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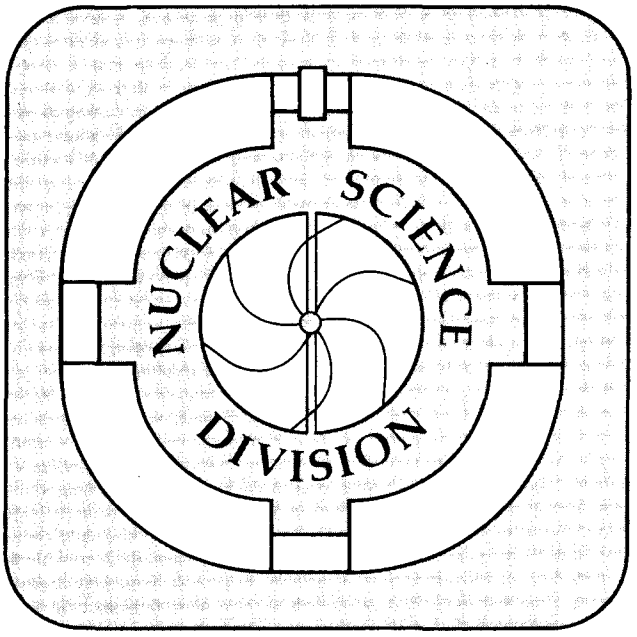

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ROTATING NEUTRON STARS AND THE EQUATION OF STATE OF DENSE MATTER*

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We investigate the properties of non-rotating as well as rotating neutron stars in the framework of Einstein's theory of general relativity. The nuclear equation of state of neutron star matter (electrically charge neutral many-baryon/lepton system) is calculated for the relativistic Hartree- and Hartree-Fock approximations. The influence of two-particle correlations on the equation of state is taken into account, too. An important result is that the limiting rotational frequency of a neutron star is set by instability modes caused by gravitational radiation reaction, which set in well below the Keplerian frequency.

1 INTRODUCTION

The number of observed pulsars has increased very rapidly during the last few years. Up to date about 500 pulsars are known. The fastest of them have ro-

tational periods in the millisecond range, e.g. P(PSR 1937+21)=1.558 msec (i.e. $\Omega_{\rm fp} \equiv \Omega_{1937+21} = 4033\ {\rm s}^{-1}$) and P(PSR 1957+20)=1.6 msec[1]. The pulsar PSR 1913+16, whose mass has been very accurately determined to be (1.442±0.003) $M_\odot$, rotates at a period of P(PSR 1913+16)=59.0 msec. Figure 1 gives an overview of the number of observed pulsars as a function of their periods[2].

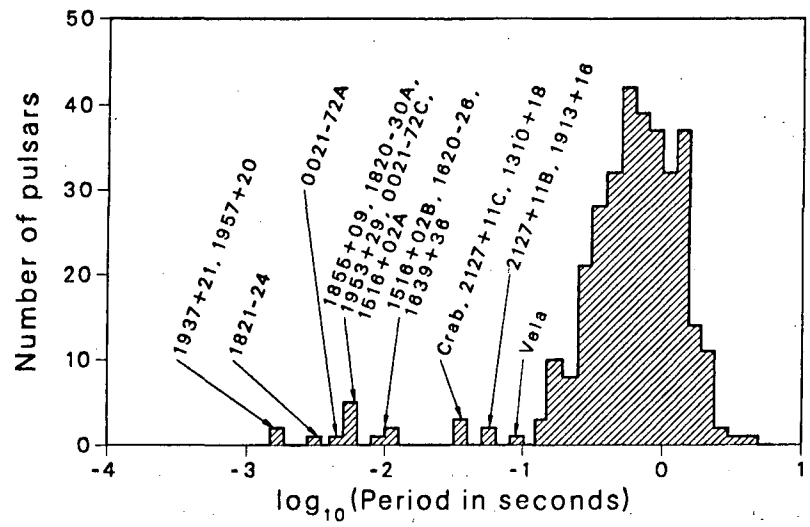

Fig. 1 Distribution of pulsar periods. The fastest pulsars have periods of 1.6 msec.

An estimate of the average energy density $\bar{\epsilon}$ of a millisecond pulsar shows that its value has to be about as large or larger than nuclear density,[3] i.e. $\bar{\epsilon} \approx \epsilon_0 \approx 2.5 \cdot 10^{14}$ g/cm^3 where ϵ_0 is the saturation density of normal nuclear matter. Since the energy density increases monotonically toward the center on the star, the central density must be even higher. Therefore the sources of pulsars are interpreted as rotating (and highly magnetized) neutron stars. The analysis of the properties of these objects is consequently of great interest.

For the construction of realistic neutron star models one has to resort to Einstein's theory of general relativity. Einstein's curvature tensor $\mathcal{G}_{\mu\nu}$ is coupled to the stress-energy density tensor $\mathcal{T}_{\mu\nu}$ of matter (G denotes the gravitational constant):

$$\mathcal{G}_{\mu\nu} = 8\pi G \mathcal{T}_{\mu\nu} . \tag{1.1}$$

The source term of Eq. (1.1), i.e. the tensor $\mathcal{T}_{\mu\nu}$, is derivable from the star's

matter Lagrangian $\mathcal{L}_m$,[3-5]

$$\frac{\delta \mathcal{L}_m}{\delta \chi} - \partial_\mu \frac{\delta \mathcal{L}_m}{\delta (\partial_\mu \chi)} = 0 , \qquad \text{for each matter field } \chi . \tag{1.2}$$

The Lagrangian $\mathcal{L}_m$ of Eq. (1.2) is a rather complicated function of various baryon ($p, n, \Sigma^{\pm,0}, \Lambda, \Xi^{0,-}, \Delta^{++,+,0,-}$), meson ($\sigma, \omega, \pi, \varrho, ...$) and lepton ($e^-, \mu^-$) fields[4-7] in the case of neutron star matter. Nuclear many-body physics and astrophysics are connected through Eqs. (1.1) and (1.2):[3] Once the equation of state (EOS), i.e. the functional dependence of pressure P on energy density ϵ, $P(\epsilon)$, of the many-body system has been calculated from the Lagrangian of Eq. (1.2), the source term of Eq. (1.1) is determined ($\mathcal{T} \equiv \mathcal{T}(\epsilon, P(\epsilon))$ and Einstein's field equations can be solved for the properties of neutron stars.

2 NEUTRON STARS

Neutron stars are objects of highly compressed matter. Typical densities in the cores of massive collapsed star models are[4,5] $\varrho < 10\,\varrho_0$. Therefore the geometry of space-time is changed considerably from flat space over the dimensions of the star ($\approx 60\%$; it changes however only by $\approx 10^{-19}$ of this over the spacing of nucleons in the star)[3].

2.1 NON-ROTATING STARS

For a static spherically symmetric star, the line element has the following form

$$ds^2 = -e^{2\,\Phi} dt^2 + e^{2\,\lambda} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right) , \tag{2.1}$$

where $\Phi(r)$ and $\lambda(r)$ are metric functions which are solutions to Einstein's equations. In this special case, the stellar structure equations have the form (Oppenheimer-Volkoff equations):[8]

$$4\pi r^2 dp(r) = -\frac{Gm(r)dm(r)}{r^2} \left[1 + \frac{P(r)}{\epsilon(r)}\right] \left[1 + \frac{4\pi r^3 P(r)}{m(r)}\right] \left[1 - \frac{2m(r)G}{r}\right]^{-1} \tag{2.2}$$

$$dm(r) = 4\pi\, r^2\, \epsilon(r)\, dr . \tag{2.3}$$

Equations (2.2) and (2.3) are to be integrated outward from the origin ($r = 0$, $m(r = 0) = 0$) of the star after picking a value of the central energy density

$\epsilon_c \equiv \epsilon(r=0)$ till the pressure $P(r)$ vanishes. This defines the radius R_s of the spherical star ($P(r=R_s)=0$) and its total gravitational mass M_s ($\equiv m(R_s)$).

2.2 ROTATING STARS

The treatment of rotating neutron stars in the framework of general relativity is much more complicated than the non-rotating case[9]. An adequate method, developed by Hartle,[10] is based on a perturbation solution on the Schwarzschild metric. Expanded through second order in the star's rotational velocity Ω, it can be written as

$$ds^2 = -e^{2\nu}dt^2 + e^{2\psi}\,(d\phi - \omega\, dt)^2 + e^{2\mu}\, d\theta^2 + e^{2\lambda}dr^2 + O\left(\frac{\Omega^3}{\Omega_c^3}\right). \tag{2.4}$$

The frequency ω is the angular velocity of the local inertial frame and depends on the radial coordinate r. It is proportional to Ω (dragging of the local inertial frame). The velocity Ω is a constant throughout the star's fluid (uniform rotation). Equation (2.4) is correct up to terms $O(\Omega^3/\Omega_c^3)$, with the "critical" velocity Ω_c given by

$$\Omega_c \equiv \sqrt{M_s\, G/R_s^3} = 36\,\sqrt{[M_s/M_\odot]/[R_s/\mathrm{km}]^3}\cdot 10^4\,\frac{1}{\mathrm{s}}\,. \tag{2.5}$$

This corresponds to the Newtonian balance of centrifuge and gravity. The metric functions in the perturbed line element of Eq. (2.4) have the form (for details, see Refs. 10-12)

$$e^{2\nu} = e^{2\Phi}[1 + 2\,(h_0 + h_2 P_2)]\,, \tag{2.6}$$

$$e^{2\psi} = r^2\,\sin^2\theta[1 + 2\,(v_2 - h_2)\,P_2]\,, \tag{2.7}$$

$$e^{2\mu} = r^2[1 + 2\,(v_2 - h_2)\,P_2]\,, \tag{2.8}$$

$$e^{2\lambda} = \left(1 + \frac{2}{r}\,\frac{m_0 + m_2 P_2}{1 - \frac{2\,m\,G}{r}}\right)\,\left(1 - \frac{2\,m\,G}{r}\right)^{-1}\,. \tag{2.9}$$

The monopole and quadrupole functions h_0, m_0 and h_2, m_2, v_2, respectively, occuring in Eqs. (2.6)-(2.9) describe the deviation from spherical symmetry. They are given as solutions of coupled sets of differential equations which will not be given here for the purpose of brevity[10-12].

An *absolute* upper limit on stable neutron star rotation is set by the Kepler frequency Ω_K, which is the maximum frequency a star can have before mass loss at the equator sets in (mass shedding). The general relativistic expression for Ω_K, denoted by Ω_K^{GR}, depends on the underlying metric of Eq. (2.4), i.e.[9]

$$\Omega_K^{GR} = e^{\nu-\psi}\ V + \omega, \qquad V \equiv \frac{\omega'}{2\psi'}\, e^{\psi-\nu} \sqrt{\frac{\nu'}{\psi'} + \left(\frac{\omega'}{2\psi'}\, e^{\psi-\nu}\right)^2}\,, \tag{2.10}$$

which is to be evaluated at the star's equator (c=1 in our units, primes denote derivatives with respect to the radial coordinate).

3 NEUTRON STAR MATTER

This Section deals with the determination of the equation of state of neutron star matter. From theoretical investigations[4-7] it is known that the basic constituents of such matter are protons, neutrons, and heavier baryons (i.e. $\Sigma^{\pm,0}$,Λ, $\Xi^{0,-}$, $\Delta^{++,\pm,0}$) interacting via the exchange of scalar-, vector-, and isovector mesons ($\sigma, \omega, \pi, \varrho, ...$). A certain number of leptons are necessary in order to keep the whole system electrically charge neutral[4]. A realistic determination of the EOS has been the subject of much effort for many years without a general consensus[13]. The models of neutron (and nuclear) matter treated so far differ mainly in the density dependence of pressure, $P(\epsilon)$. One can hope to shed light on the functional dependence of $P(\epsilon)$ by applying different equations of state for solving the stellar structure equations of Section 2 since knowledge of $P(\epsilon)$ over a large density range ϵ (i.e. $\epsilon < 10\,\epsilon_0$; $\epsilon_0 \approx 140$ MeV/fm^3 is the energy density of normal nuclear matter at saturation) is necessary, and the bulk properties of the resulting star configurations are known to depend sensitively on the EOS[3-5,13-15]. In this work, we use EOSs calculated for the relativistic Hartee and Hartree-Fock approximation for charge neutral many-baryon/lepton neutron star matter[4,5]. As a special feature, we take the influence of two-particle correlations on the EOS into account[12,15]. To achieve this goal, one has to go beyond the above approximation schemes and to introduce an effective interaction[16] (two-body potential, scattering (T) matrix) in neutron matter.

3.1 MATHEMATICAL TREATMENT

It has been shown elsewhere how the relativistic Hartree, Hartree-Fock, and so-called Λ (Brueckner-type two-body correlations) approximations can be derived from the Martin-Schwinger hierarchy[16]. The latter couples the $2n$-point Green's functions, $g_n(1,..,n;1',..,n')$, with respect to all orders of n (=1,2,..) with each other and can be truncated via an appropriate factorization procedure of higher-order Green's functions. In the case of the relativistic Λ^{00} approximation, this leads to the following integral equation for the T matrix:[16]

$$T = v - v^{ex} + \int v \; [g_1^0 \, g_1^0] \; T \, . \tag{3.1}$$

The quantity v sums the meson-exchange potentials describing the nucleon-nucleon interaction in free space (direct- and exchange term). We have used the coupling strenghts and masses of the HEA[17] and $Bonn$[18] models for the calculation of the T matrix in matter[15]. The functions g_1^0 in Eq. (3.1) denote the free two-point Green's propagator which results from the full g_1 propagator (see Eq. (3.4) below) by setting the mass-operator (self-energy) Σ equal to zero. Physically, the latter quantity plays the role of an effective single-particle potential which takes the influence of the nuclear background-medium on the propagation of a certain baryon (described by g^B) in matter into account. In the case of the Λ method, the self-energy is given by[16]

$$\Sigma^{\Lambda} = i \int [\, \mathrm{tr}\,(T \, g_1) - T \, g_1 \,] \, . \tag{3.2}$$

Because of the difficulties of the Λ approximation,[15,16] we have treated this method for $B = n$ only, i.e., restriction to neutrons as the only present baryons in neutron star matter. We will see in the discussion that this is a reasonably good approximation for matter up to[4,5] $\varrho \approx 2\,\varrho_0$. This density value therefore sets the upper limit of the applicability of the Λ method (for the determination of a *realistic* neutron star matter EOS). (Other reasons restricting the application of this method are discussed in Refs. 15,16.) The *full* many-body problem (i.e. lepton-baryon population) is treated in the framework of the relativistic Hartree-Fock method. It results formally by replacing the full T matrix of Eq. (3.1) by its "Born" term $v - v^{ex}$. Re-expressed in terms of free meson propagators, Δ^M,

the self-energy is then given by[5,15]

$$\Sigma^{B,\,\mathrm{HF}} = \sum_{M=\sigma,\omega\,\pi,\varrho} \; \sum_{B'=p,n,\Sigma^{\pm,0},\Lambda,\Xi^{0,-},\Delta^{++,+,0,-}} \int \left[\, \mathrm{tr}\,(\Delta^M\, g_1^{B'}) - \Delta^M\, g_1^{B'}\,\right] . \tag{3.3}$$

Since the coupling parameters of neither the HEA nor the $Bonn$ potential saturate nuclear matter in the Hartree-Fock description, one is forced to re-adjust them such that the bulk properties of nuclear matter at saturation (binding energy, compression modulus, effective mass, charge symmetry coefficient) can be reproduced[4,5]. The baryon propagators g_1^B in Eqs. (3.2) and (3.3) satisfy Dyson's equation,

$$\left(\gamma^\mu\, p_\mu - m_B - \Sigma^B(p)\right)\, g_1^B(p) \;=\; -\,1\,, \tag{3.4}$$

which in turn terminates the set of equations that is to be solved self-consistently, subject to additional constraints like charge neutrality[4,5]. As in the non-relativistic theory, it is sufficient for the calculations of the properties of the system to determine the so-called spectral function. One can deduce the following spectral representation for the Fourier transformed baryon propagator:[5,16]

$$g_1^B(p^\mu) = \int_{-\infty}^{+\infty} d\omega\, \frac{a^B(\omega,\vec{p})}{\omega - (p^0 - \mu^B)\,(1 + i\eta)}\,. \tag{3.5}$$

The rather complicated many-body problem is then shifted toward calculating Σ^B in combination with the spectral functions a^B. The EOS can be expressed in terms of these functions alone and follows therefore immediately once self-consistent solutions of Eqs. (3.1)-(3.5) have been found[5,12,16].

3.2 EQUATIONS OF STATE

Our equations of state, calculated as outlined in the foregoing Subsection, can be classified as follows:

(a) a relativistic Hartree EOS[4,5] (denoted by HV) calculated for electrically charge neutral neutron star matter (at zero temperature) in generalized β equilibrium, allowing for the population of hyperon/baryon states with masses lower or equal to m_Δ,

(b) a relativistic Hartree-Fock EOS[5] (denoted by HFV) calculated for matter

under the same conditions as given in (a),
(c) a relativistic EOS calculated for pure neutron matter at densities $0.02 < \varrho/\mathrm{fm}^{-3} < 0.28$ which takes into account *two − particle correlations* (in the so-called Λ^{00} approximation) using the Machleidt-Holinde-Elster *Bonn* potential and is combined with the HV EOS for $0.28 < \varrho/\ \mathrm{fm}^{-3} < 1.2$. The EOS calculated in this way is denoted by $\Lambda^{00}_{Bonn} + HV$,[15] and
(d) a relativistic EOS similar to (c), but using the Holinde-Erkelenz-Alzetta (HEA) potential. It is denoted by $\Lambda^{00}_{HEA} + HFV$ [15].

For the neutron star calculations presented here, equations of state (a)-(d) have been combined with the ones of Harrison and Wheeler for the outer surface of the star, and Negele and Vautherin for the inner surface region. Table 1 gives an overview over the EOSs:

Table 1 Equations of state of this work

Model	Equation of state (Interactions)	Mass density range [g/cm^3]	Composition
	Harrison-Wheeler	$7.8 < \epsilon < 10^{11}$	Crystalline; light metals, electron gas
	Negele-Vautherin	$10^{11} < \epsilon < 10^{13}$	Crystalline; heavy metals, relativistic electron gas
Λ^{00}_{Bonn}	Relativistic Λ^{00} approximation (T matrix); based on the Bonn meson-exchange potential $(\sigma, \omega, \pi, \varrho, \eta, \delta)$	$10^{13} < \epsilon < 5 \cdot 10^{14}$	Neutrons
Λ^{00}_{HEA}	Same; based on the HEA meson-exchange potential $(\sigma, \omega, \pi, \varrho, \eta, \delta, \phi)$	$10^{13} < \epsilon < 5 \cdot 10^{14}$	Neutrons
HV	Relativistic Hartree approximation $(\sigma, \omega, \varrho)$	$10^{13} < \epsilon < 9 \cdot 10^{15}$	p, n, Λ, $\Sigma^{\pm,0}$, $\Xi^{0,-}$; e^-, μ^-
HFV	Relativistic Hartree-Fock approximation $(\sigma, \omega, \pi, \varrho)$	$10^{13} < \epsilon < 9 \cdot 10^{15}$	p, n, Λ, $\Sigma^{\pm,0}$, $\Xi^{0,-}$, Δ^-; e^-, μ^-

4 RESULTS

The equations of state, i.e. pressure P versus energy density ϵ, calculated for the relativistic Λ^{00}, Hartree (HV) and Hartree-Fock (HFV) approximations are graphically depicted in Fig. 2 over a wide range of energy densities ϵ [5]. The

symbols ϵ_0, ϵ_{HFV}, and ϵ_{HV} denote the energy density of normal nuclear matter and those values at which the population of hyperons for the HFV (Σ^-) and

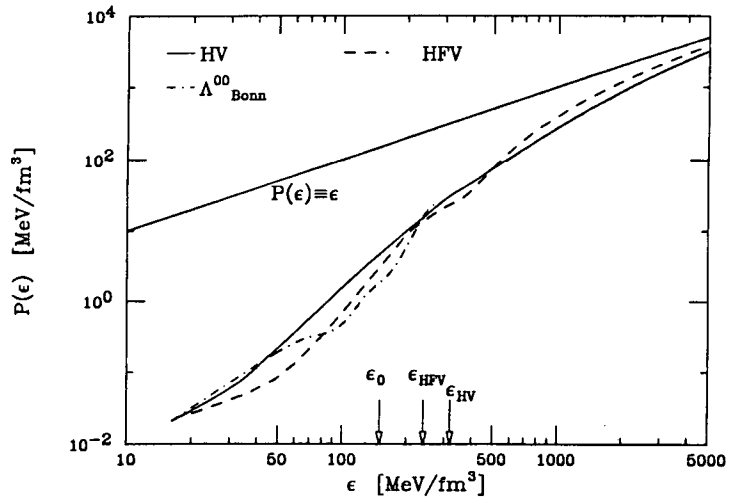

Fig. 2 Neutron star matter equations of state of this work.

HV (Λ hyperon) EOSs sets in, respectively. One sees that at large densities the EOSs stay causal ($P(\epsilon) \equiv \epsilon$ is the stiffest possible EOS). This is an important constraint on a realistic EOS. It should be noted that the constraint of causality is in general a problem for non-relativistic treatments[6]. Characteristic for the inclusion of two-particle correlations is the reduction of pressure ("softening") at a given value of ϵ relative to Hartree and Hartree-Fock EOSs[15,16]. This is demonstrated in Figure 2 for the Λ^{00}_{Bonn} EOS.

The lepton-baryon compositions of a neutron star calculated for the HV and HFV EOSs are shown in Figs. 3 and 4, respectively[4,5]. The vertical axes give the densities of leptons (ϱ^λ, $\lambda = e^-, \mu^-$) and baryons (ϱ^B, $B = p, n, ..$) relative to the total baryon density ϱ ($\equiv \sum_B \varrho^B$). As already mentioned in Section 3, at small nuclear densities neutron star matter is composed of nearly 100% neutrons. At larger ϱ values however, the lepton-baryon populations calculated for HV and HFV are radically different[5]. In the former case the Λ particle pos-

sesses the lowest threshold of the more massive baryon states (that are allowed to become populated in the framework of a self-consistent solution of Eqs. (3.3)-(3.5); universal coupling is assumed throughout[4,19]).

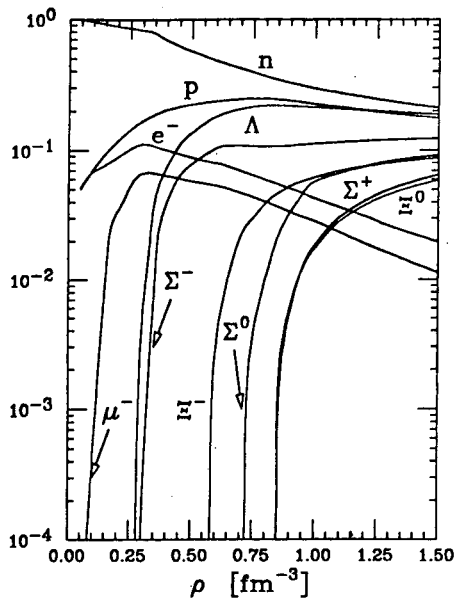

Fig. 3 Lepton-baryon population in neutron star matter calculated for HV.

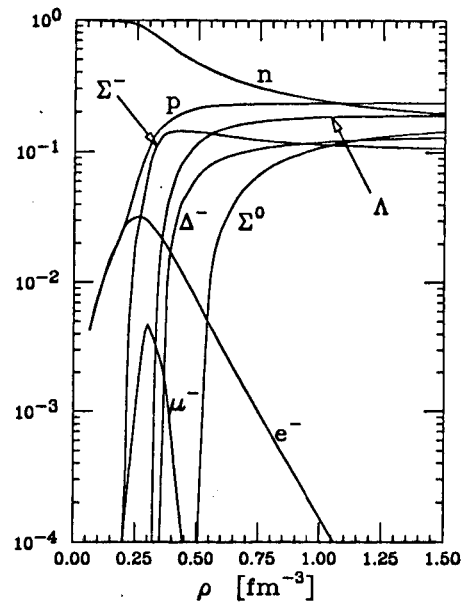

Fig. 4 Lepton-baryon population in neutron star matter calculated for HFV.

The actual threshold density is $\varrho^{\Lambda} \approx 0.28$ fm^{-3}. In contrast to this, the Σ^- state becomes populated first in the case of the HFV EOS, $\varrho^{\Sigma} \approx 0.22$ fm^{-3}. With increasing values of ϱ the more massive baryon states become successively populated, depending on charge and isospin of the particle in question. (Negatively charged baryons are "charge-favored" and baryons having the same isospin projection as the neutron are "isospin-unfavored".[4])

Table 2 summarizes the bulk parameters of our non-rotaing (spherical) neutron star studies for the four EOSs under consideration.[15] That quantity that is most sensitive against the softening caused by two-particle correlations is the star's radius (third row).

Table 2 Bulk properties of non-rotating maximum mass neutron stars (taken from Ref. 15)

	HV	$\Lambda^{00}_{Bonn} + HV$	HFV	$\Lambda^{00}_{HEA} + HFV$
$\log[\epsilon_c/(\mathrm{g}\cdot\mathrm{cm}^{-3})]$	15.37	15.37	15.38	15.38
$M_s/M_\odot$	1.976	1.969	2.198	2.195
R_s [km]	11.34	10.97	10.70	10.63
$\log[I/(\mathrm{g}\cdot\mathrm{cm}^2)]$ †	45.31	45.30	45.38	45.38
z	0.4355	0.4593	0.5945	0.6018

† The symbols I and z denote the moment of inertia and fractional redshift, respectively.

The influence of *rotation* on the properties of neutron stars is shown in Table 3 for stars rotating at their mass limits (i.e., $[dM/d\epsilon]_{\bar{\epsilon}}=0$ and $[dM/d\epsilon]_{\epsilon>\bar{\epsilon}}<0$)[15]. These values have been calculated under the assumption that the Keplerian frequency Ω_K^{GR}, defined in Eq. (2.10), determines the upper limit of stable neutron star rotation (which may however not be true, see below). An important result is that Hartle's method predicts a mass increase due to rotation at the mass limit of about $\Delta M/M_s \approx 20\%$, depending on the underlying EOS. This finding is in very good agreement with the results obtained by Friedman, Ipser, and Parker,[9] who solve Einstein's field equations exactly, and may give a hint

Table 3 Influence of rotation on the properties of neutron stars rotating at the mass limit (taken from Ref. 15).

	HV	$\Lambda^{00}_{Bonn} + HV$	HFV	$\Lambda^{00}_{HEA} + HFV$
Ω_K^{GR} $[10^4\ \frac{1}{\mathrm{s}}]$	0.92	0.98	1.18	1.19
$\log[\epsilon_c/(\mathrm{g}\cdot\mathrm{cm}^{-3})]$	15.15	15.15	15.23	15.23
$M/M_\odot$	2.26	2.25	2.52	2.51
$\Delta M/M_s$	0.20	0.20	0.18	0.18
R_{eq} [km] †	14.8	14.2	13.0	12.9
R_p [km]	10.2	9.6	9.0	8.9
$\log[I/(\mathrm{g}\cdot\mathrm{cm}^2)]$	45.36	45.35	45.39	45.39
V_{eq} ‡	0.55	0.56	0.64	0.64

† By R_{eq} and R_p we denote the equatorial and polar radius, respectively.
‡ The orbital velocity of a mass point located at the star's equator is denoted by V_{eq}.

that Hartle's formalism - developed originally for "slowly" rotating, massive objects - can be applied for the investigation of rotating star configurations with rotational frequencies of $\approx 10^4$ s^{-1}.

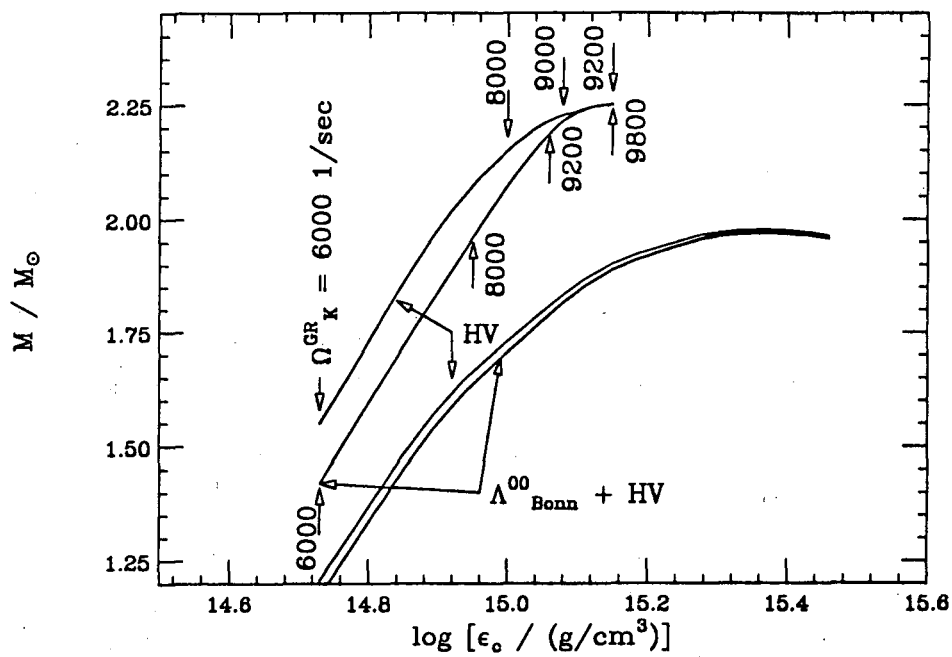

Fig. 5 Gravitational star mass vs. central energy density for star models HV and $\Lambda^{00}_{Bonn} + HV$.

Figure 5 displays the gravitational neutron star mass as a function of central energy density for both the non-rotating (the two lower lying curves) as well as rotating star models HV and $\Lambda^{00}_{Bonn} + HV$. The upper lying curves show the increase of mass due to rotation at the limiting (Keplerian) frequency. Two-particle correlations lead to neutron stars with smaller equatorial radii (5th row of Table 3). Thereby these stars will rotate at higher rotational frequencies when reaching the instability point at which mass shedding sets in.

Figure 6 exhibits the Kepler angular velocity Ω_K^{GR} as a function of gravitational mass of the rotating neutron star[15]. Different EOSs lead to a quite different dependence of Ω_K^{GR} on M.

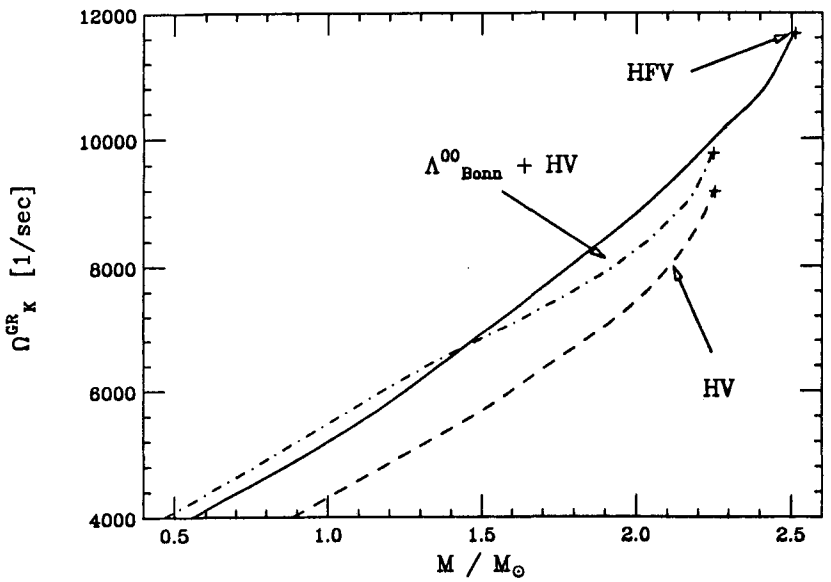

Fig. 6 Keplerian frequency vs. rotational neutron star mass.

The end point of each sequence marked by a cross denotes the maximum Keplerian frequency compatible with the underlying EOS (the corresponding values are given in the first and third row of Table 3).

Up to now we have assumed that the Keplerian frequency, determined at the neutron star's mass limit, sets the limit of stable rotation. Other types of instabilities however may already set in at lower rotational velocities. A dominant role is played by the so-called non-axisymmetric dynamical instabilities. These are excited by gravitational radiation-reaction and moderated by viscosity[20]. The corresponding instability frequencies, Ω_m^ν, depend on the order of the instability mode (subscript m) and on viscosity (superscript ν)[15,20]. The latter depends on the star's temperature. We assume values for ν in the range of $0 < \nu < 200$ $\mathrm{cm^2 s^{-1}}$. For the purpose of orientation, $\nu \approx 100$ $\mathrm{cm^2 s^{-1}}$ corresponds to a fluid temperature of $T \approx 10^9$ K. For decreasing temperatures, $\nu(T)$ increases rapidly like $\nu \propto T^{-2}$. A value of $\nu \approx 200$ $\mathrm{cm^2 s^{-1}}$ may serve as a rough upper bound on viscosity of ("cold") neutron star matter.

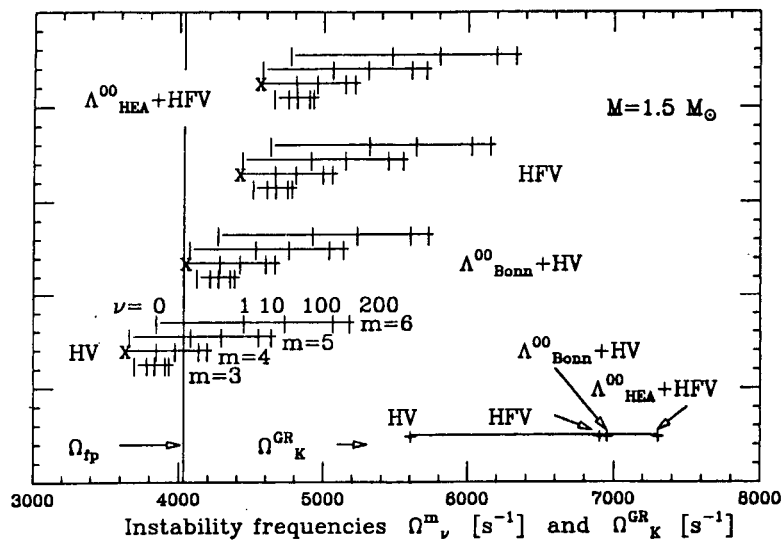

Fig. 7 Instability frequencies Ω_ν^m and Ω_K^{GR} for different EOSs.

Figure 7 shows the calculated instability frequencies Ω_m^ν in comparison with the Keplerian values for the different EOSs[15]. The underlying star model has a gravitational mass of $M = 1.5\,M_\odot$ (cf. $M_{PSR(1913+16)}$). The frequencies Ω_m^ν are shown for modes $m = 3, 4, 5, 6$. It follows that the *maximum* rotational frequency of stable neutron star rotation is clearly set by the $\Omega_{m=4}^{\nu=0}$ mode (left lying crosses), which is roughly 40% smaller than Ω_K^{GR} for the same EOS, and *not* by the Keplerian frequency. One sees that the HV EOS may not be compatible with the existence of rotating neutron stars at frequencies $\approx \Omega_{\rm fp} \equiv 1.6$ msec (Sect. 1) *and* having gravitational masses of $M \approx 1.5\,M_\odot$ as a slowly rotating star.

5 SUMMARY

We have studied the properties of non-rotating as well as rotating relativistic neutron stars. The properties of the former follow from the Oppenheimer-Volkoff equations. The treatment of rotating neutron stars is much more complicated. Our investigation is based on Hartle's formalism. This method solves Einstein's general relativistic field equations by means of a perturbation solution on the

Schwarzschild metric of spherically symmetric, non-rotating massive objects. It turns out that when using a proper method for constructing the limiting rotational frequency of a neutron star, Hartle's formalism leads to results for its bulk properties (especially its gravitational mass) that are of comparable size with those obtained by solving Einstein's equations exactly[15]. A detailed investigation of the instability modes caused by gravitational radiation-reaction shows that *this* mechanism sets the upper limit on stable neutron star rotation (for a young and therefore hot star), and *not* the Keplerian frequency.

Our investigaton is based on "realistic" neutron matter equations of state, which incorporate the essential physical features of electrically charge neutral high-density matter in β stable equilibrium (i.e., baryon/lepton population, causality). An additional basic feature of our EOSs is the inclusion of two-particle correlations over a certain density range by introducing a relativistic scattering (T) matrix in matter. The masses and coupling strenghts of the HEA and *Bonn* meson-exchange potentials are used for its calculation in the so-called Λ^{00} approximation. Our EOSs allow for a systematic investigation of the influence of different many-body treatments (i.e. Hartree, Hartree-Fock, Λ^{00}) on the bulk properties of (non-) rotating neutron stars.

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