LBL-29428

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

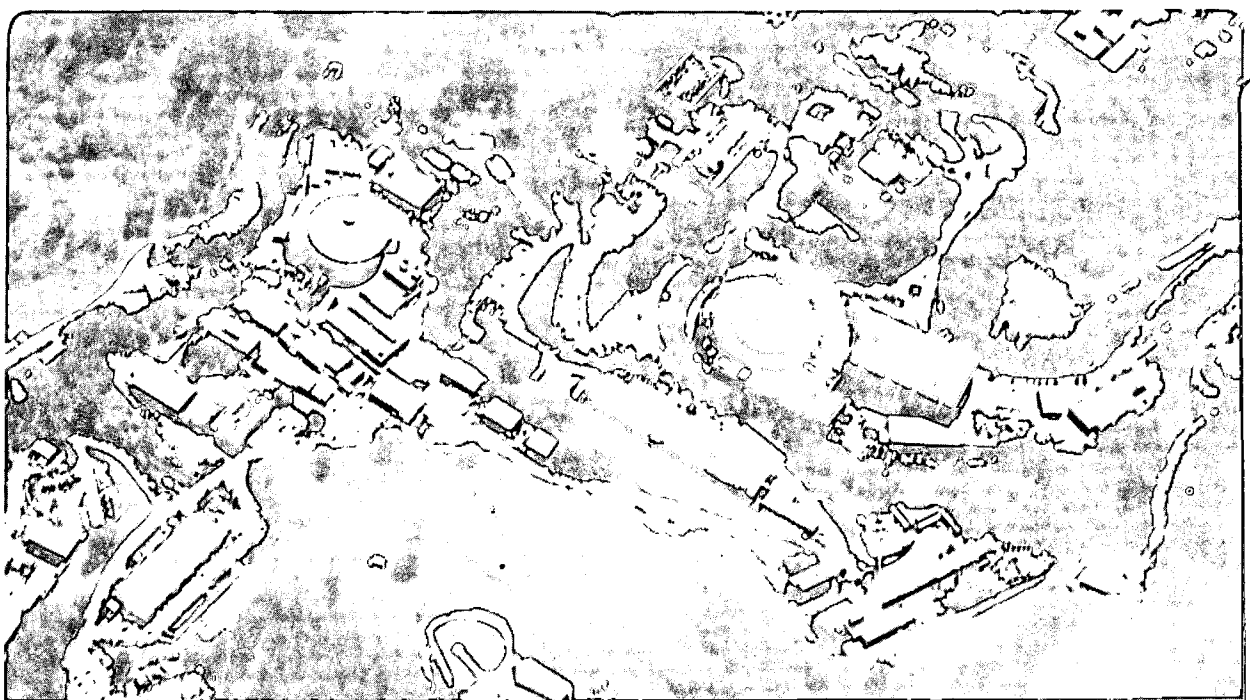
Physics Division

Presented at the 1990 International Workshop on Strong Coupling Gauge Theories and Beyond, Nagoya, Japan, July 29–August 1, 1990, and to be published in the Proceedings

Nonminimal $t\bar{t}$ Models of Composite Higgs Bosons

M. Suzuki

August 1990



LBL-29428
Copy 2
LOAN COPY
Circulates
for 4 weeks
Bldg. 50 Library.

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

UCB-PTH-90/35
LBL-29428

NONMINIMAL $t\bar{t}$ MODELS OF COMPOSITE HIGGS BOSONS†

Mahiko Suzuki

Department of Physics and Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

* Supported in part by the National Science Foundation under Research Grant PHY-85-15857 and in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

UCB-PTH-90/35
LBL-29428

NONMINIMAL $t\bar{t}$ MODELS OF COMPOSITE HIGGS BOSONS†

Mahiko Suzuki

Department of Physics and Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

Abstract. The relation between infrared-fixed point analysis and underlying fermion models of composite Higgs bosons is studied in a simple two-Higgs-doublet model and in a single-Higgs-doublet model with a singular interaction added. We examine how the infrared-fixed point analysis can be affected by a difference in fundamental interactions of constituents.

1. INTRODUCTION

In order to build Higgs particles as composites of t_R and $(t,b)_L$, one must introduce a new strong binding force at a large scale Λ.[1,2] Since electroweak and chiral symmetries are broken simultaneously by a tt condensate, both the top quark mass m_t and the Higgs boson mass m_H are related to the electroweak scale $v = (\sqrt{2}G_F)^{-1/2} = 246$ GeV through Λ. The gauged Nambu-Jona-Lasinio (N-J-L) model proves to be ideal for studying such a picture. However, the N-J-L model of composite Higgs bosons can be cast into an effective field theory, which is identical with the Standard Model in structure. The relation among m_t, m_H, and v can then be studied by the renormalization group (RG) method[3,4] beyond the chain approximation of the N-J-L model. The results are essentially the same as those obtained by the infrared-fixed point analysis of the Higgs sector.[5,6,7]

Does this mean that the explicit model of the N-J-L type has little bearing on low energy physics ? My aim here is to examine how the RG analysis of effective field theory is related and unrelated to underlying models. I find that the simplest two-doublet N-J-L model predicts all quantitative results of the RG analysis except for two overall normalizations of couplings. The reason is that the Higgs sector of this model is realized at an infrared-fixed point of a set of RG equations which is a good approximation to the complete RG equations. This exercise seems to support a unique relationship between the infrared-fixed point analysis of RG and the N-J-L model in general. When I add singular interactions of higher dimension, however, the N-J-L model leads to couplings and masses quite different from those of the minimal N-J-L model though the RG equations for effective field theory remain the same. What is going on ? I try to analysis this problem..

2. TWO-DOUBLET MODEL

For simplicity, I consider only the third generation quarks, t and b. The relevant four-fermion interaction for the N-J-L model is as follows:

$$L_{int} = G_t(\bar{Q}_L t_R)(\bar{t}_R Q_L) + G_b(\bar{Q}_L b_R)(\bar{b}_R Q_L) + G_{tb}[(\bar{Q}_L t_R)(\bar{b}_R{}^c Q_L{}^c) + \text{H.c.}] \,. \qquad (2.1)$$

To be published in Proceedings of 1990 International Workshop on Strong Coupling Gauge Theories and Beyond (World Scientific Pub., Singapore).

The couplings G_t, G_b, and G_{tb} are of order $1/\Lambda^2$ and fine-tuned such that the two eigenchannels of $(\bar{t}_R Q_L)$ and $(\bar{b}_R{}^c Q_L{}^c)$ both produce light bound states of spin-zero. The two composite Higgs doublets formed by the interaction Eq.(2.1) are denoted by ϕ_1 and ϕ_2. They interact with themselves through the diagrams depicted in Fig.1. It is convenient to introduce ϕ_t and ϕ_b, the linear combinations of ϕ_1 and ϕ_2 that couple only with t_R and with b_R, respectively. Then the effective interaction Hamiltonian involving the Higgs doublets takes the form in leading logarithm

$$H = V_q + V_H, \tag{2.2}$$

$$V_q = f(\bar{Q}_L q_{1R})\phi_1 + f(\bar{Q}_L q_{2R})\phi_1 + \text{H. c.}, \tag{2.3}$$

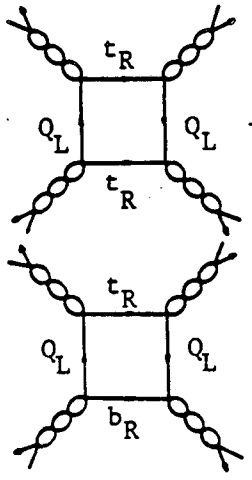
$$V_H = \mu_1{}^2\phi_1{}^\dagger\phi_1 + \mu_2{}^2\phi_2{}^\dagger\phi_2 + (\lambda/2)(|\phi_t{}^\dagger\phi_t|^2 + 2\,|\phi_t{}^{c\dagger}\phi_b|^2 + |\phi_b{}^\dagger\phi_b|^2), \tag{2.4}$$

where

$$\phi_t = \phi_1\cos\theta - \phi_2\sin\theta, \qquad \phi_b = \phi_1\sin\theta + \phi_2\cos\theta, \tag{2.5}$$

$$\bar{Q}_L q_{1R} \equiv \bar{Q}_L t_R\cos\theta + \bar{Q}_L{}^c b_R{}^c\sin\theta, \quad \bar{Q}_L q_{2R} \equiv -\bar{Q}_L t_R\sin\theta + \bar{Q}_L{}^c b_R{}^c\cos\theta, \tag{2.6}$$

$\tan 2\theta = 2G_{tb}/(G_t - G_b)$. We choose $\mu_2{}^2 - \mu_1{}^2 > 0$.



$$+ \quad (t_R \to b_R) = \tfrac{1}{2}\lambda(|\phi_t^\dagger\phi_t|^2 + |\phi_b^\dagger\phi_b|^2)$$

$$= \lambda|\phi_t^{c\dagger}\phi_b|^2$$

Fig.1. Diagrams contributing to the ϕ^4 couplings.

Since $\phi_t{}^{c\dagger}\phi_b = \phi_t{}^-\phi_b{}^o - \phi_t{}^o\phi_b{}^-$, V_H may be written as

$$V_H = \mu_1{}^2\phi_1{}^\dagger\phi_1 + \mu_2{}^2\phi_2{}^\dagger\phi_2 + (\lambda/2)[|\phi_t{}^\dagger\phi_t|^2 + 2(\phi_t{}^\dagger\phi_t)(\phi_b{}^\dagger\phi_b) - 2|\phi_t{}^\dagger\phi_b|^2 + |\phi_b{}^\dagger\phi_b|^2]. \tag{2.7}$$

Explicit evaluation of diagrams in Fig.1 gives us in leading logarithm

$$f^2 = \lambda/2 = 16\pi^2/[N_c\ln(\Lambda^2/m_q{}^2)]. \tag{2.8}$$

Note that the magnitude of the Yukawa coupling f is comnion to ϕ_1 and ϕ_2, and therefore to ϕ_t and ϕ_b. Since $\lambda > 0$, $<\phi>$ does not run away to infinity and the term $\lambda|\phi_t{}^{c\dagger}\phi_b|^2$ allows two vacuum expectation values (VEVs), if at all, to align for electric charge conservation.

Finding mass eigenvalues of physical Higgs modes from V_H is straightforward. The same results can be obtained, if we wish, by solving the qq bound state problem in the chain approximation with the gap equation as a constraint. The physical modes are denoted by $H(0^+)$, $\sigma(0^+)$, $\chi(0^-)$, and ϕ^+. The condition $m_\chi^2 > 0$ imposes a stringent constraint on the rotation angle θ defined in Eq.(2.5). Only a small region $0 < \theta < m_b/m_t$ is allowed. Within this region, the mass relations[8]

$$m_\chi^2 = m_\sigma^2, \qquad m_H^2 = 4m_t^2, \qquad m_\phi^2 = m_\sigma^2 + 2m_t^2 \tag{2.9}$$

hold in the approximation $m_b \ll m_t$. Here we call the 0^+ state made mainly of $t\bar{t}$ the H boson. When $\theta < (m_b/m_t)^3$, the both eigenchannels develop VEVs. However, in this case m_σ and m_χ are of $O(m_b)$, so the negative experimental evidence for the decay $Z \to \sigma\chi$ at LEL[9] excludes the possibility of two VEVs if χ couples to charged leptons, too.

When gauge couplings and higher orders of Higgs couplings are taken into account, the N-J-L model is no longer analytically solvable. RG analysis has been proposed to go beyond the approximation of the N-J-L model. In RG analysis the entire content of the N-J-L model is thrown into the condition that the Yukawa couplings of composite Higgs bosons blow up at the scale Λ. Only the one-loop RG analysis has so far been worked out by choosing the Yukawa couplings at Λ to be some large number rather arbitrarily.[3,4,10] Running down to the electroweak scale, one finds the renormalized values of couplings from which the masses of the Higgs bosons and the t-quark are computed. Since the values of couplings at Λ are such that their low-energy values are insensitive to the choice, the resulting Higgs sector at low energies is bound to be near an infrared-fixed point.

With the interaction Hamiltonian

$$H_{int} = V_q + V_H\,, \tag{2.10}$$

$$V_q = f_t(\bar{Q}_L t_R)\phi_t + f_b(\bar{Q}_L{}^c b_R{}^c)\phi_b + \text{H.c.}, \tag{2.11}$$

$$\begin{aligned} V_H = {} & \mu_t^2\phi_t^\dagger\phi_t + \mu_b^2\phi_b^\dagger\phi_b + \mu_{tb}^2(\phi_t^\dagger\phi_b + \text{H.c.}) + (\lambda_1/2)|\phi_t^\dagger\phi_t|^2 + (\lambda_2/2)|\phi_b^\dagger\phi_b|^2 \\ & + \lambda_3(\phi_t^\dagger\phi_t)(\phi_b^\dagger\phi_b) + \lambda_4|\phi_t^\dagger\phi_b|^2 + (\lambda_5/2)[(\phi_t^\dagger\phi_b)^2 + \text{H.c.}], \end{aligned} \tag{2.12}$$

a one-loop RG computation was performed by Luty.[10] I quote relevant results of his:

(i) The σ and χ states are degenerate within a few percent accuracy and m_H is nearly independent of θ. Futhermore, $m_\phi^2 - m_\sigma^2$ is positive and approximately constant; $[m_\phi^2 - m_\sigma^2]/m_H^2 \approx 1$.

(ii) The couplings obey $f_t = f_b$ and $\lambda_1 = \lambda_2 = \lambda_3$ both within a few percent accuracy. In addition, $\lambda_5 \approx 0$ and $\lambda_4 \approx -1.5\,\lambda_1$ hold approximately.

The mass relations are in good agreement with the N-J-L model predictions (Eq.(2.9)) except that $[m_\phi^2 - m_\sigma^2]/m_H^2 = 1/2$ in the N-J-L model. The coupling relations also resemble closely the N-J-L model predictions; in fact, the N-J-L model predicts all of them

except for λ_4, for which $\lambda_4 = -\lambda_1$ is obtained. Of course, the N-J-L model, not including gluon corrections and higher-order Higgs self-coupling effects, cannot reproduce the overall normalizations of f and λ. That is where the numerical analysis of RG plays a distinct role. Since the channel coupling between $\bar{t}_R Q_L$ and $\bar{b}_R{}^c Q_L{}^c$ is insignificant, the values of m_t and m_H resulting from the RG analysis are virtually identical with those of the minimal single-doublet model.

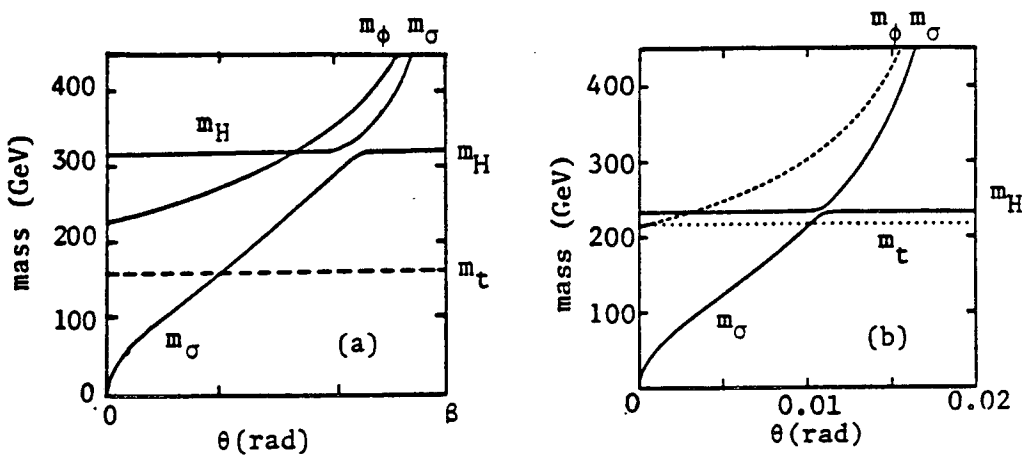


Fig.2. Masses of physical Higgs modes against the rotation angle θ in (a) the N-J-L model and (b) the one-loop RG analysis.[10] $\beta = m_b/m_t$. $\Lambda = 10^{15}$ GeV is chosen.

The one-loop RG analysis of the two-doublet model shows that the couplings are attracted towards $f_t = f_b$, $\lambda_1 = \lambda_2 = \lambda_3 = -\lambda_4$, and $\lambda_5 = 0$.[10,11] The relations $f_t = f_b$, $\lambda_1 = \lambda_2 = \lambda_3$, and $\lambda_5 = 0$ are constraints at an infrared-fixed point in the approximation that ignores the electroweak couplings, but the relation $\lambda_4 = -\lambda_1$ is not;

$$16\pi^2\, d[\ln(f_t/f_b)]/d\ln\mu \;\rightarrow\; (N_c + 1)\, f_b{}^2[(f_t/f_b)^2 - 1] \qquad (f_t \rightarrow f_b), \qquad \textit{etc}, \tag{2.13}$$

$$16\pi^2\, d[\ln(\lambda_4/\lambda_1)]/d\ln\mu \;\rightarrow\; -2\lambda_1[(\lambda_4/\lambda_1)^2 + 2] + 4N_c(f_t{}^4/\lambda_4)[(\lambda_4/\lambda_1) + 1], \tag{2.14}$$

near $f_t = f_b$, $\lambda_1 = \lambda_2 = \lambda_3$, and $\lambda_5 = 0$. Nevertheless λ_4/λ_1 converges to the value $(= -1)$ predicted by the N-J-L model apparently because the leading $1/N_c$ term dominates in Eq.(2.14). In fact, if we solve the one-loop RG equations in the approximation keeping only leading N_c terms and ignoring all gauge loops, we would obtain $f_t = f_b$, $\lambda_1 = \lambda_2 = \lambda_3 = -\lambda_4$, and $\lambda_5 = 0$ as infrared-fixed values of the approximated RG equations. In this limit the RG equations for the ratio $\lambda_1/f_t{}^2$ has an infrared-fixed point at $2N_c/(N_c - 2) \rightarrow 2$ (as $N_c \rightarrow \infty$). Since $m_H{}^2/m_t{}^2 = 2\lambda_1/f_t{}^2$, the celebrated relation $m_H{}^2 = 4m_t{}^2$ of the N-J-L model[1,2] is nothing other than a constraint at an infrared-fixed point of RG. It is amusing to notice that in the N-J-L model the relations among the Higgs self-couplings λ_i are the consequence of the two facts; (i) the dimensionless couplings of composite Higgs bosons are independent of the magnitude of the four-fermion couplings and (ii) the only diagram

inducing the ϕ_t-ϕ_b mixing in the ϕ^4 coupling is the box diagram exchanging t_R and b_R, as depicted in Fig.1. Does the N-J-L model somehow know all about vacuum stability, vacuum alignment, and an infrared-fixed point ?

If the N-J-L type models always lead to an infrared-fixed point, RG analysis would replace the models and its numerical predictions would be universally valid, irrespective of details of fundamental interactions. However, the other side of the coin is that there would be no clue at low energies to distinguish among different models built at a large scale Λ; not to mention binding mechanism of composite Higgs bosons, we are not so sure even about whether Higgs bosons are composite or not. We will study below within the general scheme of the N-J-L model how low energy parameters can be affected when fundamental interactions at Λ are modified.

3. THE N-J-L MODEL WITH MORE SINGULAR INTERACTIONS

To show how the N-J-L model predictions depend on the form of interactions, I add a new interaction of dimension eight and ten to the minimal four-fermion interaction of Eq.(2.1).[12] Since my interest is in studying the relationship between the N-J-L type models and the RG approach, I consider the single-doublet model with $m_b/m_t \to 0$ for simplicity. My new interaction Lagrangian is

$$L_{int} = J^\dagger J \quad \text{with} \quad J = (g/\Lambda)(\bar{Q}_L t_R) + (h/\Lambda^3)(\partial_\mu \bar{Q}_L \partial^\mu t_R). \tag{3.1}$$

This particular choice is motivated solely by solvability of the model. The gap equation is

$$1 = (N_c/8\pi^2\Lambda^2)\, g^2 \int_0^{\Lambda^2} x[1 - (hx/g\Lambda^2)]^2/[x + \Sigma(x)^2]\, dx, \tag{3.2}$$

the relation between the VEV and the t-quark mass becomes

$$1/f_t^2 = (v/m_t)^2/2 = (N_c/16\pi^2) \int_0^{\Lambda^2} [1-2(hx/g\Lambda^2)][1-(hx/g\Lambda^2)]/[x + \Sigma(x)^2]\, dx, \tag{3.3}$$

and the physical Higgs mass is given by

$$m_H^2 = 4m_t^2 \int [1-(hx/g\Lambda^2)]^4/[x+\Sigma(x)^2]dx \Big/ \int [1-2(hx/g\Lambda^2)][1-(hx/g\Lambda^2)]/[x+\Sigma(x)^2]dx, \tag{3.4}$$

where $\Sigma(x) = m_t[1 - (hx/g\Lambda^2)]$. For $h = 0$ and $g^2/\Lambda^2 = G$, these relations reduce to the corresponding ones of the minimal N-J-L model. In terms of a parameter $\xi = h/g$, Eqs.(3.3) and (3.4) are expressed as

$$(v/m_t)^2 = (N_c/8\pi^2)[\ln(\Lambda^2/m_t^2) - 1 - 3\xi + \xi^2], \tag{3.3'}$$

$$m_H^2 = 4m_t^2\{[\ln(\Lambda^2/m_t^2) -1- 4\xi+ 3\xi^2 -4\xi^3/3+ \xi^4/4]/[\ln(\Lambda^2/m_t^2) -1-3\xi + \xi^2]\}. \tag{3.4'}$$

By suitable choice of values for ξ and Λ, I can change the predictions of the minimal N-J-L model considerably; for instance, with $\xi = -10$ and $\Lambda = 4 \times 10^{10}$ GeV, I find $m_t = 100$ GeV and $m_H = 1$ TeV, and with $\xi = -6$ and $\Lambda = 1000$ TeV, I obtain $m_t = 150$ GeV and $m_H = 1$

TeV. The lesson I learn from this example is that we can upset radically the tight relations among m_t, m_H, and Λ of the minimal model.

What does this mean to the RG analysis of an effective renormalizable field theory ? The RG equations involve only dimensionless couplings because only those couplings run logarithmically with energy. Couplings of higher dimension are irrelevant to the RG equations below Λ, more precisely below $\ln \Lambda$. As far as the evolution below Λ is concerned, the only impact of the added interactions of higher dimension is to change the initial values of running couplings at Λ. In order to see this point more clearly, we examine the running Yukawa coupling $f_t(\mu)$. In the approximation of no gauge loops and no higher-order Higgs self-couplings, Eq.(3.3) gives us the running Yukawa coupling,

$$1/f_t(\mu)^2 = (N_c/16\pi^2)\int_{\mu^2}^{\Lambda^2} [1-2(hx/g\Lambda^2)][1-(hx/g\Lambda^2)]/[x + \Sigma(x)^2]\, dx. \tag{3.5}$$

Differentiating Eq,(3.5) with respect to $\ln \mu$, we find the RG equation for $f_t(\mu)$;

$$df_t(\mu)/d\ln \mu = (N_c/16\pi^2)f_t(\mu)^3[1 + O(\mu^2/\Lambda^2)]. \tag{3.6}$$

This is identical with the RG equation for $f_t(\mu)$ of the Standard Model (with $N_c \to \infty$) in the same approximation and its general solution is given by $1/f_t(\mu)^2 = (N_c/16\pi^2)[\ln(\Lambda^2/\mu^2) + C]$ (cf. Eq.(2.8)) *if* one ignores $O(\mu^2/\Lambda^2)$. The added coupling h/Λ^3 contributes only to terms of $O(\mu^2/\Lambda^2)$ in Eq.(3.6). However, dependence of $f_t(\mu)^2$ near $\mu = \Lambda$ is quite different whether the coupling h/Λ^3 is added or not, since the integrand in Eq.(3.5) has a steep dependence on x through $hx/g\Lambda^2$. The terms involving $hx/g\Lambda^2$ in Eq.(3.5) contribute only in the immediate (in the logarithmic scale) neighborhood of the upper limit of the integral and have no effect on $df_t(\mu)/d\ln \mu$ below it. When $\xi < 0$, the scale Λ looks superficially larger than its actual value or the initial value $f_t(\Lambda)^2$ must be adjusted to a smaller value (see Fig.3).

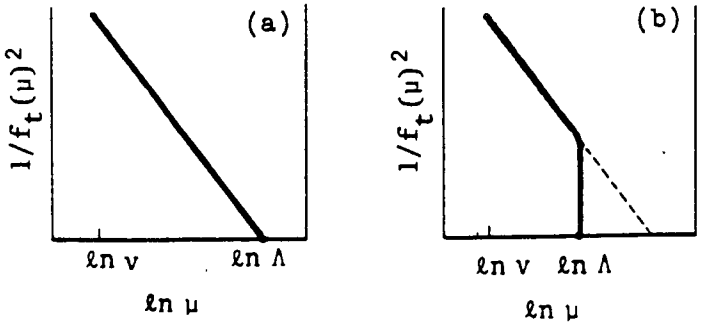


Fig.3. Schematic plot of $f_t(\mu)^2$ against $\ln \mu$ for (a) the minimal model and for (b) the model with the additonal singular interaction with $\xi < 0$.

With the new interaction added, a large portion of $f_t(\mu)^2$ arises from the singular interaction at the scale Λ which does not run logarithmically below Λ. If the running

distance ln (Λ/μ) were infinite, the low-energy coupling would be dominated by the logarithmic running. Specifically in Fig.3b, the height of the vertical line at ln Λ would be negligible as compared with the logarithmically running portion below ln Λ. However, in the electroweak theory the running distance ln(Λ/v) is finite and *can* be fairly short for a low cutoff Λ. In such a case the low energy parameters can be far from their infrared values. An important assumption in the one-loop RG analysis[3,4,10] is that physics is completely dominated by the logarithmically running effect and everything is determined by this portion. Assumptions or rules for the choice of initial values of running couplings at Λ in the RG analysis contain all the informations concerning the underlying dynamics. By restricting the ranges of the value of Λ and of the initial values of couplings at Λ, one is ruling out many classes of underlying dynamics. When the QCD coupling and higher-order Higgs self-couplings are included with $N_c \neq \infty$, we can study the RG equation for $\kappa_t(\mu) = (g_2^2/8\pi^2)[m_t(\mu)/m_W(\mu)]^2$ that Kubo *et al.*[6] and Marciano[4] discussed extensively. It leads us to the same conclusion as our approximated RG equation.

4. SUMMARY

Many different fundamental models at a scale Λ are represented by the same set of RG equations. In RG method, choice of Λ and values of couplings at Λ is all that one can adjust. If I am willing to close my eyes to some kind of naturalness, I can alter an outcome of RG analysis almost freely. Then I wonder how large error bars one should attach to the predictions of the one-loop RG analysis *if* one makes a claim to them as the general predictions of the "top-mode Higgs model".

REFERENCES

† Supported in part by National Science Foundation under Research Grant PHY-85-15857 and in part by U.S. Department of Energy under Research Contract DE-AC03-76SF00098.

1. V. A. Miransky, M. Tanabashi, and K. Yamawaki, Phys. Lett. **221B**, 177 (1989) ; Mod. Phys. Lett. **A4**, 1043 (1989) and references quoted therein.
2. Y. Nambu, Enrico Fermi Institute Report 89-08 (1989)
3. W. A. Bardeen, C. T. Hill, and M. Lindner, Phys. Rev. **D41**, 1647 (1990).
4. W. J. Marciano, Phys. Rev.**D41**, 219 (1990).
5. B. Pendleton and G. G. Ross, Phys. Lett. **98B**, 291 (1981).
6. J. Kubo, K. Sibold, and W. Zimmermann, Nucl. Phys. **B259**, 331 (1985); Phys. Lett. **221B**, 177 (1989).
7. W. J. Marciano, Phys. Re. Lett. **62**, 2793 (1989).
8. M. Suzuki, Phys. Rev. **41**, 3457 (1990).
9. D. Decamp et al. (ALEPH collaboration), Phys. Lett. **241B**, 141 (1990).
10. M. A. Luty, Phys. Rev. **D41**, 2893 (1990).
11. C. T. Hill, C. N. Leung, and S. Rao, Nucl. Phys.**B262**, 517 (1985).
12. M. Suzuki, Mod. Phys. Lett. **A5**, 1205 (1990).

LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
INFORMATION RESOURCES DEPARTMENT
BERKELEY, CALIFORNIA 94720