UC 412

LBL-29141
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Submitted to Physics Letters B

Limiting Angular Velocity of Realistic Relativistic Neutron Star Models

F. Weber and N.K. Glendenning

June 1990

For Reference

Not to be taken from this room.

NUCLEAR SCIENCE DIVISION

LBL-29141
Copy 1
Bldg. 50 Library.

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

LBL - 29141

Limiting Angular Velocity of Realistic Relativistic Neutron Star Models[†]

F. Weber[‡] and N. K. Glendenning

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720, U.S.A.

June 1990

[†] This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U. S. Department of Energy under Contract DE-AC03-76SF00098 and by the Max-Kade Foundation of New York.

[‡] Max Kade Foundation Research Fellow. Permanent address: Institute for Theoretical Physics, University of Munich, Theresienstrasse 37/III, D-8000 Munich 2, Federal Republic of Germany.

Limiting Angular Velocity of Realistic Relativistic Neutron Star Models[†]

F. Weber[‡] and N. K. Glendenning

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720, U.S.A.

Abstract

The Keplerian velocity as well as those frequencies at which instability against gravitational radiation-reaction sets in are calculated for rotating neutron star models of gravitational mass $1.5M_{\odot}$. The investigation is based on four different, realistic neutron star matter equations of state. Our results indicate that the gravitational radiation instability sets in well *below* (i.e., 63-71% of) the Keplerian frequency, and that *neutron* stars are limited to rotational periods greater than about 1 msec. In young and therefore hot ($T \approx 10^{10}$ K) neutron stars the $m = 5$ (± 1) modes and in old stars, after being spun up and reheated by mass accretion, the $m = 4$ and/or $m = 3$ modes may set the limit on stable rotation.

PACS numbers: 97.10.Kc, 97.60.Jd

[†] This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U. S. Department of Energy under Contract DE-AC03-76SF00098 and by the Max-Kade Foundation of New York.

[‡] Max Kade Foundation Research Fellow. Permanent address: Institute for Theoretical Physics, University of Munich, Theresienstrasse 37/III, D-8000 Munich 2, Federal Republic of Germany.

A fundamental problem encountered in the treatment of rotating neutron stars is the question of stability of these objects. Obviously, an *absolute upper bound* on stable rotation is set by the (1) Keplerian frequency, Ω_K^{GR}, which is defined by the balance between gravitational and centrifugal forces at the star's equator (by GR we denote its general relativistic expression, see Eq. (2) below).[1] Stars cannot rotate at velocities higher than the Keplerian value since they would shed mass at their equators. Other types of instabilities have their origin in the onset of (2) axisymmetric differential rotation of the star's matter,[2,3] and the growth of (3) non-axisymmetric instability modes that are driven by gravitational radiation-reaction.[4−8] The latter instability can be stabilized by viscosity.[9−14] Lindblom has suggested that this gravitational driven instability is probably completely damped out in sufficiently cold neutron stars ($T < 10^7$ K) by virtue of their large viscosity.[15] Below this temperature another secular instability can occur that is (4) *driven* by viscosity.

In this Letter we shall concentrate on the problem of stable rotation of newly formed (and therefore hot, i.e. $T \approx 10^{10}$ K) neutron stars as well as on estimating the critical angular velocity of rotating neutron stars that have been spun up by mass accretion from a companion and thereby reheated to $T \approx 10^8 - 10^9$ K. Our investigation is therefore based on items (1) and (3). We need not investigate items (2) and (4) which are irrelevant for young and also reheated stars having temperatures that lie in the above given range (small viscosities).

The determination of viscosity (denoted by ν) of neutron star matter is a cumbersome and not yet completely solved problem.[11,16−18] Typical values of ν discussed in the literature[7,17] are $\nu(T) \approx 100$ cm^2 s^{-1} for a temperature of $T \approx 10^9$ K. The temperature of a newly formed neutron star is expected to be about 10^{10} K after the initial burst of neutrino emission.[19,20] The cooling to about 10^9 K may take place within the first two years.[20] As the star cools, the viscosity increases rapidly like[17] $\nu(T) \propto T^{-2}$.

Our determination of Ω_K^{GR} for a given neutron star matter equation of state (EOS) has been outlined in detail in Refs. 21,22. The task is to solve Einstein's field equations of general relativity for a rotating, massive object. This can be accomplished by either a direct numerical treatment[1] or resorting to the development

of a perturbation solution on the Schwarzschild metric[23,24] (Hartle's method), i.e.

$$ds^2 = -e^{2\,\nu} dt^2 + e^{2\,\psi}\,(d\phi - \omega\, dt)^2 + e^{2\,\mu}\, d\theta^2 + e^{2\,\lambda} dr^2 + O\left(\frac{\Omega^3}{\Omega_c^3}\right). \qquad (1)$$

The quantity ω in Eq. (1) is the angular velocity of the local inertial frame. It is proportional to Ω, the star's rotational frequency. The critical velocity, $\Omega_c \equiv \sqrt{M_s G/R_s^3}$, is defined in terms of the gravitational mass and radius, M_s and R_s respectively, of the corresponding non-rotating (same central energy density) star model. In the framework of Hartle's method one makes an ansatz for the metric functions ν, ψ, μ, and λ that is based on the Minkowskian metric (valid for a spherically symmetric object) but "corrected" for deformation by the introduction of so-called monopole and quadrupole perturbation functions. The latter are solutions of sets of coupled differential equations derivable from Einstein's equations. The resulting set of relations is known as Hartle's stellar structure equations.[22–24]

The Kepler frequency, given by[1] (primes refer to derivatives with respect to the radial coordinate)

$$\Omega_K^{GR} = e^{\nu-\psi}\ V\ +\ \omega, \qquad V \equiv \frac{\omega'}{2\psi'}\, e^{\psi-\nu} \sqrt{\frac{\nu'}{\psi'} + \left(\frac{\omega'}{2\psi'}\, e^{\psi-\nu}\right)^2}\,, \qquad (2)$$

can be calculated once the metric functions are known. These are given as solutions of Hartle's stellar structure equations. The construction of star models rotating at $\Omega = \Omega_K^{GR}$ by means of a self-consistent treatment has been performed elsewhere.[21,22] An important result is that Hartle's method leads to results for the bulk properties (e.g. mass and Keplerian frequency) of the treated star models that are in good agreement with those obtained by an exact solution of Einstein's equations.[1,21,22]

The essential input quantity for constructing star models is the equation of state of neutron star matter. Since densities in the cores of neutron stars are up to ten times as large as the density of normal nuclear matter,[1,21,25–28] we employ a relativistic hadron field theory involving $p, n, \Sigma^{\pm,0}, \Lambda, \Xi^{0,-}, \Delta^{++,+,0,-}$ interacting through the exchange of $\sigma, \omega, \pi, \varrho$ mesons.[21,22,25–27] A number of leptons is necessary to achieve the important constraint of electrical charge neutrality of neutron star matter.[25] This theory is solved in the relativistic Hartree[25–28] and Hartree-Fock[21,22,26] approximations. The corresponding EOSs are denoted

by HV and HFV, respectively.[26] Furthermore we apply the relativistic ladder approximation[29] (T matrix treatment) in order to take the influence of (Brueckner-type) two-particle correlations on the EOS into account. To this the *Bonn*[30] and HEA[31] meson-exchange potentials served as an input. We denote the respective EOSs by $\Lambda^{00}_{Bonn} + HV$ and $\Lambda^{00}_{HEA} + HFV$.[21,22]

TABLE I. Rotating neutron star properties at the mass limit calculated for neutron star equations of state HV, $\Lambda^{00}_{Bonn} + HV$, HFV, $\Lambda^{00}_{HEA} + HFV$. The entries are (from top to bottom): central energy density in units of normal nuclear matter density, ϵ_0 (= 140 MeV/fm^3), Kepler frequency, gravitational mass, equatorial and polar radii, stability parameter calculated for $\Omega = \Omega_K^{GR}$, redshift in back- and forward direction, redshift at the pole.

	HV	$\Lambda^{00}_{Bonn} + HV$	HFV	$\Lambda^{00}_{HEA} + HFV$
ϵ_c/ϵ_0	5.71	5.71	6.79	6.79
$[\epsilon_c/\epsilon_0]_{\rm nonrot}$ †	9.29	9.29	9.64	9.64
Ω_K^{GR} $[10^4\ \frac{1}{s}]$	0.92	0.98	1.18	1.19
$M/M_\odot$	2.26	2.25	2.52	2.51
R_{eq} [km]	14.8	14.2	13.0	12.9
R_p [km]	10.2	9.6	9.0	8.9
t_K ‡	0.11	0.12	0.13	0.13
z_B	2.23	2.31	2.99	3.02
z_F	-0.69	-0.70	-0.74	-0.75
z_p	0.79	0.90	1.59	1.64

†) $[\epsilon_c/\epsilon_0]_{\rm nonrot}$ refers to the value of the central energy density of the non-rotating maximum mass star.
‡) The stability parameter t is defined by $t \equiv T/\mid W \mid$, $T \equiv \frac{J\Omega}{2}$, $W = M_{proper} + T - M$.[1]

Table I contains the bulk properties of rotating neutron stars at the *mass limit* calculated for our four different EOSs. The values of the limiting Keplerian frequencies are shifted due to the inclusion of two-particle correlations from 9200 s^{-1} (HV) and 11800 s^{-1} (HFV) to slightly larger values of 9800 s^{-1} ($\Lambda^{00}_{Bonn} + HV$) and 11900 s^{-1} ($\Lambda^{00}_{HEA} + HFV$), respectively.[13] The mass increase due to rotation is about 20% (18%) for HV (HFV) relative to the non-rotating value. We find maximum mass values of 2.26 $M_\odot$ (2.52 $M_\odot$) for HV (HFV) which remain nearly unchanged if correlations are included (cf. fourth row of Table I).

A characteristic feature encountered in the treatment of rotating neutron star models is the decrease of the central energy density, ϵ_c, relative to the non-rotating star configuration.[1,21,22] In our case, the ϵ_c values drop from $\epsilon_c \approx 9.3\,\epsilon_0$ and $\epsilon_c \approx 9.6\,\epsilon_0$ [21,22,25,26] to $5.7\ \epsilon_0$ (HV) and $6.8\ \epsilon_0$ (HFV), respectively, in the case of rotation at the mass limit (cf. Table I and Fig.1). Energy densities as large as $\approx 10\,\epsilon_0$ in the cores of massive non-rotating neutron star models are known to be typical.[1,25−28] However they make the use of an EOS originally derived for matter which consists of individual baryons and mesons rather questionable.[27,32,33] More likely a phase transition to quark matter in the core of the more massive stars with high central densities occurs.[34,35] The situation is less extreme for rotating neutron stars because of their somewhat smaller central energy densities and also for less massive stars lying in the range of measured masses. For stars of these masses ($\sim 1.5 M_\odot$) the central densities of our models are 3-4 nuclear density.

In Fig. 1 we exhibit the dependence of gravitational star mass on central energy density, $M(\epsilon_c)$, for the HV and $\Lambda^{00}_{Bonn} + HV$ EOSs.

For certain values of ϵ_c, the corresponding Keplerian frequencies are drawn in. The mass increase caused by rotation at $\Omega = \Omega_K^{GR}$ is shown by the two upper lying curves.

Next we turn our interest to the instability modes that correspond to item (3). These are related to star oscillation modes having angular dependence $e^{i\,m\,\phi}$, where ϕ denotes the azimuthal angle and m is a spherical harmonic index. It has been shown by Lindblom and Hiscock[12] that viscosity tends to suppress the instability modes related to (3) in sufficiently slowly rotating stars. The frequencies at which these instabilities set in, denoted by Ω_m^ν, can be obtained from (ν indicates the viscosity dependence of the frequencies)[13]

$$\Omega_m^\nu \;=\; \frac{\Omega_c}{m}\sqrt{\frac{2\,m\,(m-1)}{2\,m+1}}\;\left[\tilde{\alpha}_m(\Omega_m^\nu) + \tilde{\gamma}_m(\Omega_m^\nu)\left(\frac{\tau_{g,m}}{\tau_{\nu,m}}\right)^{\frac{1}{2\,m+1}}\right]\,, \qquad (3)$$

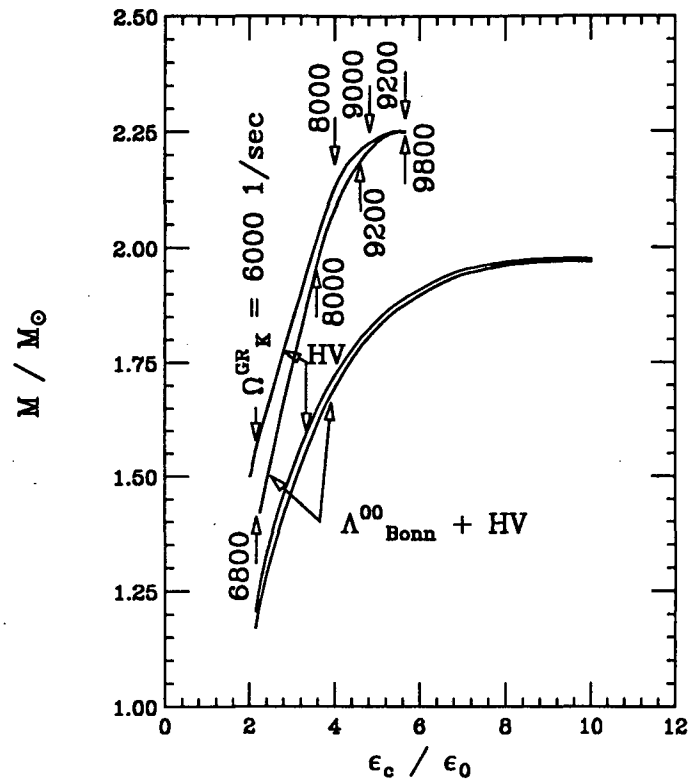

FIG. 1. Gravitational star mass as a function of central energy density (in units of ϵ_0) for star models constructed from $\Lambda^{00}_{Bonn} + HV$ and HV (i.e., with and without the inclusion of two-particle correlations, respectively). The two (lower) upper lying curves refer to (non-) rotating star models. The Keplerian frequencies are given for a number of ϵ_c values.

with

$$\tau_{g,m} = \frac{2}{3}\,\frac{(m-1)\,[(2m+1)!!]^2}{(m+1)\,(m+2)}\left(\frac{2m+1}{2m(m-1)}\right)^m \Omega_c^{-2(m+1)}\,R_s^{-(2m+1)}\,, \quad (4)$$

$$\tau_{\nu,m} = \frac{R_s^2}{(2m+1)\,(m-1)}\,\frac{1}{\nu}\,. \quad (5)$$

The two latter equations (4) and (5) give the gravitational growth time scale ($\tau_{g,m}$) of the instability mode of order m and the time scale that determines the rate at which this particular mode is damped by virtue of viscosity ($\tau_{\nu,m}$), respectively. A characteristic feature of this treatment is that Ω^{ν}_m of Eq. (3) merely depends on the spherical star properties, R_s and M_s. The functions $\tilde{\alpha}_m$ and $\tilde{\gamma}_m$ contain information about the pulsation of the rotating star models and are difficult to determine.[13,18] A reasonable first step is to replace them by their corresponding Maclaurin spheroid functions,[13,18] α_m and γ_m, i.e. $\tilde{\alpha}(\Omega_m) \approx \alpha_m(\Omega_m)$

and $\tilde{\gamma}(\Omega_m) \approx \gamma_m(\Omega_m)$. Since the latter do not depend strongly on the angular velocity Ω_m, this motivates the approximation[21] $\alpha_m(\Omega_m) \approx \alpha_m(0) = 1$ and $\gamma_m(\Omega_m) \approx \gamma_m(0) = 1$ (cf. Ref. 13)). In this Letter we do *not* apply this approximation scheme but rather take $\alpha_m(\Omega_m)$ and $\gamma_m(\Omega_m)$ as calculated in Ref. 36 (for the oscillations of rapidly rotating inhomogeneous Newtonian stellar models; polytropic index n=1) and Ref. 13 (for uniform-density Maclaurin spheroids, i.e. n=0), respectively. Managan has shown that Ω_m^ν depends much more strongly on the EOS and the mass of the neutron star model (through Ω_c and $\tau_{g,m}$, see Eqs. (3),(4)) than on the polytropic index assumed in calculating α_m.[14]

TABLE II. Properties of rotating neutron star models of gravitational mass $M = 1.5\,M_\odot$ rotating at their Keplerian frequencies, Ω_K^{GR}. The entries are explained in Table I.

	HV	$\Lambda^{00}_{Bonn} + HV$	HFV	$\Lambda^{00}_{HEA} + HFV$
ϵ_c/ϵ_0	2.09	2.42	2.75	2.85
$[\epsilon_c/\epsilon_0]_{\rm nonrot}$ †	2.93	3.11	3.61	3.64
Ω_K^{GR} $[10^4\ \frac{1}{s}]$	0.56	0.69	0.69	0.73
R_{eq} [km]	17.3	15.3	15.8	15.2
R_p [km]	11.8	10.2	10.3	9.9
t_K	0.096	0.107	0.105	0.107
z_B	1.21	1.39	1.40	1.42
z_F	-0.55	-0.58	-0.58	-0.58
z_p	0.30	0.38	0.38	0.40

†) $[\epsilon_c/\epsilon_0]_{\rm nonrot}$ refers to the non-rotating 1.5 $M_\odot$ star model.

The numerical outcome of this treatment is shown in Fig. 2 and Tables II and III for a rotating neutron star model of $M = 1.5\,M_\odot$ (cf.[37] $M_{PSR\,1913+16}$).
One clearly sees from Fig. 2 that all rotational instability periods, P_m^ν ($\equiv 2\pi/\Omega_m^\nu$), referring to modes $m = 3,4,5,6$ and viscosities ranging from ≈ 0 (hot star) up to 200 cm^2s^{-1} (which may serve as a rough upper bound on $\nu(T)$ of a somewhat cooled neutron star, i.e. $T \approx 10^9$ K) are located well *above* the Keplerian periods, P_K^{GR}; specifically it follows that for $\nu \approx 0$ the $m = 5$ modes (periods $P_{m=5}^{\nu=0}$) possess the largest periods (lowest frequencies) and therefore are excited first. The numerical outcome is summarized in the second through fourth rows of Table

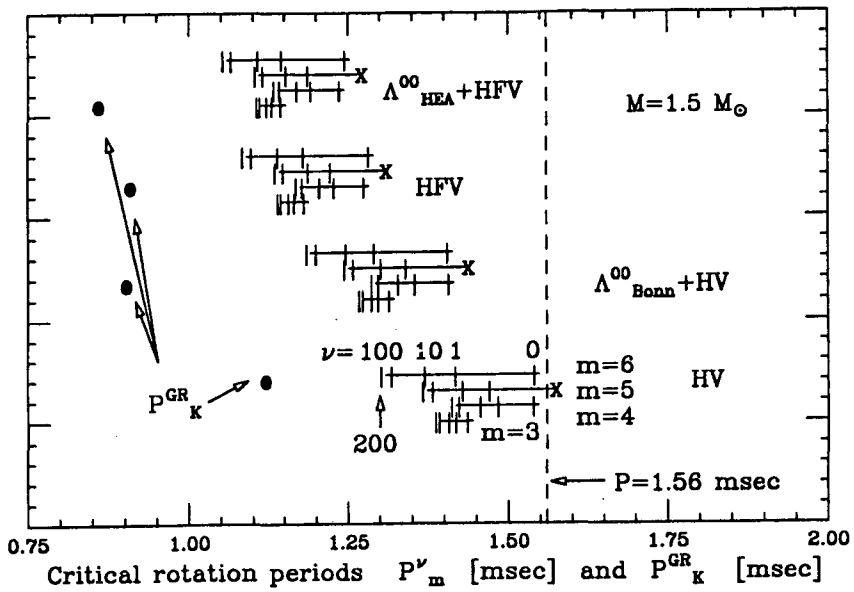

FIG. 2. Critical rotational periods P^ν_m and P^{GR}_K at which instability against gravitational radiation-reaction and mass shedding sets in, respectively. The periods P^ν_m are shown for instability modes $m = 3, 4, 5, 6$ for the four equations of state of this work. The right lying crosses denote the *minimum* stable rotational periods (maximum frequencies) calculated from our EOSs for young and therefore hot neutron stars. The tick marks refer to viscosities ν =0,1,10,100, and 200 cm^2 s^{-1} (in this order from right to left). The dots give P^{GR}_K.

III. According to this, the $m = 5\,(\pm 1)$ modes can be expected to *set the limit* on stable neutron star rotation for all EOSs under consideration. *That is to say, hot neutron stars cannot rotate at periods shorter than this limit* (marked by crosses in Fig. 2). Of the four realistic models studied, three can rotate stably at periods as small as 1.56 msec (PSR 1913+16). The model that is stable at the smallest period, 1.27 msec, is $\Lambda^{00}_{HEA} + HFV$.

The Keplerian periods are clearly smaller than $P^{\nu=0}_{m=5}$ for all EOSs under consideration and irrelevant as far as the discussion of stable rotation is concerned. One finds for the corresponding rotational frequencies $\Omega^{\nu=0}_{m=5} \approx$(0.63-0.71) Ω^{GR}_K. For the purpose of comparison, we have also calculated the limiting rotational periods for the EOSs used by Lindblom.[13] It turned out that the $P^{\nu=0}_{m=5}$ periods set again the limit on stable rotation. The critical rotational frequencies $\Omega^{\nu=0}_{m=5}$ are in this case related to the corresponding Keplerian velocities by $\approx$(0.70-0.80) Ω^{GR}_K. Finally we make a comparison with the empirical formula $\Omega(T) \approx 0.86\ \Omega_{\max}$ (for $T \approx 10^{10}$ K) of Ipser and Lindblom[36] for estimating $\Omega^{\nu=0}_{m=4,5}$.[15] Here, $\Omega_{\max}$ denotes a max-

imum frequency that is related to the Keplerian velocity by $\Omega_{\max} \approx 0.93\ \Omega_K^{GR}$. From this one gets $\Omega_{m=4,5}^{\nu=0} \approx (0.86)\cdot(0.93)\ \Omega_K^{GR} \approx 0.80\ \Omega_K^{GR}$. In other words, the maximum stable rotational frequency that a newly born neutron star can have is roughly 20% smaller than the Keplerian value, for the models studied by these authors. Taking into account the fact that the amount of the reduction depends rather sensitively on the assumed initial temperature of the neutron star ($T = 5\cdot 10^{10}$ K leads to a $\approx 77\%$ reduction) as well as on the underlying EOS, it seems likely that the actual limiting frequency of a newly born star may lie in the range $0.65 < \Omega_{m=4,5}^{\nu=0}/\Omega_K^{GR} < 0.85$. Our investigation, based on the equations of state we use, as described earlier and in ref. [21], suggests that the limit lies closer to the lower part of this range.

The instability parameter, t (defined in Table I), corresponding to the above m=5 mode is rather insensitive to different equations of state. It has values of $0.043 \leq t_{m=5}^{\nu=0} \leq 0.046$ (see Table III).

TABLE III. Properties of a neutron star of gravitational mass $M = 1.5\,M_\odot$ rotating at its maximum possible angular velocity, $\Omega_{m=5}^{\nu=0}$, calculated for the four different equations of state of this work.

	HV	$\Lambda_{Bonn}^{00} + HV$	HFV	$\Lambda_{HEA}^{00} + HFV$
ϵ_c/ϵ_0	2.50	2.65	3.35	3.43
$\Omega_{m=4}^{\nu=0}$ $[\frac{1}{s}]$ †	4063	4442	4900	5053
$\Omega_{m=5}^{\nu=0}$ $[\frac{1}{s}]$	3987	4364	4789	4936
$\Omega_{m=6}^{\nu=0}$ $[\frac{1}{s}]$	4060	4448	4872	5020
$t_{m=5}^{\nu=0}$	0.043	0.046	0.045	0.045
$\log[I/(g\cdot cm^2)]$	45.25	45.22	45.18	45.16
R_{eq} [km]	15.36	13.98	13.44	13.09
R_p [km]	13.11	11.97	11.50	11.22
z_B	1.44	1.57	1.64	1.69
z_F	-0.59	-0.61	-0.62	-0.63
z_p	0.25	0.28	0.29	0.30

†) The frequencies $\Omega_m^{\nu=0}$ refer to zero-viscosity instability modes of order $m = 4, 5, 6$.

It should be noted that one obtains for rotation *at the mass limit* $t_K \approx 0.12$ (cf. seventh row of Table I), and $t_K \approx 0.10$ for the $M = 1.5\,M_\odot$ mass model (cf. sixth row of Table II). Only in the former case the stability parameter would come

close to that value at which instability against a bar mode (i.e., $m = 2$) is expected to set in (≈ 0.14).

In our earlier investigation[21] the above mentioned approximation scheme, $\alpha_m \equiv 1$ and $\gamma_m \equiv 1$, was applied for the calculation of P_m^ν. A comparison of both cases shows that the changes with respect to $P_m^{\nu=0}|_{\min}$ (crosses in Fig. 2) are smaller than 9%. One obtains however deviations for larger viscosities which are the larger the larger ν is. This behavior becomes clear from Eqs. (3)-(5), since then the angular velocity dependence of both $\alpha(\Omega_m)$ *and* $\gamma(\Omega_m)$ becomes important.

Up to now we have discussed the onset of instability modes of young and therefore hot neutron stars ($\nu \approx 0$). The limits derived apply therefore to any pulsar whose thermal history since birth has involved only cooling, since while hot it would have spun down by gravitational radiation until viscosity, at its present or some intermediate temperature, damped the instability. For colder neutron stars ($T \approx 10^9$ K, $\nu \approx 200$ cm^{2}s^{-1}) - whose present angular velocities are reached by the spin-up process of old neutron stars driven by mass accretion from a companion - it follows from Fig. 2 that the $m = 4$ and/or $m = 3$ instability modes, $P_{m=3,4}^{\nu=200}$, possess the *largest* rotational periods and thus set the limit on stable rotation in this case. We stress that this conclusion however may only be true if the above mentioned axisymmetric differential rotation instability, item (2), occurs at smaller rotational periods than those given by $P_{m=3,4}^{\nu=200}$.

In summary, we have presented an investigation of stable neutron star rotation by assuming that the limiting frequency is determined by either the Keplerian velocity (Ω_K^{GR}) or instability modes (Ω_m^ν) caused by the emission of gravitational radiation that are damped by the presence of viscosity. We find that in the case of a newly formed (hot) rotating neutron star for all four equations of state under consideration the maximum rotational frequency is set by the gravitationally excited $m = 5$ instability mode. These are (at most) between 63 and 71% of the Keplerian frequencies and *set therefore the limit on stable neutron star rotation.* For neutron stars that have been spun up and reheated to temperatures $T \approx 10^9$ K by the accretion of mass from a companion, we find that the limit on stable rotation is set by the $m = 4$ and/or $m = 3$ instability modes.

The indication of this work is that gravitation radiation-reaction instabilities set a lower limit of a little more than 1 msec (1.27 msec for our models) on the

rotation period of realistic relativistic neutron star models. This has possibly very important implications for the nature of any pulsar that is found to have a shorter period.[38] For example Sawyer[39] has calculated that the viscosity of quark matter is many orders of magnitude greater than that of neutron matter, so that for a quark star the smaller Kepler period would set the lower limit.

Acknowledgement

One of us (F. W.) thanks the President of the Max Kade Foundation of New York for a grant.

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U. S. Department of Energy under Contract DE-AC03-76SF00098 and the Max Kade foundation.

References

1. J. L. Friedman, J. R. Ipser, and L. Parker, Ap. J. **304**, 115 (1986), and references contained therein.
2. J. P. Ostriker and P. Bodenheimer, Ap. J. **151**, 1089 (1968).
3. R. H. Durisen, Ap. J. **199**, 179 (1975).
4. S. Chandrasekhar, Phys. Rev. Lett. **24**, 611 (1970); Ap. J. **161**, 561 (1970).
5. J. L. Friedman and B. F. Schutz, Ap. J. **222**, 281 (1978).
6. J. L. Friedman, Comm. Math. Phys. **62**, 247 (1978).
7. J. L. Friedman, Phys. Rev. Lett. **51**, 11 (1983).
8. R. V. Wagoner, Ap. J. **278**, 345 (1984).
9. L. Lindblom and S. L. Detweiler, Ap. J. **211**, 565 (1977).
10. S. L. Detweiler and L. Lindblom, Ap. J. **213**, 193 (1977).
11. N. Comins, Mon. Not. Roy. Astron. Soc. **189**, 233, 255 (1979).
12. L. Lindblom and W. A. Hiscock, Ap. J. **267**, 384 (1983).
13. L. Lindblom, Ap. J. **303**, 146 (1986).
14. R. A. Managan, Ap. J. **309**, 598 (1986).
15. L. Lindblom, Ap. J. **317**, 325 (1987).
16. G. M. Ewart, R. A. Guyer, and G. Greenstein, Ap. J. **202**, 238 (1975).
17. E. Flowers and N. Itoh, Ap. J. **206**, 218 (1976); **230**, 847 (1979).
18. C. Cutler and L. Lindblom, Ap. J. **314**, 234 (1987).
19. G. Glen and P. Sutherland, Ap. J. **239**, 671 (1980).
20. K. Nomoto and S. Tsuruta, Ap. J. **312**, 711 (1987), and references contained therein.
21. F. Weber, N. K. Glendenning, and M. K. Weigel, *Structure and Stability of Rotating Relativistic Neutron Stars* (LBL-28845, 1990).
22. F. Weber, N. K. Glendenning, and M. K. Weigel, *Rotating Relativistic Neutron Stars* (LBL-29106, 1990).
23. J. B. Hartle, Ap. J. **150**, 1005 (1967).
24. J. B. Hartle and K. S. Thorne, Ap. J. **153**, 807 (1968).
25. N. K. Glendenning, Ap. J. **293**, 470 (1985), and references contained therein.
26. F. Weber and M. K. Weigel, Nucl. Phys. **A505**, 779 (1989), and references contained therein.

27. N. K. Glendenning, *Neutron Stars, Fast Pulsars, Supernovae and the Equation of State of dense Matter*, in: NATO Advanced Study Institute, "The Nuclear Equation of State", ed. by W. Greiner, Peniscola, Spain, May 21 - June 3, 1989, to be published by Plenum Press, London (LBL-27293).
28. N. K. Glendenning, Nucl. Phys. **A493**, 521 (1989).
29. P. Poschenrieder and M. K. Weigel, Phys. Rev. **C38**, 471 (1988).
30. R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).
31. K. Holinde, K. Erkelenz, and R. Alzetta, Nucl. Phys. **A194**, 161 (1972); **A198**, 598 (1972).
32. N. K. Glendenning, *Supernovae, Compact Stars and Nuclear Physics*, in: Proc. Int. Nucl. Phys. Conf., Sao Paulo, Brasil, August 20-26, 1989, (World Scientific, Singapore, 1990).
33. N. K. Glendenning, *Strange – Quark – Matter Stars*, in: Proc. Int. Workshop on Relativ. Aspects of Nucl. Phys., ed. by T. Kodama, K. C. Chung, S. J. B. Duarte, and M. C. Nemes (World Scientific, Singapore, 1990).
34. G. Baym and S. Chin, Phys. Lett. **62B**, 241 (1976).
35. G. Chapline and M. Nauenberg, Nature (London) **264**, 235 (1976); Phys. Rev. **D16**, 456 (1977).
36. J. R. Ipser and L. Lindblom, Phys. Rev. Lett. **24**, 2777 (1989); Ap. J. **355**, 226 (1990).
37. J. H. Taylor and J. M. Weisberg, Ap. J. **345**, 434 (1989).
38. N. K. Glendenning, Phys. Rev. Lett. **63** (1989) 2629.
39. R. F. Sawyer, Phys. Lett. **233** (1989) 412.

Postal Addresses

Norman K. Glendenning, Lawrence Berkeley Laboratory, Nuclear Science Division, University of California, 1 Cyclotron Road, Berkeley, California 94720, U.S.A.

Fridolin Weber, Lawrence Berkeley Laboratory, Nuclear Science Division, University of California, 1 Cyclotron Road, Berkeley, California 94720, U.S.A.; permanent address: Institute for Theoretical Physics, Ludwig-Maximilians University of Munich, Theresienstrasse 37/III, D-8000 Munich 2, F.R.G.

LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
INFORMATION RESOURCES DEPARTMENT
BERKELEY, CALIFORNIA 94720