LBL-29276
Preprint

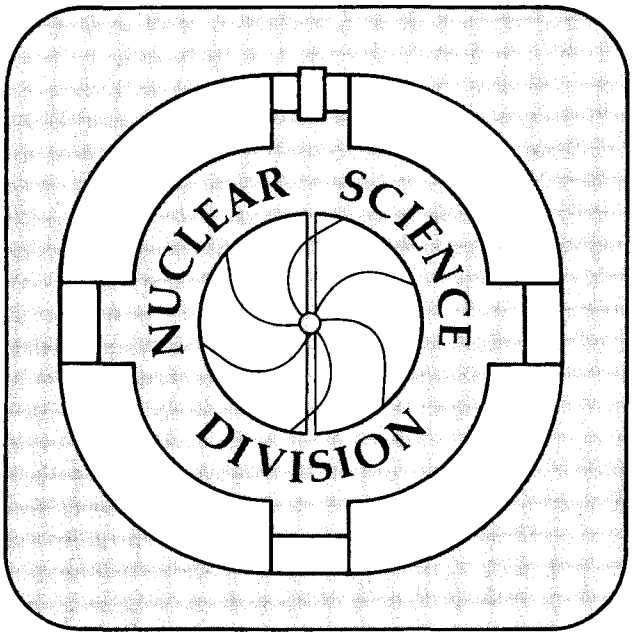
Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Submitted to Nuclear Physics B

Quark Damping and Energy Loss in the High Temperature QCD

M.H. Thoma and M. Gyulassy

July 1990



LOAN COPY
Circulates
for 2 weeks
Bldg. 50 Library.
LBL-29276
Copy 2

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

QUARK DAMPING AND ENERGY LOSS IN THE HIGH TEMPERATURE QCD*

Markus H. Thoma and Miklos Gyulassy

Nuclear Science Division
Lawrence Berkeley Laboratory
Berkeley, CA 94720

Abstract

The energy loss per unit length of high energy quarks in a quark – gluon plasma is calculated via PQCD. Unlike the damping rate, the energy loss is infrared finite and surprisingly small.

* This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 and the Deutsche Forschungsgemeinschaft.

1. Introduction

The problem of quark damping and energy loss is of interest in connection with possible jet signatures of the formation of quark – gluon plasmas (QGP) in ultrarelativistic nuclear collisions [1,2]. The first estimates of dE/dx by Bjorken [1] were based on kinetic arguments in which the infrared divergences were cut off by hand at a reasonable scale $k_D \simeq 0.5$ GeV. Taking $\alpha_s \simeq 0.2$, as consistent with lattice QCD [3] for $T \gtrsim T_c \simeq 200$ MeV, that estimate led to a surprisingly small value of $dE/dx \ll 1$ GeV/fm. In ref.[2] it was pointed out that such a small dE/dx may under favorable conditions lead to one of the signatures (jet unquenching) for the onset of QCD deconfinement for $T > T_c$.

The aim of the present paper is to calculate dE/dx in an alternate way combining techniques of plasma physics [4] and high temperature QCD [5]. The induced chromoelectric field in the wake of a high energy quark is used to calculate dE/dx (section 2). That induced field is related to the longitudinal and transverse dielectric functions, which can be expressed in turn in terms of the gluon self energy (section 3). One of the advantages of the present approach is that infrared singularities are automatically self regulated by the Debye mass of order gT. We show, in particular, that no additional magnetic mass is needed to regulate the transverse part of dE/dx because only non – static gluons ($\omega > 0$) contribute to the energy loss [6]. In contrast, we show that the quark damping rate for high energy quarks remains logarthmically divergent without introducing a non – perturbative mechanism for magnetic screening (section 5). Our final result for dE/dx, when extrapolated down to $T \gtrsim T_c$, agrees with the small value obtained from the kinetic estimates by Bjorken [1] (section 4).

We use the following notations throughout the paper: Four momenta are denoted by capital letters, $K \equiv (k_0, \vec{k})$, $\omega \equiv k_0$ and $k \equiv |\vec{k}|$. The Minkowski metric is used, $K^2 = \omega^2 - k^2$. Space – time indices are denoted by Greek symbols μ, ν,..., space indices by i, j,... and color indices by a, b,.... Summation over equal indices is assumed.

2. Energy Loss of a Quark in the Quark-Gluon Plasma

In this section we extend the result for the energy loss of a charged, non – relativistic particle in a plasma by Ichimaru [4] to the case of a relativistic color charge in the QGP. We restrict ourselves to the classical, abelian approximation for the quark dynamics, which is equivalent to the one – loop approximation. This limits the validity of the present appropach to the high temperature behaviour of QCD above the phase transition, for which perturbation theory is applicable. Quantum and non – abelian effects are included in the dielectric functions via the gluon self energy only (section 3). Then, analogous to ref.[4], an incident quark induces a chromoelectric field in the QGP. The induced field reacts on the incident quark by the Lorentz force, causing an energy loss of the quark. Assuming a small energy loss compared to the energy of the quark, the energy loss per unit length is given by [4]

$$\frac{dE}{dx} = \frac{\vec{v}}{v}\, q^a\, Re\vec{E}^a_{ind}(\vec{x} = \vec{v}t, t). \tag{2.1}$$

Here, $\vec{v}$ denotes the velocity of the quark, supposed to be constant, $v = |\vec{v}|$, and q^a its color charge defined by $q^a q^a = C_F \alpha_s$ with the strong coupling constant $\alpha_s = g^2/4\pi$ and the Casimir invariant $C_F = 4/3$ of the fundamental representation of the SU(3). (In the case of a high energy gluon, we have to substitute C_F by the Casimir invariant of the adjoint representation $C = 3$ in the following.) In the abelian approximation, the total chromoelectric field in the QGP can be related to the external current $\vec{j}^a_{ext}$ of the quark by using the Maxwell equations and the continuity equation in momentum space [4]:

$$\left[\epsilon_{ij}(\omega,k) - \frac{k^2}{\omega^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\right] E^a_{tot,j}(\omega,\vec{k}) = \frac{4\pi}{i\omega} j^a_{ext,i}(\omega,\vec{k}), \tag{2.2}$$

where ϵ_{ij} is the dielectric tensor of the QGP. For an isotropic and homogenous medium it can be decomposed by using the longitudinal and transverse dielectric functions:

$$\epsilon_{ij}(\omega,k) = \epsilon_L(\omega,k)\frac{k_i k_j}{k^2} + \epsilon_T(\omega,k)\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right). \tag{2.3}$$

The quark current of a color point charge in the momentum space reads

$$\vec{j}^a_{ext} = 2\pi q^a \vec{v}\delta(\omega - \vec{v}\cdot\vec{k}). \tag{2.4}$$

Combining (2.2), (2.3) and (2.4), we find the induced electric field:

$$\begin{aligned}\vec{E}^a_{ind}(\vec{x},t) = -i\frac{q^a}{2\pi^2}\int d\omega d^3k \frac{1}{k^2\omega}\Big\{&\left[\epsilon_L^{-1} - 1\right](\vec{v}\cdot\vec{k})\vec{k} + [k^2\vec{v} - (\vec{v}\cdot\vec{k})\vec{k}] \\ &\left[(\epsilon_T - k^2/\omega^2)^{-1} - (1 - k^2/\omega^2)^{-1}\right]\Big\}\delta(\omega - \vec{v}\cdot\vec{k})\exp(i\vec{k}\cdot\vec{x} - i\omega t).\end{aligned} \tag{2.5}$$

Substituting (2.5) into (2.1) gives

$$\frac{dE}{dx} = -\frac{C_F\alpha_s}{2\pi^2 v}\int d^3k \left\{\frac{\omega}{k^2}\left[Im\epsilon_L^{-1} + (v^2k^2 - \omega^2)Im(\omega^2\epsilon_T - k^2)^{-1}\right]\right\}_{\omega=\vec{v}\cdot\vec{k}}. \tag{2.6}$$

The dielectric functions of the QGP will be determined in the next section.

3. Dielectric Functions, Spectral Densities and Gluon Self Energy

In this section we want to express the dielectric functions of the QGP by the gluon self energy. For this purpose, we compare the most general expression for the gauge and Lorentz invariant action

$$\Gamma = \frac{1}{2}\int \frac{d^4K}{(2\pi)^4}\left[\epsilon(\omega,k)E^a_i(K)E^a_i(-K) - \mu^{-1}(\omega,k)B^a_i(K)B^a_i(-K)\right] \tag{3.1}$$

with the effective action $\Gamma_{eff} = \Gamma_0 + \Gamma_1$ [7]. Here,

$$\Gamma_0 = \frac{1}{2}\int \frac{d^4K}{(2\pi)^4}\left[E_i^a(K)E_i^a(-K) - B_i^a(K)B_i^a(-K)\right] \tag{3.2}$$

is the classical action, and

$$\Gamma_1 = \frac{1}{2}\int \frac{d^4K}{(2\pi)^4} A_\mu^a(K)\Pi_{ab}^{\mu\nu}(K)A_\nu^b(-K) \tag{3.3}$$

contains the quantum effects through the gluon self energy $\Pi_{\mu\nu}^{ab}(K)$. In order to relate the dielectric function ϵ and the magnetic permeability μ in (3.1) to the gluon self energy, we express the chromoelectric and chromomagnetic fields by the gauge potential $A_\mu^a(K)$ in the abelian approximation:

$$\begin{aligned} E_i^a(K) &= i\omega A_i^a(K) - ik_i A_0^a(K), \\ B_i^a(K) &= i\varepsilon_{ijk}k_j A_k^a(K). \end{aligned} \tag{3.4}$$

It is convenient to introduce the longitudinal and transverse gluon self energies Π_L and Π_T, defined by Pisarski [5]:

$$\begin{aligned} \Pi_L &= -\Pi_{00} = -\frac{k_i}{\omega}\Pi_{0i}, \\ \Pi_{ij} &= \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\Pi_T - \frac{k_i k_j}{k^2}\Pi_L, \end{aligned} \tag{3.5}$$

where $\Pi^{\mu\nu}$ obeys the transversality condition $k_\mu \Pi^{\mu\nu} = 0$, which holds at least in the high temperature approximation [8]. Substituting (3.4) into (3.1) and (3.2), and (3.5) into (3.3), we achieve by comparing Γ with Γ_{eff}:

$$\begin{aligned} \epsilon(\omega,k) &= 1 - \frac{\Pi_L(\omega,k)}{k^2}, \\ \mu^{-1}(\omega,k) &= 1 - \frac{1}{k^2}\left[\Pi_T(\omega,k) + \frac{\omega^2}{k^2}\Pi_L(\omega,k)\right]. \end{aligned} \tag{3.6}$$

The connection between ϵ_L, ϵ_T and ϵ, μ is given by [4]

$$\epsilon_L = \epsilon, \qquad \epsilon_T = \epsilon - \frac{k^2}{\omega^2}\left(\mu^{-1} - 1\right). \tag{3.7}$$

Combining (3.6) and (3.7) results in

$$\begin{aligned} \epsilon_L(\omega,k) &= 1 - \frac{\Pi_L(\omega,k)}{k^2}, \\ \epsilon_T(\omega,k) &= 1 + \frac{\Pi_T(\omega,k)}{\omega^2}. \end{aligned} \tag{3.8}$$

Now, we have to specify our gluon self energy Π_L and Π_T. First, since we are interested in the perturbative regime, we restrict ourselves to the one – loop approximation (fig.1). Secondly, we only use the leading order of the high temperature expansion, $T \gg \omega, k$, for which the gluon self energy turns out to be gauge invariant [9,10] and is given by [5]

$$\begin{aligned}
\Pi_L &= -3m_g^2 \left\{ 1 - \frac{\omega}{2k} \left[\ln\left|\frac{\omega+k}{\omega-k}\right| - i\pi\theta(k-\omega)\theta(\omega) \right] \right\}, \\
\Pi_T &= -\frac{3}{2} m_g^2 \frac{\omega^2}{k^2} \left\{ 1 - \left(1 - \frac{k^2}{\omega^2}\right) \frac{\omega}{2k} \left[\ln\left|\frac{\omega+k}{\omega-k}\right| - i\pi\theta(k-\omega)\theta(\omega) \right] \right\},
\end{aligned} \tag{3.9}$$

where the gluon "mass" is defined by $m_g^2 \equiv g^2T^2(1+N_f/6)/3$. (3.8) and (3.9) agree with the dielectric functions obtained from the semiclassical gluon transport equation in the abelian dominance approximation [11].

From (3.8) and (3.9) we can easily calculate the imaginary parts of ϵ_L^{-1} and $(\omega^2\epsilon_T - k^2)^{-1}$ which appear in the energy loss (2.6). From (3.8) follows:

$$\begin{aligned}
Im\epsilon_L^{-1} &= k^2 Im \frac{1}{k^2 - \Pi_L} \equiv k^2 Im\Delta_L \equiv \pi k^2 \rho_L^{disc}, \\
Im(\omega^2\epsilon_T - k^2)^{-1} &= Im \frac{1}{\omega^2 - k^2 + \Pi_T} \equiv -Im\Delta_T \equiv -\pi\rho_T^{disc},
\end{aligned} \tag{3.10}$$

where we have introduced the discontinous part of the longitudinal and transverse spectral density ρ_L^{disc} and ρ_T^{disc} defined as the imaginary part of the longitudinal and transverse effective gluon propagator Δ_L and Δ_T used by Pisarski [5]. This effective propagator can be obtained from the Dyson – Schwinger equation, shown in fig.1, containing an infinite set of one – loop diagrams in the high temperature approximation. The use of this propagator within an effective perturbation theory was proposed by Braaten and Pisarski [12] for the calculation of thermodynamical quantities in the QGP in a gauge independent way. This will be used in section 5 for the evaluation of the quark damping rate, which is closely related to the energy loss.

Using the high temperature approximation (3.9) for the gluon self energy in (3.10) yields ($0 \le \omega < k$):

$$\begin{aligned}
Im\epsilon_L^{-1}(\omega,k) &= -\frac{3\pi m_g^2}{2}\omega k \left\{ \left[k^2 + 3m_g^2 - \frac{3m_g^2}{2}\frac{\omega}{k}\ln\frac{k+\omega}{k-\omega} \right]^2 + \left[\frac{3\pi m_g^2}{2}\frac{\omega}{k} \right]^2 \right\}^{-1}, \\
Im[\omega^2\epsilon_T(\omega,k) - k^2]^{-1} &= -\frac{3\pi m_g^2}{4}\frac{\omega(k^2-\omega^2)}{k^3} \\
&\left\{ \left[k^2 - \omega^2 + \frac{3m_g^2}{2}\frac{\omega^2}{k^2}\left(1 + \frac{k^2-\omega^2}{2\omega k}\ln\frac{k+\omega}{k-\omega}\right) \right]^2 + \left[\frac{3\pi m_g^2}{4}\frac{\omega(k^2-\omega^2)}{k^3} \right]^2 \right\}^{-1}.
\end{aligned} \tag{3.11}$$

It is instructive to approximate (3.11) by neglecting the gluon "mass" m_g in the denominator of (3.11). By using (3.9) and (3.10) it can be shown that this approximation corresponds to an expansion of $Im\epsilon_L^{-1}$ and $Im(\omega^2\epsilon_T - k^2)^{-1}$ for small Π_L and

Π_T. It is also equivalent to the use of the one – loop propagator, as can be seen from (3.10) e.g., $1/(k^2 - \Pi_L) \simeq 1/k^2 + \Pi_L/k^4 = \Delta_L^{(0)} + \Delta_L^{(0)}\Pi_L\Delta_L^{(0)} \equiv \Delta_L^{(1)}$ and therefore $Im\epsilon_L \simeq k^2 Im\Delta_L^{(1)} = Im\Pi_L/k^2$. The one – loop propagator $\Delta_L^{(1)}$ differs from the propagator Δ_L of (3.10) and fig.1 by using only free propagators $\Delta_L^{(0)}$ on the right hand side of the Dyson – Schwinger equation and does not contain an infinite set of one – loop diagrams therefore. Using (3.9), we see that $Im\Pi_L/k^2$ agrees with (3.11), if we set $m_g = 0$ in the denominator of (3.11).

4. Results for the Energy Loss

Now, we end up with the final, gauge independent result for the energy loss of a high energy quark in the QGP by substituting (3.11) into (2.6). The gauge independence is due to the fact that the dielectric functions in the high temperature limit are gauge independent as well as the chromoelectric field in the abelian approximation. It is possible to perform the integration over the momentum of the electric field k analytically, but the integral over the angle $\cos\theta \equiv (\vec{v}\cdot\vec{k})/(vk)$ has to be done numerically. The integral over k is infrared finite due to the screening factor m_g in the denominator of (3.11), but logarthmically ultraviolet divergent, as it is expected for the energy loss. Therefore, we have to choose a maximal momentum k_{max}, which cannot exceed the initial momentum of the quark, of course. The choice of k_{max} is not crucial, because it appears only under the logarithm. For the case of a light, high energy quark travelling through the QGP, we set $v = 1$ and $k_{max} = p$, where p is the momentum of the quark. Just above the phase transition at $T = 0.25$ GeV, as it can be expected in ultrarelativistic heavy ion collisions, and for typical values of $\alpha_s = 0.1 - 0.3$, predicted by lattice calculations [3] and used in ref.[2], we obtain the energy loss in a QGP of two flavors ($N_f = 2$) depending on the quark momentum as shown in fig.2. From this we read off an surprisingly small energy loss e.g., $dE/dx \simeq 0.2$ GeV/fm for a 20 GeV light quark at $\alpha_s = 0.2$, compared to an energy loss of about 1 GeV/fm in hadronic matter [2].

In the case of an heavy quark of mass M, the energy loss is even reduced further by two factors. The first factor is due to the velocity $v = p/\sqrt{p^2 + M^2} < 1$ of the quark and the second due to the kinematical reduction of the maximal momentum transfer k_{max}. Assuming the maximal momentum transfer to come from elastic forward scattering of the quark by the particles of the QGP of typical momenta $q \simeq 2T$, much smaller than the quark momentum ($p \gg T$), yields $k_{max} \simeq 4Tp/(\sqrt{p^2 + M^2} - p + 4T)$. The details are given in the appendix A1. The energy loss of a charm quark ($M \simeq 1.5$ GeV) and a bottom quark ($M \simeq 5.2$ GeV), compared with the massless case, is shown in fig.3 for $T = 0.25$ GeV and $\alpha_s = 0.2$. Especially for a bottom quark a significant reduction of the energy loss can be observed.

Now, we use the approximation to (3.11), mentioned at the end of section 3 i.e, we set $m_g = 0$ in the denominator of (3.11). Then, the integral in (2.6) can be evaluated analytically, but an infrared cutoff has to be introduced, as in [1]. For this we choose the Debye momentum $k_D = \sqrt{3}m_g$ and obtain:

$$\frac{dE}{dx} = \frac{4\pi}{3} C_F \alpha_s^2 T^2 \ln\left(\frac{k_{max}}{k_D}\right) \frac{1}{v^2}\left(v + \frac{v^2 - 1}{2}\ln\frac{1+v}{1-v}\right). \tag{4.1}$$

(4.1) is displayed in fig.4 (dashed line) and compared with the full expression (solid line) for $M = 0$ (i.e, $v = 1$ and $k_{max} = p$), $T = 0.25$ GeV and $\alpha_s = 0.2$. The deviation is about 10%, indicating the validity of the perturbation theory, since (4.1) corresponds to the one – loop approximation to the gluon propagator, while (3.11) contains an infinite set of diagrams, as discussed at the end of section 3. Therefore, the simple formula (4.1) serves as an useful estimation for the energy loss of a high energy quark in the QGP.

In fig.4 there are two additional curves represented. They come from a different approach by Bjorken [1]. He calculated the energy loss of a massless quark due to elastic scattering by the particles of the QGP by averaging the cross section for elastic scattering times the mean energy transfer over the thermal distrbution ($N_f = 2$):

$$\frac{dE}{dx} = \frac{4\pi}{3} C_F \, \alpha_s^2 \, T^2 \, \ln\left(\frac{4pT}{k_D^2}\right) \left[\exp(-\frac{k_D}{T}) \left(1 + \frac{k_D}{T}\right)\right]. \tag{4.2}$$

(4.2) is represented by the dotdashed line in fig.4. The difference in the argument of the logarithm compared to (4.1) comes from a different choice of the minimal energy transfer (infrared cutoff), but is numerically not essential since $k_D \sim T$. The last factor in (4.2) reduces the energy loss in our example of a 20 GeV quark about a factor of two. It stems from the introduction of an effective mass, $m_{eff} = k_D$, for the particles of the thermal distribution. This was not considered in our case in (2.6), since distribution functions for massless particles in the QGP were used in the gluon self energy (3.9). The introduction of an effective mass, coming from higher order effects in the Dyson – Schwinger equation of fig.1 e.g., from a mean field approximation [13], does not allow a closed form for the gluon self energy in the high temperature approximation. Only in the case $\omega = 0$ we proved that the longitudinal part of the gluon self energy by quark polarization $\Pi_L^f(0,k)$ is reduced by a factor 1.7 for an effective quark mass $k_D \simeq 2T$. Assuming this to hold approximately for the imaginary part of Π_L and Π_T in (3.11), we can understand the reduction of the energy loss by higher order effects.

In fig.4 we also displayed (4.2) without the effective mass factor (dotted line). The remarkably good agreement with (4.1) is due to the fact that there is no energy loss by Cerenkov radiation in the QGP at high temperatures, because the index of refraction is smaller than one [9]. Therefore, only elastic scattering contributes to (4.1) and (2.6) with (3.11).

It should be noted that the full expression for the energy loss (2.6) with (3.11), for which no infrared cutoff is necessary, contains the so called density effect [14]. It comes from the dielectric polarization of the medium, while (4.1) and (4.2) take only close collisions ($k > k_D$) into account.

5. Quark Damping Rate in the Quark – Gluon Plasma

In this section we investigate the damping rate of a light, high energy quark and compare it to the energy loss.

Perturbation theory should be applicable for calculating observables of the quark – gluon plasma at high temperatures, such as the screening length k_D^{-1} or the dispersion

relations of quarks and gluons. But, as it turned out, the results for quantities which have to be calculated beyond the leading order g^2T^2 e.g., damping rates, are gauge dependent in general [15]. In order to avoid this problem, Pisarski and Braaten [12] suggested an effective perturbation expansion which leads to gauge independent results. According to this, the propagators and vertices for small momenta and energies of the order gT, have to be modified by resumming a certain class of one – loop diagrams with hard loop momenta ("hard thermal loops").

Pisarski [16] used this effective perturbation theory for calculating the damping rate of a heavy quark by considering the quark self energy. In this case, only an effective gluon propagator is required, since the energy of a heavy quark ($M \gg T$) is large. The same holds for a light, high energy quark with momentum $p \gg T$. Therefore, at high temperatures, we have to consider the one – loop diagram of fig.5 of the quark self energy. The effective gluon propagator coincides with the one of fig.1, because the hard thermal loop in the gluon propagator agrees with the gauge invariant gluon self energy in the high temperature approximation [12,16].

The physical process involved in this damping can be studied by cutting the diagram of fig.5 in such a way that fig.6 results. According to fig.6, the damping is caused by the emission or absorption of a virtual collective gluon mode. Real gluons are forbidden, because there is no Cerenkov radiation i.e., the cut must go through the polarization part of the gluon self energy of fig.1 and not through a single gluon line. This collective gluon mode contains an imaginary part for $\omega < k$ (Landau damping in (3.9)), by which it can decay into a quark or gluon pair. This process is also responsible for the energy loss (2.6), since it corresponds to elastic scattering via the exchange of a collective gluon described by the dielectric constant (3.11).

From fig.6 we can easily estimate the order of magnitude of the damping rate $\gamma \equiv -Imp_0$. It is proportional to the inverse of the collision time and therefore to the cross section of the reaction of fig.6 times the density of the particles in the QGP:

$$\gamma \sim \frac{1}{\tau} \sim \rho\,\sigma = \int d^3q\, n(q) \int dt\, \frac{d\sigma}{dt}, \tag{5.1}$$

where $n(q)$ is the momentum distribution of the quarks and gluons in the QGP. Since $q \sim T$ and $d\sigma/dt \sim \alpha_s^2/(t+\mu^2)^2$ [17], where we have introduced a quantity $\mu^2 \sim \alpha_s T^2$ exemplifying the collective effect of the gluon self energy, we obtain $\gamma \sim \alpha_s^2 T^3/\mu^2 = \alpha_s T$. This agrees with Pisarski's result [16] and with the damping rate of a light, high energy quark, calculated in the appendix A2. The final result reads:

$$\gamma = -\frac{C_F \alpha_s T}{\pi^2} \int d^3k \left\{ \frac{1}{\omega k^2} \left[Im\epsilon_L^{-1} - (k^2-\omega^2) Im(\omega^2\epsilon_T - k^2)^{-1} \right] \right\}_{\omega = k\cos\theta}. \tag{5.2}$$

In order to obtain the result for the energy loss (2.6) in the case $v = 1$ from it, one has to multiply (5.2) with $\omega^2/2T$ under the integral. One factor of ω is expected, because ω is the energy transfer by the exchanged gluon to the particles of the QGP. The additional factor ω/T comes from the high temperature expansion of the gluon distribution function $n_B(\omega) \simeq T/\omega$. For the damping rate we have to integrate over $n_B(\omega)$ (see (A.9)), but

this is not the case for the energy loss. There the distribution functions appear only in the gluon self energy. The additional factor 1/2 is due to the two delta functions in the damping rate (A.13), while there is only one in the energy loss (2.5).

Inserting (3.11) into (5.2), we are able to integrate over k analytically and afterwards over θ numerically. For the longitudinal part of (5.2) we obtain $\gamma_L = 0.732\alpha_s T$, but for the transverse part we encounter a logarthmically infrared divergence. This was already observed in the case of a heavy quark [16]. This singularity occurs at $\omega = 0$ and is therefore not present in the energy loss because of the additional factor ω^2 (dynamical screening [6]). The reason for the divergence of the damping rate is the missing magnetic screening in the leading order of the transverse gluon self energy in the static limit $\omega = 0,\ k \to 0$. This deficiency can be cured by the introduction of a non – perturbative magnetic gluon mass [16].

Of course, neglecting Debye screening i.e., $m_g = 0$ in the denominator of (3.11), results in a quadratic infrared singularity in the longitudinal and transverse damping rate.

6. Summary

In this paper we have investigated the energy loss and the damping rate of a high energy quark travelling through the QGP. The energy loss was calculated by considering the Lorentz force on the incident quark by the induced chromoelectric field in the QGP. The damping rate was obtained by evaluating the quark self energy within the effective perturbation theory developed by Pisarski and Braaten [12,16]. In both cases the high temperature approximation for the gluon self energy, which determines the dielectric functions of the QGP, was used. Obtaining a gauge independent, simple result for the dielectric functions (3.11) is the great advantage of the high temperature approximation. These dielectric functions also agree with the one derived from the semiclassical approximation of the gluon transport theory in the abelian dominance limit [11].

Extrapolating the high temperature formula down to $T \simeq 0.25$ GeV (fig.2), expected in ultrarelativistic heavy ion collisions, we confirm the surprisingly small value $dE/dx \simeq 0.2$ GeV/fm for a 20 GeV quark, first pointed out by Bjorken [1]. Higher order effects, like the consideration of effective masses for the particles of the QGP, reduce the energy loss further. This lends support to the idea [2] that unquenching of hard jets may provide under favorable condtions one of the signatures for the production of a QGP phase in ultrarelativistic heavy ion collisions.

Furthermore, the reduction of the energy loss for heavy flavor jets could result in an observable enhancement of jets producing D or B's as compared to jets without heavy quarks.

Our formula for the energy loss (2.6) and (3.11) is free of infrared singularities including non – perturbative (static and dynamic) chromoelectric screening effects via the dielectric functions (density effect). We showed also that the simple pocket formula (4.1) provides a useful approximation (at the 10 – 20 % level) to the numerical results.

Elastic scattering dominates the energy loss, because of the absence of Cerenkov radiation at high temperatures [9]. The magnitude of the contribution of bremsstrahlung to

dE/dx remains an open problem in QCD. However, at least for QED the radiative energy loss is suppressed by the Landau – Pomeranchuk effect [2].

The damping rate of a light, high energy quark in the QGP, calculated by the effective perturbative expansion of Pisarski and Braaten [12,16], is closely related to the energy loss (2.6) together with (3.11). The damping mechanism is caused by elastic scattering on quarks and gluons in the QGP via the exchange of a Landau damped, collective gluon mode. Unlike the energy loss, however, the transverse part of the damping rate shows a logarthmically infrared sigularity due to the absence of magnetic screening in this approximation. Dynamical screening [6] in this case is not enough to regulate the infrared divergence.

Appendices

A1. Maximal momentum transfer for the energy loss of a massive quark

The maximal momentum transfer occurs for elastic forward scattering of the quark of momentum p and energy $p_0 = \sqrt{p^2 + M^2}$ by the massless particles of the QGP of momentum $-q$ and energy $q_0 = q$. The scattered states are denoted by a prime. Energy conservation

$$\nu \equiv \sqrt{p^2 + M^2} - \sqrt{p'^2 + M^2} = q' - q \tag{A.1}$$

and momentum conservation

$$p' = p - q' - q = p - 2q - \nu \tag{A.2}$$

give

$$\nu = \frac{2pq - 2q^2}{\sqrt{p^2 + M^2} - p + 2q}. \tag{A.3}$$

For $p \gg q \simeq 2T$ follows

$$\nu = \frac{4Tp}{\sqrt{p^2 + M^2} - p + 4T}. \tag{A.4}$$

Since the gluon is on mass shell in the case of forward scattering, $\nu = \omega_{max} = k_{max} \cos\theta = k_{max}$, we obtain the maximal momentum transfer claimed in section 4. (A.4) agrees with our assumption $k_{max} = p$ in the massless limit, $M = 0$.

A2. Evaluation of the Damping Rate of a light, high energy quark

Here, we want to calculate the damping rate of a light quark with a high momentum $p \gg T$, for which the approximation of fig.5 holds and the quark mass can be neglected. The dispersion relation of a massless quark with four momentum $P = (p_0, \vec{p})$ is given by [18]

$$p_0 = p - \frac{b(p_0, p)}{1 + a(p_0, p)}, \qquad p = |\vec{p}|, \tag{A.5}$$

where

$$\begin{aligned} a &= \frac{1}{4p^2}\left[Tr(\not{P}\Sigma) - p_0\, Tr(\gamma_0 \Sigma)\right], \\ b &= \frac{1}{4p^2}\left[P^2\, Tr(\gamma_0 \Sigma) - p_0\, Tr(\not{P}\Sigma)\right] \end{aligned} \tag{A.6}$$

and $\Sigma = \Sigma(P)$ denotes the quark self energy. If there is no overdamping i.e., $\gamma \equiv -Im p_0 \ll Re\, p_0$, and since $Re\, p_0 \simeq p$ for high momentum p, we obtain for the leading order in the high temperature approximation

$$\gamma \simeq Im\, b(p,p) = -\frac{1}{4p}\, Im\, Tr(\not{P}\Sigma)\Big|_{p_0=p}. \tag{A.7}$$

(The function $a(p_0, p)$ in the denominator of (A.5) contributes to higher order only.) Therefore, we have to caculate the imaginary part of the quark self energy on the mass shell at finite temperature, for which we prefer the real – time formalism [19]. Since the result is gauge independent, we are free to choose any gauge. Using the Coulomb gauge, it is convenient to divide the quark self energy into a longitudinal and a transverse part $\Sigma = \Sigma_L + \Sigma_T$ [16], where

$$\begin{aligned} \Sigma_L &= i\, C_F\, g^2 \int \frac{d^4k}{(2\pi)^4}\, [\gamma_0\, S(Q)\, \gamma_0\, \Delta_L(K)], \\ \Sigma_T &= -i\, C_F\, g^2 \int \frac{d^4k}{(2\pi)^4} \left[\gamma_i\, S(Q)\, \gamma_j \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \Delta_T(K)\right]. \end{aligned} \tag{A.8}$$

Here, $\Delta_L(K)$ and $\Delta_T(K)$ are the gauge invariant, effective longitudinal and transverse gluon propagators of (3.10) and $S(Q)$, $(Q \equiv P + K)$, the quark propagator. The thermal part of $\Delta_{L,T}$ can be expressed by the spectral densities $\rho_{L,T}$ and the Bose – Einstein distribution n_B [10,16,20]:

$$\Delta_{L,T}(K) = -2\pi\, i \int_0^\infty d\omega\, (2\omega)\, \rho_{L,T}(\omega, k)\, n_B(\omega)\, \delta(k_0^2 - \omega^2). \tag{A.9}$$

Since we are interested in the imaginary part of the quark self energy, we take into account the contribution of the discontinuities due to Landau damping to the spectral densities at $\omega < k$ only, which are given by (3.10) and (3.11). At high temperatures $T \gg \omega$, we can

approximate the distribution function by $n_B(\omega) = T/\omega$. The quark propagator is given by the zero temperature propagator

$$S(Q) = \frac{\not{Q}}{Q^2 + i\epsilon}, \tag{A.10}$$

because $k \sim gT$ [16] and therefore $q \sim p \gg T$. The traces in (A.7) yield:

$$\begin{aligned} Tr[P\gamma_0 S(Q)\gamma_0] &= \frac{4}{Q^2 + i\epsilon}\,(p_0 q_0 + \vec{p}\cdot\vec{q}), \\ Tr\left[P\gamma_i S(Q)\gamma_j\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\right] &= \frac{8}{Q^2 + i\epsilon}\left[p_0 q_0 - \frac{(\vec{p}\cdot\vec{k})(\vec{q}\cdot\vec{k})}{k^2}\right]. \end{aligned} \tag{A.11}$$

Using (A.7), (A.8), (A.9) and (A.11), we obtain after integrating over k_0:

$$\begin{aligned} \gamma_L = Im\Bigg\{ &-\frac{C_F g^2 T}{p}\int\frac{d^3k}{(2\pi)^3}\int_0^\infty \frac{d\omega}{\omega}\,\rho_L^{disc}(\omega,k) \\ &\left[\frac{p(p+\omega) + p^2 + \vec{p}\cdot\vec{k}}{(p+\omega)^2 - (\vec{p}+\vec{k})^2 + i\epsilon} + \frac{p(p-\omega) + p^2 + \vec{p}\cdot\vec{k}}{(p-\omega)^2 - (\vec{p}+\vec{k})^2 + i\epsilon}\right]\Bigg\}, \\ \gamma_T = Im\Bigg\{ &\frac{2C_F g^2 T}{p}\int\frac{d^3k}{(2\pi)^3}\int_0^\infty \frac{d\omega}{\omega}\,\rho_T^{disc}(\omega,k) \\ &\left[\frac{p(p+\omega) - (\vec{p}\cdot\vec{k})^2/k^2 - \vec{p}\cdot\vec{k}}{(p+\omega)^2 - (\vec{p}+\vec{k})^2 + i\epsilon} + \frac{p(p-\omega) - (\vec{p}\cdot\vec{k})^2/k^2 - \vec{p}\cdot\vec{k}}{(p-\omega)^2 - (\vec{p}+\vec{k})^2 + i\epsilon}\right]\Bigg\}. \end{aligned} \tag{A.12}$$

Expanding the square brackets in (A.12) for $p \gg \omega, k$ and defining $\cos\theta \equiv (\vec{p}\cdot\vec{k})/(pk)$ leads to

$$\begin{aligned} \gamma_L &= -C_F\, g^2 T Im\left\{\int\frac{d^3k}{(2\pi)^3}\int_0^\infty\frac{d\omega}{\omega}\rho_L^{disc}(\omega,k)\left[\frac{1}{\omega - k\cos\theta + i\epsilon} + \frac{1}{-\omega - k\cos\theta + i\epsilon}\right]\right\}, \\ \gamma_T &= C_F\, g^2 T Im\Bigg\{\int\frac{d^3k}{(2\pi)^3}\int_0^\infty\frac{d\omega}{\omega}\rho_T^{disc}(\omega,k)(1-\cos^2\theta) \\ &\qquad\left[\frac{1}{\omega - k\cos\theta + i\epsilon} + \frac{1}{-\omega - k\cos\theta + i\epsilon}\right]\Bigg\}. \end{aligned} \tag{A.13}$$

Using the principal value description, $Im(\pm\omega - k\cos\theta + i\epsilon)^{-1} = \pi\,\delta(\pm\omega - k\cos\theta)$, and performing the integration over ω results in

$$\gamma = -\frac{C_F\alpha_s T}{\pi}\int d^3k\left\{\frac{1}{\omega k^2}\left[k^2\rho_L^{disc}(\omega,k) - (k^2-\omega^2)\rho_T^{disc}(\omega,k)\right]\right\}_{\omega = k\cos\theta}. \tag{A.14}$$

(A.14) together with (3.10) is equivalent to (5.2).

References

[1] J.D Bjorken, Fermilab preprint Pub-82/59-THY (1982)
[2] M. Gyulassy and M. Plümer, LBL preprint 28531 (1990)
[3] M. Gao (Columbia Univ. Group), Nucl. Phys. B (Proc. Suppl.) 9 (1989) 368
F. Karsch and H.W. Wyld, Phys. Lett B 213 (1988) 505
F. Karsch, Nucl. Phys. B (Proc. Suppl.) 9 (1989) 357
[4] S. Ichimaru, Basic Principles of Plasma Physics (W.A. Benjamin, Reading, 1973)
[5] R.D. Pisarski, Physica A158 (1989) 146
[6] G. Baym, H. Monien and C.J. Pethick, Proc. 16th International Workshop on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Austria. H. Feldmeier, ed. (GSI and Institut für Kernphysik, Darmstadt, 1988), p.128
C.J. Pethick, G. Baym and H. Monien, Nucl. Phys. A498 (1989) 313c
[7] T.H. Hansson and I. Zahed, Nucl. Phys. B292 (1987) 725
[8] U. Heinz, K. Kajantie and T. Toimela, Ann. Phys. (N.Y.) 176 (1987) 218
[9] V.V. Klimov, Sov. Phys. JETP 55 (1982) 199
[10] H.A. Weldon, Phys. Rev. D26 (1982) 1394
[11] H.T. Elze and U.Heinz, Phys. Rep. 183 (1989) 81
[12] E. Braaten and R. D. Pisarski, Nucl. Phys. B337 (1990) 569
[13] K. Kajantie and K. Kapusta, Ann. Phys. (N.Y.) 160 (1985) 477
[14] J.D. Jackson, Classical Electrodynamics, 2nd edition (John Wiley, New York,1975), chap. 13
[15] M. E. Carrington, T.H. Hansson, H. Yamagishi and I. Zahed, Ann. of Phys. (N.Y.) 190 (1989) 373
[16] R.D. Pisarski, Phys. Rev. Lett. 63 (1989) 1129
[17] J.E. Leader and E. Predazzi, An Introduction to Gauge Theories and the "New Physics" (Cambridge University Press, Cambridge, 1982), Appendix A
[18] H.A. Weldon, Phys. Rev. D26 (1982) 2789
[19] L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320
[20] H.A. Weldon, Phys. Rev. D40 (1989) 2410

Figure Captions

Fig.1: Gluon propagator of the effective perturbation theory.

Fig.2: Energy loss of a light quark for different values of the strong coupling constant.

Fig.3: Energy loss for different quark masses.

Fig.4: Full expression for the energy loss (solid line), one – loop approximation for the energy loss (dashed line), energy loss by elastic scattering with (dotdashed line) and without (dotted line) effective masses of the particles in the QGP.

Fig.5: Quark self energy in the effective perturbation theory.

Fig.6: Elastic scattering by the exchange of a collective gluon.

Fig.1

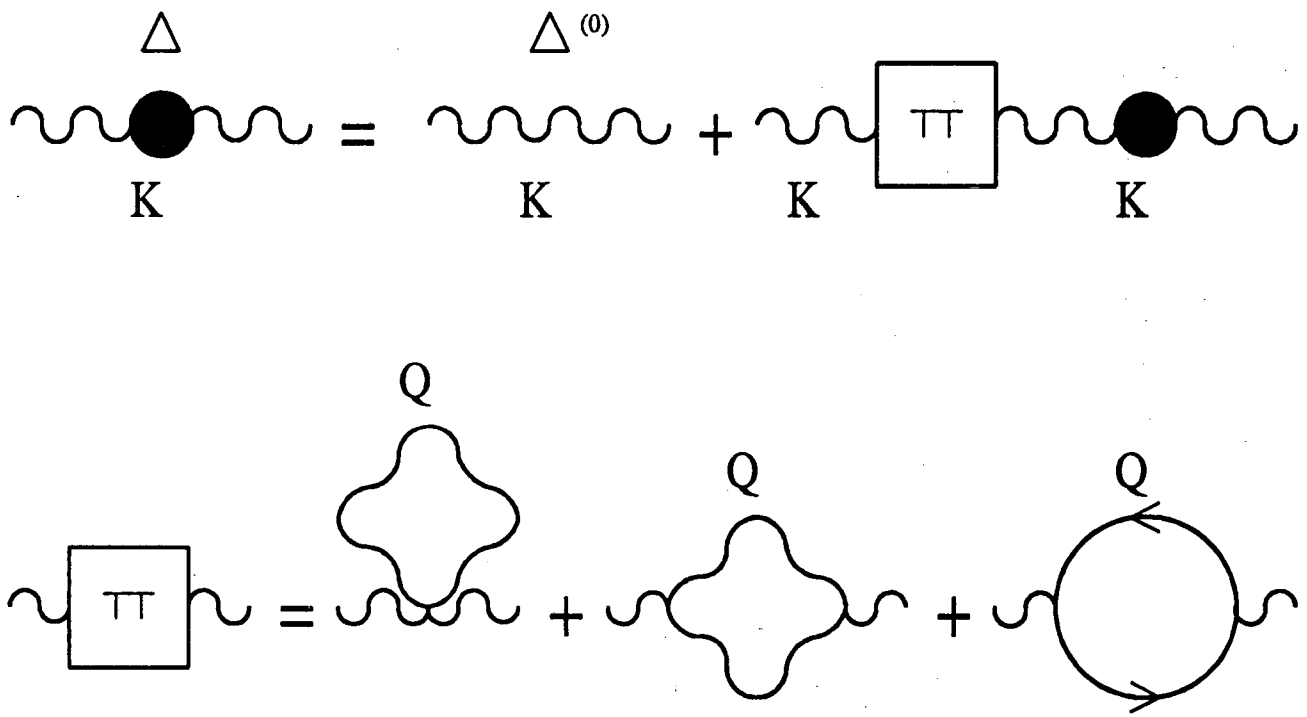


Fig. 2

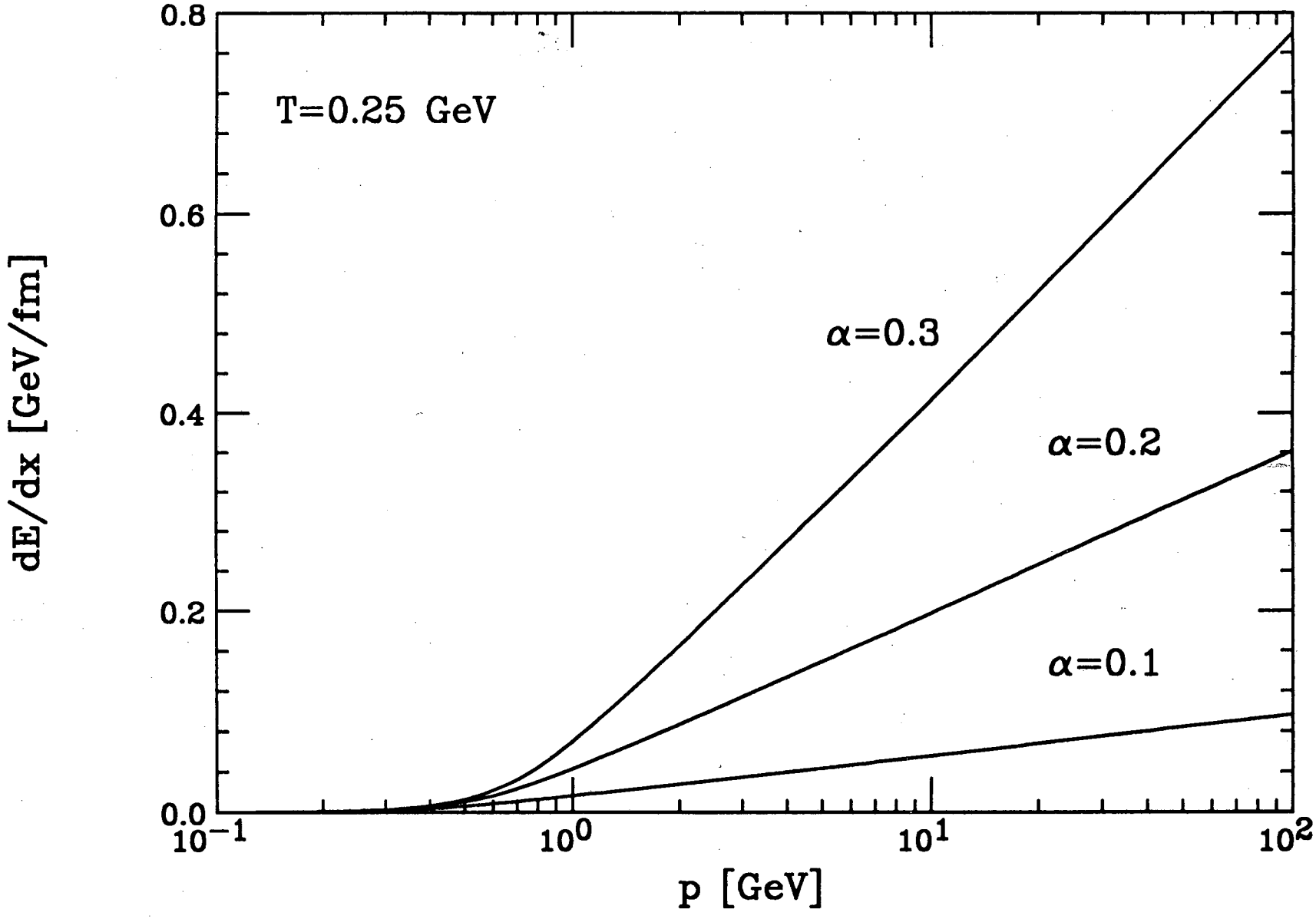

T=0.25 GeV
α=0.3
α=0.2
α=0.1
0.8
0.6
0.4
0.2
0.0
dE/dx [GeV/fm]
10^-1
10^0
10^1
10^2
p [GeV]

Fig. 3

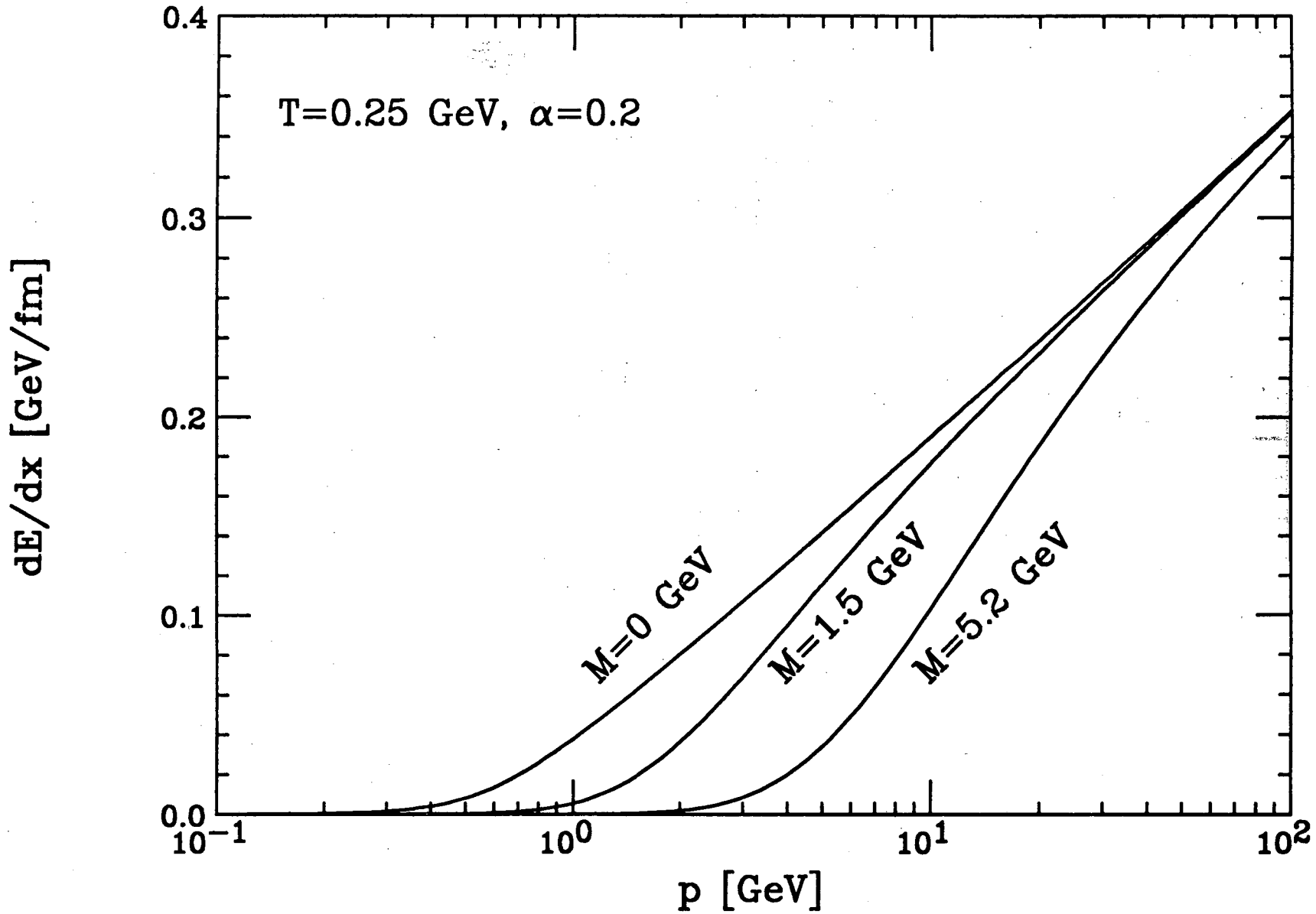

T=0.25 GeV, α=0.2
M=0 GeV
M=1.5 GeV
M=5.2 GeV
dE/dx [GeV/fm]
p [GeV]
0.0
0.1
0.2
0.3
0.4
10^-1
10^0
10^1
10^2

Fig. 4

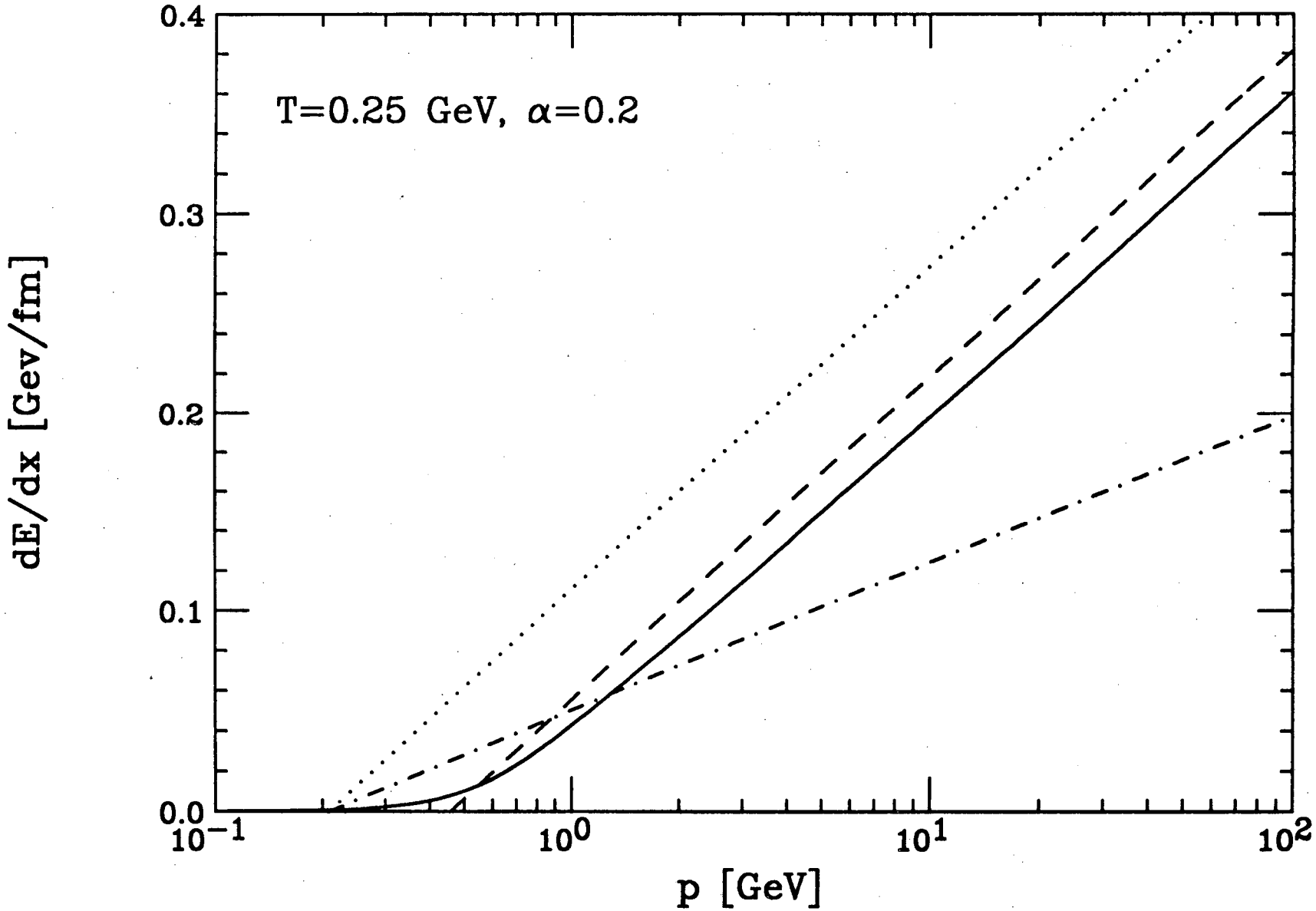


Fig.5

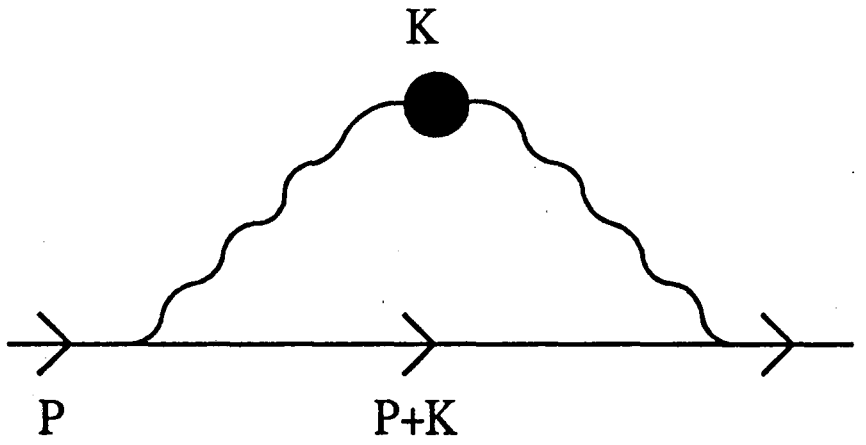
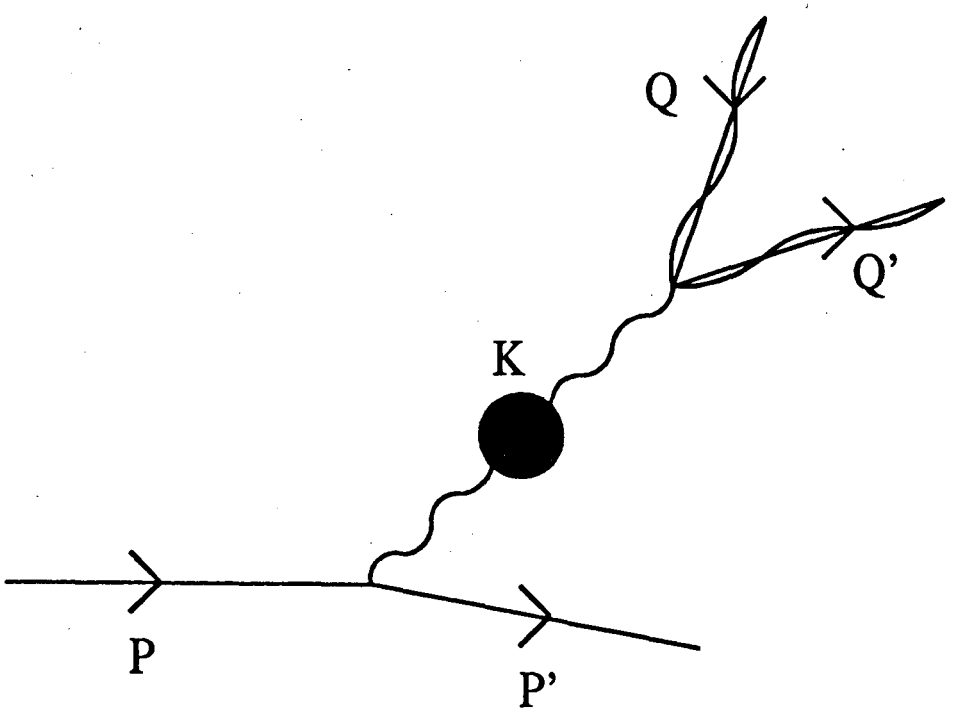


Fig.6



LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
INFORMATION RESOURCES DEPARTMENT
BERKELEY, CALIFORNIA 94720