LBL-29543
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Submitted to Physical Review Letters

Spin Alignment Quantization, Berry's Phase Quantization, Stationary Condition and Twinned Superdeformed Bands

S.J. Wang

August 1990

LOAN COPY
Circulates
for 2 weeks
Bldg. 50 Library.
LBL-29543
Copy 2

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Spin Alignment Quantization, Berry's Phase Quantization, Stationary Condition And Twinned Superdeformed Bands

Shun-Jin Wang

Center of Theoretical Physics, CCAST(World Lab.) Beijing and

Department of Modern Physics, Lanzhou University, Lanzhou 730001, PR China
and
Nuclear Science Division, Lawrence Berkeley Laboratory,

University of California, Berkeley, CA 94720, USA

Striking regularities of the twinned superdeformed bands is discussed in terms of non-adiabatic Berry's phase quantization and stationary condition within the framework of cranking shell model. It is shown that the rotating deformed mean field will generate a Berry's phase for nucleons subjected to this potential.The non-adiabatic Berry's phase is related to spin alignment. The stationary condition of the rotational states requires Berry's phase quantization and in turn leads to spin alignment quantization. For nucleons which are weakly coupled to the deformed field, spin alignment quantization will take place as the rotational frequency reaches a critical value. Those nucleons whose spin alignment is quantized and keeps constant do not contribute to the rotational energy and the moment of inertia. For nuclei at extreme conditions of large elongation and high spins, the normal parity states possess pseudo-SU(3)- pseudo-SU(2) dynamical symmetry which provides a weak pseudo spin-orbit coupling scheme. If the pseudo-angular momentum is stuck to the large deformation, the critical frequency for spin alignment quantization can be calculated to yield reasonable values. the main aspects of the regularities of the twinned superdeformed bands can thus be understood.

This work was supported in part by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract No.DE-AC03-76SF00098 and by the Natural Science Foundation of China.

Striking regularities in the rotational bands were observed recently in superdeformed nuclei in both the mass-150 and mass-190 regions [1-4]. The surprising feature of the phenomenon is that rotational bands in different superdeformed nuclei have been found to have equivalent transition energies within 1 or 2 kev, which implies an identical moment of inertia for these different superdeformed nuclei. Another unexpected aspect of the rotational superdeformed bands is that the spin alignment is quantized in units of 1/2 or 1 $\hbar$ and the spin quantization begins at a critical rotational frequency. Since a considerable number of nuclei exhibite such a puzzling behaviour, it is probably not accidental, but rather a generic property of superdeformed nuclei. A general underlying principle may govern this phenomenon. In this note, we make an effort to understand the above generic phenomenon in terms of Berry's phase quantization which is required by the stationary condition of the populated rotational high spin states.

Berry's phase[5] is a generic property of a quantum system subjected to a time-dependent external field. In nuclear physics, a heavy ion colliding off-center with a target nucleus will result in a rapidly rotating compound nuclear system. Nucleons which are coupled to the deformed mean field will feel a time-dependent (rotating) potential and Berry's phase will be induced in their wave functions. The non-adiabatic effect on Berry's phase (while in this case appears as a Coriolis force) will cause spin alignment for each nucleon driven by the rotating potential. As indicated in previous papers[6,7], for SU(2) dynamical group (rotation generators belong to su(2)), non-adiabatic Berry's phase is related to spin alignment and Berry's phase quantization leads to spin alignment quantization. Since stationary states have only dynamical phase, but no non-quantized Berry's phase, to populate stationary rotational states requires quantization of their Berry's phase, and this in turn leads to spin alignment quantization. For the whole deformed nucleus,each nucleon feeling the rotating deformed mean field will contribute a certain amount of spin alignment, which may or may not be quantized. However to make the rotational high spin state stationary, the total Berry's phase (summation of each nucleon's contribution in an independent particle model) must be quantized. If some nucleons are weakly coupled to the

deformed potential (through for instance spin-orbit coupling, since the orbit is strongly coupled to the deformation) and the rotational frequency reaches a critical value to break the weak coupling, such nucleons will have spin alignment quantization.As soon as a nucleon has realized spin alignment quantization, it will not contribute to the rotation energy and the related moment of inertia. In what follows, we will formulate the above ideas in detail.

Suppose a nucleus has an axially symmetric deformation, the Hamiltonian for a nucleon in the deformed mean field is H_o , which has the following eigen-solutions

$$H_o \varphi_m = \epsilon_m \varphi_m . \qquad (1)$$

Since deformation breaks rotation invariance, a rotational mode is generated to restore the rotation symmetry. This rotation-symmetry restoration procedure can be carried out by first cranking the system and then quantizing the resulting classical rotational Hamiltonian. Let us crank the deformed nucleus through a periodic rotation along the z-axis,

$$H(t) = \exp\{-ij_z \omega t\} H_o \exp\{ij_z \omega t\} . \qquad (2)$$

The rotating system obeys the time-dependent Schrodinger equation

$$i \frac{\partial \psi(t)}{\partial t} = H(t) \psi(t) . \qquad (3)$$

Let

$$\psi(t) = \exp\{-ij_z \omega t) \eta(t) . \qquad (4)$$

Equation of motion for $\eta(t)$ is

$$i \frac{\partial \eta(t)}{\partial t} = H(\omega) \eta(t) , \qquad (5)$$

where

$$H(\omega) = H_o - \omega j_z . \qquad (6)$$

Solutions of eqs.(3) and (5) are

$$\eta(t) = \exp\{-iH(\omega)t\} \eta(0) , \qquad (7)$$

$$\psi(t) = U(t)\,\psi(0) \ , \tag{8}$$

where evolution operator U(t) is defined as

$$U(t) = \exp\{-ij_z\omega t\}\,\exp\{-iH(\omega)t\} \ . \tag{9}$$

Consider solutions in one period T ($T = 2\pi/\omega$),

$$U(T) = \exp\{-ij_z 2\pi\}\,\exp\{-iH(\omega)T\} \ . \tag{10}$$

Because of the symmetry

$$\exp\{-ij_z 2\pi\}\,H(\omega)\,\exp\{ij_z 2\pi\} = H(\omega) \ , \tag{11}$$

U(T) and $H(\omega)$ are commuting and have common eigen-states,

$$H(\omega)\,\eta_m = E_m\,\eta_m \ , \tag{12}$$

$$U(T)\,\eta_m = \exp\{-i\,\phi_m\}\,\eta_m \ . \tag{13}$$

The total phase ϕ_m will be given later. Consider cyclic or recurrent solutions whose initial states are eigen-solutions of $H(\omega)$, i.e.,

$$\psi_m(0) = \eta_m \ . \tag{14}$$

Assume a complete set of bases $\{\xi_m\}$, which are also eigen-states of j_z , i.e.,

$$j_z\,\xi_m = m\,\xi_m \ . \tag{15}$$

The eigen-state η_m can be expanded in terms of ξ_m,

$$\eta_m = \sum_{m'} C_{m'm}\,\xi_{m'} \ . \tag{16}$$

The cyclic solution ψ_m is

$$\begin{aligned}\psi_m(T) &= \exp\{-ij_z 2\pi\}\,\exp\{-iH(\omega)T\}\,\eta_m = \exp\{-iE_m T\}\,\exp\{-ij_z 2\pi\}\,\eta_m \\ &= \exp\{-iE_m T\}\,\sum_{m'} C_{m'm}\,\exp\{-i2m'\pi\}\,\xi_{m'} \\ &= \exp\{-iE_m T - i2m\pi\}\,\psi_m(0) \ .\end{aligned} \tag{17}$$

In the derivation of eq.(17), eqs.(14) and (15) have been used. From eq.(17), we obtain the total phase,

$$\Phi_m = E_m T + 2m\pi \ . \tag{18}$$

The expectation value of H(t) is

$$\mathcal{E}_m(t) = \langle \psi_m(t) | H(t) | \psi_m(t) \rangle = \langle \eta_m | H_0 | \eta_m \rangle$$

$$= E_m(\omega) + \omega \, \overline{j_z^m} \ , \tag{19}$$

where the expectation value of spin along the rotating axis is

$$\overline{j_z^m} = \langle \eta_m | j_z | \eta_m \rangle = \langle \psi_m(t) | j_z | \psi_m(t) \rangle \ . \tag{20}$$

From eq.(19), we can calculate dynamical phase,

$$\Phi_m^d = \int_0^T \mathcal{E}_m(t) \, dt = E_m T + 2m\pi \ , \tag{21}$$

and Berry's phase

$$\Phi_m^B = -2m\pi \left(1 - \overline{j_z^m}/m \right) \ . \tag{22}$$

Eq.(22) indicates that Berry's phase is related to expectation value of spin along the rotating axis and gives Berry's phase a physical explanation besides its gauge geometric interpretation. The expression (22) provides as well an algorithm to calculate non-adiabatic Berry's phase. In the code of cranking shell model[8], calculation of the spin alignment $\overline{j_z^m}$ is a straightforward job.

We proceed to exploit relationship between spin alignment quantization and Berry's phase quantization. Spin alignment quantization means that $\overline{j_z^m}$ takes integer or half-integer values,

$$\overline{j_z^m} = m', \quad m' = \text{integer or half-integer} \ . \tag{23}$$

This makes Berry's phase quantized as well,

$$\Phi_m^B = -2\pi(m - m') = -2N\pi \ , \qquad N = m - m' = \pm \text{integer} \ . \tag{24}$$

Therefore, for SU(2) dynamical group, Berry's phase quantization is related to spin alignment quantization.

Let us consider the machanism which conducts spin alignment (or Berry's

phase) quantization. Since spin alignment quantization is a dynamical phenomenon, the specific Hamiltonian is needed. For high spin physics, the most popular theory available is the cranking shell model[8] . For nuclei at extreme conditions of large elongations and high spins, the Hamiltonian of normal parity states possesses pseudo-SU(3) ⊗ pseudo-SU(2) dynamical symmetry [9-11], namely,

$$H_0 = \hbar\Omega (\tilde{N} + 5/2) - \varepsilon \tilde{Q}_{20} - \tilde{C}\, \tilde{l}\cdot\tilde{s} + \tilde{D}\, \tilde{l}^2 \qquad (25)$$

where { $\tilde{Q}_{2\mu}$, $\tilde{l}_i$ } constitute pseudo-SU(3) algebra and { $\tilde{s}_i$ } constitute pseudo-SU(2) algebra. Inserting eq.(25) in eqs.(6), (12) and (19), we have

$$E_m = \hbar\Omega (\tilde{N} + 5/2) + \langle \eta_m | (- \varepsilon \tilde{Q}_{20} - \tilde{C}\, \tilde{l}\cdot\tilde{s} + \tilde{D}\, \tilde{l}^2) | \eta_m \rangle - \omega \langle \eta_m | j_z | \eta_m \rangle , \qquad (26)$$

$$\mathcal{E}_m(t) = \mathcal{E}_m(\omega) = E_m + \omega \langle \eta_m | j_z | \eta_m \rangle = \mathcal{E}_m(0) + \Delta \mathcal{E}_m(\omega) , \qquad (27)$$

where

$$\mathcal{E}_m(0) = \hbar\Omega (\tilde{N} + 5/2) + \langle \eta_m(0) | (- \varepsilon \tilde{Q}_{20} - \tilde{C}\, \tilde{l}\cdot\tilde{s} + \tilde{D}\, \tilde{l}^2) | \eta_m(0) \rangle , \qquad (28a)$$

$$\Delta \mathcal{E}_m(\omega) = \langle \eta_m(\omega) | (- \varepsilon \tilde{Q}_{20} - \tilde{C}\, \tilde{l}\cdot\tilde{s} + \tilde{D}\, \tilde{l}^2) | \eta_m(\omega) \rangle - \langle \eta_m(0) | (- \varepsilon \tilde{Q}_{20} - \tilde{C}\, \tilde{l}\cdot\tilde{s} + \tilde{D}\, \tilde{l}^2) | \eta_m(0) \rangle . \qquad (28b)$$

Now we discuss the consequences of nucleonic spin alignment quantization. From eqs.(6) and (12), we have

$$\frac{\partial H(\omega)}{\partial \omega} = - j_z , \qquad (29)$$

and

$$\frac{\partial E_m(\omega)}{\partial \omega} = - \langle \eta_m | j_z | \eta_m \rangle . \qquad (30)$$

On the other hand, from eqs.(19) and (30) we have

$$\frac{\partial \mathcal{E}_m(\omega)}{\partial \omega} = \frac{\partial E_m}{\partial \omega} + \langle \eta_m | j_z | \eta_m \rangle + \omega \frac{\partial \langle \eta_m | j_z | \eta_m \rangle}{\partial \omega}$$

$$= \omega \frac{\partial \langle \eta_m | j_z | \eta_m \rangle}{\partial \omega}, \qquad (31)$$

and

$$\mathcal{E}_m(\omega) = \mathcal{E}_m(0) + \int_0^{\omega} \omega \frac{\partial \langle \eta_m | j_z | \eta_m \rangle}{\partial \omega} d\omega \qquad (32)$$

Comparing eq.(32) with eq.(27), we obtain

$$\Delta\mathcal{E}_m(\omega) = \int_0^{\omega} \omega \frac{\partial\langle\eta_m|j_z|\eta_m\rangle}{\partial\omega} d\omega . \qquad (33)$$

If a nucleon's spin alignment is quantized and keeps constant, i.e., $\overline{j_z^m} = m$, then

$$\frac{\partial\langle\eta_m|j_z|\eta_m\rangle}{\partial\omega} = 0 , \qquad (34)$$

and

$$\mathcal{E}_m(\omega) = \mathcal{E}_m(0) . \qquad (35a)$$

$$\Delta\mathcal{E}_m(\omega) = \langle\eta_m(\omega)|(-\varepsilon\tilde{Q}_{20} - \tilde{C}\,\tilde{l}\cdot\tilde{s} + \tilde{D}\,\tilde{l}^2)|\eta_m(\omega)\rangle - \langle\eta_m(0)|(-\varepsilon\tilde{Q}_{20} - \tilde{C}\,\tilde{l}\cdot\tilde{s} + \tilde{D}\,\tilde{l}^2)|\eta_m(0)\rangle = 0. \qquad (35b)$$

Eqs.(35a,b) imply double meanings: If a nucleon's spin alignment gets quantized, (i) there is a complete cancellation among the deformation effect $-\varepsilon\tilde{Q}_{20}$, the Coriolis effect $-\tilde{C}\,\tilde{l}\cdot\tilde{s}$, and the centrifugal effect $\tilde{D}\,\tilde{l}^2$; (ii) this nucleon does not contribute to the rotational energy and the moment of inertia. The former conclusion is obvious from eq.(35b). The latter conclusion can be made clear as follows. In fact, for the whole nucleus, the total rotational energy is

$$E(\omega) = \sum_{m=1}^{A} \mathcal{E}_m(\omega) = E(0) + \sum_{m=1}^{A} \Delta\mathcal{E}_m(\omega)$$

$$= E(0) + \sum_m{}' \Delta\mathcal{E}_m(\omega) = E(0) + \Delta E(\omega) . \qquad (36)$$

Where $\Delta E(\omega)$ comes from only those nucleons whose spin alginments are not quantized and do not keep constant. Suppose for these nucleons,

$$\overline{j_z^m} \doteq \omega\vartheta_m , \qquad (37)$$

then

$$E(\omega) = E(0) + 1/2\,\vartheta\omega^2$$

$$= E(0) + \{1/(2\vartheta)\}\,R^2 , \qquad R = \vartheta\omega , \qquad (38)$$

where the total moment of inertia is contributed from the those nucleons whose spin alignments are not quantized,

$$\vartheta = \sum_m{}' \vartheta_m . \qquad (39)$$

Thus those nucleons whose spin alignments are quantized and keep constant do not contribute to the rotational energy and the moment of inertia.

Now we apply the above results to understand the superdeformed bands of ^{192}Hg(1), ^{194}Hg(2) and ^{194}Hg(3)[4]. For ^{192}Hg(1), since the total pseudo-spin $\tilde{S} = 0$, the total angular momentum I is equal to the pseudo-angular momentum of the top, $\tilde{R}$, and the rotational energy is (assume $\tilde{R} = I$ = even, since its signature is even),

$$E_I^{(1)} = \frac{\hbar^2}{2\mathcal{J}_+(I)} I(I+1) \quad . \tag{40}$$

The ^{194}Hg(2) has two extra neutrons and its core (^{192}Hg(1)) has an even signature. Suppose the extra two neutrons get their pseudo spin alignment quantized and together contribute $\tilde{S} = 1$, which is maximum, and can not be increased any more, therefore keeps constant. The pseudo-angular momentum of the core, $\tilde{R}$, takes the values of even integers because of its even signature. According to pseudo-spin and pseudo-angular momentum coupling scheme, the total angular momentum of ^{194}Hg(2) is $\tilde{R} + \tilde{S} = I + 1$ (we assume $\tilde{R} = I$ for comparison with ^{192}Hg(1)). From the above results, the extra two neutrons do not contribute to the rotational energy and the momemt of inertia, and the rotational energy of ^{194}Hg(2) is thus

$$E_{I+1}^{(2)} = \frac{\hbar^2}{2\mathcal{J}_+(I)} I(I+1) \quad . \tag{41}$$

Now consider ^{194}Hg(3). Its core (^{192}Hg) has negative signature and the pseudo-angular momentum of the core takes the values of odd integers, namely $\tilde{R} = I-1$. The extra two neutrons also got their pseudo-spin alignments quantized and contribute $\tilde{S} = 1$. The total angular momentum of ^{194}Hg(3) is $\tilde{R} + \tilde{S} = I$. The rotational energy of ^{194}Hg(3) is thus contributed only from its core according to the above results, namely,

$$E_I^{(3)} = \frac{\hbar^2}{2\mathcal{J}_-(I-1)} I(I-1) \quad . \tag{42}$$

Where $\mathcal{J}_+$ and $\mathcal{J}_-$ denote the moments of inertia for ^{192}Hg with positive and negative signatures respectively. Since from numerical calculations and theoretical considerations of the cranking shell model[8,9],

$$\frac{\partial \langle m\alpha = \frac{1}{2} | j_z | m\alpha = \frac{1}{2} \rangle}{\partial \omega} \doteq \frac{\partial \langle m\alpha = -\frac{1}{2} | j_z | m\alpha = -\frac{1}{2} \rangle}{\partial \omega} \tag{43}$$

and

$$\mathcal{J}(m,\alpha) = \frac{\partial \langle m\alpha | j_z | m\alpha \rangle}{\partial \omega} \qquad (44)$$

it follows that

$$\mathcal{J}(\alpha=+1/2) \doteq \mathcal{J}(\alpha=-1/2), \qquad (45)$$

or

$$\mathcal{J}_+ = \mathcal{J}_- = \mathcal{J} \quad . \qquad (46)$$

From the energy spetra (40)-(41), we obtain the corresponding gamma energies as follows,

$$^{192}\mathrm{Hg}(1):\quad E_\gamma^{(1)}(I) = \frac{\hbar^2}{\mathcal{J}(I)}(2I-1) + \hbar^2/2\ [\frac{1}{\mathcal{J}(I)} - \frac{1}{\mathcal{J}(I-2)}](I-2)(I-1), \qquad (47a)$$

$$^{194}\mathrm{Hg}(2):\quad E_\gamma^{(2)}(I+1) = \frac{\hbar^2}{\mathcal{J}(I)}(2I-1) + \hbar^2/2\ [\frac{1}{\mathcal{J}(I)} - \frac{1}{\mathcal{J}(I-2)}](I-2)(I-1), \qquad (47b)$$

$$^{194}\mathrm{Hg}(3):\quad E_\gamma^{(3)}(I) = \frac{\hbar^2}{\mathcal{J}(I-1)}(2I-3) + \hbar^2/2\ [\frac{1}{\mathcal{J}(I-1)} - \frac{1}{\mathcal{J}(I-3)}](I-2)(I-3), \qquad (47c)$$

which can reproduce the observed gamma spectra[4].

Finally, let us calculate the critical frequency of the nucleonic spin alignment quantization. As pointed out by A.Bohr et al.[9-11], at extreme conditions of large elongations and high spins, the Hamiltonian of normal parity states possesses pseudo-SU(3)-SU(2) dynamical symmetry, and the pseudo-orbital angular momenta are strongly coupled to the large deformation.In the Hamiltonian (25), the pseudo spin-orbit coupling $-\tilde{C}\,\tilde{l}\cdot\tilde{s}$ responsible for spin alignment, is very weak and provides a machanism to realize pseudo-spin alignment quantization. If the pseudo-orbital angular momenta are stuck to the deformation, the $\tilde{C}\,\tilde{l}\cdot\tilde{s}$ term is like a Hamiltonian which describes a spin-1/2 particle in a magnetic field. The deformation carrying the orbits plays the role of a magnetic field whose strength depends on the value of the orbital angular momentum.As the deformed nucleus rotates, the deformed potential acting as a magnetic field rotates as well. The corresponding Hamiltonian describes a spin-1/2 particle in a rotating magnetic field and will cause spin alignment quantization. Now let us consider the problem quantatively. The corresponding Hamiltonian responsible for spin alignment is the pseudo spin-orbit coupling, namely

$$\mathcal{H}_\sigma = -\hbar\omega_0\,\tilde{v}_{ls}\,\tilde{l}\cdot\tilde{s} = -\tilde{B}\cdot\tilde{s}\ , \qquad (48)$$

where the effective "magnetic field" is

$$\tilde{B} = B\ (1\ ,\ 0\ ,\ 0\ )\ ,\qquad B = \hbar\Omega = \hbar\omega_0 \tilde{\Lambda}\, \tilde{v}_{\ell s}\ . \tag{49}$$

As the deformed nucleus rotates, the cranked Hamiltonian is

$$\mathcal{H}(t) = \exp\{-i\tilde{s}_z\,\omega t\}\, \mathcal{H}_0 \exp\{i\tilde{s}_z\,\omega t\} = -\,\tilde{B}(t)\cdot\tilde{s}\ , \tag{50}$$

where the rotating "magnetic field" is

$$\tilde{B}(t) = B\ (\cos\omega t,\ \sin\omega t\ ,\ 0\ )\ . \tag{51}$$

The equation of motion for the pseudo-spin in a rotating deformed system is the same as the equation of motion for spin-1/2 particle in a rotating magnetic field, namely,

$$i\hbar\frac{\partial\psi(t)}{\partial t} = \mathcal{H}(t)\,\psi(t)\,. \tag{52}$$

This problem has been solved analytically in ref. 7 . The critical frequency for spin-1/2 alignment quantization is

$$\omega_c = \Omega\ , \tag{53}$$

$$\Omega = \omega_0 \tilde{\Lambda}\, \tilde{v}_{\ell s}\ . \tag{54}$$

For Hg, $82 \leq N \leq 126$, $\tilde{v}_{\ell s} = 0.019$ and $\hbar\omega_0 = 41/\ A^{1/3}$ Mev = 7 Mev,

$$\hbar\Omega = 0.133\ \tilde{\Lambda}\ \text{Mev}, \tag{55}$$

which yields

$$\hbar\omega_c = 0.133\ \tilde{\Lambda}\ \text{Mev} = \begin{cases} 0.133\ \text{Mev}\ ,\ \text{for}\ \ \tilde{\Lambda} = 1, & (56a) \\ 0.266\ \text{Mev}\ ,\ \text{for}\ \ \tilde{\Lambda} = 2, & (56b) \\ 0.399\ \text{Mev}\ ,\ \text{for}\ \ \tilde{\Lambda} = 3. & (56c) \end{cases}$$

For ^{194}Hg(2) and ^{194}Hg(3), $\hbar\omega_c = 0.2$ Mev. Thus the calculated $\hbar\omega_c$ look reasonable.

Finally we discuss the twinned SD bands of the odd-A nuclei. For the twinned bands of ^{151}Tb and ^{152}Dy, ^{150}Gd and ^{151}Tb, as pointed out in Refs.3 and 9, the pseudo SU(3) limit yields a decoupling constant near 1, and this in turn leads to the rotational energy spectra of the odd-A nuclei identical to those of the corres-

ponding cores if the last odd particle (hole) does'nt modify the moment of inertia. Since the last odd particle's(hole's) spin alignment takes the value of 1/2 $\hbar$, namely quantized and constant, according to the above discussion, it does not contribute the rotational energy and the moment of inertia, namely its different polarization effects cancelled each other.

We conclude with the following remarks. The twinned SD bands can be understood in terms of the limit pseudo SU(3)-SU(2) dynamical symmetry and the spin alignment quantization which is required by the quantum mechanical stationary condition. The pesudo SU(3)-SU(2) symmetry is essential for yielding decoupling constant near 1 and for providing a weak $\tilde{l}\cdot\tilde{s}$ coupling. The spin alignment quantization is crucial for the cancellation of different polarization effects and the identical moment of inertia. In our calculation of the critical frequency of spin alignment quantization,we have assumed that the pseudo orbits of the extra nucleons are strongly coupled to the deformation and do not contribute to their spin alignment quantization. Although this assumption is consistent with the experimental data which indicate that just the pseudo spin alignment is quantized, the effects of the pseudo orbits of the extra nucleons are still a challenging problem which is left out in this note and deserves further investigation.

I am grateful to F.S.Stephens, R.M.Diamond, M.A.Deleplanque and W.J. Swiatecki for Many illuminating discussions and valuable comments. this work was supported in part by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S.Department of Energy under Contract No.DE-AC03-76SF00098 and by the Natural Science Foundation of China.

References

1. Talks given by P.J.Twin, F.S.Stephens, and I.Ragnarsson respectively at the Workshop on the Nucleus at High Spin, Niels Bohr Institute, Copen-Hamgen, Denmark, Oct.2-27, 1989.
2. T.Byrski et al., Phys.Rev.Lett. 64(1990)1650.
3. W.Nazarewicz, et al., Phys.Rev.Lett. 64(1990)1654.
4. F.S.Stephens, et al., Phys.Rev.Lett. 64(1990)2623;
   F.S.Stephens, et al., Phys.Rev.Lett. 65(1990)301.
5. M.V.Berry, Proc.R.Lond. A329(1984)45; Nature Vol.326 (1987)277;
   B.Simon, Phys.Rev.Lett. 51(1983)2167;
   Y.Aharanov and J.Anandan, Phys.Rev.Lett. 58(1987)1593;
   J.Samuel and R.Bhandari, Phys.Rev.Lett. 60(1988)2339.
6. S.J.Wang, Non-adiabatic effect on Berry's phase for light propagating in an optical fiber, LBL Preprint-29235, May,1990; submitted to Phys.Rev.A.
7. S.J.Wang, Non-adiabatic Berry's phase for a spin particle in a rotating magnetic field, LBL Preprint-29330, July,1990; accepted by Phys.Rev.A.
8. R.Bengtsson and J.D.Garrett, The Cranking Model-Theoretical and Experimental Basis, Lund Mph-84/18,1984; to be published by World Scientific Publishing Co., Singapore.
9. A.Bohr, I.Hamamoto and Ben R. Mottelson, Physica Scripta, Vol.26(1982)267.
10. K.T.Hecht and A.Adler, Nucl.Phys.A137(1969)129;
    R.D. Ratna Raju, J.P.Draayer, and K.T.Hecht, Nucl.Phys.A202(1973)433.
11. A.Arima, M.Harvey, and K.Shimizu, Phys.Lett. 30B(1969)517.

LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
INFORMATION RESOURCES DEPARTMENT
BERKELEY, CALIFORNIA 94720