}\, f_{ab} \\
&+ i\bar{\psi}_\mu \gamma^\mu \chi^i_L e^{\mathcal{G}/2}\mathcal{G}_i - \bar{\psi}_\mu \gamma^\nu \gamma^\mu \chi^j_L \mathcal{G}_{ji} D_\nu z^i + \textit{Hermitian conjugate}. \qquad (1.11)
\end{aligned}
$$

The large number of degrees of freedom in even this toy model makes evaluation of the one-loop potential a daunting project. In Chapter 2 we will see how the fermionic one-loop contribution to the effective action of a simpler theory may be calculated, thus outlining the procedure without too much obscuring algebra. (This result is due to Gaillard [12], and will be used freely in Chapter 4, where we will consider spin-$\frac{1}{2}$ loops in the context of our full theory.) Chapter 3 will consider the case of gravitino, or spin-$\frac{3}{2}$, loops, and develop machinery for computing the Dirac γ-matrix traces for such loops. Chapters 4 and 5 will generalize this calculation to the full fermionic Lagrangian, (1.11), and then work out several terms explicitly for the case of no-scale models.

Finally, Chapter 6 will collate and summarize the results obtained in the previous three chapters, and combine them with the results of Gaillard and Jain [18–20]. This will yield the full one-loop effective scalar Lagrangian for no-scale models with canonical vector kinetic energy.

In the explicit calculations of Chapters 3–5 we use a double subtraction procedure to regulate divergent integrals, consistent with the procedure used elsewhere [20,21] for the bosonic loops. Our final results in Chapter 6 will be presented in a prescription-independent form, and we will comment on additional terms that may be present [13] when the regularization prescription is made fully consistent with local supersymmetry. Finally, we make some brief remarks about the application of our results to the problem of determining the mass scales in a no-scale model, such as the toy model whose Kähler potential takes the form of equation (1.1).

CHAPTER 2

Fermion Loops: a Primer

The evaluation of effective potentials induced by fermion loops has a long and venerable history in the literature of particle physics. As early as 1936, Weisskopf considered the theory of nonlinear photon couplings through electron loops in quantum electrodynamics [30], and (without the benefit of Feynman diagrams, or even modern four-vector notation) obtained an analytic result for the sum over all one-loop amplitudes, in the limit where the photons are soft compared to the electron mass.

This sort of limit is exactly where effective field theories are most useful: since the perturbative expansion is in loops (or, more intuitively, in powers of $\hbar$) rather than in the coupling constants of the theory, the sum over all configurations of external lines—in our case, of scalar fields—can be evaluated to $O(\hbar)$ in one step. Assume for simplicity that each vertex around the loop contributes one external line to the diagram. Since the nth one-loop diagram has n internal propagators, and a symmetry factor of $1/n$ from the $(n-1)!$ inequivalent configurations of external lines, the sum is simply the logarithm of the internal propagator. Thus:

$$
\Gamma^{(n)}(0,\ldots,0) = -\left(\frac{n!}{n}\right) \times i\int \frac{d^4p}{(2\pi)^4}\left(i\Delta_F(p)\right)^n, \qquad (2.1)
$$

where $\Delta_F(p)$ is the appropriate internal propagator, and the overall minus sign on the loop comes from Fermi statistics. The one-loop effective potential then becomes

$$
\begin{aligned}
V_{1\text{-loop}}(\phi_c) &= -i\int \frac{d^4p}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{n}\left(\phi_c\Delta_F(p)\right)^n \\
&= -i\int \frac{d^4p}{(2\pi)^4}\ln\left(1 - i\phi_c\Delta_F(p)\right).
\end{aligned} \qquad (2.2)
$$

From this schematic result we can observe two salient features of effective Lagrangians. First, the integral in (2.2) is divergent and must be regulated; and

second, the masses of the particles in the loop (represented here by the implied mass in Δ_F) will determine the momentum scale at which they contribute to the effective potential. The latter point should be emphasized: since we are far from having an ultimate point field theory of everything, and indeed have no strong reason to believe that such a thing exists, it is reassuring to know that the physics below a given energy scale can in general be accurately described by an effective theory in which the unobserved heavy modes have been integrated out and appear only as point couplings. To be sure, a momentum cutoff cannot protect us entirely from our own ignorance, since large logarithmic corrections from scales far above the regularization scale can contribute to the low-energy physics. Still, the ability that the effective Lagrangian formulation gives us to view the universe at a hierarchy of mass scales, from Weisskopf's soft photons below m_e right up to the Planck scale, is in essence what makes it a useful tool.

With these observations in mind, we now consider a calculation by Gaillard [12] that contains all the essential features of the general fermion-loop result we hope to obtain. This is the effective Lagrangian for the case of a single Dirac spinor ψ, coupled to a scalar background by the general tree-level Lagrangian

$$\mathcal{L} = \bar{\psi}(iZ(\phi)\partial\!\!\!/ - B(\phi))\psi, \tag{2.3}$$

where $Z(\phi)$ is a derivative coupling (which we could take to be a matrix in fermion space if ψ has internal quantum numbers), and $B(\phi)$ may have nontrivial γ-matrix content. The inverse propagator is given by

$$\Delta^{-1}(x,y) \equiv \frac{\delta^2 S[\psi]}{\delta\psi(y)\delta\psi(x)} = Z(x)(i\partial\!\!\!/_x + C(x))\delta^4(x-y), \tag{2.4}$$

where $C \equiv Z^{-1}B$.

As we saw in (2.2), the one-loop effective Lagrangian (not really an effective potential, since now we are allowing derivative couplings), will turn out to be given

by the logarithm of an inverse propagator; more precisely, by [31],

$$\int d^4x \mathcal{L}_{1-\text{loop}} = -i\,\mathrm{Tr}\ln Z^{-1}\Delta^{-1} = -i\,\mathrm{Tr}\ln(i\partial\!\!\!/ - C), \tag{2.5}$$

with the trace taken over spacetime points in the Taylor series for the logarithm, as well as over Dirac-matrix space. A Fourier transform gives

$$(i\partial\!\!\!/_x + C(x))\delta^4(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\Big(p\!\!\!/ - C\big(-i\frac{\partial}{\partial p}\big)\Big)e^{ip\cdot y} \tag{2.6}$$

so that now we can expand out the logarithm, shift the argument of C, and recontract, finally writing

$$\mathcal{L}_{1-\text{loop}} = -i\int \frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\ln\Big(p\!\!\!/ - C\big(x - i\frac{\partial}{\partial p}\big)\Big) \equiv -i\int \frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\ln(p\!\!\!/ - \bar{C}), \tag{2.7}$$

with the uncapitalized trace now acting only on the Dirac matrices.

If $C(x) = \gamma^\mu A_\mu(x) + M(x)$ (which, as we shall see in Chapters 3 and 4, is not by any means the most general form of physical interest), we can write

$$\begin{aligned}\mathrm{tr}\ln(p\!\!\!/ - \bar{A\!\!\!/} - \bar{M}) &= \mathrm{tr}\ln(-p\!\!\!/ + \bar{A} - \bar{M}) \\ &= \frac{1}{2}\,\mathrm{tr}\ln\big(-(p\!\!\!/ - \bar{A\!\!\!/})^2 + \bar{M}^2 + [p\!\!\!/ - \bar{A\!\!\!/}, \bar{M}]\big).\end{aligned} \tag{2.8}$$

Since we know that

$$[p\!\!\!/ - \bar{A\!\!\!/}, \bar{M}] = i(\overline{\partial\!\!\!/ M} + i[\bar{A\!\!\!/}, \bar{M}]) \equiv i\overline{D\!\!\!/ M}, \tag{2.9}$$

with

$$\overline{\partial\!\!\!/ M} \equiv e^{-i\partial_x\cdot\partial_y}(\partial\!\!\!/ M)e^{i\partial_x\cdot\partial_y} \tag{2.10}$$

defined in the natural way, and with

$$D_\mu(x)f(x) \equiv [\partial_\mu + iA_\mu(x), f(x)] \equiv [d_\mu(x), f(x)], \tag{2.11}$$

we have implicitly solved for the one-loop effective Lagrangian of the theory. It only remains to expand out the Taylor series for each barred (*i.e.*, shifted) function of the spacetime coordinates.

However, the appearance of a covariant derivative, $\mathcal{D}_\mu$, in our answer suggests that (2.8) can be cast in a fully covariant form, thus presumably simplifying the algebra. (When we tackle the full fermion sector of the theory, this simplification will be our only hope of extracting physics from the result.) Since $\mathcal{L}_{1-\text{loop}}(x)$ is to be integrated over all space, we are free to operate on the argument of the momentum integral with a unitary operator,

$$\operatorname{tr}\ln B \to \operatorname{tr}\ln(UBU^{-1}), \tag{2.12}$$

provided that $U(i\partial/\partial p, \partial_x)$ equals one at the origin. To see this, merely consider the series expansion of U and U^{-1}: the latter operates to the right on nothing, and the former can be integrated by parts n times, so as to operate to the left on nothing.

If we take

$$\begin{aligned} U &= \exp\Big[-i(\partial_\mu + iA_\mu(x))\frac{\partial}{\partial p_\mu}\Big]\exp\Big[i\partial_\mu\frac{\partial}{\partial p_\mu}\Big] \\ &= \exp\Big[id_\mu(x)\frac{\partial}{\partial p_\mu}\Big]\exp\Big[i\partial_\mu\frac{\partial}{\partial p_\mu}\Big], \end{aligned} \tag{2.13}$$

we will have the following simplifying identities:

$$\begin{aligned} U(i\not{p})U^{-1} &= i\not{p} - \not{\partial}_x, \qquad \text{and} \\ Uf(x)U^{-1} &= e^{-id(x)\cdot\partial_p} f(x) e^{id(x)\cdot\partial_p}. \end{aligned} \tag{2.14}$$

After applying these, equation (2.8) becomes

$$\operatorname{tr}\ln B = \frac{1}{2}\operatorname{tr}\ln\big(-(\not{p}+\not{j})^2 + \hat{M}^2 + i\widehat{\not{\mathcal{D}}M}\big), \tag{2.15}$$

where

$$\hat{M} = U\bar{M}U^{-1} = \sum_{n=0}^{\infty}\frac{1}{n!}(\partial_{\mu_1}\cdots\partial_{\mu_n}M^2)\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}}, \tag{2.16}$$

and similarly for $\widehat{\not{\mathcal{D}}M}$, and where

$$\hat{j}_\mu \equiv \hat{j}_{\nu\mu}\frac{\partial}{\partial p_\nu}, \tag{2.17}$$

and

$$\begin{aligned} \hat{j}_{\nu\mu} &\equiv \sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}(\partial_{\mu_1}\cdots\partial_{\mu_n}[d_\nu, d_\mu])\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}} \\ &= \sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}(\partial_{\mu_1}\cdots\partial_{\mu_n}j_{\nu\mu})\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}}. \end{aligned} \tag{2.18}$$

Our expression for the effective Lagrangian now depends only on M, $j_{\mu\nu}$, and their covariant derivatives. Furthermore, we can square out the first term of (2.15):

$$(\not{p}+\not{j})^2 = p^2 + j^2 + 2p^\mu j_\mu - i\hat{k}^\mu{}_\mu + i\sigma^{\mu\nu}(j_{\mu\nu} - i\hat{k}_{\mu\nu}), \tag{2.19}$$

where $\hat{k}_{\mu\nu} \equiv i([j_\nu, p_\mu] - j_{\mu\nu})$, and so

$$\hat{k}_{\mu\nu} = \sum_{n=0}^{\infty}\frac{(n+1)(n+2)}{(n+3)!}(\partial_{\mu_1}\cdots\partial_{\mu_n}\partial_\mu j_{\sigma\nu})\frac{(-i)^n\partial^{n+1}}{\partial p_\sigma\partial p_{\mu_1}\cdots\partial p_{\mu_n}}. \tag{2.20}$$

To evaluate the divergent contributions to $\mathcal{L}_{1-\text{loop}}$, the only parts we can hope to get right without a real understanding of the short-distance physics, we need a regularization scheme for the momentum integration that will allow us to extract both quadratic and logarithmic divergences. For consistency with earlier results [12,18,19,21] we choose a double subtraction scheme, introducing a new mass scale, which we denote by μ:

$$\operatorname{tr}\ln B \to \operatorname{tr}\ln B + \operatorname{tr}\ln(B - 2\mu^2) - 2\operatorname{tr}\ln(B - \mu^2). \tag{2.21}$$

Since the task of picking out the quadratic and logarithmic divergences in the regulated integral depends on the detailed form of B, which in the more general problem will differ from the result we have obtained here, there is little pedagogical value in carrying on. Instead, we now proceed directly to the actual fermionic Lagrangian of our $N = 1$ supergravity model, equation (1.11), and try to apply what we have learned from this chapter's single-fermion model to the full fermionic spectrum of the supersymmetric theory.

CHAPTER 3

Pure Gravitino Loops

Because the models we wish to study possess a local supersymmetry generator, they must have a supersymmetric partner for the graviton, and so the fermionic Lagrange density given in (1.11) contains both gravitino self-couplings and mixing terms between the gravitinos and the chiral fields. (Indeed, as mentioned in Chapter 1, it is the fact that the tree-level gravitino mass is a free parameter of the theory that makes the no-scale models so attractive as candidates for describing the non-supersymmetric world we observe at accelerator energies). Since the gravitino transforms as both a vector and a spinor under the gauge group of the theory, however, we appear to have the freedom to eliminate at least some of these couplings.

To evaluate the one-loop contribution from spin-$\frac{3}{2}$ fermions, ψ_μ, the obvious first step is to find some gauge in which these fields decouple completely from the spin-$\frac{1}{2}$ chiral fields, designated by $\chi_L^j = \frac{1}{2}(1-\gamma_5)\chi^j$. (The gaugino fields λ^a decouple at once if we set $\gamma^\mu\psi_\mu = 0$.) As we shall see, however, this explicit diagonalization does not lead to a genuine simplification, because the Fadeev-Popov gauge ghosts introduce new mixing between the fermions. Thus we will first perform the functional integration over the gravitino degrees of freedom only (also for the moment neglecting background Yang-Mills fields, so that we may replace D_μ by ∂_μ), and then, in Chapter 4, consider all fermion-mixing mass terms in a consistent way. (Alternatively, we could work in a "smeared" gauge, defined by setting $\gamma\cdot\psi = f(x)$, where $f(x)$ is some suitably damped function. This approach has not yet been adequately explored.)

The portion of the fermion Lagrangian, equation (1.11), which depends on the

gravitinos is:

$$\begin{aligned}\mathcal{L}_\psi = {} & \frac{1}{4}\bar\psi_\mu\epsilon^{\mu\nu\rho\sigma}\gamma_\nu\gamma_5\partial_\rho\psi_\sigma + \frac{1}{8}\bar\psi_\mu\epsilon^{\mu\nu\rho\sigma}\gamma_\nu\psi_\rho\mathcal{G}_i\partial_\sigma z^i + \frac{i}{4}\bar\psi_\mu\sigma^{\mu\nu}(1+\gamma_5)\psi_\nu e^{\mathcal{G}/2} \\ & - \frac{1}{2}\bar\psi_\mu\gamma^\mu\lambda_R^a\mathcal{G}_i T^i_{aj}z^j - \bar\psi_\mu\gamma^\nu\gamma^\mu\chi_L^j\mathcal{G}_{j\bar l}\partial_\nu z^{\bar l} + i\bar\psi_\mu\gamma^\mu\chi_L^i e^{\mathcal{G}/2}\mathcal{G}_i \\ & + \textit{Hermitian conjugate.}\end{aligned} \tag{3.1}$$

In the gauge where $\gamma^\mu\psi_\mu = 0$, we can simplify this, using

$$\bar\psi_\mu\epsilon^{\mu\nu\rho\sigma}\gamma_\nu\gamma_5 = i(\bar\psi^\rho\gamma^\sigma - \bar\psi^\sigma\gamma^\rho) \tag{3.2}$$

and

$$\bar\psi_\mu\sigma^{\mu\nu} = -i\bar\psi^\nu, \tag{3.3}$$

so that $\mathcal{L}$ reduces to

$$\begin{aligned}\mathcal{L}\big|_{\gamma\cdot\psi=0} &= \bar\psi^\mu\left(-\frac{i}{4}\partial\!\!\!/ + \frac{i}{8}\mathcal{G}_i\partial\!\!\!/ z^i\gamma_5 + \frac{1}{4}e^{\mathcal{G}/2}(1+\gamma_5)\right)\psi_\mu \\ &\quad - 2\bar\psi^\mu\chi_L^j\mathcal{G}_{j\bar l}\partial_\mu z^{\bar l} + H.c. \\ &= \bar\psi^\mu\left(-\frac{i}{2}\partial\!\!\!/ - \frac{i}{8}(\mathcal{G}_{\bar l}\partial\!\!\!/ z^{\bar l} - \mathcal{G}_i\partial\!\!\!/ z^i)\gamma_5 + \frac{1}{2}e^{\mathcal{G}/2}\right)\psi_\mu \\ &\quad - 2\bar\psi^\mu\mathcal{G}_{j\bar l}\partial_\mu z^{\bar l}\chi_L^j - 2\bar\chi_R^j\mathcal{G}_{j\bar k}\partial^\mu z^{\bar k}\psi_\mu \\ &= \frac{1}{2}\bar\psi^\mu(-i(\partial\!\!\!/ + i\Gamma\!\!\!/\gamma_5) + M)\psi_\mu - 2(\bar\psi^\mu\chi_\mu + \bar\chi^\mu\psi_\mu),\end{aligned} \tag{3.4}$$

where:

$$\begin{aligned}\Gamma_\mu \equiv \frac{i}{4}(\mathcal{G}_i\partial_\mu z^i - \mathcal{G}_{\bar l}\partial_\mu z^{\bar l}), \qquad M \equiv e^{\mathcal{G}/2}, \\ \text{and} \quad \chi_\mu \equiv \mathcal{G}_{j\bar l}\partial_\mu z^{\bar l}\chi_L^j.\end{aligned} \tag{3.5}$$

We could now redefine ψ_μ in order to complete the square:

$$\psi^\mu_{\text{new}} = \psi^\mu_{\text{old}} - 4\Delta_M\chi^\mu. \tag{3.6}$$

If we require that Δ_M satisfy

$$(-i(\partial\!\!\!/ + i\Gamma\!\!\!/\gamma_5) + M)\Delta_M = 1, \tag{3.7}$$

the $\bar{\psi}\cdot\chi$ cross-term will just drop out. In terms of the new ψ-field, the gravitino Lagrangian would then be:

$$\mathcal{L}_{\psi_{\text{new}}} = \frac{1}{2}\bar{\psi}^{\mu}\Delta_M^{-1}\psi_\mu + 8\bar{\chi}^{\mu}\Delta_M\chi_\mu. \tag{3.8}$$

However, as mentioned before, new $\psi\chi$-mixing terms will appear in the ghost integration, and in any case we shall see in Chapter 4 that it is more natural to treat the $\bar{\psi}^{\mu}\chi_\mu$ term as a generalized mass, and to carry our result for the ψ_μ-field over to an extended spinor containing all the fermion degrees of freedom. For the time being, therefore, we shall simply ignore the remaining χ^i- and λ^a-terms in the action.

With this simplification, equation (3.8) resembles the fermionic Lagrangian of Chapter 2, equation (2.3). However, the Dirac γ-matrix content of the inverse propagator prevents us from simply writing Δ_M out explicitly, and so we must resort to less elegant methods, which we present below. The calculation will be carried out in considerable detail, even at the cost of some redundancy, so as to make the parallel with the previous chapter easier to follow through the ensuing thicket of spin-matrices.

The variation of ψ_μ under a gauge transformation is given by [28]:

$$\begin{aligned} i\delta_\epsilon\psi_{L\mu} &= (\partial_\mu - i\Gamma_\mu)\epsilon_L + \frac{i}{2}\gamma_\mu M\epsilon_R + O(\bar{\psi}\chi, \bar{\chi}\chi, \bar{\lambda}\lambda), \\ i\delta_\epsilon\psi_{R\mu} &\equiv (i\delta_\epsilon\psi_{L\mu})_{\text{ch. conj.}} \\ &= (\partial_\mu + i\Gamma_\mu)\epsilon_R + \frac{i}{2}\gamma_\mu M\epsilon_L + O(\bar{\chi}\psi, \bar{\chi}\chi, \bar{\lambda}\lambda); \end{aligned} \tag{3.9}$$

for an infinitesimal change of gauge parametrized by ϵ. (This definition of the gauge variation actually differs from that of reference [28] by a factor of i, as is evident from the way we take the charge conjugate. Our choice gives the gauge determinant the same sign as the inverse propagators, with the sign conventions of reference [29].) Since our gauge condition is $\gamma\cdot\psi = 0$, the Fadeev-Popov determinant (which

parametrizes our overcounting of states in the functional integral) is

$$\frac{1}{\epsilon}\det\delta_\epsilon(\gamma\cdot\psi_{\text{odd}}) = \det\left(-i(\bar{\partial} + i\bar{\not{d}}\gamma_5) + 2M\right). \tag{3.10}$$

Thus the (one-loop) effective action from gravitino loops is

$$\mathcal{S}^1_{\text{eff}} = -i\ln\int[d\psi]\,\delta[\gamma\cdot\psi]\det{}^{-1}(-i\not{\partial} + 2M)e^{-i\int d^4x\,\mathcal{L}_\psi}, \tag{3.11}$$

where $\delta[\gamma\cdot\psi] = \int[d\alpha]\,\exp(i\int d^4x\,\bar{\alpha}\gamma\cdot\psi)$ is a functional delta-function, and

$$d_\mu \equiv \partial_\mu + i\Gamma_\mu\gamma_5. \tag{3.12}$$

Writing the functional integral over α out explicitly,

$$\begin{aligned} \mathcal{S}^1_{\text{eff}} &= -i\ln\Big(\det{}^{-1}(-i\not{\partial} + 2M)\int[d\psi][d\alpha]\,e^{-i\int d^4x\,(\frac{1}{2}\bar{\psi}\cdot\Delta^{-1}\psi - \bar{\alpha}\gamma\cdot\psi)}\Big) \\ &= -i\ln\Big(\det{}^{-1}(-i\not{\partial} + 2M)\int[d\psi]\,e^{-i\int d^4x\,\frac{1}{2}\bar{\psi}\cdot\Delta^{-1}\psi}\int[d\alpha]\,e^{-i\int d^4x\,\frac{1}{2}\bar{\alpha}\gamma\cdot\Delta\gamma\alpha}\Big), \end{aligned} \tag{3.13}$$

after a shift of $\psi_\mu \to \psi_\mu + \Delta_M\gamma_\mu\alpha$. Integrating over ψ_μ and α, we get:

$$\begin{aligned} \mathcal{S}^1_{\text{eff}} &= -i\ln\left(\det{}^{-1}(-i\not{\partial} + 2M)\det{}^{4/2}(-i\not{\partial} + M)\det{}^{1/2}\gamma^\mu\Delta_M\gamma_\mu\right) \\ &= -i\,\mathrm{Tr}\left(-\ln(-i\not{\partial} + 2M) + 2\ln(-i\not{\partial} + M) + \frac{1}{2}\ln\gamma^\mu\Delta_M\gamma_\mu\right) \\ &= -i\,\mathrm{Tr}\left(-\ln\Delta^{-1}_{2M} + 2\ln\Delta^{-1}_M + \frac{1}{2}\ln\nabla^{-1}_M\right), \end{aligned} \tag{3.14}$$

where

$$\nabla_M \equiv (\gamma^\mu\Delta_M\gamma_\mu)^{-1} = \left(\gamma^\mu(-i\not{\partial} + M)^{-1}\gamma_\mu\right)^{-1}, \tag{3.15}$$

and where the (capitalized) trace is understood to be taken over both the spacetime coordinates and the Dirac-matrix indices.

After making a Fourier transform, using the sign convention of [12], we see that

$$\begin{aligned} \mathcal{S}^1_{\text{eff}} = -i\int d^4x\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\big(-\ln(-\not{p} + \bar{\not{d}}\gamma_5 + 2\bar{M}) + 2\ln(-\not{p} + \bar{\not{d}}\gamma_5 + \bar{M}) \\ + \frac{1}{2}\ln\gamma^\mu(-\not{p} + \bar{\not{d}}\gamma_5 + \bar{M})^{-1}\gamma_\mu\big), \end{aligned} \tag{3.16}$$

where the trace is now only over Dirac indices, and

$$\begin{aligned}\tilde{f}(x) &\equiv f(x - i\frac{\partial}{\partial p}) \\ &= e^{-i(\partial/\partial p_\mu)(\partial/\partial x^\mu)} f(x) e^{i(\partial/\partial p_\mu)(\partial/\partial x^\mu)}.\end{aligned} \tag{3.17}$$

Since $S^1_{eff} \equiv \int d^4x\, \mathcal{L}^1_\psi(x)$, we have found the one-loop Lagrangian, and are done in principle.

Before evaluating the momentum integral, however, we can make it covariant by getting rid of the shifted arguments. First picking a representation for the Dirac algebra that diagonalizes the γ_5-matrix [29]:

$$\begin{aligned}\gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^i = -\gamma_i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \\ \text{and} \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};\end{aligned} \tag{3.18}$$

we can write

$$\gamma^0 \not{A} = A_0 - \gamma^0\gamma\cdot\mathbf{A} \equiv \begin{pmatrix} \tilde{\not{A}} & 0 \\ 0 & \not{A} \end{pmatrix}, \tag{3.19}$$

where $\not{A} = A_0 + \sigma\cdot\mathbf{A}$ and $\tilde{\not{A}} = A_0 - \sigma\cdot\mathbf{A}$ are 2-dimensional matrices.

Now, for any 4-dimensional matrix of the form

$$\mathcal{M}_4 = \begin{pmatrix} \tilde{\not{A}} & m \\ m^* & \not{B} \end{pmatrix} \tag{3.20}$$

it is true that $\mathrm{tr}\, f(\mathcal{M}_4) = \mathrm{tr}\, f(\tilde{\mathcal{M}}_4)$, where

$$\tilde{\mathcal{M}}_4 \equiv \mathcal{M}_4\big|_{\substack{\sigma\to-\sigma \\ m\to-m}} = \begin{pmatrix} \not{A} & -m \\ -m^* & \tilde{\not{B}} \end{pmatrix}. \tag{3.21}$$

So $\mathrm{tr}\ln\mathcal{M}_4 = \frac{1}{2}(\mathrm{tr}\ln\mathcal{M}_4 + \mathrm{tr}\ln\tilde{\mathcal{M}}_4) = \frac{1}{2}\mathrm{tr}\ln\mathcal{M}_4\tilde{\mathcal{M}}_4$, because odd powers of σ^i are traceless, and odd powers of m do not occur on the diagonal.

But if we consider the 8-dimensional matrices

$$\begin{aligned}\mathcal{M}_8 &= \begin{pmatrix} \not{A} & m \\ m^* & \not{B} \end{pmatrix} \quad \text{and} \\ \tilde{\mathcal{M}}_8 &= \mathcal{M}_8\big|_{m\to-m} = \begin{pmatrix} \not{A} & -m \\ -m^* & \not{B} \end{pmatrix},\end{aligned} \tag{3.22}$$

we see that $\mathrm{tr}\, f(\mathcal{M}_8) = \mathrm{tr}\, f(\tilde{\mathcal{M}}_8)$ as well, again because odd terms are traceless. Now, $\mathrm{tr}\, f(\mathcal{M}_4\tilde{\mathcal{M}}_4)$ gets σ-matrices only in pairs like $\not{F}\tilde{\not{G}}$ and $\tilde{\not{F}}\not{G}$, and

$$\not{F}\tilde{\not{G}} = F^\mu G_\mu - \sigma^i(F_iG_0 - F_0G_i) - i\epsilon^{ij}{}_k\sigma^k F_iG_j; \tag{3.23}$$

whereas $\mathrm{tr}\, f(\mathcal{M}_8\tilde{\mathcal{M}}_8)$ gets γ-matrices only in

$$\begin{aligned}\not{F}\not{G} &= F^\mu G_\mu + \frac{1}{2}F_\mu G_\nu[\gamma^\mu, \gamma^\nu] \\ &= F^\mu G_\mu - \begin{pmatrix} \sigma^i & 0 \\ 0 & -\sigma^i \end{pmatrix}(F_iG_0 - F_0G_i) - i\epsilon^{ij}{}_k \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix} F_iG_j.\end{aligned} \tag{3.24}$$

And since $\mathcal{M}_4\tilde{\mathcal{M}}_4$ has the same form in $\not{A}$ and $\not{B}$ that $\mathcal{M}_8\tilde{\mathcal{M}}_8$ has in $\not{A}$ and $\not{B}$, it follows that

$$\begin{aligned}\frac{1}{2}\mathrm{tr}\ln\mathcal{M}_4\tilde{\mathcal{M}}_4 &= \frac{1}{4}\mathrm{tr}\ln\mathcal{M}_8\tilde{\mathcal{M}}_8 \\ &= \frac{1}{4}\mathrm{tr}\ln\begin{pmatrix} A^2 - m^2 & -\not{A}m + m\not{B} \\ m\not{A} - \not{B}m & -m^2 + B^2 \end{pmatrix}.\end{aligned} \tag{3.25}$$

By putting a γ^0 in front of each of our inverse propagators (thus introducing an irrelevant $\det^n\gamma^0 = 1$ into the final result), we can make direct use of this trick, writing:

$$\mathcal{M}_4 = \gamma^0(-\not{p} + \not{\bar{q}}\gamma_5 + \bar{M}) = \begin{pmatrix} -\not{k} + \not{\bar{K}} & -\bar{M} \\ -\bar{M} & -\not{k} - \not{\bar{K}} \end{pmatrix} \tag{3.26}$$

and

$$\mathcal{M}_8\tilde{\mathcal{M}}_8 = \begin{pmatrix} (\not{p} - \not{\bar{q}})^2 - \bar{M}^2 & -[\not{p}, \bar{M}] + 2\not{\bar{q}}\bar{M} \\ -[\not{p}, \bar{M}] - 2\not{\bar{q}}\bar{M} & (\not{p} + \not{\bar{q}})^2 - \bar{M}^2 \end{pmatrix}. \tag{3.27}$$

Now that the two chiralities in the inverse propagator have been explicitly separated, we covariantize the action by making the shift $\mathcal{M}_s(p, x - i\partial/\partial p) \to \mathcal{U}\mathcal{M}_s(p, x - i\partial/\partial p)\mathcal{U}^{-1}$, where

$$\mathcal{U} = e^{-i(\partial_\mu + i\Gamma_\mu(x)\gamma_5)(\partial/\partial p_\mu)} e^{i\partial_\mu(\partial/\partial p_\mu)} = e^{-id(x)\cdot\partial_p} e^{i\partial_x\cdot\partial_p}. \tag{3.28}$$

In the chiral representation, $d_\mu = \mathrm{diag}(\partial_\mu + i\Gamma_\mu, \partial_\mu - i\Gamma_\mu) \equiv \mathrm{diag}(d_\mu^+, d_\mu^-)$, and the shifted, squared inverse propagator becomes

$$(\mathcal{U}\mathcal{M}_s\mathcal{U}^{-1})(\mathcal{U}\tilde{\mathcal{M}}_s\mathcal{U}^{-1}) = \begin{pmatrix} (\not{p} + \not{j})^2 - \hat{M}^2 & e^{-id^+\cdot\partial_p}(-i\not{\partial}M + 2\not{\Gamma}M)e^{id^-\cdot\partial_p} \\ e^{-id^-\cdot\partial_p}(-i\not{\partial}M - 2\not{\Gamma}M)e^{id^+\cdot\partial_p} & (\not{p} - \not{j})^2 - \hat{M}^2 \end{pmatrix}, \tag{3.29}$$

with:

$$\hat{M}^2 \equiv \sum_{n=0}^{\infty} \frac{1}{n!}(\partial_{\mu_1}\cdots\partial_{\mu_n}M^2)\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}}, \tag{3.30}$$

$$\hat{j}_\mu \equiv \hat{J}_{\nu\mu}\frac{\partial}{\partial p_\nu}, \quad \text{and} \tag{3.31}$$

$$\begin{aligned} \hat{J}_{\nu\mu} &\equiv \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!}(\partial_{\mu_1}\cdots\partial_{\mu_n}[d_\nu^+, d_\mu^+])\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}} \\ &= \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!}(\partial_{\mu_1}\cdots\partial_{\mu_n}J_{\nu\mu})\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}}. \end{aligned} \tag{3.32}$$

We may introduce the covariant derivative operators,

$$D_\mu^\pm X \equiv [d_\mu^\pm, X], \tag{3.33}$$

which act as ordinary derivatives on functions $X(x)$ of the spacetime coordinates (like M^2 or $J_{\nu\mu} = i\partial_\nu\Gamma_\mu - i\partial_\mu\Gamma_\nu$) that connect states of the same helicity. We then generalize this to a chiral covariant derivative, $D_\mu = \mathrm{diag}(D_\mu^+, D_\mu^-)$, which is defined on the helicity-flipping mass term as well:

$$D_\mu^\pm M \equiv \partial_\mu M \pm i\{\Gamma_\mu, M\} = \partial_\mu M \pm 2i\Gamma_\mu M. \tag{3.34}$$

Thus both the diagonal and off-diagonal terms in (3.29) are actually covariant.

The role of D_μ, the covariant derivative defined above, becomes more transparent if we write the gravitino mass term as

$$\bar{\psi}^\mu M\psi_\mu = \bar{\psi}_R^\mu m_{RL}\psi_{L\mu} + H.\,c., \tag{3.35}$$

and define

$$D_\mu m_{RL} = \partial_\mu m_{RL} + i\{\Gamma_\mu, m_{RL}\} \tag{3.36}$$

and

$$D_\mu m_{LR} = D_\mu m_{RL}^\dagger = (D_\mu m_{RL})^\dagger = \partial_\mu m_{RL}^\dagger - i\{\Gamma_\mu, m_{RL}^\dagger\}, \tag{3.37}$$

and also

$$\begin{aligned} D_\mu m_{LR}m_{RL} &= \partial_\mu m_{LR}m_{RL} - i[\Gamma_\mu, m_{LR}m_{RL}]; \\ D_\mu m_{RL}m_{LR} &= \partial_\mu m_{RL}m_{LR} + i[\Gamma_\mu, m_{RL}m_{LR}]. \end{aligned} \tag{3.38}$$

In other words, $\bar{\psi}_R$ transforms in the same way as $\psi_L \equiv \psi_R^c$, and opposite to $\psi_R = \psi_L^c \sim \bar{\psi}_L$. (Note that the reality property $m_{LR} = m_{RL}^\dagger$ is unique to the gravitino. For spin-$\frac{1}{2}$ fermions, m_{RL} is a general complex matrix, and Γ_μ is also in general a matrix. We will make use of this added generality when we incorporate the other fermions into our calculation in Chapter 4.) With the definitions (3.33) and (3.34) of the covariant derivative, the expression (3.29) is equivalent to equation (4.21) of reference [12]. (There is a factor i missing in front of the $(\not{\partial}M)^2$ term in equation (4.21) of reference [12].)

A more straightforward, but perhaps less rigorous, way to obtain the result (3.29) is to write [27] the gravitino Lagrangian in terms of the eight-component spinor $\Psi \equiv (\psi_R, \psi_L)^T$, perform the functional integration (treating Ψ and $\bar{\Psi}$ as independent variables), and divide the final result by two to compensate for the doubling of real degrees of freedom.

Returning to (3.29), since only even powers of these off-diagonal terms will appear in the trace, we can work directly with their product; that is,

$$
\begin{aligned}
&(e^{-i\partial^{\pm}\cdot\partial_p}(\partial\!\!\!/ M \pm 2i\not{p}M)e^{i\partial^{\mp}\cdot\partial_p})(e^{-i\partial^{\mp}\cdot\partial_p}(\partial\!\!\!/ M \mp 2i\not{p}M)e^{i\partial^{\pm}\cdot\partial_p}) \\
&\quad = e^{-i\partial\cdot\partial_p}(\partial\!\!\!/ M \pm 2i\not{p}M)(\partial\!\!\!/ M \mp 2i\not{p}M)e^{i\partial\cdot\partial_p} \\
&\quad = \sum_{n=0}^{\infty}\frac{1}{n!}(\partial_{\mu_1}\cdots\partial_{\mu_n}(\partial^\mu M\partial_\mu M + 4\Gamma^\mu\Gamma_\mu M^2))\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}} \\
&\quad \equiv \not{p}M\widehat{\not{p}M}^\dagger \equiv |\widehat{\not{p}M}|^2.
\end{aligned}
\tag{3.39}
$$

It will also be convenient to square out the diagonal terms explicitly, writing

$$
\begin{aligned}
(\not{p} \pm \not{j})^2 &= p^2 + \hat{j}^\mu\hat{j}_\mu \pm \{\not{p},\not{j}\} \\
&= p^2 + \hat{j}^\mu\hat{j}_\mu \pm 2p^\mu\hat{j}_\mu \pm [\hat{j}_\mu, p^\mu] \pm i\sigma^{\mu\nu}[\hat{j}_\nu, p_\mu] \\
&= p^2 + \hat{j}^\mu\hat{j}_\mu \pm 2p^\mu\hat{j}_\mu \mp i\hat{K}^\mu{}_\mu \pm i\sigma^{\mu\nu}(\hat{j}_{\mu\nu} - i\hat{K}_{\mu\nu}),
\end{aligned}
\tag{3.40}
$$

where $\hat{K}_{\mu\nu} \equiv i([\hat{j}_\nu, p_\mu] - \hat{j}_{\mu\nu})$, and so

$$
\hat{K}_{\mu\nu} = \sum_{n=0}^{\infty}\frac{(n+1)(n+2)}{(n+3)!}(\partial_{\mu_1}\cdots\partial_{\mu_n}\partial_\mu J_{\sigma\nu})\frac{(-i)^n\partial^{n+1}}{\partial p_\sigma\partial p_{\mu_1}\cdots\partial p_{\mu_n}}. \tag{3.41}
$$

Continuing as in the simple fermion case, we write

$$
\hat{\mathbf{B}} \equiv \mathcal{U}\mathcal{M}_B\tilde{\mathcal{M}}_B\mathcal{U}^{-1}, \tag{3.42}
$$

and introduce a renormalization scale μ. The twice-subtracted contribution from each term $i\,\mathrm{Tr}\ln\Delta_M^{-1}$ in $\mathcal{L}^1_{\mathrm{reg}}$ is then

$$
\begin{aligned}
\mathcal{L}^1_{\mathrm{reg}}(\mu^2) &= \frac{i}{4}\int\frac{d^4p}{(2\pi)^4}\left(\mathrm{tr}\ln\hat{\mathbf{B}} + \mathrm{tr}\ln(\hat{\mathbf{B}} - 2\mu^2) - 2\,\mathrm{tr}\ln(\hat{\mathbf{B}} - \mu^2)\right) \\
&= \frac{i\mu^2}{4}\int_0^1 d\lambda\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\Big(\frac{1}{\hat{\mathbf{B}} - \lambda\mu^2} - \frac{1}{\hat{\mathbf{B}} - (1+\lambda)\mu^2}\Big) \\
&= -\frac{i\mu^4}{4}\int_0^1 d\xi\int_0^1 d\lambda\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\Big(\frac{1}{\hat{\mathbf{B}} - \lambda\mu^2 - \xi\mu^2}\Big)^2.
\end{aligned}
\tag{3.43}
$$

To expand this in powers of p^μ, we write

$$
\begin{aligned}
\hat{\mathbf{B}} = (p^2 + \hat{j}^\mu\hat{j}_\mu - \hat{M}^2)\mathbf{I} - \frac{i}{2}\widehat{\not{p}M}\mathbf{S}_+ + \frac{i}{2}\widehat{\not{p}M}^\dagger\mathbf{S}_- \\
+ (2p^\mu\hat{j}_\mu - i\hat{K}^\mu{}_\mu + i\sigma^{\mu\nu}(\hat{j}_{\mu\nu} - i\hat{K}_{\mu\nu}))\mathbf{S}_s,
\end{aligned}
\tag{3.44}
$$

where $\mathbf{I}$ is the 8-dimensional identity matrix, and $\mathbf{S}_s$ and $\mathbf{S}_\pm$ are direct products of the 4-dimensional identity matrix with the Pauli matrices σ_s and $\sigma_\pm \equiv \sigma_x \pm i\sigma_y$. Then, by defining $\hat{\mathbf{A}}$ such that

$$
\hat{\mathbf{B}} \equiv (p^2 - M^2)\mathbf{I} - \hat{\mathbf{A}}, \tag{3.45}
$$

we can use the expansion

$$
\frac{1}{\hat{\mathbf{B}} - \lambda\mu^2 - \xi\mu^2} = \frac{\mathbf{I}}{p^2 - M^2 - \lambda\mu^2 - \xi\mu^2}\sum_{n=0}^{\infty}\Big(\hat{\mathbf{A}}\frac{1}{p^2 - M^2 - \lambda\mu^2 - \xi\mu^2}\Big)^n, \tag{3.46}
$$

and, after squaring this, we shall be able to pick out the divergences of $\mathcal{L}^1_{\mathrm{reg}}(\mu^2)$ by power-counting.

We are only interested in the divergent terms, which are at most of fourth order in derivatives of z^i and z^t; therefore, by (3.32) and (3.41), we can take

$$
\hat{j}_{\mu\nu} \to \frac{1}{2}J_{\mu\nu} - \frac{i}{3}\partial_\sigma J_{\mu\nu}\frac{\partial}{\partial p_\sigma} - \frac{1}{8}\partial_\rho\partial_\sigma J_{\mu\nu}\frac{\partial}{\partial p_\rho}\frac{\partial}{\partial p_\sigma}, \tag{3.47}
$$

and

$$
\hat{K}_{\mu\nu} \to \frac{1}{3}\partial_\mu J_{\sigma\nu}\frac{\partial}{\partial p_\sigma} - \frac{i}{4}\partial_\rho\partial_\mu J_{\sigma\nu}\frac{\partial}{\partial p_\rho}\frac{\partial}{\partial p_\sigma}, \tag{3.48}
$$

since $J_{\mu\nu} = [d^+_\mu, d^+_\nu]$ is already of second order in derivatives of the scalar fields. So only terms at most quadratic in $\hat{j}$ or in $\hat{K}$ will survive. As for the mass terms, we have only $\hat{N}^2 \equiv \hat{M}^2 - M^2$ left on the diagonal, and, since we are only interested in the divergent part, which is of order $\geq p^{-2}$ in $\hat{\mathbf{B}}$, as defined in (3.42), or of order $\geq p^{-6}$ in the expansion of (3.46), we need only keep

$$
\begin{aligned}
\hat{N}^2 &= \sum_{n=1}^{\infty}\frac{(-i)^n}{n!}(\partial_{\mu_1}\cdots\partial_{\mu_n}M^2)\frac{\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}} \\
&\longrightarrow -i\partial_\mu M^2\frac{\partial}{\partial p_\mu} - \frac{1}{2}\partial_\mu\partial_\nu M^2\frac{\partial}{\partial p_\mu}\frac{\partial}{\partial p_\nu}.
\end{aligned}
\tag{3.49}
$$

And $|\widehat{\not{p}M}|^2$, which appears in the trace only at order p^{-6} (from $\hat{\mathbf{A}}^2$ terms), need not be expanded at all:

$$
|\widehat{\not{p}M}|^2 \to \partial^\mu M\partial_\mu M + 4\Gamma^\mu\Gamma_\mu M^2 \tag{3.50}
$$

Putting all of these approximations together into $\hat{\mathbf{A}}$, we can write expansion (3.46) as:

$$\frac{1}{\hat{\mathbf{B}} - \lambda\mu^2 - \xi\mu^2} = \frac{\mathbf{I}}{p^2 - M^2 - \lambda\mu^2 - \xi\mu^2} + \frac{\mathcal{A}}{(p^2 - M^2 - \lambda\mu^2 - \xi\mu^2)^2} + \frac{\mathcal{B}_\mu p^\mu + \mathcal{C}}{(p^2 - M^2 - \lambda\mu^2 - \xi\mu^2)^3} + \frac{\mathcal{D}_{\mu\nu} p^\mu p^\nu}{(p^2 - M^2 - \lambda\mu^2 - \xi\mu^2)^4} + O\Big(\frac{1}{p^7}\Big), \tag{3.51}$$

where, if we neglect terms that cannot contribute to the traces:

$$\mathcal{A} \equiv \frac{i}{2}\not{p}\widehat{M}\mathbf{S}_+ - \frac{i}{2}\not{p}\widehat{M}^\dagger\mathbf{S}_- - \frac{1}{2}i\sigma^{\mu\nu}J_{\mu\nu}\mathbf{S}_s, \tag{3.52}$$

$$\mathcal{B}_\mu \equiv 2i\partial_\mu M^2\mathbf{I} + \frac{2}{3}i(\partial^\nu J_{\mu\nu} - i\sigma^{\nu\rho}(\partial_\mu J_{\nu\rho} + \partial_\nu J_{\mu\rho}))\mathbf{S}_s, \tag{3.53}$$

$$\mathcal{C} \equiv (\partial^2 M^2 - \partial^\mu M\partial_\mu M - 4\Gamma^\mu\Gamma_\mu M^2 + \frac{1}{2}J^{\mu\nu}J_{\mu\nu} - \frac{1}{4}\sigma^{\mu\nu}J_{\mu\nu}\sigma^{\rho\sigma}J_{\rho\sigma})\mathbf{I} - \frac{1}{4}i\sigma^{\mu\nu}(\partial^2 J_{\mu\nu} + 2\partial^\rho\partial_\mu J_{\rho\nu})\mathbf{S}_s, \tag{3.54}$$

and

$$\mathcal{D}_{\mu\nu} \equiv (-4\partial_\mu\partial_\nu M^2 - 2J_\mu{}^\rho J_{\nu\rho})\mathbf{I} + (2\partial^\rho\partial_\mu J_{\rho\nu} + i\sigma^{\rho\sigma}(\partial_\mu\partial_\nu J_{\rho\sigma} + 2\partial_\mu\partial_\rho J_{\nu\sigma}))\mathbf{S}_s. \tag{3.55}$$

Squaring this (again keeping only the divergent parts, and dropping terms that are odd in p^μ) and Wick-rotating, we have

$$\mathcal{L}^1_{\rm reg} = -\frac{\mu^4}{4}\int_0^1 d\xi \int_0^1 d\lambda \int \frac{d^4p}{(2\pi)^4} \times \Big(\frac{8}{(p^2 + M^2 + \lambda\mu^2 + \xi\mu^2)^2} - \frac{2\,\mathrm{tr}\,\mathcal{A}}{(p^2 + M^2 + \lambda\mu^2 + \xi\mu^2)^3} + \frac{2\,\mathrm{tr}\,\mathcal{C} + \mathrm{tr}\,\mathcal{A}^2}{(p^2 + M^2 + \lambda\mu^2 + \xi\mu^2)^4} + \frac{2\,\mathrm{tr}\,\mathcal{D}_{\mu\nu}p^\mu p^\nu}{(p^2 + M^2 + \lambda\mu^2 + \xi\mu^2)^5} + O\Big(\frac{1}{p^{10}}\Big)\Big), \tag{3.56}$$

which can be integrated to yield

$$\mathcal{L}^1_{\rm quad} = \frac{\mu^2 \ln 2}{32\pi^2}(8M^2 + \mathrm{tr}\,\mathcal{A}) \tag{3.57}$$

for the quadratically divergent part, and

$$\mathcal{L}^1_{\rm log} = -\frac{\ln\mu^2}{64\pi^2}(4M^4 + M^2\,\mathrm{tr}\,\mathcal{A} + \frac{1}{3}\mathrm{tr}\,\mathcal{C} + \frac{1}{6}\mathrm{tr}\,\mathcal{A}^2 + \frac{1}{24}\mathrm{tr}\,\mathcal{D}^\mu{}_\mu) \tag{3.58}$$

for the logarithmically divergent part. The traces are:

$$\begin{aligned} \mathrm{tr}\,\mathcal{A} &= 0, \\ \mathrm{tr}\,\mathcal{C} &= 8\partial^\mu\partial_\mu M^2 - 8\partial^\mu M\partial_\mu M - 32\Gamma^\mu\Gamma_\mu M^2, \\ \mathrm{tr}\,\mathcal{D}^\mu{}_\mu &= -32\partial^\mu\partial_\mu M^2 - 16J^{\mu\nu}J_{\mu\nu}, \\ \mathrm{tr}\,\mathcal{A}^2 &= -8\partial^\mu M\partial_\mu M - 32\Gamma^\mu\Gamma_\mu M^2 - 4J^{\mu\nu}J_{\mu\nu}. \end{aligned} \tag{3.59}$$

Using (3.57) and (3.58), we can evaluate the first two terms of (3.16) at once. Thus, the gravitino contribution to the divergent part of the 1-loop Lagrangian is minus twice the expression (3.43) or (3.56):

$$\mathcal{L}^1_{3/2} = \frac{\mu^2 M^2 \ln 2}{2\pi^2} + \frac{\ln\mu^2}{8\pi^2}(\partial^\mu M\partial_\mu M + 4\Gamma^\mu\Gamma_\mu M^2 + \frac{1}{3}J^{\mu\nu}J_{\mu\nu} - \frac{1}{3}\partial^\mu\partial_\mu M^2 - M^4). \tag{3.60}$$

The Fadeev-Popov determinant is just the expression (3.56), but with $M \to 2M$, giving

$$\mathcal{L}^1_{FP} = -\frac{\mu^2 M^2 \ln 2}{\pi^2} - \frac{\ln\mu^2}{4\pi^2}(\partial^\mu M\partial_\mu M + 4\Gamma^\mu\Gamma_\mu M^2 + \frac{1}{12}J^{\mu\nu}J_{\mu\nu} - \frac{1}{3}\partial^\mu\partial_\mu M^2 - 4M^4). \tag{3.61}$$

The M^2 terms in these two expressions are the well-known contributions to the Coleman-Weinberg scalar potential [32], and the J^2 terms are the analogue [12] of the fermion contribution to the β-function in gauge theories; the coefficients are in agreement with previous results.

Moving on to the contribution from the auxiliary field α, we note that the covariant derivative $d_\mu(x) = \partial_\mu + i\Gamma_\mu\gamma_5$ commutes with $\gamma^0\gamma^\mu$. So we are free to

write:

$$\begin{aligned}\mathrm{Tr}\ln\nabla_M^{-1} &= \mathrm{Tr}\ln\gamma^\mu\Delta_M\gamma_\mu \\ &= \int d^4x\int\frac{d^4p}{(2\pi)^4}\ \mathrm{tr}\ln\gamma^0\gamma^\mu\mathcal{M}_4^{-1}\gamma^0\gamma_\mu \\ &= \int d^4x\int\frac{d^4p}{(2\pi)^4}\ \mathrm{tr}\ln\gamma^0\gamma^\mu\mathcal{U}\mathcal{M}_4^{-1}\mathcal{U}^{-1}\gamma^0\gamma_\mu.\end{aligned} \tag{3.62}$$

If we now expand $\mathcal{U}\mathcal{M}_4^{-1}\mathcal{U}^{-1}$ about $\mathcal{M}_0^{-1}$, with

$$\mathcal{M}_0 \equiv \gamma^0(-\not{p}+M) \tag{3.63}$$

and

$$\delta\mathcal{M} \equiv \mathcal{U}\mathcal{M}_4\mathcal{U}^{-1} - \mathcal{M}_0\gamma^0(-\not{\hat{p}}\gamma_5+\hat{N}), \tag{3.64}$$

we have

$$\mathcal{U}\mathcal{M}_4^{-1}\mathcal{U}^{-1} = \sum_{n=0}^{\infty}(-\mathcal{M}_0^{-1}\delta\mathcal{M})^n\mathcal{M}_0^{-1}. \tag{3.65}$$

Here $\hat{J}_\mu$ is defined by (3.31) and (3.32), and

$$\begin{aligned}\gamma^0\hat{N} \equiv \gamma^0(\hat{M}-M) &= \sum_{n=1}^{\infty}\frac{1}{n!}[d_{\mu_1},[d_{\mu_2},\cdots[d_{\mu_n},\gamma^0M]]]\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}} \\ &= \gamma^0\sum_{n=1}^{\infty}\frac{1}{n!}(D_{\mu_1}\cdots D_{\mu_n}M)\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}},\end{aligned} \tag{3.66}$$

where as before $d_\mu = \partial_\mu + i\Gamma_\mu\gamma_5$, and D_μ is the covariant derivative as defined by (3.33) and (3.34) in the chiral representation. At first sight this appears much more complicated than equations (3.18) through (3.25), which would have resulted in a similar expression, involving an infinite series in $D_\mu M$; i.e., in Γ_μ and its derivatives, had we applied the transformation (3.28) directly to the propagator Δ_M. The fact that Γ_μ appears only in second order in (3.43) reflects the fact that the mass M appears in the final trace only in the combination $MM^\dagger$, which satisfies (3.38). However, no way to cast the auxiliary field determinant (3.62) in such a compact form has yet been found.

But since all the terms in $\delta\mathcal{M}$ contain spatial derivatives, we can halt the expansion at the fourth term, and then commute γ^μ through to the right explicitly, removing the initial γ^0 as before (since $\det\gamma^0 = 1$). Thus:

$$\begin{aligned}&\mathrm{tr}\ln\gamma^0\gamma^\mu\mathcal{U}\mathcal{M}_4^{-1}\mathcal{U}^{-1}\gamma^0\gamma_\mu = \\ &\qquad \mathrm{tr}\ln\gamma^0\gamma^\mu\mathcal{M}_0^{-1}(1-\delta\mathcal{M}\mathcal{M}_0^{-1}+(\delta\mathcal{M}\mathcal{M}_0^{-1})^2)\gamma^0\gamma_\mu,\end{aligned} \tag{3.67}$$

and by a Hausdorff expansion of the logarithm, we can write

$$\begin{aligned}\ln A(1-B) = \ln A + \ln(1-B) &+ \frac{1}{2}[\ln A,\ln(1-B)] \\ &+ \frac{1}{12}[\ln A,[\ln A,\ln(1-B)]] \\ &- \frac{1}{12}[\ln(1-B),[\ln A,\ln(1-B)]] + \textit{finite},\end{aligned} \tag{3.68}$$

where

$$A \equiv \gamma^\mu\mathcal{M}_0^{-1}\gamma^0\gamma_\mu = \gamma^\mu\frac{-\not{p}-M}{p^2-M^2}\gamma_\mu, \tag{3.69}$$

and

$$B \equiv A^{-1}\gamma^\nu\mathcal{M}_0^{-1}(\delta\mathcal{M}\mathcal{M}_0^{-1}-(\delta\mathcal{M}\mathcal{M}_0^{-1})^2)\gamma^0\gamma_\nu, \tag{3.70}$$

and where all higher terms in the commutator expansion can be neglected, since they give only finite contributions to the p-integral.

The first term is just

$$\begin{aligned}\mathrm{tr}\ln\gamma^\mu\frac{-\not{p}-M}{p^2-M^2}\gamma_\mu &= \mathrm{tr}\ln\frac{2\not{p}-4M}{p^2-M^2} \\ &= 2\ln(p^2-4M^2)-4\ln(p^2-M^2)+4\ln 2\end{aligned} \tag{3.71}$$

The constant divergence, $4\ln 2$, precisely cancels the bosonic divergence given in references [20] and [21], as the underlying supersymmetry of the Lagrangian requires. To evaluate the remaining terms in the expansion, we push momentum derivative operators to the right in the expression for B above, and retain only terms that give logarithmically divergent contributions to the effective action. Terms linear in the antisymmetric tensor $J_{\mu\nu}$ vanish in the trace because they carry a factor γ_5; in

fact, even without this factor they would contribute no divergent terms due to their antisymmetry. Similarly, terms with an odd number of (covariant) derivatives on M will not appear in the effective action. These terms can be explicitly eliminated in the expansion of B, equation (3.70), because they appear with either an odd number of γ-matrices or an odd power of p, and so must vanish either in the trace or upon integration over all momentum.

The other surviving terms are then

$$\begin{aligned}\operatorname{tr}\ln(1-B) &= -\operatorname{tr}B - \frac{1}{2}\operatorname{tr}B^2 \\ &= \frac{1}{p^4}(2J^{\mu\nu}J_{\mu\nu} - 8MD_\mu D_\nu M^\dagger - 8D^\mu M D_\mu M^\dagger) \\ &\quad + \frac{p^\mu p^\nu}{p^6}(-4J_\mu{}^\rho J_{\nu\rho} + 8MD_\mu D_\nu M^\dagger + 32D_\mu M D_\nu M^\dagger);\end{aligned} \tag{3.72}$$

$$\begin{aligned}\frac{1}{2}\operatorname{tr}[\ln A, \ln(1-B)] &= -\frac{1}{4}\operatorname{tr}A^{-2}[A^2, B+\frac{1}{2}B^2] - \frac{1}{8}\operatorname{tr}A^{-4}[A^2,[A^2,B+\frac{1}{2}B^2]] \\ &= \frac{1}{p^4}(-\frac{1}{4}J^{\mu\nu}J_{\mu\nu} + 14MD^\mu D_\mu M^\dagger + 2D^\mu M D_\mu M^\dagger) \\ &\quad + \frac{p^\mu p^\nu}{p^6}(J_\mu{}^\rho J_{\nu\rho} - 38MD_\mu D_\nu M^\dagger - 10D_\mu M D_\nu M^\dagger);\end{aligned} \tag{3.73}$$

$$\begin{aligned}\frac{1}{12}\operatorname{tr}[\ln A, [\ln A, \ln(1-B)]] &= -\frac{1}{48}\operatorname{tr}A^{-4}[A^2,[A^2,B+\frac{1}{2}B^2]] \\ &= \frac{p^\mu p^\nu}{p^6}(-\frac{1}{12}J_\mu{}^\rho J_{\nu\rho} + 10MD_\mu D_\nu M^\dagger + \frac{2}{3}D_\mu M D_\nu M^\dagger);\end{aligned} \tag{3.74}$$

and

$$\begin{aligned}-\frac{1}{12}\operatorname{tr}[\ln(1-B), [\ln A, \ln(1-B)]] &= -\frac{1}{24}\operatorname{tr}A^{-2}[B,[A^2,B]] \\ &= \frac{1}{p^4}(\frac{1}{12}J^{\mu\nu}J_{\mu\nu} + \frac{4}{3}D^\mu M D_\mu M^\dagger) \\ &\quad + \frac{p^\mu p^\nu}{p^6}(-\frac{1}{2}J_\mu{}^\rho J_{\nu\rho} - \frac{20}{3}D_\mu M D_\nu M^\dagger).\end{aligned} \tag{3.75}$$

In evaluating the commutator terms, we have used the identity

$$[\ln A, X] = \sum_{n=1}^{\infty}\frac{1}{n}A^{-n}\overbrace{[A,\cdots[A,[A,X]]]}^{n\text{ commutators}} \tag{3.76}$$

repeatedly, truncating all expansions at $O(1/p^4)$. Note that the log expansion above does not yield increasing powers of $1/p$, since there is an A^{-1} in front for each additional A in the commutators. But all terms with more than two commutators vanish identically, because for divergent pieces there are at most two p-derivatives acting within each term; thus, we have fully evaluated the nonfinite part of the trace.

Only (3.71) contributes to the quadratic divergence, giving

$$\mathcal{L}^{\text{aux}}_{\text{quad}} = \frac{\mu^2 M^2 \ln 2}{4\pi^2} \tag{3.77}$$

when we integrate it in the familiar twice-subtracted prescription, while the sum of the logarithmic divergences from (3.72), (3.73), (3.74), and (3.75) is

$$\mathcal{L}^{\text{aux}}_{\log} = -\frac{\ln\mu^2}{16\pi^2}(7M^4 - \frac{15}{32}J^{\mu\nu}J_{\mu\nu} - \frac{1}{2}MD^\mu D_\mu M^\dagger + \frac{1}{3}D^\mu M D_\mu M^\dagger). \tag{3.78}$$

For the gravitinos, of course, we have $M = M^\dagger$, and so $MD^\mu D_\mu M^\dagger = M\partial^2 M - 4\Gamma^2 M^2$. But the more general result was worth the effort, as we shall see in the following chapter.

Adding these results to (3.60) and (3.61), we get the total sum of the quadratic and logarithmic divergences arising from pure gravitino loops:

$$\begin{aligned}\mathcal{L}^1_\psi = -\frac{\mu^2 M^2 \ln 2}{4\pi^2} - \frac{\ln\mu^2}{16\pi^2}(&\partial^\mu M\partial_\mu M - \frac{11}{6}M\partial^2 M \\ &+ \frac{34}{3}\Gamma^\mu\Gamma_\mu M^2 - \frac{77}{96}J^{\mu\nu}J_{\mu\nu} - 7M^4).\end{aligned} \tag{3.79}$$

At this point we can eliminate a total divergence, writing $M\partial^2 M \to -\partial^\mu M\partial_\mu M$. The divergent Lagrangian then becomes

$$\begin{aligned}\mathcal{L}^1_\psi = -\frac{\mu^2 M^2 \ln 2}{4\pi^2} + \frac{\ln\mu^2}{16\pi^2}(&7M^4 + \frac{77}{96}J^{\mu\nu}J_{\mu\nu} \\ &- \frac{17}{6}(\partial^\mu M\partial_\mu M + 4\Gamma^\mu\Gamma_\mu M^2)).\end{aligned} \tag{3.80}$$

Rewriting this in terms of the Kähler potential $\mathcal{G}(z, z^\dagger)$, we find that

$$\mathcal{L}^1_\psi = -\frac{\mu^2 \ln 2}{4\pi^2} e^{\mathcal{G}} + \frac{\ln \mu^2}{16\pi^2}\Big(7e^{2\mathcal{G}} + \frac{77}{96} J^{\mu\nu} J_{\mu\nu} - \frac{17}{6}\mathcal{G}_i \mathcal{G}_j \partial^\mu z^i \partial_\mu z^j e^{\mathcal{G}}\Big) \tag{3.81}$$

$$= -\frac{\mu^2 \ln 2}{4\pi^2} e^{\mathcal{G}} + \frac{\ln \mu^2}{16\pi^2}\Big(7e^{2\mathcal{G}} - \frac{17}{6}\mathcal{G}_i \mathcal{G}_j \partial^\mu z^i \partial_\mu z^j e^{\mathcal{G}} + \frac{77}{192}\mathcal{G}_{i,j}\mathcal{G}_{kl}\partial^\mu z^i \partial^\nu z^j (\partial_\mu z^k \partial_\nu z^l - \partial_\nu z^k \partial_\mu z^l)\Big). \tag{3.82}$$

since

$$J^{\mu\nu} = \frac{1}{2}\mathcal{G}_{i,j}(\partial^\mu z^i \partial^\nu z^j - \partial^\nu z^i \partial^\mu z^j). \tag{3.83}$$

This is the complete result for contributions from pure spin-$\frac{3}{2}$ loops. The remaining task, to generalize (3.82) by including the fermion-mixing mass terms we neglected, will be carried out, in straightforward but exhausting detail, in the next chapter.

CHAPTER 4

The Total Fermion Contribution

Now, at last, we return to the spin-$\frac{1}{2}$ fermions in the theory, and at the same time consider the effects of background Yang-Mills fields. When this result is combined with those of Gaillard and Jain [20,21], we shall have the complete one-loop corrected scalar and gauge Lagrangian (for canonical gauge kinetic energy) of our simple no-scale supergravity model. This result is then easily generalized to other $N = 1$ SUGRA models. Looking once more at the full fermionic Lagrangian, (1.11), in the gauge $\gamma \cdot \psi = 0$, we write the kinetic energy piece as

$$\mathcal{L}^{KB}_{\psi,\chi,\lambda} = -\frac{i}{2}\bar{\psi}^\mu \eta_{\mu\nu} \not{d}^\nu{}_\rho \psi^\rho + i\bar{\chi}^I \mathcal{G}_{I,J} \not{d}^J{}_K \chi^K + \frac{i}{2}\bar{\lambda}^a \operatorname{Re} f_{ab} \not{d}^b{}_c \lambda^c. \tag{4.1}$$

Here $\not{d} \equiv \gamma^\mu d_\mu$ stands for the fully gauge- and reparametrization-covariant derivative, which takes the form

$$\begin{aligned}
(d_\mu)^\nu{}_\rho &= \eta^\nu{}_\rho(\partial_\mu + i\Gamma_\mu \gamma_5), \\
(d_\mu)^J_K &= \delta^J_K(\partial_\mu - i\Gamma_\mu\gamma_5) + ie(\tilde{A}_\mu)^J_K + \Gamma^J{}_{LK}\partial_\mu z^L, \qquad \text{and} \\
(d_\mu)^b_c &= \delta^b_c(\partial_\mu + i\Gamma_\mu\gamma_5) - e\epsilon^b{}_{ca}\tilde{A}^a_\mu + \frac{i}{2}(1/\operatorname{Re} f)^{ba}\tilde{\mathcal{D}}_\mu(\operatorname{Im} f_{ac})\gamma_5,
\end{aligned} \tag{4.2}$$

when acting on the gravitinos, chiral fermions, and gauginos respectively. The labels $I, J, K, \ldots$ are shorthand for $i, \bar{\imath}, j, \bar{\jmath}, k, \bar{k}, \ldots$, where it is to be understood that χ^i_L and $\bar{\chi}^i_R$ both transform like $\hat{z}^i$, and that $A^{\bar{\imath}}_{\bar{\jmath}} = (A^i_j)^* = A^j_i$. The $\epsilon^b{}_{ca}$ are the totally antisymmetric structure constants of the gauge group. As always, the only nonvanishing reparametrization connections are $\Gamma^i{}_{kj}$ and its complex conjugate. The Γ_μ pieces are just the chiral $U(1)$ connections [33,34], excluding the fermion-dependent pieces. The relative minus sign for the chiral fermions is just due to the fact that the left-handed χ's transform like the right-handed ψ_μ and λ's under the chiral $U(1)$. In the presence of background gauge fields, we must covariantize all

spatial derivatives, so that, from (3.5),

$$\Gamma_\mu = \frac{i}{4}(\mathcal{G}_i \tilde{D}_\mu z^i - \mathcal{G}_i \tilde{D}_\mu z^i). \tag{4.3}$$

The formalism is somewhat simplified if we rescale the gaugino fields, so as to put their kinetic energy into canonical form. More precisely, we make the transformation

$$\lambda^a \to \left(\frac{1}{\sqrt{\mathrm{Re}\, f}}\right)^a_b \lambda^b, \tag{4.4}$$

so that the gaugino kinetic energy is just

$$\mathcal{L}^{kin}_\lambda = \frac{i}{2}\bar{\lambda}^a \delta_{ac} \not{D}^c_b \lambda^c, \tag{4.5}$$

where now the covariant derivative is

$$(d_\mu)^b_c = \delta^b_c(\partial_\mu + i\Gamma_\mu \gamma_5) - e\epsilon^b_{cm}\tilde{A}^a_\mu + i(L_\mu)^b_c \gamma_5, \tag{4.6}$$

with

$$(L_\mu)^b_c = \frac{1}{2}(1/\sqrt{\mathrm{Re}\, f})^b_a \delta^{ad}(\tilde{D}_\mu \,\mathrm{Im}\, f_{dc})(1/\sqrt{\mathrm{Re}\, f})^a_c. \tag{4.7}$$

In the rest of this chapter we shall work in these rescaled coordinates, which leave the final result for the effective action invariant.

The rest of the quadratic terms in $\mathcal{L}_F$ can be written in the form

$$\begin{aligned}\mathcal{L}^M_{\psi,\chi,\lambda} = &-\frac{1}{2}m^\chi_{ij}\bar{\chi}^i_R\chi^j_L + \frac{1}{2}m^\psi_{\mu\nu}\bar{\psi}^\mu_R\psi^\nu_L - \frac{1}{2}m^\lambda_{ab}\bar{\lambda}^a_R\lambda^b_L \\ &+ m^{\psi\chi}_{\mu j}\bar{\psi}^\mu_R\chi^j_L - (m^{\lambda\chi}_{aj} + \tilde{m}^{\lambda\chi}_{aj})\bar{\lambda}^a_R\chi^j_L + H.\,c.,\end{aligned} \tag{4.8}$$

with the masses given by:

$$\begin{aligned}
m^\chi_{ij} &= 2(\mathcal{G}_{ij} + \mathcal{G}_i\mathcal{G}_j - \mathcal{G}_k\Gamma^k_{ij})e^{\mathcal{G}/2}, \\
m^\psi_{\mu\nu} &= \eta_{\mu\nu}e^{\mathcal{G}/2} \qquad \text{(This is our old } M.\text{)} \\
m^\lambda_{ab} &= -\frac{1}{2}(1/\sqrt{\mathrm{Re}\, f})^c_a(1/\sqrt{\mathrm{Re}\, f})^d_b \frac{\partial f_{cd}}{\partial z^k}\mathcal{G}_l\mathcal{G}^{lk}e^{\mathcal{G}/2}, \\
m^{\psi\chi}_{\mu j} &= -2\mathcal{G}_{jk}(\tilde{D}_\mu z^k), \\
m^{\lambda\chi}_{aj} &= \frac{i}{2}(1/\sqrt{\mathrm{Re}\, f})^c_a(1/\mathrm{Re}\, f)^{db}\frac{\partial f_{bc}}{\partial z^j}\mathcal{G}_i T^i_{dk}z^k \\
&\quad - 2i(1/\sqrt{\mathrm{Re}\, f})^c_a \mathcal{G}_{ij}T^j_{ck}z^k, \qquad \text{and} \\
\tilde{m}^{\lambda\chi}_{aj} &= \frac{i}{4}(1/\sqrt{\mathrm{Re}\, f})^c_a \frac{\partial f_{bc}}{\partial z^j}\sigma^{\mu\nu}\tilde{F}^b_{\mu\nu}.
\end{aligned} \tag{4.9}$$

We have written $m^{\lambda\chi}$ and $\tilde{m}^{\lambda\chi}$ out separately, since the latter term has nontrivial Dirac-matrix content and will require careful treatment.

There is one more piece quadratic in the fermions. This is the term in the fermionic Lagrange density, (1.11), that involves ψ^μ and λ^a. This term mixes only fermions of the same handedness, since it contains an odd number of γ-matrices; thus, it is not a mass term. It can, however, be treated as a connection, and we shall treat it so in what follows. In the gauge $\gamma \cdot \psi = 0$, the relevant term becomes

$$\mathcal{L}_{\psi\lambda} = \frac{i}{2}\bar{\lambda}^a \not{\Gamma}_{\mu a}\psi^\mu + H.\,c., \tag{4.10}$$

where $\not{\Gamma}_{\mu a} \equiv \gamma^\nu(\sqrt{\mathrm{Re}\, f})_{ab}\tilde{F}^b_{\mu\nu}$.

To apply the results of the pure-gravitino calculation above to this full fermion Lagrangian, we define a big $(4 + (N+2) + N_G)$-component spinor θ, with the flavor-space entries:

$$\theta^\mu = \psi^\mu, \qquad \theta^i = \chi^i, \quad \text{and} \quad \theta^a = \lambda^a. \tag{4.11}$$

The fermionic Lagrangian, equations (4.1) and (4.8), can then be written as

$$\mathcal{L}_F = \frac{1}{2}\bar{\theta}Z(i\not{d} - M_\theta)\theta \equiv \frac{1}{2}\bar{\theta}\Delta^{-1}_\theta\theta, \tag{4.12}$$

where Δ^{-1}_θ is a $(4 + (N+2) + N_G)$-dimensional matrix in flavor space. Z is the metric with block-diagonal entries,

$$Z_{\mu\nu} = -\eta_{\mu\nu}, \quad Z_{ij} = 2\mathcal{G}_{ij}, \quad Z_{ab} = \delta_{ab}, \tag{4.13}$$

which is used to lower indices, while Z^{-1} is used to raise them. The mass matrix has the γ-matrix space decomposition

$$M_\theta = \frac{1}{2}(1-\gamma_5)\otimes M + \frac{1}{2}(1+\gamma_5)\otimes M^\dagger, \tag{4.14}$$

where $M = Z^{-1}(ZM)$ is defined implicitly by

$$ZM \equiv \begin{pmatrix} m^{\psi}_{\mu\nu} & m^{\psi\chi}_{\mu j} & 0 \\ m^{\psi\chi}_{i\nu} & m^{\chi}_{ij} & m^{\lambda\chi}_{ib} \\ 0 & m^{\lambda\chi}_{aj} & m^{\lambda}_{ab} \end{pmatrix}, \tag{4.15}$$

with the individual entries given by equation (4.9). We have left out the contribution from the last mass piece of (4.9): as mentioned before, the $\sigma^{\mu\nu}$-dependence introduces a slight complication, and so we shall first derive the one-loop results without this masslike term, and then modify them to find the full answer.

With these definitions, the decomposition of the inverse propagator Δ_θ^{-1} in flavor space is, with the convention that $M^{\psi\chi} = \frac{1}{2}(1-\gamma_5)m^{\psi\chi} + \frac{1}{2}(1+\gamma_5)(m^{\psi\chi})^\dagger$ and so on,

$$\Delta_\theta^{-1} \equiv \begin{pmatrix} (\Delta_\psi^{-1})_{\mu\nu} & -M^{\psi\chi}_{\mu j} & i\not{d}^{\psi\lambda}_{\mu b} \\ -M^{\psi\chi}_{i\nu} & (\Delta_\chi^{-1})_{ij} & -M^{\lambda\chi}_{ib} \\ i\not{d}^{\psi\lambda}_{a\nu} & -M^{\lambda\chi}_{aj} & (\Delta_\lambda^{-1})_{ab} \end{pmatrix}. \tag{4.16}$$

We write its inverse with upper indices. Note that $\Delta_\theta^{\mu\nu}$ does not equal $\Delta_\psi^{\mu\nu}$, since inversion mixes the off-diagonal elements with the diagonal ones.

We now gauge-fix. The ghost contribution has already been evaluated. The auxiliary field contribution is just

$$\mathcal{L}_\alpha = -\bar{\alpha}\gamma_\mu\psi^\mu = -\bar{\alpha}\gamma_\mu\theta^\mu. \tag{4.17}$$

As before, we shift the θ-field to remove the cross-term involving the auxiliary field:

$$\theta^\mu \to \theta^\mu + \Delta_\theta^{\mu\nu}\gamma_\nu\alpha, \tag{4.18}$$

after which the full fermionic Lagrangian is

$$\mathcal{L}_F = \frac{1}{2}\bar{\theta}\Delta_\theta^{-1}\theta - \frac{1}{2}\bar{\alpha}\gamma_\mu\Delta_\theta^{\mu\nu}\gamma_\nu\alpha. \tag{4.19}$$

We will come back to the auxiliary field later. To compute the contributions from the other fermions we decompose the fermions into left-handed and right-handed parts. In particular, we write

$$\theta_L = \begin{pmatrix} \psi^\mu_L \\ \chi^i_L \\ \lambda^a_L \end{pmatrix}, \tag{4.20}$$

with a corresponding right-handed spinor θ_R. Then, in the γ-matrix chiral representation, the total (quadratic) fermionic Lagrangian can be written as

$$\begin{aligned} \mathcal{L}_F &\equiv \frac{1}{2}\bar{\Theta}Z\begin{pmatrix} i\not{d}^+ & -M \\ -M^\dagger & i\not{d}^- \end{pmatrix}\Theta \\ &= (\bar{\theta}_R \quad \bar{\theta}_L)Z\left[\begin{pmatrix} i\not{d}^+ & 0 \\ 0 & i\not{d}^- \end{pmatrix} - \begin{pmatrix} 0 & M \\ M^\dagger & 0 \end{pmatrix}\right]\begin{pmatrix} \theta_R \\ \theta_L \end{pmatrix}, \end{aligned} \tag{4.21}$$

where $\Theta^T \equiv (\theta_R^T, \theta_L^T)$ and its transpose are spinors in the chiral representation. Our covariant derivatives are:

$$\begin{aligned} (d^\pm_\mu)^\nu{}_\rho &= \eta^\nu{}_\rho(\partial_\mu \pm i\Gamma_\mu); \\ (d^\pm_\mu)^i_j &= \delta^i_j(\partial_\mu \mp i\Gamma_\mu) \pm ie(\tilde{A}_\mu)^i_j + \Gamma^i{}_{\kappa j}\tilde{D}_\mu z^\kappa; \\ (d^\pm_\mu)^a_b &= \delta^a_b(\partial_\mu \pm i\Gamma_\mu) - e\epsilon^a{}_{bc}\tilde{A}^c_\mu \pm i(L_\mu)^a_b; \end{aligned} \tag{4.22}$$

and also, from the $\lambda\psi$-term, equation (4.10):

$$\begin{aligned} (d^\pm_\mu)^\nu_a &= (\sqrt{\mathrm{Re}\, f})_{ab}(\tilde{F}^\nu{}_\mu)^b; \\ (d^\pm_\mu)^a_\nu &= (\sqrt{\mathrm{Re}\, f})^a_b(\tilde{F}_{\nu\mu})^b. \end{aligned} \tag{4.23}$$

The fermionic one-loop correction is given by:

$$S^1_{\mathrm{eff}} = -i\,\mathrm{Tr}\ln\begin{pmatrix} i\not{d}^+ & -M \\ -M^\dagger & i\not{d}^- \end{pmatrix}. \tag{4.24}$$

In this form it is easy to see how to use the results of the gravitino loop calculation presented above. As before, by using $\mathrm{Tr}\, f(\gamma_\mu) = \mathrm{Tr}\, f(-\gamma_\mu)$, and doubling the dimensions of our matrices, we can write:

$$S^1_{\rm eff} = -\frac{i}{4}\,\mathrm{Tr}\ln\begin{pmatrix} \not{d}^+\not{d}^+ + MM^\dagger & -i\not{D}^+M \\ -i\not{D}^-M^\dagger & \not{d}^-\not{d}^- + M^\dagger M \end{pmatrix}, \tag{4.25}$$

where $D^\pm_\mu$ is defined by generalizing equation (3.34) so as to include background Yang-Mills and background reparametrization invariance:

$$\begin{aligned}\begin{pmatrix} 0 & D^+_\mu M \\ D^-_\mu M^\dagger & 0 \end{pmatrix} &= \left[\begin{pmatrix} d^+_\mu & 0 \\ 0 & d^-_\mu \end{pmatrix}, \begin{pmatrix} 0 & M \\ M^\dagger & 0 \end{pmatrix}\right] \\ &= \begin{pmatrix} 0 & d^+_\mu M - M d^-_\mu \\ d^-_\mu M^\dagger - M^\dagger d^+_\mu & 0 \end{pmatrix},\end{aligned} \tag{4.26}$$

so that, from (4.21) and (4.22),

$$\begin{aligned} D^+_\mu M &= \partial_\mu M + i\{\Gamma_\mu, M\} + i\Lambda^*_\mu M + iM\Lambda_\mu + i\{L_\mu, M\}, \\ D^-_\mu M^\dagger &= \partial_\mu M^\dagger - i\{\Gamma_\mu, M^\dagger\} - i\Lambda_\mu M^\dagger - iM^\dagger\Lambda^*_\mu - i\{L_\mu, M^\dagger\}, \end{aligned} \tag{4.27}$$

where now Γ_μ stands for a matrix with elements

$$(\Gamma_\mu)^\nu{}_\rho = \eta^\nu{}_\rho\Gamma_\mu, \qquad (\Gamma_\mu)^i_j = -\delta^i_j\Gamma_\mu, \quad \text{and} \quad (\Gamma_\mu)^a_b = \delta^a_b\Gamma_\mu, \tag{4.28}$$

L_μ stands for a matrix whose only nonzero elements are given by $(L_\mu)^a_b$, as defined in equation (4.7), and Λ_μ is the matrix with elements

$$\begin{aligned} &(\Lambda_\mu)^\nu{}_\rho = 0, \qquad (\Lambda_\mu)^i_j = eA^i_{\mu j} + i\Gamma^i{}_{kj}\mathcal{D}_\mu z^k, \qquad (\Lambda_\mu)^a_b = -ie\epsilon^a{}_{bc}\tilde{A}^c_\mu, \\ &(\Lambda^\pm_\mu)^\nu_a = i(\sqrt{\mathrm{Re}\, f})_{ab}(\tilde{F}^{\nu}{}_\mu)^b, \quad \text{and} \quad (\Lambda^\pm_\mu)^a_\nu = i(\sqrt{\mathrm{Re}\, f})^a_b(\tilde{F}_{\nu\mu})^b, \end{aligned} \tag{4.29}$$

From $\not{A}\not{B} = A\cdot B - i\sigma_{\mu\nu}A^\mu B^\nu$, equation (4.25) becomes

$$\begin{aligned} S^1_{\rm eff} &\equiv -\frac{i}{4}\,\mathrm{Tr}\ln(d^\mu d_\mu + \mathrm{M}^2) \\ &= -\frac{i}{4}\,\mathrm{Tr}\ln\begin{pmatrix} d^+_\mu d^{+\mu} + MM^\dagger - \frac{i}{2}\sigma^{\mu\nu}F^+_{\mu\nu} & -i\not{D}^+M \\ -i\not{D}^-M^\dagger & d^-_\mu d^{-\mu} + M^\dagger M - \frac{i}{2}\sigma^{\mu\nu}F^-_{\mu\nu} \end{pmatrix}. \end{aligned} \tag{4.30}$$

where $F^\pm_{\mu\nu} \equiv [d^\pm_\mu, d^\pm_\nu]$. Now, treating everything but the $d_\mu d^\mu$ term as a single mass-squared piece, M^2, we can use the results of either the gravitino loop calculation or of reference [12] to find the divergent one-loop corrections: we get

$$\mathcal{L}^1_{\rm reg} = \frac{1}{256\pi^2}\,\mathrm{Tr}\Big(\big(\mathrm{M}^4 + \frac{1}{6}(F^+_{\mu\nu}F^{+\mu\nu} + F^-_{\mu\nu}F^{-\mu\nu})\big)\ln(\mu_0^2/\rho\mu^2) + 2\mathrm{M}^2\eta\mu^2\Big), \tag{4.31}$$

where the trace is over both internal labels and γ-matrix space. This has an overall minus sign relative to the scalar case, and we have divided by four to compensate for squaring the propagator and doubling its dimensionality. Using the identities $\mathrm{Tr}\,\gamma_\mu = 0$, $\mathrm{Tr}\,\sigma_{\mu\nu} = 0$, $\mathrm{Tr}\,\sigma_{\mu\nu}\sigma_{\rho\sigma} = 4(\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho})$, and $\mathrm{Tr}\,\mathbf{1} = 4$, the γ-matrix trace can be easily evaluated, and we have

$$\begin{aligned} \mathcal{L}^1_{\rm reg} = \frac{1}{32\pi^2}\,\mathrm{Tr}\Big(&\big[(MM^\dagger)^2 - D^+_\mu M D^{-\mu}M^\dagger \\ &- \frac{1}{6}(F^+_{\mu\nu}F^{+\mu\nu} + F^-_{\mu\nu}F^{-\mu\nu})\big]\ln(\mu_0^2/\rho\mu^2) + 2MM^\dagger\eta\mu^2\Big), \end{aligned} \tag{4.32}$$

which is just a modification of equation (3.60), with the trace over internal indices only.

Now, returning to the last mass term in (4.9), we introduce the variable $\tilde{M}_{\mu\nu}$, also a $(4 + (N+2) + N_G)$-dimensional matrix in flavor space. We use the decompositions

$$\begin{aligned} (M^\dagger + \tilde{M}^\dagger_{\mu\nu}\sigma^{\mu\nu})(M + \tilde{M}_{\mu\nu}\sigma^{\mu\nu}) &= M^\dagger M + 2\tilde{M}^\dagger_{\mu\nu}\tilde{M}^{\mu\nu} + i\epsilon^{\mu\nu\rho\sigma}\gamma_5\tilde{M}^\dagger_{\rho\sigma} \\ &+ (M^\dagger\tilde{M}_{\mu\nu} + \tilde{M}^\dagger_{\mu\nu}M - 4i\eta^{\rho\lambda}\eta^\sigma{}_\mu\eta^\tau{}_\nu\tilde{M}^\dagger_{\rho\sigma}\tilde{M}_{\lambda\tau})\sigma^{\mu\nu} \end{aligned} \tag{4.33}$$

and

$$\not{D}^+(M + \tilde{M}_{\mu\nu}\sigma^{\mu\nu}) = \not{D}^+M - 2i\gamma_\mu D^+_\nu\tilde{M}^{\mu\nu} + \epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_5 D^+_\nu\tilde{M}_{\rho\sigma}. \tag{4.34}$$

Then, to let $M \to M + \tilde{M}_{\mu\nu}\sigma^{\mu\nu}$, we make the replacement

$$\mathrm{Tr}\, M^\dagger M \to \mathrm{Tr}\, M^\dagger M + 2\,\mathrm{Tr}\,\tilde{M}^\dagger_{\mu\nu}\tilde{M}^{\mu\nu}, \tag{4.35}$$

after taking the Dirac-matrix trace. The $\ln\mu^2$-dependent pieces come from the $(M^\dagger M - \frac{1}{2}i\sigma^{\mu\nu}F^{\pm}_{\mu\nu})^2$ term, and also from the $\not{D}^- M^\dagger \not{D}^+ M^\dagger$ term, in equation (4.30). The replacement $M \to M + \tilde{M}_{\mu\nu}\sigma^{\mu\nu}$ thus also results in the modification

$$\begin{aligned}\mathrm{Tr}(M^\dagger M)^2 \to\ & \mathrm{Tr}(M^\dagger M)^2 + 4\,\mathrm{Tr}\, M^\dagger M \tilde{M}^\dagger_{\mu\nu}\tilde{M}^{\mu\nu} \\ & + 4\,\mathrm{Tr}(\tilde{M}^\dagger_{\mu\nu}\tilde{M}^{\mu\nu})^2 - \mathrm{Tr}(\epsilon^{\mu\nu\rho\sigma}\tilde{M}^\dagger_{\mu\nu}\tilde{M}_{\rho\sigma})^2 \\ & + \textstyle\sum_\pm \mathrm{Tr}(\tilde{M}^\dagger_{\mu\nu}M + M^\dagger\tilde{M}_{\mu\nu} - \frac{i}{2}F^\pm_{\mu\nu} - 4i\eta^{\rho\lambda}\eta^\sigma{}_\mu\eta^\tau{}_\nu\tilde{M}^\dagger_{\rho\sigma}\tilde{M}_{\lambda\tau}) \\ & \quad \times (\tilde{M}^{\dagger\mu\nu}M + M^\dagger\tilde{M}^{\mu\nu} - \frac{i}{2}F^{\pm\mu\nu} - 4i\eta^{\alpha\gamma}\eta^{\beta\mu}\eta^{\delta\nu}\tilde{M}^\dagger_{\alpha\beta}\tilde{M}_{\gamma\delta}).\end{aligned} \tag{4.36}$$

The $\pm$ refers to the two different contributions that arise from the square of the last matrix in (4.30). Finally,

$$\begin{aligned}\mathrm{Tr}\, D^-_\mu M^\dagger D^{+\mu} M \to\ & \mathrm{Tr}\, D^-_\mu M^\dagger D^{+\mu} M - 16\,\mathrm{Tr}\, D^-_\nu \tilde{M}^{\dagger\mu\nu} D^{+\rho}\tilde{M}_{\mu\rho} \\ & - \epsilon^{\mu\nu\rho\sigma}\epsilon_\mu{}^{\alpha\beta\tau}\,\mathrm{Tr}\, D^-_\nu \tilde{M}^\dagger_{\rho\sigma} D^+_\alpha \tilde{M}_{\beta\tau} - 2i\,\mathrm{Tr}\, D^-_\mu \tilde{M}^{\dagger\mu\nu} D^+_\nu M \\ & - 2i\,\mathrm{Tr}\, D^-_\mu M^\dagger D^+_\nu \tilde{M}^{\mu\nu}.\end{aligned} \tag{4.37}$$

To use these results to incorporate the last mass term in (4.9), we set

$$Z\tilde{M}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (\tilde{m}^{\lambda\chi})_{\mu\nu} \\ 0 & (\tilde{m}^{\lambda\chi})_{\mu\nu} & 0 \end{pmatrix}. \tag{4.38}$$

From this, of course, we can find $\tilde{M}^\dagger_{\mu\nu}$:

$$\gamma_0 Z\tilde{M}^\dagger_{\mu\nu}\sigma^{\mu\nu}\gamma_0 = (Z\tilde{M}_{\mu\nu}\sigma^{\mu\nu})^\dagger. \tag{4.39}$$

Since these additional traces will take on a much simpler form when we set $f_{ab}(z) = \delta_{ab}s$, we will defer their flavor-space expansion to Chapter 5, where we shall specialize to the no-scale models.

Let us now compute the flavor-space traces for the terms not involving $\tilde{M}_{\mu\nu}$. From (4.13) and (4.15), we find for the matrices M and $M^\dagger$ (with one upper and

one lower index each):

$$M = \begin{pmatrix} -\eta^{\mu\rho}m^\psi_{\rho\nu} & -\eta^{\mu\rho}m^{\psi\chi}_{\rho i} & 0 \\ \frac{1}{2}\mathcal{G}^{jk}m^{\psi\chi}_{\nu k} & \frac{1}{2}\mathcal{G}^{jk}m^\chi_{ki} & \frac{1}{2}\mathcal{G}^{jk}m^{\lambda\chi}_{bk} \\ 0 & \delta^{ac}m^{\lambda\chi}_{ci} & \delta^{ac}m^\lambda_{cb} \end{pmatrix}; \tag{4.40}$$

$$M^\dagger \equiv Z^{-1}(ZM)^\dagger = \begin{pmatrix} -\eta^{\mu\rho}m^{\dagger\psi}_{\rho\nu} & -\eta^{\mu\rho}m^{\dagger\psi\chi}_{\rho i} & 0 \\ \frac{1}{2}\mathcal{G}^{jk}m^{\dagger\psi\chi}_{\nu k} & \frac{1}{2}\mathcal{G}^{jk}m^{\dagger\chi}_{ki} & \frac{1}{2}\mathcal{G}^{jk}m^{\dagger\lambda\chi}_{bk} \\ 0 & \delta^{ac}m^{\dagger\lambda\chi}_{ci} & \delta^{ac}m^{\dagger\lambda}_{cb} \end{pmatrix}; \tag{4.41}$$

and from (4.28), we find that

$$\{\Gamma_\rho, M\} = 2\Gamma_\rho \begin{pmatrix} -\eta^{\mu\rho}m^\psi_{\rho\nu} & 0 & 0 \\ 0 & -\frac{1}{2}\mathcal{G}^{jk}m^\chi_{ki} & 0 \\ 0 & 0 & \delta^{ac}m^\lambda_{cb} \end{pmatrix}. \tag{4.42}$$

Of course, this last relation is just a reflection of the fact that since, for example, ψ_L transforms like $\bar{\psi}_R$ under the chiral $U(1)$, the mass m^ψ must transform by twice as much as ψ_R, but that, since χ_L transforms in the opposite sense from $\bar{\psi}_R$, the mass $m^{\psi\chi}$ does not transform. We also observe that the only nonzero components of the matrix $\{L_\rho, M\}$ are:

$$\begin{aligned}\{L_\rho, M\}^a_b &= (L_\rho)^c_b\delta^{ad}m^\lambda_{dc} + (L_\rho)^a_c\delta^{cd}m^\lambda_{db}, \\ \{L_\rho, M\}^j_b &= (L_\rho)^a_b(m^{\lambda\chi})^j_a, \\ \{L_\rho, M\}^a_i &= (m^{\lambda\chi})^j_a(L_\rho)^a_b.\end{aligned} \tag{4.43}$$

From $F^+_{\mu\nu} = F^-_{\mu\nu} = [d^\pm_\mu, d^\pm_\nu]$, we find:

$$\begin{aligned}(F^+_{\mu\nu})^\rho{}_\sigma &= ((\sqrt{\mathrm{Re}\, f})_{ab}(\tilde{F}^\rho{}_\mu)^b(\sqrt{\mathrm{Re}\, f})^a_c(\tilde{F}_{\sigma\nu})^c - (\mu \leftrightarrow \nu)) + \eta^\rho{}_\sigma J_{\mu\nu}; \\ (F^+_{\mu\nu})^i_j &= -(R_{\mu\nu})^i_j + ie(f_{\mu\nu})^i_j - \delta^i_j J_{\mu\nu};\end{aligned}$$

$$
\begin{aligned}
(F^+_{\mu\nu})^a_b &= ((\sqrt{\mathrm{Re}\, f})_{bd}(\tilde{F}^\rho{}_\mu)^d(\sqrt{\mathrm{Re}\, f})^a_c(\tilde{F}_{\rho\nu})^c - (\mu \leftrightarrow \nu)) \\
&\quad - ie(\tilde{F}_{\mu\nu})^a_b + \delta^a_b J_{\mu\nu} + (K_{\mu\nu})^a_b; \\
(F^+_{\mu\nu})^a_c &= (\delta^b_a \partial_\mu + i(L_\mu)^b_a - e\epsilon^b{}_{ac}\tilde{A}^c_\mu)(\sqrt{\mathrm{Re}\, f})_{bd}(\tilde{F}^\rho{}_\nu)^d - (\mu \leftrightarrow \nu); \\
(F^+_{\mu\nu})^a_\rho &= (\delta^a_b \partial_\mu - i(L_\mu)^a_b + e\epsilon^a{}_{bc}\tilde{A}^c_\mu)(\sqrt{\mathrm{Re}\, f})^b_d(\tilde{F}_{\rho\nu})^d - (\mu \leftrightarrow \nu); \qquad (4.44)
\end{aligned}
$$

where $R_{\mu\nu}$ and $f_{\mu\nu}$ are curvature terms, defined in terms of the scalar covariant derivative and the background gauge field by

$$
\tilde{\mathcal{D}}_\mu \tilde{\phi}^k \tilde{\mathcal{D}}_\nu \tilde{\phi}^l R^i{}_{jlk} + e(\tilde{F}_{\mu\nu})^i{}_j + e\Gamma^i{}_{kj}(\tilde{F}_{\mu\nu}\tilde{\phi})^k \equiv (R_{\mu\nu} + ef_{\mu\nu})^i{}_j, \qquad (4.45)
$$

and where $J_{\mu\nu} = i(\partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu)$ is the chiral $U(1)$ curvature defined in the previous chapter. The curvature term $K_{\mu\nu}$ is:

$$
(K_{\mu\nu})^a_b = i(\delta^a_c \partial_\mu - \epsilon^a{}_{cd}\tilde{A}^d_\mu)(L_\nu)^c_b - (\mu \leftrightarrow \nu). \qquad (4.46)
$$

It is useful to note that when f_{ab} is proportional to the unit matrix, as is the case for our toy model, this curvature term takes on a very simple form. We will make use of this simplification later on.

The traces are:

$$
\begin{aligned}
\mathrm{Tr}\, M^\dagger M &= 4(m^\psi)^2 + \frac{1}{4}\mathcal{G}^{il}\mathcal{G}^{jj} m^{\dagger\chi}_{lj} m^\chi_{ij} + \delta^{ac}\delta^{bd} m^{\dagger\lambda}_{ab} m^\lambda_{cd} \\
&\quad - \eta^{\mu\nu}\mathcal{G}^{il} m^{\dagger\psi\chi}_{\mu i} m^{\psi\chi}_{\nu i} + \delta^{ab}\mathcal{G}^{il} m^{\dagger\lambda\chi}_{al} m^{\lambda\chi}_{bi}; \qquad (4.47) \\
\mathrm{Tr}(M^\dagger M)^2 &= 4(m^\psi)^4 + \frac{1}{16}\mathcal{G}^{il}\mathcal{G}^{jj}\mathcal{G}^{kk}\mathcal{G}^{ll} m^{\dagger\chi}_{lj} m^\chi_{jk} m^{\dagger\chi}_{kl} m^\chi_{li} \\
&\quad + \delta^{ac}\delta^{bd}\delta^{ef}\delta^{gh} m^{\dagger\lambda}_{ab} m^\lambda_{de} m^{\dagger\lambda}_{fg} m^\lambda_{hc} - 2(m^\psi)^2 \eta^{\mu\nu}\mathcal{G}^{il} m^{\dagger\psi\chi}_{\mu i} m^{\psi\chi}_{\nu i} \\
&\quad - \frac{1}{2} m^\psi (\eta^{\mu\nu}\mathcal{G}^{il}\mathcal{G}^{jj} m^{\psi\chi}_{\nu i} m^{\dagger\chi}_{lj} m^{\psi\chi}_{\mu j} + H.\ c.) \\
&\quad + \frac{1}{2}\eta^{\mu\rho}\eta^{\mu'\rho'}\mathcal{G}^{il}\mathcal{G}^{jj} m^{\dagger\psi\chi}_{\rho i} m^{\psi\chi}_{\mu' i} m^{\dagger\psi\chi}_{\rho' j} m^{\psi\chi}_{\mu j} \\
&\quad + \frac{1}{2}\delta^{ab}\delta^{a'b'}\mathcal{G}^{il}\mathcal{G}^{jj} m^{\dagger\lambda\chi}_{bi} m^{\lambda\chi}_{a'i} m^{\dagger\lambda\chi}_{b'j} m^{\lambda\chi}_{aj} \\
&\quad - \frac{1}{2}\eta^{\mu\nu}\mathcal{G}^{il}\mathcal{G}^{jj}\mathcal{G}^{kk} m^{\dagger\psi\chi}_{\nu i} m^\chi_{ij} m^{\dagger\chi}_{jk} m^{\psi\chi}_{k\mu}
\end{aligned}
$$

$$
\begin{aligned}
&\quad - \eta^{\mu\nu}\mathcal{G}^{il}\mathcal{G}^{jj}\delta^{ab} m^{\dagger\psi\chi}_{\nu i} m^{\lambda\chi}_{bi} m^{\dagger\lambda\chi}_{aj} m^{\psi\chi}_{j\mu} \\
&\quad + \frac{1}{2}\delta^{ab}\mathcal{G}^{il}\mathcal{G}^{jj}\mathcal{G}^{kk} m^{\dagger\lambda\chi}_{bl} m^\chi_{ij} m^{\dagger\chi}_{jk} m^{\lambda\chi}_{ka} \\
&\quad + 2\delta^{ab}\delta^{cd}\delta^{ef}\mathcal{G}^{il} m^{\dagger\lambda}_{bc} m^{\lambda\chi}_{di} m^{\dagger\lambda\chi}_{el} m^\lambda_{fa} \\
&\quad + \frac{1}{2}(\delta^{ab}\delta^{cd}\mathcal{G}^{il}\mathcal{G}^{jj} m^{\dagger\lambda\chi}_{bl} m^\chi_{ij} m^{\dagger\lambda\chi}_{cj} m^\lambda_{da} + H.\ c.); \qquad (4.48) \\
\mathrm{Tr}\, D^-_\mu M^\dagger D^{+\mu} M &= 4 D^-_\mu m^\psi D^{+\mu} m^\psi + \frac{1}{4} D^-_\mu(\mathcal{G}^{il} m^{\dagger\chi}_{lj}) D^{+\mu}(\mathcal{G}^{jj} m^\chi_{ij}) \\
&\quad + \delta^{ac}\delta^{bd} D^-_\mu m^{\dagger\lambda}_{ab} D^{+\mu} m^\lambda_{cd} - \eta^{\nu\rho} D_\mu(\mathcal{G}^{il} m^{\dagger\psi\chi}_{\nu i}) D^\mu m^{\psi\chi}_{\rho i} \\
&\quad + \delta^{ab} D^-_\mu(\mathcal{G}^{il} m^{\dagger\lambda\chi}_{al}) D^{+\mu} m^{\lambda\chi}_{bi}; \qquad (4.49)
\end{aligned}
$$

where, by equations (4.26–4.28) and (4.40–4.42),

$$
\begin{aligned}
D^+_\mu m^\psi &= (\partial_\mu + 2i\Gamma_\mu) m^\psi, \\
D^+_\mu (m^\chi)^i_j &= \delta^i_k(\partial_\mu - 2i\Gamma_\mu)(m^\chi)^k_j + i(\Lambda^*_\mu)^i_k (m^\chi)^k_j + i(m^\chi)^i_k (\Lambda_\mu)^k_j, \qquad (4.50) \\
D^+_\mu (m^\lambda)^a_b &= \delta^c_b(\partial_\mu + 2i\Gamma_\mu)(m^\lambda)^a_c + i(\Lambda^*_\mu)^c_b (m^\lambda)^a_c + i(m^\lambda)^c_b (\Lambda_\mu)^a_c \qquad (4.51) \\
&\quad + i(L_\mu)^c_b (m^\lambda)^a_c + i(m^\lambda)^c_b (L_\mu)^a_c, \qquad (4.52)
\end{aligned}
$$

and

$$
D^+_\mu (m^{\lambda\chi})^a_i = \partial_\mu (m^{\lambda\chi})^a_i + i(m^{\lambda\chi})^b_i (L_\mu)^a_b + i(m^{\lambda\chi})^a_k (\Lambda_\mu)^k_i, \qquad (4.53)
$$

and the D_μ's in the second to last term of (4.49) are just the appropriate gauge- and reparametrization-covariant derivatives, since there are no chiral $U(1)$ or L_μ connection terms. The first term in each of equations (4.47–4.49) is just the pure gravitino contribution, as given by equations (3.60) and (3.61). Finally, from (4.44), it is possible to find the field-strength dependent traces needed to evaluate (4.32) and the subsequent modifications explicitly. However, the general expressions are rather cumbersome (even by the standards of the present work), and not particularly illuminating. Instead, we give the results for $f_{ab} = s\delta_{ab}$, as is appropriate for our toy model. In this case we find, after setting $(K_{\mu\nu})^a_b = \delta^a_b K_{\mu\nu}$ and $(L_\mu)^a_b = \delta^a_b L_\mu$,

the following traces:

$$(F^+_{\mu\nu})^\rho{}_\sigma(F^{+\mu\nu})^\sigma{}_\rho = 4J_{\mu\nu}J^{\mu\nu} - 2(\mathrm{Re}\, s)^2(\tilde{F}^\rho{}_\mu)^a(\tilde{F}_{\sigma\nu})_a((\tilde{F}^\sigma{}_\mu)^b(\tilde{F}_{\rho\nu})_b - (\mu \leftrightarrow \nu)), \tag{4.54}$$

$$\begin{aligned}(F^+_{\mu\nu})^i_j(F^{+\mu\nu})^j_i &= (R_{\mu\nu})^i_j(R^{\mu\nu})^j_i - 2ie(R_{\mu\nu})^i_j(f^{\mu\nu})^j_i + (N+2)J_{\mu\nu}J^{\mu\nu} \\ &\quad + 2J_{\mu\nu}(R^{\mu\nu} - ief^{\mu\nu})^i_i.\end{aligned} \tag{4.55}$$

$$\begin{aligned}(F^+_{\mu\nu})^a_b(F^{+\mu\nu})^b_a &= N_G(J_{\mu\nu} + K_{\mu\nu})(J^{\mu\nu} + K^{\mu\nu}) - e^2(\tilde{F}_{\mu\nu})^a_b(\tilde{F}^{\mu\nu})^b_a \\ &\quad - 2(\mathrm{Re}\, s)^2(\tilde{F}^\rho{}_\mu)^b(\tilde{F}_{\rho\nu})_a((\tilde{F}^{\sigma\mu})^a(\tilde{F}^\nu_\sigma)_b - (\mu \leftrightarrow \nu)) \\ &\quad - 4ie\,\mathrm{Re}\, s(\tilde{F}^\rho{}_\mu)_b(\tilde{F}_{\rho\nu})^a(\tilde{F}^{\mu\nu})^a_b \\ &\quad + 4(J^{\mu\nu} + K^{\mu\nu})\,\mathrm{Re}\, s(\tilde{F}^\rho{}_\mu)_a(\tilde{F}_{\rho\nu})^a,\end{aligned} \tag{4.56}$$

$$\begin{aligned}(F^+_{\mu\nu})^\rho_\sigma(F^{+\mu\nu})^\sigma_\rho &= 2(\tilde{\mathcal{D}}^\mu\sqrt{\mathrm{Re}\, s}\tilde{F}^{\rho\nu})_a(\tilde{\mathcal{D}}_\mu\sqrt{\mathrm{Re}\, s}\tilde{F}_{\rho\nu})^a \\ &\quad - 2(\tilde{\mathcal{D}}^\nu\sqrt{\mathrm{Re}\, s}\tilde{F}^{\rho\mu})_a(\tilde{\mathcal{D}}^\mu\sqrt{\mathrm{Re}\, s}\tilde{F}_{\rho\nu})^a \\ &\quad + 2L_\mu L^\mu\,\mathrm{Re}\, s(\tilde{F}^{\rho\nu})^a(\tilde{F}_{\rho\nu})_a - 2L_\mu L^\nu\,\mathrm{Re}\, s(\tilde{F}^{\rho\mu})^a(\tilde{F}_{\rho\nu})_a,\end{aligned} \tag{4.57}$$

where in writing the last expression we have used the fact that the gauge-covariant derivative $\tilde{\mathcal{D}}_\mu$ automatically accounts for the gauge-field dependent connections given explicitly in the last two lines of (4.44). Of course, this last trace and the trace $(F^+_{\mu\nu})^\sigma_\rho(F^{+\mu\nu})^\rho_\sigma$ are identical.

We now return to the auxiliary field whose one-loop contribution is, from equation (4.19),

$$\mathcal{S}^1_{\mathrm{aux}} = -\frac{i}{2}\,\mathrm{Tr}\ln(\gamma_\mu\Delta^{\mu\nu}_\theta\gamma_\nu). \tag{4.58}$$

To evaluate this using the covariant derivative expansion technique we will need to use the following substitution rules: for a background-covariant derivative operator

d_μ, we take

$$\begin{aligned}d_\mu &\to ip_\mu + i\hat{G}_{\nu\mu}\frac{\partial}{\partial p_\nu} \\ &\equiv ip_\mu - \sum_{n=1}^{\infty}\frac{n}{(n+1)!}(d_{\mu_1}\cdots d_{\mu_n}G_{\nu\mu})\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}},\end{aligned} \tag{4.59}$$

with $G_{\mu\nu} = [d_\mu, d_\nu]$, and for a matrix-valued function of spacetime coordinates $F(x)$, we take

$$F \to \hat{F} = \sum_{n=1}^{\infty}\frac{1}{n!}(d_{\mu_1}\cdots d_{\mu_n}F)\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}}. \tag{4.60}$$

This is similar to the procedure we used in Chapter 2, where we made the analogous transformations (2.18) and (2.16). However, we must be careful, because, for example, $d_\mu = \partial_\mu + i\Gamma_\mu\gamma_5$ does not commute with γ^μ. This will cause problems, since in order to use this substitution rule we need to bring an operator $e^{-id(x)\cdot\partial_p}$, applied from the left on the argument of the log in equation (4.58), through any γ-matrices to act on quantities such as the momentum and the masses. Even though moving d_μ through a γ^ν only changes the sign on some of the connections, a nice way of finding the appropriate substitution rule is by noting that d_μ commutes with $\gamma^0\gamma^\nu$. Then, since $\ln\gamma^0\gamma^0 = \ln 1 = 0$, we can write

$$\mathrm{Tr}\ln\gamma_\mu\Delta^{\mu\nu}_\theta\gamma_\nu = \mathrm{Tr}\ln\gamma^0\gamma_\mu((\gamma^0\Delta^{-1}_\theta)^{-1})^{\mu\nu}\gamma^0\gamma_\nu. \tag{4.61}$$

Now, from (4.16), we see immediately that $\gamma^0\Delta^{-1}_\theta$ contains terms like $\gamma^0\not{d} = \gamma^0\gamma^\mu d_\mu$ and $\gamma^0 M$. Our operator $e^{-id(x)\cdot\partial_p}$ can move through all the $\gamma^0\gamma^\nu$ pairs freely so that the first part of the rule, equation (4.59), remains unaltered. However, the second part, equation (4.60), must be applied not to M but rather to $\gamma^0 M$. Now, after the shift, we can get rid of all the γ^0's by introducing yet another γ^0, as we did in (4.61). The result of all of this is to evaluate (4.58) by using the substitution rule (4.59) for the covariant derivatives, but instead of (4.60) for the masses we use

$$M \to \hat{M} = \gamma^0(\gamma^0 M - i(d_{\mu_1}\gamma^0 M)\frac{\partial}{\partial p_{\mu_1}} - \frac{1}{2}(d_{\mu_1}d_{\mu_2}\gamma^0 M)\frac{\partial^2}{\partial p_{\mu_1}\partial p_{\mu_2}} + \cdots). \tag{4.62}$$

This just amounts to changing the sign on, for example, all the chiral connections in the covariant derivatives acting on M.

From (4.16) we find the expansion

$$\begin{aligned}\Delta_\theta^{\mu\nu} = {} & \Delta_\psi^{\mu\nu} + \Delta_\psi^{\mu\sigma} M_{\sigma l}^{\psi\chi} \Delta_\chi^{lj} M_{j\rho}^{\psi\chi} \Delta_\psi^{\rho\nu} + \Delta_\psi^{\mu\sigma} M_{\sigma l}^{\psi\chi} \Delta_\chi^{lj} M_{j\alpha}^{\psi\chi} \Delta_\psi^{\alpha\beta} M_{\beta\kappa}^{\psi\chi} \Delta_\chi^{\kappa L} M_{L\rho}^{\psi\chi} \Delta_\psi^{\rho\nu} \\ & + \Delta_\psi^{\mu\sigma} M_{\sigma l}^{\psi\chi} \Delta_\chi^{lj} M_{j\alpha}^{\chi\lambda} \Delta_\lambda^{ab} M_{b\kappa}^{\chi\lambda} \Delta_\chi^{\kappa L} M_{L\rho}^{\psi\chi} \Delta_\psi^{\rho\nu} - \Delta_\psi^{\mu\sigma} \not{F}_{\sigma a}^{\psi\lambda} \Delta_\lambda^{ab} \not{F}_{b\rho}^{\psi\lambda} \Delta_\psi^{\rho\nu} \\ & + i\Delta_\psi^{\mu\sigma} M_{\sigma l}^{\psi\chi} \Delta_\chi^{lj} M_{j\alpha}^{\chi\lambda} \Delta_\lambda^{ab} \not{F}_{b\rho}^{\psi\lambda} \Delta_\psi^{\rho\nu} + i\Delta_\psi^{\mu\sigma} \not{F}_{\sigma a}^{\psi\lambda} \Delta_\lambda^{ab} M_{b\kappa}^{\chi\lambda} \Delta_\chi^{\kappa L} M_{L\rho}^{\psi\chi} \Delta_\psi^{\rho\nu} \\ & - \Delta_\psi^{\mu\sigma} M_{\sigma l}^{\psi\chi} \Delta_\chi^{lj} M_{j\alpha}^{\psi\chi} \Delta_\psi^{\alpha\beta} \not{F}_{\beta a}^{\psi\lambda} \Delta_\lambda^{ab} \not{F}_{b\rho}^{\psi\lambda} \Delta_\psi^{\rho\nu} \\ & - \Delta_\psi^{\mu\sigma} \not{F}_{\sigma a}^{\psi\lambda} \Delta_\lambda^{ab} \not{F}_{b\alpha}^{\psi\lambda} \Delta_\psi^{\alpha\beta} M_{\beta\kappa}^{\psi\chi} \Delta_\chi^{\kappa L} M_{L\rho}^{\psi\chi} \Delta_\psi^{\rho\nu} \\ & - \Delta_\psi^{\mu\sigma} \not{F}_{\sigma a}^{\psi\lambda} \Delta_\lambda^{ab} M_{bl}^{\chi\lambda} \Delta_\chi^{lj} M_{jc}^{\chi\lambda} \Delta_\lambda^{cd} \not{F}_{d\rho}^{\psi\lambda} \Delta_\psi^{\rho\nu} \\ & + \Delta_\psi^{\mu\sigma} \not{F}_{\sigma a}^{\psi\lambda} \Delta_\lambda^{ab} \not{F}_{b\alpha}^{\psi\lambda} \Delta_\psi^{\alpha\beta} \not{F}_{\beta c}^{\psi\lambda} \Delta_\lambda^{cd} \not{F}_{d\rho}^{\psi\lambda} \Delta_\psi^{\rho\nu} + O(\partial^7/\partial x^7). \end{aligned} \tag{4.63}$$

After the substitution rule, now given by equations (4.59) and (4.62), is used,

$$(\Delta_\psi^{-1})_{\mu\nu} = -\eta_{\mu\nu}(i\not{\partial} - M_\psi) \to \eta_{\mu\nu}(\not{p} - \tilde{C}), \tag{4.64}$$

where

$$\tilde{C} = \hat{M}_\psi - \gamma^\mu (\tilde{G}_\psi)_{\nu\mu} \frac{\partial}{\partial p_\nu} \equiv \hat{M}_\psi - \tilde{\not{\Phi}}_\psi, \tag{4.65}$$

and

$$(G_\psi)_{\mu\nu} \equiv i(\partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu)\gamma_5 = J_{\mu\nu}\gamma_5, \tag{4.66}$$

with $(\tilde{G}_\psi)_{\mu\nu}$ as in (4.59)—this is just like $\tilde{J}_{\mu\nu}$ in Chapter 3—and $\hat{M}_\psi$ as in (4.60). In addition, since $\not{F}^{\psi\lambda}$ is part of a covariant derivative, its substitution rule is

$$i\not{F}^{\psi\lambda} \to -\not{\Phi}^{\psi\lambda}, \tag{4.67}$$

which contains at least one p derivative, so that the last four terms of (4.63) give only finite corrections and can be ignored. We then have the expansion

$$\begin{aligned}\gamma_\mu \Delta_\psi^{\mu\nu} \gamma_\nu &= \gamma_\mu \frac{1}{\not{p}} \gamma^\mu + \gamma_\mu \frac{1}{\not{p}} \Big(\tilde{C} + \tilde{C} \frac{1}{\not{p}} \tilde{C} + \cdots \Big) \frac{1}{\not{p}} \gamma^\mu \\ &= -\frac{2}{\not{p}} + \gamma_\mu \frac{1}{\not{p}} K \frac{1}{\not{p}} \gamma^\mu, \end{aligned} \tag{4.68}$$

where

$$K \equiv \tilde{C} + \tilde{C} \frac{1}{\not{p}} \tilde{C} + \tilde{C} \frac{1}{\not{p}} \tilde{C} \frac{1}{\not{p}} \tilde{C} + \cdots . \tag{4.69}$$

We also need to expand the last three terms in (4.63). The last two terms, being of order M^4, will yield only log-divergent corrections. The second term, being only of order M^2, will give both quadratically and logarithmically divergent corrections. Again, after using the substitution rule, remembering that $(\Delta_\chi^{-1})_{lj} = Z_{lK}(\Delta_\chi^{-1})_j^K$,

$$(\Delta_\chi^{-1})_j^l = (i\not{\partial}_j^l - (M_\chi)_j^l) \to -(\not{p}\delta_j^l - (\hat{M}_\chi)_j^l + \tilde{\not{\Phi}}_{\chi j}^l), \tag{4.70}$$

where again $(\tilde{Q}_\chi)_j^l$ and $(\hat{M}_\chi)_j^l = Z^{lK}\widehat{M}_{Kj}^\chi$ are as specified by the substitution rule. We find, up to log-divergent terms,

$$\begin{aligned}\gamma_\mu \Delta_\theta^{\mu\nu} \gamma_\nu = {} & -\frac{2}{\not{p}} + \gamma_\mu \frac{1}{\not{p}} K \frac{1}{\not{p}} \gamma^\mu \\ & - \gamma^\sigma \Big(\frac{1}{\not{p}} + \frac{1}{\not{p}} K \frac{1}{\not{p}} \Big) \hat{M}_{\sigma l}^{\psi\chi} \Big(\frac{1}{\not{p}} + \frac{1}{\not{p}} (\hat{M}^\chi - \tilde{\not{\Phi}}^\chi + \hat{M}^\chi \frac{1}{\not{p}} \hat{M}^\chi) \frac{1}{\not{p}} \Big)_\kappa^l \\ & \qquad \times Z^{\kappa j} \widehat{M}_{j\rho}^{\psi\chi} \Big(\frac{1}{\not{p}} + \frac{1}{\not{p}} K \frac{1}{\not{p}} \Big) \gamma^\rho \\ & + \gamma^\sigma \frac{1}{\not{p}} M_{\sigma l}^{\psi\chi} \frac{Z^{lj}}{\not{p}} \Big(M_{j\mu}^{\psi\chi} \frac{\eta^{\mu\nu}}{\not{p}} M_{\nu\kappa}^{\psi\chi} - M_{j\alpha}^{\chi\lambda} \frac{\delta^{ab}}{\not{p}} M_{b\kappa}^{\chi\lambda} \Big) \frac{Z^{\kappa L}}{\not{p}} M_{L\rho}^{\psi\chi} \frac{1}{\not{p}} \gamma^\rho . \end{aligned} \tag{4.71}$$

We expand $\Delta_\theta^{\mu\nu}$ up to $O(M^4)$ and $O(\partial^4/\partial x^4)$. Dropping terms that cannot contribute to the effective action due to their Lorentz transformation properties, we find, for (4.71), that

$$\begin{aligned}\gamma_\mu \Delta_\theta^{\mu\nu} \gamma_\nu = {} & -\frac{2}{\not{p}} + \gamma_\mu \frac{1}{\not{p}} K \frac{1}{\not{p}} \gamma^\mu + \mathcal{M}_4 + \mathcal{M}_3 + \mathcal{M}_2 \\ & + \mathcal{N}_4 + \mathcal{N}_3 + \mathcal{N}_2 + \mathcal{N}_2' + \mathcal{F}_1 + \mathcal{F}_1' + \mathcal{F}_2 + \cdots , \end{aligned} \tag{4.72}$$

where

$$\begin{aligned}\gamma_\mu \frac{1}{\not{p}} K \frac{1}{\not{p}} \gamma^\mu = {} & \frac{4M_\psi}{p^2} - \frac{2M_\psi^2}{\not{p}p^2} + \frac{4M_\psi^3}{p^4} - \frac{2M_\psi^4}{\not{p}p^6} \\ & + \gamma_\mu \frac{1}{\not{p}} (N_\psi - \tilde{\not{\Phi}}_\psi) \frac{1}{\not{p}} \gamma^\mu - \frac{2M_\psi}{p^2} \gamma^0 N_\psi \gamma^0 \frac{1}{\not{p}} \\ & - \frac{2M_\psi}{\not{p}} N_\psi \frac{1}{p^2} + \gamma_\mu \frac{1}{\not{p}} N_\psi \frac{1}{\not{p}} N_\psi \frac{1}{\not{p}} \gamma^\mu + \gamma_\mu \frac{1}{\not{p}} \tilde{\not{\Phi}}_\psi \frac{1}{\not{p}} \tilde{\not{\Phi}}_\psi \frac{1}{\not{p}} \gamma^\mu , \end{aligned} \tag{4.73}$$

and

$$\mathcal{M}_4 = \frac{1}{p^4}\gamma^\sigma\frac{1}{\not p}\gamma^\rho\Big(M^{\psi\chi}_{\sigma I}Z^{IJ}M^{\psi\chi}_{J\mu}\eta^{\mu\nu}M^{\psi\chi}_{\nu K}Z^{KL}M^{\psi\chi}_{L\rho}$$
$$- M^{\psi\chi}_{\sigma I}Z^{IJ}M^{\chi\lambda}_{Ja}\delta^{ab}M^{\chi\lambda}_{bK}Z^{KL}M^{\psi\chi}_{L\rho}$$
$$- M^{\psi\chi}_{\sigma I}Z^{IL}M^{\chi}_{LK}Z^{MK}M^{\chi}_{MJ}Z^{JN}M^{\psi\chi}_{N\rho} - 2M^2_\psi M^{\psi\chi}_{\sigma J}Z^{JK}M^{\psi\chi}_{K\rho}$$
$$- M^2_\psi M^{\psi\chi}_{\sigma J}Z^{JK}M^{\psi\chi}_{K\rho} - M_\psi M^{\psi\chi}_{\sigma I}Z^{IK}M^{\chi}_{KJ}Z^{JL}M^{\psi\chi}_{L\rho}$$
$$- M_\psi M^{\psi\chi}_{\sigma I}Z^{IK}M^{\chi}_{KJ}Z^{JL}M^{\psi\chi}_{L\rho}\Big), \tag{4.74}$$

$$\mathcal{M}_3 = -\frac{1}{p^4}\gamma^\sigma\gamma^\rho\Big(M^{\psi\chi}_{\sigma I}Z^{IK}M^{\chi}_{KJ}Z^{JL}M^{\psi\chi}_{L\rho} + M_\psi M^{\psi\chi}_{\sigma I}Z^{IJ}M^{\psi\chi}_{J\rho}$$
$$+ M_\psi M^{\psi\chi}_{\sigma I}Z^{IJ}M^{\psi\chi}_{J\rho}\Big), \tag{4.75}$$

$$\mathcal{M}_2 = -\frac{1}{p^2}\gamma^\sigma\frac{1}{\not p}\gamma^\rho M^{\psi\chi}_{\sigma I}Z^{IJ}M^{\psi\chi}_{J\rho}, \tag{4.76}$$

and

$$\mathcal{N}_4 = -\gamma^\sigma\frac{1}{\not p}N_{\sigma J}\frac{1}{\not p}N'_\rho\frac{1}{\not p}\gamma^\rho, \tag{4.77}$$

$$\mathcal{N}_3 = -\gamma^\sigma\frac{1}{\not p}N_{\sigma I}\frac{1}{\not p}Z^{IJ}M^{\psi\chi}_{J\rho}\frac{1}{\not p}\gamma^\rho - \gamma^\sigma\frac{1}{\not p}M_{\sigma I}\frac{1}{\not p}N'_\rho\frac{1}{\not p}\gamma^\rho, \tag{4.78}$$

$$\mathcal{N}_2 = M^{\psi\chi}_{\sigma I}M^{\psi\chi}_{\rho J}Z^{IJ}\gamma\Big(\frac{1}{\not p}\tilde{\Phi}_\psi\frac{\not p}{p^4} + \frac{\not p}{p^4}\tilde{\Phi}_\psi\frac{1}{\not p}\Big)\gamma^\rho, \tag{4.79}$$

$$\mathcal{N}'_2 = M^{\psi\chi}_{\sigma I}M^{\psi\chi}_{\rho J}\gamma^\sigma\frac{1}{p^2}(\tilde{\Phi}^\chi)^{IJ}\frac{1}{p^2}\gamma^\rho, \tag{4.80}$$

with

$$N_\psi = \hat{M}_\psi - M_\psi,$$
$$N_{\sigma J} = (\hat{M}^{\psi\chi} - M^{\psi\chi})_{\sigma J}, \tag{4.81}$$
$$N'_\sigma = (\hat{M}^{\psi\chi} - M^{\psi\chi})'_\sigma,$$

and also

$$\mathcal{F}_1 = -\gamma^\mu\frac{1}{\not p}M^{\psi\chi}_{\mu I}\frac{1}{\not p}(M^{\chi\lambda})^{Ia}\frac{1}{\not p}\tilde{\Phi}^{\psi\lambda}_{a\nu}\frac{1}{\not p}\gamma^\nu, \tag{4.82}$$

$$\mathcal{F}'_1 = -\gamma^\mu\frac{1}{\not p}\tilde{\Phi}^{\psi\lambda}_{a\mu}\frac{1}{\not p}(M^{\chi\lambda})^{Ia}\frac{1}{\not p}M^{\psi\chi}_{\nu I}\frac{1}{\not p}\gamma^\nu, \tag{4.83}$$

$$\mathcal{F}_2 = \gamma^\mu\frac{1}{\not p}\tilde{\Phi}^{\psi\lambda}_{a\mu}\frac{\delta^{ab}}{\not p}\tilde{\Phi}^{\psi\lambda}_{b\nu}\frac{1}{\not p}\gamma^\nu. \tag{4.84}$$

Here, we have used $\gamma_\mu M = M^\dagger\gamma_\mu$, and the fact that M_ψ is real. To evaluate the divergent corrections that we are interested in, we write (4.72) as

$$\gamma_\nu\Delta^{\mu\nu}_\theta\gamma_\mu = -\frac{2}{\not p}\Big(1 - \frac{1}{2}\not p\gamma_\mu\frac{1}{\not p}K\frac{1}{\not p}\gamma^\mu - \frac{1}{2}\not p(\mathcal{M}_4 + \mathcal{M}_3 + \mathcal{M}_2$$
$$+ \mathcal{N}_4 + \mathcal{N}_3 + \mathcal{N}_2 + \mathcal{N}'_2 + \mathcal{F}_1 + \mathcal{F}'_1 + \mathcal{F}_2 + \cdots)\Big) \tag{4.85}$$
$$\equiv -\frac{2}{\not p}(1 + X),$$

and then use the Hausdorff expansion,

$$\ln AB = \ln A + \ln B + \frac{1}{2}[\ln A, \ln B] + \frac{1}{12}[\ln A, [\ln A, \ln B]]$$
$$+ \frac{1}{12}[\ln B, [\ln B, \ln A]] + \cdots, \tag{4.86}$$

and the identity $\ln(1/\not p) = -\frac{1}{2}\ln p^2$, to find

$$\mathrm{Tr}\ln\gamma_\mu\Delta^{\mu\nu}_\theta\gamma_\nu = \mathrm{Tr}\ln(-2) - \frac{1}{2}\mathrm{Tr}\ln p^2 + \mathrm{Tr}\ln(1 + X)$$
$$- \frac{1}{4}\mathrm{Tr}[\ln p^2, \ln(1 + X)] + \cdots. \tag{4.87}$$

The field-independent pieces are the same as those in the pure gravitino calculation, equation (3.71). Since they cancel with the field-independent graviton sector contribution [20], we will now drop them. Furthermore, we use the identity

$$[\ln p^2, X] = \frac{1}{p^2}[p^2, X] + \frac{1}{2p^4}[p^2, [p^2, X]] + \cdots, \tag{4.88}$$

and the fact that only the $\mathcal{N}$ and $\mathcal{F}$ pieces contribute to the commutator terms (because they have momentum derivatives) to find the relevant terms in (4.87). The terms that give divergent contributions are:

$$\ln\gamma_\mu\Delta^{\mu\nu}_\theta\gamma_\nu = -\frac{1}{2}\not p(\mathcal{M}_4 + \mathcal{M}_2 + \mathcal{N}_4 + \mathcal{N}_3 + \mathcal{N}_2 + \mathcal{N}'_2 + \mathcal{F}_1 + \mathcal{F}'_1 + \mathcal{F}_2)$$
$$- \frac{1}{2}M_\psi\{\frac{1}{\not p}, \not p\mathcal{M}_3\} + \frac{1}{2\not p}M^2_\psi\mathcal{M}_2$$
$$- \frac{1}{8}(\not p\mathcal{M}_2)^2 - \frac{2}{3}M^2_\psi(\frac{2}{\not p}\mathcal{M}_2 + \mathcal{M}_2\frac{1}{\not p})$$
$$+ \frac{\not p}{8p^2}[p^2, (\mathcal{N}_4 + \mathcal{N}_3 + \mathcal{N}_2 + \mathcal{N}'_2 + \mathcal{F}_1 + \mathcal{F}'_1 + \mathcal{F}_2)]$$
$$+ \frac{5\not p}{96p^4}[p^2, [p^2, (\mathcal{N}_4 + \mathcal{N}_3 + \mathcal{F}_2)]]. \tag{4.89}$$

Here we have dropped terms that depend only on K, corrections from which have already been computed in the previous chapter. Some algebra shows that

$$\mathcal{N}_4 \ni \frac{1}{p^4}(\gamma^0 d_\mu \gamma^0 M^{\psi\chi})_{\sigma\rho}(d_\nu M^{\dagger\psi\chi})'_\rho \gamma^\sigma \frac{1}{\not p}\Big(\gamma^\mu\gamma^\nu + 8\frac{p^\mu p^\nu}{p^2} - \frac{4}{\not p}\gamma^\nu p^\mu - 2\eta^{\mu\nu}\Big)\gamma^\rho, \quad (4.90)$$

$$\frac{1}{p^2}[p^2,\mathcal{N}_4] \ni \frac{1}{p^4}(\gamma^0 d_\mu \gamma^0 M^{\psi\chi})_{\sigma\rho}(d_\nu M^{\dagger\psi\chi})'_\rho \gamma^\sigma \frac{1}{\not p}\Big(8\frac{p^\mu p^\nu}{p^2} - \frac{2}{\not p}\gamma^\nu p^\mu - 2\eta^{\mu\nu}\Big)\gamma^\rho, \quad (4.91)$$

$$\frac{1}{p^4}[p^2,[p^2,\mathcal{N}_4]] \ni \frac{1}{p^4}(\gamma^0 d_\mu \gamma^0 M^{\psi\chi})_{\sigma\rho}(d_\nu M^{\dagger\psi\chi})'_\rho \gamma^\sigma \frac{8}{\not p}\frac{p^\mu p^\nu}{p^2}\gamma^\rho, \quad (4.92)$$

and

$$\begin{aligned}\{\mathcal{N}_3 \text{ and } \frac{1}{p^2}[p^2,\mathcal{N}_3]\} \ni{}& \frac{1}{2}M^{\dagger\psi\chi}_{\rho\sigma}(\gamma^0 d_\mu d_\nu \gamma^0 M)'_\sigma \gamma^\sigma \frac{\not p}{p^6}\Big(8\frac{p^\mu p^\nu}{p^2} - 2\eta^{\mu\nu}\Big)\gamma^\rho \\ &+ \frac{1}{2}M^{\psi\chi}_{\rho\sigma}(d_\mu d_\nu M^\dagger)'_\rho \gamma^\sigma \frac{1}{p^6}\Big(8\frac{p^\mu p^\nu}{\not p} - 2\eta^{\mu\nu}\not p - 2\gamma^\nu p^\mu - 2\gamma^\mu p^\nu\Big)\gamma^\rho\end{aligned} \quad (4.93)$$

$$\begin{aligned}\frac{1}{p^4}[p^2,[p^2,\mathcal{N}_3]] \ni{}& \frac{1}{2}M^{\dagger\psi\chi}_{\rho\sigma}(\gamma^0 d_\mu d_\nu \gamma^0 M)'_\sigma \gamma^\sigma \frac{\not p}{p^6}\Big(8\frac{p^\mu p^\nu}{p^2}\Big)\gamma^\rho \\ &+ \frac{1}{2}M^{\psi\chi}_{\rho\sigma}(d_\mu d_\nu M^\dagger)'_\rho \gamma^\sigma \frac{1}{p^6}\Big(8\frac{p^\mu p^\nu}{\not p}\Big)\gamma^\rho,\end{aligned} \quad (4.94)$$

and

$$\mathcal{N}_2 \ni M^{\psi\chi}_{\sigma\tau} M^{\dagger\psi\chi}_{\rho\tau} Z'' \gamma^\sigma G^\psi_{\mu\nu}\Big(\frac{1}{\not p}\gamma^\mu\gamma^\nu\frac{1}{p^4} - \frac{3}{\not p}\gamma^\mu\frac{p^\nu \not p}{p^6}\Big)\gamma^\rho, \quad (4.95)$$

$$\frac{1}{p^2}[p^2,\mathcal{N}_2] \ni 5M^{\psi\chi}_{\sigma\tau} M^{\dagger\psi\chi}_{\rho\tau} Z'' \gamma^\sigma G^\psi_{\mu\nu}\frac{1}{\not p}\gamma^\mu\frac{p^\nu \not p}{p^6}\gamma^\rho, \quad (4.96)$$

and also

$$\mathcal{N}_2' \text{ and } \frac{1}{p^2}[p^2,\mathcal{N}_2'] \ni -M^{\psi\chi}_{\sigma\tau} M^{\dagger\psi\chi}_{\rho\tau}\gamma^\sigma (G''_\chi)_{\mu\nu}\gamma^\mu p^\nu \frac{1}{p^6}\gamma^\rho, \quad (4.97)$$

and finally

$$\begin{aligned}\mathcal{F}_1 - \frac{1}{4p^2}[p^2,\mathcal{F}_1] &\ni \frac{1}{2}M^{\psi\chi}_{\sigma\tau}(M^{\dagger\chi\lambda})'^{\alpha}(G^{\chi\lambda}_{\mu\nu})_{\alpha\sigma}\frac{\gamma^\rho}{p^4}\Big(\frac{1}{\not p}\gamma^\mu\gamma^\nu - \frac{3}{2\not p}\gamma^\mu\frac{1}{\not p}p^\nu\Big)\gamma^\rho, \\ \mathcal{F}_1' - \frac{1}{4p^2}[p^2,\mathcal{F}_1'] &\ni \frac{1}{2}(G^{\chi\lambda}_{\mu\nu})_{\alpha\rho}(M^{\chi\lambda})'^{\alpha}M^{\dagger\psi\chi}_{\sigma\tau}\frac{\gamma^\rho}{p^4}\Big(\frac{1}{\not p}\gamma^\mu\gamma^\nu - \frac{7}{2\not p}\gamma^\mu\frac{1}{\not p}p^\nu\Big)\gamma^\sigma,\end{aligned} \quad (4.98)$$

and

$$\begin{aligned}\mathcal{F}_2 - \frac{1}{4p^2}[p^2,\mathcal{F}_2] - \frac{5}{48p^2}[p^2,[p^2,\mathcal{F}_2]] \ni{}& \\ (G^{\chi\lambda}_{\mu\nu})_{\alpha\rho}(G^{\chi\lambda}_{\alpha\beta})^{\sigma}\frac{\gamma^\rho}{p^6}\not p\gamma^\mu\Big(\gamma^\nu\gamma^\alpha\gamma^\beta - \frac{3}{2\not p}\gamma^\alpha\gamma^\nu p^\beta - \frac{7}{2\not p}\gamma^\alpha\gamma^\beta p^\nu - \frac{3}{2}\gamma^\nu\gamma^\alpha\frac{1}{\not p}p^\beta \\ - \frac{3}{2}\eta^{\nu\beta}\frac{1}{\not p}\gamma^\alpha\not p + \frac{49}{6}\frac{1}{\not p}\gamma^\alpha\frac{1}{\not p}p^\nu p^\beta\Big)\Big)\gamma^\sigma,\end{aligned} \quad (4.99)$$

where we have dropped total momentum derivatives. The $p^{-4}[p^2,[p^2,\mathcal{A}]]$ terms, where $\mathcal{A} \in \{\mathcal{N}_2,\mathcal{N}_2',\mathcal{F}_1,\mathcal{F}_1'\}$, do not contribute to either the $O(\mu^2)$ or the $O(\ln\mu^2)$ effective action. Also, the derivation of the expressions involving $\mathcal{N}_2$ is complicated by the fact that $G^\psi_{\mu\nu}$ contains a γ_5.

To evaluate the momentum integrals we again follow reference [27]. The regulated integrals that we require are (after a Wick rotation):

$$\begin{aligned}&i\int\frac{d^4p}{(2\pi)^4}\frac{p_{\mu_1}\cdots p_{\mu_{2l}}A^{\mu\cdots\mu_{2l}}}{p^{2(l+k)}} \\ &\longrightarrow \begin{cases}\eta_{\mu_1\mu_2}\cdots\eta_{\mu_{(2l-1)}\mu_{2l}}A^{\mu\cdots\mu_{2l}}\dfrac{(2l)!}{(l!2^l)^2}\Big(\dfrac{2\mu^2\ln 2}{16\pi^2}\Big), & k=1; \\ \eta_{\mu_1\mu_2}\cdots\eta_{\mu_{(2l-1)}\mu_{2l}}A^{\mu_1\cdots\mu_{2l}}\dfrac{(2l)!}{l!(l+1)!2^{2l}}\Big(\dfrac{-\ln(\mu^2/2\mu_0^2)}{16\pi^2}\Big), & k=2;\end{cases}\end{aligned} \quad (4.100)$$

where $A^{\mu_1\cdots\mu_2}$ is understood to be totally symmetric in its indices.

It is now straightforward, if tedious, to obtain the divergent corrections. The only quadratically divergent correction arises from the $\mathcal{M}_2$ term. We find that

$$i\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\Big(-\frac{1}{2}\not p\mathcal{M}_2\Big) = -2\Big(\frac{2\mu^2\ln 2}{16\pi^2}\Big)\mathrm{Tr}(m^{\dagger\psi\chi}m^{\psi\chi}). \quad (4.101)$$

The $O(M^4)$ log-divergent corrections are

$$\begin{aligned}i\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\Big(-\frac{1}{2}\not p\mathcal{M}_4\Big) = {}& \\ -\Big(\frac{\ln(\mu^2/2\mu_0^2)}{16\pi^2}\Big)\mathrm{Tr}\Big(&m^{\dagger\psi\chi}m^{\psi\chi}m^{\dagger\psi\chi}m^{\psi\chi} - m^{\dagger\psi\chi}m^{\chi\lambda}m^{\dagger\chi\lambda}m^{\psi\chi} \\ &- m^{\dagger\psi\chi}m^{\chi}m^{\dagger\chi}m^{\psi\chi} - 3m_\psi^2 m^{\dagger\psi\chi}m^{\psi\chi} - m_\psi(m^{\dagger\psi\chi}m^{\chi}m^{\dagger\psi\chi} + H.c.)\Big),\end{aligned} \quad (4.102)$$

$$i\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\,(-\frac{1}{2\not p}M_\psi^2\mathcal{M}_2) = -\Big(\frac{\ln(\mu^2/2\mu_0^2)}{16\pi^2}\Big)m_\psi^2\,\mathrm{Tr}(m^{\dagger\psi\chi}m^{\psi\chi}), \tag{4.103}$$

$$i\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\,(-\frac{2}{3}M_\psi^2(\frac{2}{\not p}\mathcal{M}_2+\mathcal{M}_2\frac{1}{\not p})) = 4\Big(\frac{\ln(\mu^2/2\mu_0^2)}{16\pi^2}\Big)m_\psi^2\,\mathrm{Tr}(m^{\dagger\psi\chi}m^{\psi\chi}), \tag{4.104}$$

$$\begin{aligned} i\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\,(-\frac{1}{8}\not p\mathcal{M}_2\not p\mathcal{M}_2) &= \\ \frac{1}{6}\Big(\frac{\ln(\mu^2/2\mu_0^2)}{16\pi^2}\Big)\Big(&\mathrm{Tr}(2m^{\dagger\psi\chi}m^{\psi\chi}m^{\dagger\psi\chi}m^{\psi\chi}-m^{\dagger\psi\chi}m^{\psi\chi}m^{\psi\chi}m^{\dagger\psi\chi}) \\ &+\mathrm{Tr}(m^{\dagger\psi\chi}m^{\psi\chi})^2\Big) \end{aligned} \tag{4.105}$$

and

$$\begin{aligned} i\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\,(-\frac{1}{2}M_\psi\{\frac{1}{\not p},\not p\mathcal{M}_3\}) &= \\ -2\Big(\frac{\ln(\mu^2/2\mu_0^2)}{16\pi^2}\Big)\Big(&m_\psi\,\mathrm{Tr}(m^{\dagger\psi\chi}m^{\chi}m^{\dagger\psi\chi}+m^{\psi\chi}m^{\dagger\chi}m^{\psi\chi}) \\ &+4m_\psi^2\,\mathrm{Tr}(m^{\dagger\psi\chi}m^{\psi\chi})\Big). \end{aligned} \tag{4.106}$$

In all these expressions the trace refers to a contraction of Lorentz indices, so that, for example,

$$\mathrm{Tr}(m^{\dagger\psi\chi}m^{\psi\chi}) \equiv \eta^{\mu\nu}(m^{\dagger\psi\chi}m^{\psi\chi})_{\mu\nu} = \frac{1}{2}\eta^{\mu\nu}\mathcal{G}^{ij}m^{\dagger\psi\chi}_{\mu j}m^{\psi\chi}_{\nu i}. \tag{4.107}$$

The casual reader is also reminded that we must use $Z^{ij}\to\frac{1}{2}\mathcal{G}^{ij}$ to contract the chiral fermion indices.

The log-divergent corrections from the derivative terms are

$$\begin{aligned} i\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\,(-\frac{1}{2}\not p\mathcal{N}_4+\frac{\not p}{8p^2}[p^2,\mathcal{N}_4]+\frac{5\not p}{96p^4}[p^2,[p^2,\mathcal{N}_4]]) &= \\ \frac{1}{72}\Big(\frac{\ln(\mu^2/2\mu_0^2)}{16\pi^2}\Big)\mathcal{G}^{ij}\Big(&19(d_\mu^-m^{\dagger\psi\chi}_{\nu j})\eta^{\nu\rho}(d^{+\mu}m^{\psi\chi}_{\rho i})-68(d^{-\mu}m^{\dagger\psi\chi}_{\mu j})(d^{+\nu}m^{\psi\chi}_{\nu i}) \\ &+4(d^{-\mu}m^{\dagger\psi\chi}_{\nu j})(d^{+\nu}m^{\psi\chi}_{\mu i})\Big), \end{aligned} \tag{4.108}$$

$$\begin{aligned} i\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\,(-\frac{1}{2}\not p\mathcal{N}_3+\frac{\not p}{8p^2}[p^2,\mathcal{N}_3]+\frac{5\not p}{96p^4}[p^2,[p^2,\mathcal{N}_3]]) &= \\ -\frac{1}{144}\Big(\frac{\ln(\mu^2/2\mu_0^2)}{16\pi^2}\Big)\mathcal{G}^{ij}\Big(&8(d_\mu^-d^{-\mu}m^{\dagger\psi\chi}_{\nu j})\eta^{\nu\rho}m^{\psi\chi}_{\rho i}-31(d_\nu^-d^{-\mu}m^{\dagger\psi\chi}_{\mu j})m^{\psi\chi}_{\nu i} \\ &-31(d_\nu^-d^{-\mu}m^{\dagger\psi\chi}_{\nu j})m^{\psi\chi}_{\mu i}-46m^{\dagger\psi\chi}_{\rho j}\eta^{\nu\rho}(d_\mu^+d^{+\mu}m^{\psi\chi}_{\nu i}) \\ &+23m^{\dagger\psi\chi}_{\mu j}(d_\nu^+d^{+\mu}m^{\psi\chi}_{\nu i})+23m^{\dagger\psi\chi}_{\nu j}(d_\nu^+d^{+\mu}m^{\psi\chi}_{\mu i})\Big), \end{aligned} \tag{4.109}$$

and

$$\begin{aligned} i\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\,(-\frac{1}{2}\not p\mathcal{N}_2+\frac{\not p}{8p^2}[p^2,\mathcal{N}_2]+\frac{5\not p}{96p^4}[p^2,[p^2,\mathcal{N}_2]]) &= \\ \Big(\frac{\ln(\mu^2/2\mu_0^2)}{16\pi^2}\Big)J^{\mu\nu}\mathcal{G}^{ij}m^{\dagger\psi\chi}_{\mu j}m^{\psi\chi}_{\nu i}. \end{aligned} \tag{4.110}$$

Finally, it is easy to show that the contribution to the $O(\ln\mu^2)$ effective action from $\mathcal{N}_2'$ vanishes due to the antisymmetry of $(G_\chi^{ij})_{\mu\nu}$ in its Lorentz indices, and that the log-divergent contributions from $\mathcal{F}_1$ and $\mathcal{F}_1'$ cancel each other.

The contributions from $\mathcal{N}_4$ and $\mathcal{N}_3$ to the auxiliary-field effective action of equation (4.58) add up very neatly to

$$\begin{aligned} -\frac{i}{2}\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\,(-\frac{1}{2}\not p(\mathcal{N}_4+\mathcal{N}_3)+\frac{\not p}{8p^2}[p^2,(\mathcal{N}_4+\mathcal{N}_3)] &\\ +\frac{5\not p}{96p^4}[p^2,[p^2,(\mathcal{N}_4+\mathcal{N}_3)]]) &\\ =\Big(\frac{\ln(\mu^2/2\mu_0^2)}{32\pi^2}\Big)G^{ij}(d^{-\mu}m^{\dagger\psi\chi}_{\mu j})(d^{+\nu}m^{\psi\chi}_{\nu i}) &\\ =\Big(\frac{\ln(\mu^2/2\mu_0^2)}{32\pi^2}\Big)(4G_{ij}(d^\mu)^i_k\tilde{\mathcal{D}}_\mu z^k(d^\nu)^j_l\tilde{\mathcal{D}}_\nu z^l), & \end{aligned} \tag{4.111}$$

where we have made use of integration by parts, neglected total divergences, and used equation (4.9) to write the final result. This term has a bosonic analogue in the contribution from mixed scalar and graviton loops, given by the first two terms of equation (2.68) in reference [20], and exactly cancels that contribution. Notice that in (4.111) the derivatives are both gauge- and reparametrization-covariant, and contain no chiral $U(1)$ connections, by virtue of equation (4.42).

CHAPTER 5

Results for no-scale supergravity

We now specialize our results to the no-scale supergravity model discussed in Chapter 1. Taking $h = 0$ and writing out the s-field dependent terms explicitly, we find, from (4.9), that

$$m^{\lambda}_{ab} = \delta_{ab} e^{\mathcal{G}/2}, \qquad m^{\psi\chi}_{\mu a} = -\frac{1}{2(\mathrm{Re}\, s)^2}\partial_\mu \bar{s},$$

$$m^{\psi\chi}_{\mu i} = -2\mathcal{G}_{ij}\tilde{D}_\mu z^j, \qquad m^{\lambda\chi}_{ai} = \frac{i}{2(\mathrm{Re}\, s)^{3/2}}\mathcal{G}_i (T_a z)^i,$$

$$m^{\lambda\chi}_{ai} = -\frac{2i}{\sqrt{\mathrm{Re}\, s}}\mathcal{G}_{ij}(T_a z)^j, \qquad \bar{m}^{\lambda\chi}_{ab} = \frac{i}{4\sqrt{\mathrm{Re}\, s}}\sigma_{\mu\nu}\tilde{F}^{\mu\nu}_a, \quad \text{and} \quad \bar{m}^{\lambda\chi}_{ai} = 0, \tag{5.1}$$

which we can use to rewrite the mass-term traces, equations (4.47–4.49). For example, equation (4.47) becomes

$$\begin{aligned}\mathrm{Tr}\, M^\dagger M = {} & (|\mathcal{G}^{ij}(\mathcal{G}_{ij} + \mathcal{G}_i\mathcal{G}_j - \mathcal{G}_k\Gamma^k_{ij})|^2 + 4 + N_G + \mathcal{G}_s\mathcal{G}_{\bar{s}} + \mathcal{G}_i\mathcal{G}_{\bar{i}})e^{\mathcal{G}} \\ & - \frac{4}{(s+\bar{s})^2}\partial_\mu s\partial^\mu \bar{s} - 4\mathcal{G}_{i\bar{i}}\tilde{D}_\mu z^i \tilde{D}^\mu z^{\bar{i}} \\ & + \Big(\frac{2}{s+\bar{s}}\mathcal{G}_i\mathcal{G}_j + 8\mathcal{G}_{ij}\Big)(T_a z)^i (T^a z)^j.\end{aligned} \tag{5.2}$$

The full expressions for $\mathrm{Tr}(M^\dagger M)^2$ and $\mathrm{Tr}\, D^-_\mu M^\dagger D^{+\mu} M$ are unwieldy; we shall include them in the total one-loop result at the conclusion of this paper, instead of expanding them here. The additional traces from terms involving $\tilde{M}_{\mu\nu}$, from equations (4.35–4.37), must still be calculated. Using the no-scale masses, we get the following results:

$$\mathrm{Tr}'M^\dagger M = \frac{1}{4}(s+\bar{s})\tilde{F}^a_{\mu\nu}\tilde{F}^{\mu\nu}_a; \tag{5.3}$$

$$\begin{aligned}\mathrm{Tr}'(M^\dagger M)^2 = {} & \Big(-\frac{1}{16(s+\bar{s})}\partial_\mu s\partial^\mu \bar{s} + 2(s+\bar{s})e^{\mathcal{G}} + \frac{1}{6}e^{2\mathcal{G}/3}W_i W^i \\ & + \frac{1}{2}\mathcal{G}_i\mathcal{G}_j(T_a z)^i(T^a z)^j\Big)\tilde{F}^a_{\mu\nu}\tilde{F}^{\mu\nu}_b \\ & - (s+\bar{s})\Big(\frac{1}{2}\tilde{F}^{\mu\nu}_a\partial^\rho(i\bar{s} - is) + \tilde{\mathcal{D}}^\mu \tilde{F}^{\nu\rho}_a\Big)(\tilde{\mathcal{D}}_\mu \tilde{F}^a_{\nu\rho} - \tilde{\mathcal{D}}_\nu \tilde{F}^a_{\mu\rho}) \\ & + \frac{1}{32}(s+\bar{s})^2(\tilde{F}^a_{\mu\nu}\tilde{F}^{\mu\nu}_a\tilde{F}^b_{\rho\sigma}\tilde{F}^{\rho\sigma}_b + 10\tilde{F}^a_{\mu\nu}\tilde{F}^{b\mu\nu}\tilde{F}_{a\rho\sigma}\tilde{F}^{\rho\sigma}_b \\ & \qquad + 12\tilde{F}^a_{\mu\nu}\tilde{F}^{\nu\rho}_a\tilde{F}^b_{\rho\sigma}\tilde{F}^{\sigma\mu}_b - 38\tilde{F}^a_{\mu\nu}\tilde{F}^{b\nu\rho}\tilde{F}_{a\rho\sigma}\tilde{F}^{\sigma\mu}_b) \\ & + \frac{5}{3}\mathcal{G}_i\mathcal{G}_j(T^a z)^i(T^b z)^j\tilde{F}_{a\mu\nu}\tilde{F}^{\mu\nu}_b - \frac{1}{2}(N + N_G + 6)J_{\mu\nu}J^{\mu\nu} \\ & - \frac{1}{4}\,\mathrm{Tr}\, R^{\mu\nu}R_{\mu\nu} + \frac{i}{2}e\,\mathrm{Tr}\, R_{\mu\nu}f^{\mu\nu} + \frac{1}{4}e^2\,\mathrm{Tr}\, f^{\mu\nu}f_{\mu\nu} \\ & - \frac{1}{2}N_G(e^2\tilde{F}^a_{\mu\nu}\tilde{F}^{\mu\nu}_a + K_{\mu\nu}K^{\mu\nu} + 2J_{\mu\nu}K^{\mu\nu});\end{aligned} \tag{5.4}$$

$$\begin{aligned}\mathrm{Tr}'D^-_\mu M^\dagger D^{+\mu} M = {} & (s+\bar{s})\Big(\frac{1}{8}\tilde{\mathcal{D}}_\mu \tilde{F}^a_{\nu\rho}\tilde{\mathcal{D}}^\mu \tilde{F}^{\nu\rho}_a + \frac{1}{4}\tilde{\mathcal{D}}_\mu \tilde{F}^a_{\nu\rho}\tilde{\mathcal{D}}^\rho \tilde{F}^{\mu\nu}_a - \tilde{\mathcal{D}}^\nu \tilde{F}^a_{\mu\nu}\tilde{\mathcal{D}}_\rho \tilde{F}^{\mu\rho}_a\Big) \\ & + \frac{1}{8(s+\bar{s})}(\tilde{F}^{\mu\nu}_a\tilde{F}^a_{\mu\nu}\eta^\rho_\rho - 10\tilde{F}^{\mu\rho}_a\tilde{F}^a_{\mu\rho}) \times \\ & \qquad (5\partial_\rho s\partial^\rho \bar{s} - 2\partial_\rho s\partial^\rho s - 2\partial_\rho \bar{s}\partial^\rho \bar{s}) \\ & + \Big(\frac{1}{8}\tilde{F}^a_{\mu\nu}\tilde{\mathcal{D}}^\rho \tilde{F}^{\mu\nu}_a - \frac{1}{4}\tilde{F}^a_{\mu\nu}\tilde{\mathcal{D}}^\nu \tilde{F}^{\mu\rho}_a - \tilde{F}^{\mu\rho}_a\tilde{\mathcal{D}}^\nu \tilde{F}^a_{\mu\nu}\Big)\partial_\rho(s+\bar{s}) \\ & + \frac{3}{2(s+\bar{s})}\tilde{\mathcal{D}}_\mu(\tilde{F}^{\mu\nu}_a\mathcal{G}_i(T^a z)^i)\partial_\nu(i\bar{s} - is).\end{aligned} \tag{5.5}$$

In (5.4) the traces are over both barred and unbarred scalar-field indices, and, to make this result notationally consistent with the bosonic results of references [18] and [20], we should take $e \to 1$. In Chapter 6, we adopt this convention.

We now have all the terms that contribute quadratic or logarithmic divergences to the fermionic Lagrangian. After adding the direct and auxiliary-field contributions, as we did for the pure gravitino loops in Chapter 3, we can combine the total with the bosonic contributions derived by Gaillard and Jain [20,21]. We shall present this inclusive result below.

CHAPTER 6

Conclusions

In Chapters 3-5, we successfully identified all divergent one-loop fermionic contributions to the effective scalar Lagrangian for a general supergravity theory with canonical kinetic energy for the gauge fields, and outlined the simplified results for a real Kähler potential of the no-scale form. We now specialize to the prototype supergravity theory from superstrings [8,9] that is defined by (1.1-1.4), with $h = 0$ in equation (1.1), and, combining our results with the bosonic contributions calculated elsewhere [20,21] we use the special properties of the Kähler potential G of equation (1.4), as given in Appendix B of reference [3], as well as the gauge invariance of both the Kähler potential G and the superpotential W, to obtain the following imposing result for the total divergent part of the one-loop contribution to the effective Lagrangian:

$$
\begin{aligned}
\mathcal{L}^1_{\rm reg} = \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a\Big[&\frac{\mathrm{Re}\,s}{8\pi^2}\Big(13\mu^2\eta - \ln(\rho\mu^2)\big(\frac{132}{3}e^{\mathcal{G}} + 9\hat{V} + \frac{88}{3}\mathcal{D} + 16\mathcal{K}\big)\Big) \\
&+ \frac{\ln(\rho\mu^2)}{16\pi^2}\Big(\frac{3}{k_G} - \frac{1}{k}\Big)\Big] \\
&+ \frac{\partial_\mu s\partial^\mu\bar{s}}{(s+\bar{s})^2}\Big[\frac{\mu^2\eta}{4\pi^2} + \frac{\ln(\mu^2\rho)}{32\pi^2}\Big(\big(\frac{79}{3} + N_G\big)e^{\mathcal{G}} + \frac{8}{3}(\hat{V} + \mathcal{D}) + \mathcal{W} + 4\mathcal{K}\Big)\Big] \\
&+ G_{mn}\tilde{\mathcal{D}}_\mu z^m\tilde{\mathcal{D}}^\mu z^n\Big[\frac{\mu^2\eta}{48\pi^2}(N-8) + \frac{\ln(\mu^2\rho)}{32\pi^2}\Big(\big(13 - \frac{1}{3}N\big)e^{\mathcal{G}} - \frac{1}{3}\big(5 + \frac{2}{3}N\big)\hat{V} \\
&\qquad - \frac{1}{3}\big(8 + \frac{2}{3}N\big)\mathcal{D}\Big)\Big] \\
&+ \frac{\ln(\mu^2\rho)}{32\pi^2}\Big(G_mG_n\tilde{\mathcal{D}}_\mu z^m\tilde{\mathcal{D}}^\mu z^n\Big(\big(10 + N_G\big)e^{\mathcal{G}} + \frac{55}{27}\hat{V} + \frac{40}{9}\mathcal{D} + \frac{1}{9}\mathcal{W} - \frac{2}{9}\mathcal{K}\Big) \\
&\quad + \tilde{\mathcal{D}}^\mu\phi^j\tilde{\mathcal{D}}_\mu\phi^i\Big(e^{\mathcal{G}}\Big[\big(\frac{47}{3} + N_G\big)W_i\overline{W}_j + \frac{19}{3}W_{ik}\overline{W}^k_j + W_{ikl}\overline{W}^{kl}_j\Big] \\
&\qquad + \frac{1}{s+\bar{s}}(T^a)^k_i(T_a)^l_j\Big(\frac{142}{3}G_kG_l - 4G_{kl}\Big) \\
&\qquad - \frac{12}{s+\bar{s}}(T^a)_{ij}(T_a)^{kl}G_kG_l\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+ \Big[G_n\tilde{\mathcal{D}}^\mu z^n\tilde{\mathcal{D}}_\mu\phi^i\Big(e^{\mathcal{G}}\Big[\big(\frac{47}{3} + N_G\big)W_i\overline{W} + \frac{70}{9}W_{ik}\overline{W}^k + \frac{1}{3}W_{ikl}\overline{W}^{kl}\Big] \\
&\qquad + \frac{G_k}{s+\bar{s}}\Big[\frac{103}{3}(T^a)^k_i\mathcal{D}_a + \frac{20}{3}K^k_i\Big]\Big) \\
&\quad - G_n\tilde{\mathcal{D}}^\mu z^n\frac{\partial_\mu\bar{s}}{(s+\bar{s})}\Big[2\big(\frac{25}{3} + N_G\big)e^{\mathcal{G}} + \frac{32}{9}\hat{V} + \frac{40}{9}\mathcal{D} + \frac{1}{3}\mathcal{W} + \frac{4}{3}\mathcal{K}\Big] \\
&\quad - \frac{\partial^\mu\bar{s}}{(s+\bar{s})}\tilde{\mathcal{D}}_\mu\phi^i\Big(e^{\mathcal{G}}\Big[2\big(\frac{25}{3} + N_G\big)W_i\overline{W} + \frac{28}{3}W_{ik}\overline{W}^k + W_{ikl}\overline{W}^{kl}\Big] \\
&\qquad + \frac{G_k}{s+\bar{s}}\Big[\frac{64}{3}(T^a)^k_i\mathcal{D}_a + 8K^k_i\Big]\Big) \\
&\quad + G_mG_n\tilde{\mathcal{D}}_\mu z^m\tilde{\mathcal{D}}^\mu z^n\Big(\frac{88}{9}e^{\mathcal{G}} - \frac{32}{27}\hat{V} + \frac{38}{27}\mathcal{D}\Big) - 2N_Ge^{\mathcal{G}}\frac{\partial^\mu s\partial_\mu s}{(s+\bar{s})^2} \\
&\quad + \tilde{\mathcal{D}}^\mu\phi^j\tilde{\mathcal{D}}_\mu\phi^i\Big(e^{\mathcal{G}}\big(18W_{ij}\overline{W} - 2W_{ijk}\overline{W}^k\big) + \frac{19}{3}\frac{G_kG_l}{s+\bar{s}}(T^a)^k_i(T_a)^l_j\Big) \\
&\quad + G_n\tilde{\mathcal{D}}^\mu z^n\tilde{\mathcal{D}}_\mu\phi^i\Big(e^{\mathcal{G}}\big(24W_i\overline{W} - \frac{4}{3}W_{ik}\overline{W}^k\big) + \frac{44}{9}\frac{G_k}{s+\bar{s}}(T^a)^k_i\mathcal{D}_a\Big) \\
&\quad + G_n\tilde{\mathcal{D}}^\mu z^n\frac{\partial_\mu s}{(s+\bar{s})}\Big(\big(\frac{146}{3} + N_G\big)e^{\mathcal{G}} + \frac{16}{9}\hat{V} + \frac{11}{3}\mathcal{D}\Big) \\
&\quad + \frac{\partial^\mu s}{(s+\bar{s})}\tilde{\mathcal{D}}_\mu\phi^i\Big(e^{\mathcal{G}}\big((48 + N_G)W_i\overline{W} + 3W_{ik}\overline{W}^k\big) \\
&\qquad + 12\frac{G_k}{s+\bar{s}}(T^a)^k_i\mathcal{D}_a\Big) + H.\,c.\Big]\Big) \\
&- \frac{\mu^2\eta}{16\pi^2}\big[(N - N_G - 12)e^{\mathcal{G}} + \frac{1}{3}(2N - 29)\hat{V} + \frac{2}{3}(N - 13)\mathcal{D}\big] \\
&+ \frac{\ln(\mu^2\rho)}{32\pi^2}\Big[(N - N_G - 44)e^{2\mathcal{G}} + \frac{4}{3}(N - 2)e^{\mathcal{G}}\hat{V} + \frac{1}{9}(4N + 59)\hat{V}^2 \\
&\quad + \frac{2}{9}(2N + 93)\mathcal{D}^2 + \frac{8}{3}(N - 4)\mathcal{D}e^{\mathcal{G}} + \frac{8}{9}(N + 73)\hat{V}\mathcal{D} \\
&\quad + 2\mathcal{W}\big(3e^{\mathcal{G}} + \frac{2}{3}\hat{V} + \frac{8}{3}\mathcal{D}\big) + 2\mathcal{K}\big(\frac{4}{3}\hat{V} + \frac{1}{3}\mathcal{D} - 6e^{\mathcal{G}}\big) \\
&\quad - \frac{4}{k_G(s+\bar{s})}\mathcal{D} + e^{2\mathcal{G}}\Big(W_{ijk}W_l\overline{W}^{ijl}\overline{W}^k - \frac{20}{3}W_{ij}W_k\overline{W}^{ik}\overline{W}^j \\
&\qquad + (W_{ijk}W\overline{W}^{ij}\overline{W}^k + H.\,c.) - \frac{28}{3}W_{ij}W\overline{W}^i\overline{W}^j\Big) \\
&\quad + \frac{4}{(s+\bar{s})^2}\mathcal{D}_a\big(\mathcal{D}_bG_k(-6T^aT^b\phi)^k + G_j(KT^a)^j\big) \\
&\quad + 4\frac{e^{\mathcal{G}}}{(s+\bar{s})}\Big(20\mathcal{D}_aW_i(T^a)^i_jW^j - 5W_iK^i_jW^j + 2W_{ij}(T^a)^i_kW^{kj} \\
&\qquad - \big((3W_i\overline{W} + W_{ij}\overline{W}^j)(K\phi)^j + H.\,c.\big)\Big)\Big] + (\mathcal{L}^1_{\rm reg})', \qquad (6.1)
\end{aligned}
$$

where $(\mathcal{L}^1_{\rm reg})'$ includes all the terms which contain three or four derivatives with

respect to the spacetime coordinates:

$$
\begin{aligned}
(\mathcal{L}^1_{\rm reg})' = \frac{\ln(\mu^2)\rho}{32\pi^2}\Big[&\frac{1}{9}(N-101)(G_{mn}\tilde{D}_\mu z^m\tilde{D}^\mu z^n)^2 + \frac{1}{12}\Big(\frac{579}{8} - 5N - 45N_G\Big)\frac{(\partial_\mu s\partial^\mu\bar{s})^2}{(s+\bar{s})^4} \\
&+ \frac{1}{12}\Big(\frac{2429}{8} + 5N + 45N_G\Big)\frac{|\partial_\mu s\partial^\mu s|^2}{(s+\bar{s})^4} \\
&+ \frac{1}{36}\Big(\frac{5751}{8} + 23N + 15N_G\Big)G_{mn}\tilde{D}_\mu z^m\tilde{D}_\nu z^n G_{mn}\tilde{D}^\mu z^m\tilde{D}^\nu z^n \\
&+ \frac{1}{36}\Big(\frac{4457}{8} - 23N - 15N_G\Big)G_{mn}\tilde{D}_\mu z^m\tilde{D}_\nu z^n G_{mn}\tilde{D}^\nu z^m\tilde{D}^\mu z^n \\
&+ \frac{\partial_\mu s\partial_\nu\bar{s}}{6(s+\bar{s})^2}\Big(\Big(\frac{365}{8} + 5N + 15N_G\Big)G_{mn}\tilde{D}^\mu z^m\tilde{D}^\nu z^n \\
&\qquad - \Big(\frac{749}{8} + 5N + 15N_G\Big)G_{mn}\tilde{D}^\nu z^m\tilde{D}^\mu z^n \\
&\qquad - 8\eta^{\mu\nu}G_{mn}\tilde{D}_\rho z^m\tilde{D}^\rho z^n\Big) \\
&- \frac{1}{32}(s+\bar{s})^2(3F^a_{\mu\nu}F^{\mu\nu}_a F^b_{\rho\sigma}F^{\rho\sigma}_b + 2F^a_{\mu\nu}F^{b\mu\nu}F_{a\rho\sigma}F^{\rho\sigma}_b \\
&\qquad - 4F^a_{\mu\nu}F^{\nu\rho}_a F^b_{\rho\sigma}F^{\sigma\mu}_b - 30F^a_{\mu\nu}F^{b\mu\nu}F_{a\rho\sigma}F^{\sigma\mu}_b) \\
&+ \frac{1}{36}(s+\bar{s})\Big(\frac{317}{2}\tilde{D}_\mu F^a_{\nu\rho}\tilde{D}^\mu F^{\nu\rho}_a \\
&\qquad - 45\tilde{D}_\mu F^a_{\nu\rho}\tilde{D}^\rho F^{\mu\nu}_a + 95\tilde{D}^\nu F^a_{\mu\nu}\tilde{D}_\rho F^{\mu\rho}_a\Big) \\
&- F^a_{\mu\nu}F^{\mu\nu}_a\Big(\frac{1}{72(s+\bar{s})}\Big(43\partial_\mu s\partial^\mu s + 43\partial_\mu\bar{s}\partial^\mu\bar{s} - \frac{775}{2}\partial_\mu s\partial^\mu\bar{s}\Big) \\
&\qquad - 4(s+\bar{s})G_{mn}\tilde{D}_\mu z^m\tilde{D}^\mu z^n\Big) \\
&+ F^a_{\mu\rho}(F_\nu{}^\rho)_a\Big(\frac{1}{72(s+\bar{s})}\Big(\frac{859}{4}\partial^\mu s\partial^\nu s + \frac{859}{4}\partial^\mu\bar{s}\partial^\nu\bar{s} + \frac{563}{2}\partial^\mu s\partial^\nu\bar{s}\Big) \\
&\qquad + 16(s+\bar{s})G_{mn}\tilde{D}^\mu z^m\tilde{D}^\nu z^n\Big) \\
&+ iF^{\mu\nu}_a\Big(\Big[\frac{1}{6}\Big(\frac{461}{8} + \frac{23}{3}N + 5N_G\Big) + \frac{1}{9}G_r z^r\Big(\frac{397}{8} + 5N + 5N_G\Big)\Big] \\
&\qquad \times \mathcal{D}^a G_{mn}\tilde{D}_\mu z^m\tilde{D}_\nu z^n \\
&\quad + \frac{4}{9}(G_m(T^a z)^k + 3(T^a)^k_i)G_{kj}\tilde{D}^\mu z^i\tilde{D}^\nu z^j \\
&\quad + \Big(1 + \frac{2}{3}G_r z^r\Big)\Big(\frac{1351}{24} + \frac{65}{9}N + 15N_G\Big)\frac{\partial_\mu s\partial_\nu\bar{s}}{6(s+\bar{s})^2}\mathcal{D}^a\Big) \\
&+ 9i\tilde{D}_\mu F^{\mu\nu}_a G_{ij}(\tilde{D}_\nu z^i(T^a\bar{z})^j - \tilde{D}_\nu z^j(T^a z)^i) \\
&- \frac{3i\partial_\mu(\bar{s}-s)}{2(s+\bar{s})}\tilde{D}_\nu(F^{\mu\nu}_a\mathcal{D}^a)\Big].
\end{aligned}
\tag{6.2}
$$

In the above expressions the indices labeled $m,n,p,q = 0,\ldots,N$ run over both the t and the ϕ fields, while $i,j,k,l = 1,\ldots,N$ run only over the ϕ degrees of freedom. The scalar reparametrization-covariant derivative d_μ is defined in the usual way,

$$
(d^\mu)^i{}_j = (\tilde{D}^\mu)^i{}_j + \Gamma^i{}_{kj}(\tilde{\phi})\tilde{D}^\mu\tilde{\phi}^k(x). \tag{6.3}
$$

The matrices T^a and K represent the gauge group generators and Casimir operator, respectively, on the chiral supermultiplets; and $k^{-1} = \mathrm{Tr}\,K/N_G$ and $k_G^{-1} = \mathrm{Tr}\,K_G/N_G$, where N_G is the number of gauge degrees of freedom and K_G represents the gauge Casimir on the gauge supermultiplets. The gauge coupling constants are assumed to be unified at the scale at which we are working, but the chiral multiplet representation of the gauge group is in general reducible, so K is not proportional to the unit matrix. However, we have assumed that the T^a are traceless.

The classical scalar potential V defined by (1.1) and (1.4) with $h = 0$ is

$$
V = \mathcal{V} + \mathcal{D} = e^{\mathcal{G}} + \hat{V} + \mathcal{D}, \tag{6.4}
$$

with

$$
\hat{V} = e^{\mathcal{G}}W_i\overline{W}^i, \quad \text{and} \quad \mathcal{D} = \frac{1}{(s+\bar{s})}\mathcal{D}_a\mathcal{D}^a, \tag{6.5}
$$

where $\mathcal{D}^a = G_i(T^a\phi)^i = G_i(T^a\phi)^t$ by gauge invariance. In addition we have introduced the dimension-two operators $\mathcal{W} = e^{\mathcal{G}}W_{ij}\overline{W}^{ij}$ and $\mathcal{K} = 2G_i(K\phi)^j/(s+\bar{s})$. As throughout this work, the index-raising operator on scalar-field matrices is the inverse scalar metric; thus $\overline{W}^i = \mathcal{G}^{ij}\overline{W}_j$, and so forth. The nonderivative terms in $\mathcal{L}^1_{\rm reg}$ differ from those given elsewhere [3] by the inclusion of the graviton "mass" contribution, $m_G^2 = -2V$.

Finally, the field strength $F^a_{\mu\nu}$ is normalized as in (1.8) with $f_{ab}(z) = s\delta_{ab}$, *i.e.*, with noncanonical kinetic energy. As stressed in the text, we have not included

the loop corrections arising from this latter coupling; these will contribute terms of order N_G involving derivative couplings of the s-field, as well as terms involving the dual field strength $\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}_a$, and will be presented elsewhere.

The complete one-loop corrections for the simpler case we have considered, where $f_{ab}(z) = constant$, can be obtained from the results of the previous chapter; the full fermion-loop corrections for nonconstant $f_{ab}(z)$ have been evaluated there, and all such terms, except for those proportional to $F^a_{\mu\nu}\partial_\rho(s+\bar{s})$, are included in (6.2). The rationale behind this exception is the following: when we modified the results of the previous bosonic calculation [20,21] by setting $e = 1/\sqrt{\mathrm{Re}\, s}$ and then rescaling (that is, letting $F^a_{\mu\nu} \to \sqrt{\mathrm{Re}\, s}F^a_{\mu\nu}$), the covariant derivatives $\tilde{\mathcal{D}}_\rho$ acting on $F^a_{\mu\nu}$ generated terms like $F^a_{\mu\nu}\partial_\rho(s+\bar{s})$, which we have neglected. Since we do not have all the terms of this form, it would be inconsistent to include only the fermionic terms in our total.

In (6.1) and (6.2), η and ρ parametrize the uncertainties in threshold effects and finite terms that are dependent on the regularization prescription (in the double-subtraction scheme used in our explicit calculations, $\eta = 2\ln 2$, and $\rho = \frac{1}{2}$). Determining these parameters requires a knowledge of the details of the underlying short-distance physics that serves to damp the apparently divergent integrals. However, many qualitative results found [16,17,3] by studying the one-loop effective potential are prescription-independent.

On the other hand, treatment of the quadratically divergent terms requires more care, since they do not scale uniformly [13,27] with threshold effects, so it would seem that a different uncertainty factor η_i should be introduced for each quadratically divergent term. However, the symmetries of the theory can be used to reduce this uncertainty. The approach taken in references [13] and [27] was to assume that the underlying theory is finite, and to use a Pauli-Villars regulation to parametrize the

effects of the heavy modes. When these modes are introduced in a manner consistent with supersymmetry, there are additional terms, due to mass-dependent couplings, that are not quadratically divergent, but scale as does the cutoff, μ^2. In this way it was possible to fully determine the leading-N contributions from chiral multiplet loops to the part of the one-loop effective Lagrangian that scales as μ^2. Note that mass-dependent couplings do not induce additional terms in $\ln\mu^2$, so in principle the full leading-N "divergent" one-loop Lagrangian (*i.e.*, the part that grows with μ^2) is known. A similar treatment of the leading-N_G (number of gauge multiplets) contribution will be given elsewhere, where additional divergent contributions to the s-field kinetic energy, which arise from the noncanonical form of the classical gauge kinetic-energy term, will also be included.

Before physics can be extracted from the results we have obtained, we must find a similar procedure to regulate all the quadratic divergences. Note, however, that it may not be necessary to fully regulate the theory in the sense that the regulated theory including massive Pauli-Villars modes is actually completely finite. To identify correctly the "divergent" part (which is all we can hope to do without a complete understanding of the short-distance physics), we need only cancel the quadratic divergences, since the coefficients of the log divergences are not prescription-dependent. For example, in regulating [13] the leading-N part, massive modes were introduced in a way that cannot be easily generalized to the nonleading gauge- and superpotential-dependent couplings of the gauge-nonsinglet chiral multiplets. On the other hand, many of the quadratically divergent contributions arising from these couplings cancel among scalar and fermion loops.

In addition, all the results presented here must be generalized so as to include background fermion fields [3,27] and a nonflat background metric [27], in order to determine the wavefunction renormalizations and Weyl transformation [13,27] needed

to recast the terms quadratic in Kähler and spacetime derivatives into standard form [28]. Then, for example, once the full effective one-loop Lagrangian has been determined for scales above the scale of hidden gaugino condensation—$h = 0$ in equation (1.4)—an analysis similar to that of Dine *et al.* [9], following Affleck *et al.* [35], can be performed to determine [13] the effects of one-loop corrections from the unconfined hidden Yang-Mills regime on the effective theory below the scale of hidden gaugino condensation. In particular, one will be able to address questions such as the stability of the potential [36,37,3] and soft supersymmetry-breaking terms [17,38–40,10,3,41].

As stressed in Chapter 1, the model [8,9] we are studying here is a prototype, not a realistic model for particle physics. Nevertheless, the various techniques developed in this paper can easily be generalized to more realistic superstring-inspired [42,43] or superstring-derived [44–48] models. Moreover, the symmetry structure [41] of the prototype model is similar to that of many more realistic models. It has recently been conjectured [41] that an exact classical noncompact, nonlinear symmetry of the model is responsible for the cancellation of observable soft supersymmetry-breaking effects found by explicit calculation [3] at one loop in perturbation theory for the effective theory below the scale of condensation. Indeed, when effects due to symmetry-breaking by anomalies at the quantum level of the hidden gauge sector are included, a nonvanishing contribution to observable-sector gaugino masses is found [41]. Its magnitude is such that a sufficiently large gauge hierarchy is generated even if the gravitino mass and the scales Λ_{GUT} and Λ_C are only a few orders of magnitude below the Planck scale, as suggested by a numerical analysis [3] of the effective one-loop potential. It is important to determine whether or not any other, potentially larger, observable supersymmetry-breaking effects are generated by purely perturbative loop effects. If realistic models can be constructed in which

there are no such effects, they may provide the best candidates for a natural and fundamental explanation of the enormous disparity between the natural scales of gravity and of observable particle physics.

Notes and References

[1] In accord with the traditional practice of particle physics, units of measure in which both c and $\hbar$ equal one will be assumed throughout this work, and so, for example, "short-distance" and "large-mass" will be treated as synonymous terms. Strenuous efforts will be made, however, not to set two equal to one.

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