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Electroweak Symmetry Breaking

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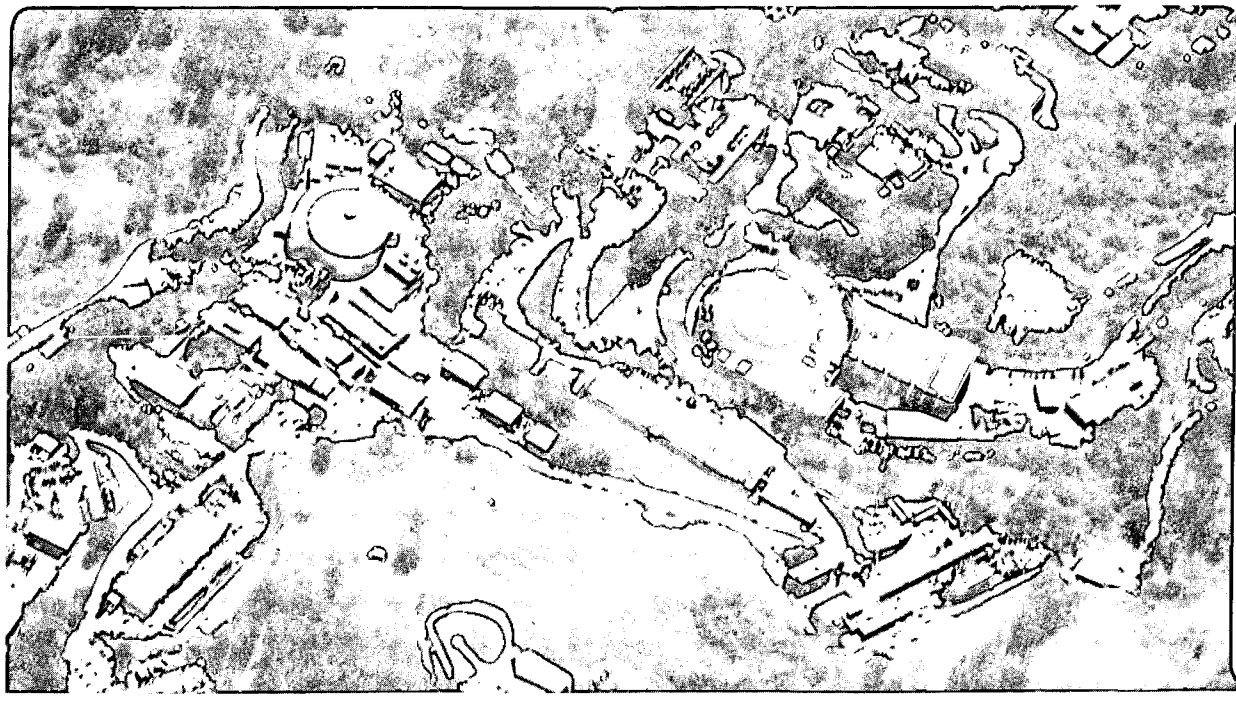

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ELECTROWEAK SYMMETRY BREAKING *

Lectures presented at the Workshop/Symposium on TeV Physics
China Center of Advanced Science and Technology
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ELECTROWEAK SYMMETRY BREAKING

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Abstract The Higgs mechanism is reviewed in its most general form, requiring the existence of a new symmetry-breaking force and associated particles, which need not however be Higgs bosons. The first lecture reviews the essential elements of the Higgs mechanism, which suffice to establish low energy theorems for the scattering of longitudinally polarized W and Z gauge bosons. An upper bound on the scale of the symmetry-breaking physics then follows from the low energy theorems and partial wave unitarity. The second lecture reviews particular models, with and without Higgs bosons, paying special attention to how the general features discussed in lecture 1 are realized in each model. The third lecture focuses on the experimental signals of strong WW scattering that can be observed at the SSC above 1 TeV in the WW subenergy, which will allow direct measurment of the strength of the symmetry-breaking force.

1. INTRODUCTION

In these three lectures I will discuss electroweak symmetry breaking from a general perspective, stressing model independent properties that follow just from the assumption that the electroweak interactions are described by a spontaneously broken gauge theory.[1)] This means I assume the Higgs mechanism[2)] though not necessarily the existence of Higgs bosons. This framework requires the existence of a new force

of nature and new associated quanta which may or may not be Higgs bosons.

I will refer generically to the required new dynamical system as

$$\mathcal{L}_{SB} = \mathcal{L}_{\text{Symmetry Breaking}}, \tag{1.1}$$

the lagrangian of the still unknown symmetry breaking sector. We will see that the general framework is sufficient to tell us a good deal about the range of possibilities for $\mathcal{L}_{SB}$. In particular, general symmetry properties together with unitarity imply that the new physics of $\mathcal{L}_{SB}$ must emerge at or below ~ 1.8 TeV in the scattering of longitudinally polarized gauge bosons, $W_L W_L \to W_L W_L$.[2] If the quanta of $\mathcal{L}_{SB}$ are much lighter than 1 TeV, then there are narrow Higgs bosons and $\mathcal{L}_{SB}$ has a weak interaction strength that is amenable to perturbation theory. If the new quanta lie above 1 TeV then $\mathcal{L}_{SB}$ is a strongly interacting system with a rich spectrum, there are no narrow Higgs bosons and perhaps none at all, the theory cannot be analyzed perturbatively, and we say that the Higgs mechanism is implemented "dynamically".

The SSC is a minimal collider with the assured capability to allow us to determine which possibility is realized in nature. The point is that the SSC is (just) sufficient to observe the signal of strong WW scattering that occurs if $\mathcal{L}_{SB}$ lives above 1 TeV. Therefore we will learn from the presence or absence of the signal in SSC experiments. If the signal does not occur it means that the physics lies below 1 TeV, in contrast to the more typical situation in high energy physics where a negative search at a given energy leaves open the possibility that still higher energies may be needed. This is the sense in which the SSC is a "no-lose" facility for the study of the symmetry breaking mechanism. Of course the technical challenges to realize this potential are enormous, both in accelerator physics (luminosity of $10^{33}\text{cm}^{-2}\text{s}^{-1}$ is essential) and especially in the experimental physics of the detectors. In the final lecture I will discuss some of the signals and backgrounds that must be mastered.

The first lecture (Sections 1-4) presents the general framework of a spontaneously broken gauge theory:

- the Higgs mechanism in its most general form, with or without Higgs boson(s) (Section 2)
- the implications of symmetry and unitarity for the mass scale and interaction strength of the new physics that the Higgs mechanism requires (Section 3)

In addition I will review a "softer" theoretical argument based on the "naturalness" problem (Section 4) which leads to a prejudice against Higgs bosons unless they are supersymmetric. This is a prejudice, not a theorem, and it could be overturned in the future by a clever new idea. This is a good place to remember the slogan: **all theorists presumed guilty until proven innocent.**

In the second lecture I will illustrate the general framework by reviewing some specific models (Section 5):

- the Weinberg-Salam model of the Higgs sector
- the minimal supersymmetric extension of the Weinberg-Salam model
- technicolor as an example of the Higgs mechanism without Higgs bosons.

The third lecture concludes with a discussion of strong WW scattering (Section 6), that must occur if $\mathcal{L}_{SB}$ lives above 1 TeV. In particular I will describe some of the experimental signals and backgrounds at the SSC. A brief summary is presented in Section 7.

A more complete review and more extensive bibliography can be found in reference 4.

2. THE GENERIC HIGGS MECHANISM

In this section we review the Higgs mechanism in its most general form. The basic ingredients are a gauge sector and a symmetry breaking sector,

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{SB}. \tag{2.1}$$

$\mathcal{L}_{\text{gauge}}$ is an unbroken *locally symmetric = gauge invariant* theory, describing massless gauge bosons that are transversely polarized, just like the photon. For instance, for $SU(2)_L \times U(1)_Y$ gauge symmetry the gauge bosons are a triplet $\vec{W} = W_1, W_2, W_3$ corresponding to the generators $\vec{T}_L$ and a singlet gauge boson X corresponding to the hypercharge generator Y. If there were no $\mathcal{L}_{SB}$, the unbroken $SU(2)_L$ nonabelian symmetry would give rise to a force that would confine quanta of nonvanishing $\vec{T}_L$ charge, such as left-handed electrons and neutrinos.

In the generic Higgs mechanism $\mathcal{L}_{SB}$ breaks the *local* (or *gauge*) symmetry of $\mathcal{L}_{\text{gauge}}$. To do so $\mathcal{L}_{SB}$ must possess a *global* symmetry G that breaks spontaneously to a subgroup H,

$$G \rightarrow H. \tag{2.2}$$

In the electroweak theory we do not yet know either of the groups G or H,

$$G = ? \tag{2.3a}$$

$$H = ? \tag{2.3b}$$

We want to discover what they are and beyond that we want to discover the symmetry breaking sector

$$\mathcal{L}_{SB} = ? \tag{2.4}$$

including the mass scale of its spectrum

$$M_{SB} = ? \tag{2.5}$$

and the interaction strength

$$\lambda_{SB} = ? \tag{2.6}$$

Eq. (2.4) used to be the 64 $\times 10^8$ dollar question (in then-year dollars, moreor less); with a revised design it has become something like an 80 $\times 10^8$ dollar question.

We do already know one fact about G and H. The $SU(2)_L \times U(1)_Y$ gauge invariance of $\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{SB}$ is *a local* symmetry, meaning that it is an invariance under transformations that depend on space-time,

$$e^{i\vec{f}(x)\cdot\vec{T}_L} e^{if_0(x)Y} \tag{2.7}$$

G and H are *global* symmetries of $\mathcal{L}_{SB}$, meaning that they are symmetries which do not depend on space-time (i.e., as eq. (2.7) would be if $\vec{f}$ and f_0 were constants rather than functions of $x = \vec{x}, t$). Therefore G must be at least as big as $SU(2)_L \times U(1)_Y$ or $\mathcal{L}_{SB}$ would *explicitly* (as opposed to *spontaneously*) break the $SU(2)_L \times U(1)_Y$ gauge symmetry. Similarly H must be at least as big as $U(1)_{EM}$ or the theory after spontaneous breakdown will not accommodate the unbroken gauge symmetry of QED. That is, in order to be consistent with the desired pattern of breaking for the *local* symmetry

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM} \tag{2.8}$$

the spontaneous breaking of the *global* symmetry of $\mathcal{L}_{SB}$

$$G \rightarrow H \tag{2.9}$$

is constrained by

$$G \supset SU(2)_L \times U(1)_Y \tag{2.10a}$$

$$H \supset U(1)_{EM} \tag{2.10b}$$

<u>Step I:</u>

There are two steps in the Higgs mechanism. The first has nothing to do with gauge symmetry—it is just the spontaneous breaking of a global symmetry as explained by the Goldstone theorem.[5,6] By *spontaneous* symmetry breaking $G \rightarrow H$ we mean that

$$G = \text{ global symmetry of } \textit{interactions} \text{ of } \mathcal{L}_{SB} \tag{2.11a}$$

while

$$H = \text{ global symmetry of the } \textit{ground-state} \text{ of } \mathcal{L}_{SB}. \tag{2.11b}$$

That is, the dynamics of $\mathcal{L}_{SB}$ are such that the state of lowest energy (the *vacuum* in quantum field theory) has a smaller symmetry group than the symmetry of the force laws of the lagrangian. Goldstone's theorem tells us that for each broken generator of G the spectrum of $\mathcal{L}_{SB}$ contains a massless spin zero particle or Goldstone boson,

$$\begin{aligned} &\# \text{ of massless scalars} \\ &\quad = \# \text{ of broken symmetry axes} \\ &\quad = \text{dimension } G - \text{dimension H} \\ &\quad = \# \text{ of energetically flat directions in field space.} \end{aligned} \tag{2.12}$$

The last line is the clue to the proof of the theorem: masses arise from terms that are quadratic in the fields,

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}m^2\phi^2, \tag{2.13}$$

so a field direction that is locally flat in energy (i.e., goes like ϕ^n with $n \geq 3$) corresponds to a massless mode.

The classic example is the hypersombrero* potential. Consider a triplet of scalars

$$\vec{\varphi} = \varphi_1, \varphi_2, \varphi_3 \tag{2.14}$$

with interactions described by the potential $V(\varphi)$:

$$\begin{aligned} V(\varphi) &= \lambda(\vec{\varphi}^2 - v^2)^2 \\ &= \lambda(\vec{\varphi}^2)^2 - 2\lambda v^2\vec{\varphi}^2 + \lambda v^4 \end{aligned} \tag{2.15}$$

λ is the dimensionless coupling constant and v is a real constant with dimension of a mass. The global symmetry group is

$$G = O(3), \tag{2.16}$$

*A sombrero is a big Mexican hat with a very broad curved rim; you would recognize the potential as a sombrero if you plotted it in four dimensions with three axes for the $\vec{\varphi}$ and the fourth for V.

like the symmetry of ordinary space. There are three symmetry axes, i.e., generators, so

$$\dim G = 3 \tag{2.17}$$

(in general for $O(N)$ the dimension in $N(N-1)/2$). Since $\mathcal{L} \propto -V$ we see comparing eqs. (2.15) and (2.13) that our scalars are tachyons,

$$m^2_{\vec{\varphi}} = -4\lambda v^2 \tag{2.18}$$

However (2.18) is not a true description of the spectrum because we have not identified the ground state of the system. Equation (2.18) is expressed relative to the state $\vec{\varphi} = \vec{0}$, but we see that (2.15) has its ground state (in lowest order) at

$$\vec{\varphi}^2 = v^2. \tag{2.19}$$

The classical ground state breaks the $O(3)$ symmetry, since one component of $\vec{\varphi}$ is singled out to be nonvanishing. We define the axes so that the special component is φ_3, and the classical ground state is given by

$$\varphi_3 = v \tag{2.20a}$$

$$\varphi_1 = \varphi_2 = 0. \tag{2.20b}$$

The ground state settles (spontaneously) on one of the infinity of possible equivalent directions. The fact that it could have equivalently picked any other direction means that the potential is locally flat under rotations that would carry φ_3 into a different direction, i.e., that there are massless modes associated with the axes (generators) of those rotations. These latter are precisely the broken generators, which are no longer symmetries in the ground state. Goldstone's theorem then follows.

For our hypersombrero the remaining symmetry is

$$H = O(2) \tag{2.21}$$

the rotations about the $\hat{n}_3$ axis, so

$$\text{dimension } H = 1 \tag{2.22}$$

and from (2.12) we expect $3-1=2$ massless particles. We easily check this by redefining φ_3 to vanish in the ground state:

$$\varphi_3 \to \varphi_3 + v. \tag{2.23}$$

In terms of the new field with $\varphi_3 = 0$ the potential V is

$$V(\varphi) = \lambda(\vec{\varphi}^2)^2 + 2\lambda v\varphi_3\vec{\varphi}^2 + 4\lambda v^2\varphi_3^2. \tag{2.24}$$

Notice that (2.24) clearly lacks the full $O(3)$ symmetry because of the last two terms but is only invariant under the $O(2)$ rotations that mix up φ_1 and φ_2. Notice also the absence of mass terms for φ_1 and φ_2, so that $m_1 = m_2 = 0$ as expected. Finally notice that φ_3 has a mass term with the correct sign (in contrast to the tachyonic masses in (2.15)), given by

$$m_3^2 = 8\lambda v^2 \tag{2.25}$$

PLEASE DO NOT BE DECEIVED by the previous example however. The essential features are the symmetries of the lagrangian (G) and the ground state (H). Elementary scalars are *not* essential: if it is necessary to make J. Goldstone happy, Nature makes composite scalars. She has (almost) already done so on at least one occasion. That is, we believe on the basis of strong theoretical and experimental evidence that QCD with two massless quarks is an example of Her cooperation in this regard. The initial global (flavor) symmetry is

$$G = SU(2)_L \times SU(2)_R \tag{2.26}$$

since we can perform separate isospin rotations on the right and left chirality u and d quarks. The ground state is believed to have a non-vanishing expectation value for the bilinear operator

$$\langle \bar{u}_L u_R + \bar{d}_L d_R + h.c.\rangle_0 \neq 0 \tag{2.27}$$

where $h.c.$ = hermitean conjugate. Equation (2.27) breaks the global symmetry spontaneously, $G \to H$, where

$$H = SU(2)_{L+R} \tag{2.28}$$

is the ordinary isospin group of nuclear and hadron physics. That is, (2.27) is not invariant under independent rotations of left and right helicity quarks but only under rotations that act equally on left and right helicities. In this example,
dim $G = 6$ and dim $H = 3$ so we expect $6-3 = 3$ Goldstone bosons. In nature we believe they are the pion triplet, π^+, π^-, π^0, which are much lighter than typical hadrons because the u and d quark masses are very small,[7] of order 10 MeV. (I refer to the "current" quark masses, the parameters that appear in the QCD lagrangian.)

Step II:
In the first step we considered only the global symmetry breakdown induced by $\mathcal{L}_{SB}$ — Goldstone's territory. Now we consider the interplay of $\mathcal{L}_{SB}$ with $\mathcal{L}_{\text{gauge}}$. The essential point of the Higgs mechanism is that when a spontaneously broken generator of $\mathcal{L}_{SB}$ coincides with a generator of a gauge invariance of $\mathcal{L}_{\text{gauge}}$, the associate Goldstone boson w and massless gauge boson W mix to form a massive gauge boson. The number of degrees of freedom are preserved, since the Goldstone boson disappears from the physical spectrum while the gauge boson acquires a third (longitudinal) polarization state. We will see how this occurs in general, without assuming the existence of elementary scalar particles.

By assumption the Goldstone boson w couples to one of the gauge currents, with a coupling strength f that has the dimension of a mass,

$$\langle 0|J^{\mu}_{\text{gauge}}|w(p)\rangle = \frac{i}{2} f p^{\mu} \tag{2.29}$$

f is analogous to F_π, the pion decay constant. Equation (2.29) means that an effective representation of the current contains a term linear in w,

$$J^{\mu}_{\text{gauge}}(x) = \frac{1}{2} f \partial^{\mu} w(x) + \cdots \tag{2.30}$$

In the lagrangian J^{μ}_{gauge} is by definition coupled to the gauge boson W^{μ},

$$\mathcal{L}_{\text{gauge}} = g W_{\mu} J^{\mu}_{\text{gauge}} + \cdots \tag{2.31}$$

where g is the dimensionless gauge coupling constant. Substituting eq. (2.30) we find

$$\mathcal{L}_{\text{gauge}} = \frac{1}{2} g f W_\mu (\partial^\mu w) \ldots \tag{2.32}$$

which shows that W_μ mixes in the longitudinal (parallel to $\vec{p}$) direction with the would–be Goldstone boson w.

We can use (2.32) to compute the W mass. Before symmetry breaking the W is massless and transversely polarized. Therefore as in QED we can write its propagator in Landau gauge,

$$D_0^{\mu\nu} = \frac{-i}{k^2}(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}) \tag{2.33}$$

In higher orders the propagator is the sum of the geometric series due to "vacuum polarization", i.e., all states that mix with the gauge current. The vacuum polarization tensor is defined as

$$\begin{aligned} \Pi^{\mu\nu}(k) &= -\int d^4k e^{-ik\cdot x} \langle T J^\mu(x) J^\nu(0) \rangle_0 \\ &= i\frac{g^2 f^2}{4}(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}) + \cdot \end{aligned} \tag{2.34}$$

In (2.34) I have indicated explicitly the contribution from the Goldstone boson pole: the factor $1/k^2$ is just the massless propagator and the factor $(gf/2)^2$ can be recognized from eq. (2.32). The only subtle point is the $g^{\mu\nu}$ in (2.34). It is present since gauge invariance requires current conservation, $k_\mu \Pi^{\mu\nu} = 0$. Since it is a constant term with no absorptive part, its presence does not change the spectrum of the theory. (In theories with elementary scalars it arises automatically from the "seagull" interaction given by the Feynman rules.)

Finally we compute the W propagator from the geometric series (figure 2.1):

$$\begin{aligned} D^{\mu\nu} &= (D_0 + D_0 \Pi D_0 + \ldots)^{\mu\nu} \\ &= -\frac{i}{k^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\left(1 + \frac{g^2 f^2}{4k^2} + \cdots\right) \\ &= -i\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\frac{1}{k^2}\frac{1}{1 - \frac{g^2 f^2}{4k^2}} \end{aligned}$$

$$= -i\frac{g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}}{k^2 - \frac{g^2 f^2}{4}}. \tag{2.35}$$

The massless Goldstone boson pole induces a pole in the gauge boson propagator,

$$M_W = \frac{1}{2}gf. \tag{2.36}$$

From the measured value of the Fermi constant,

$$G_F = \frac{g^2}{4\sqrt{2}M_W^2} = \frac{1}{\sqrt{2}f^2} \tag{2.37}$$

we learn that

$$f \simeq 250GeV. \tag{2.38}$$

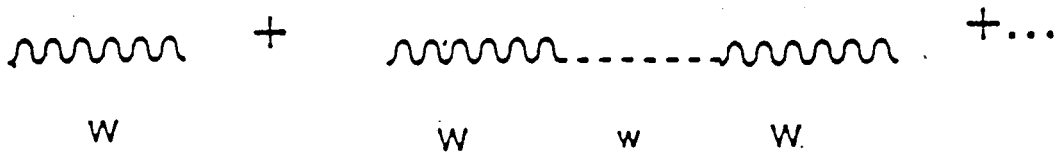

Figure 2.1: Geometric series for the W propagator corresponding to eq. (2.35).

Customarily instead of f we refer to $v \equiv f$, the so–called vacuum expectation value. This custom, which I will also follow (though it is in general not appropriate), derives from theories with elementary scalar fields (see Section 5) where $v \equiv f$ is both the coupling strength of the Goldstone boson w to J_{gauge}, as in (2.29), and is also the value of the Higgs boson field in the ground state as in (2.19). However the above derivation shows that there is no need for any physical Higgs scalar to exist. The condensate that breaks the symmetry may in general be of a composite operator, as in (2.27), and may have no simple relationship to the parameter $f \equiv v$ defined in (2.29). For instance, in QCD there is no trivial relationship between F_π and $\langle \bar{u}u + \bar{d}d \rangle_0$ (though there is a nontrivial relation involving also the quark and pion masses.[8)]

I will conclude this discussion of the Higgs mechanism with two more topics:

1. The significance of the ρ parameter for the global symmetries of $\mathcal{L}_{SB}$.

2. The equivalence theorem which allows us to connect the Goldstone boson dynamics of $\mathcal{L}_{SB}$ with the scattering of longitudinal gauge bosons in the laboratory.

First, what do we learn from the experimental observation that to within a few percent

$$\rho \equiv \left(\frac{M_W}{M_Z \cos\theta_W}\right)^2 = 1. \tag{2.39}$$

In deriving (2.36) I was careless with the T_{3L} indices and did not discuss the Z mass. More carefully, instead of (2.29) I should have written

$$\langle 0|J_a^\mu|w_b\rangle = ip^\mu \frac{f_a}{2}\delta_{ab} \tag{2.40}$$

where $a, b = 1, 2, 3$. Choosing

$$w^\pm = \frac{1}{\sqrt{2}}(w_1 \pm iw_2) \tag{2.41}$$

we see that $U(1)_{EM}$ rotates the 1 and 2 components into one another, so that $U(1)_{EM}$ invariance implies

$$f_1 = f_2. \tag{2.42}$$

What about f_3? Is there an analogy to the isospin symmetry of hadron physics that ensures $f_1 = f_2 = f_3$?

As in the derivation of (2.36) we find that

$$M_{W^\pm} = \frac{1}{2}gf_1 \tag{2.43}$$

but for the W_3 and X bosons (associated with T_{3L} and Y) we find with an analogous calculation the mass matrix

$$\begin{pmatrix} M^2_{W_3} & M^2_{W_3-X} \\ M^2_{W_3-X} & M^2_X \end{pmatrix} = \frac{1}{4}f_3^2 \begin{pmatrix} g^2 & gg' \\ gg' & g'^2 \end{pmatrix} \tag{2.44}$$

where g and g' are the $SU(2)_L$ and $U(1)_Y$ couplings. The diagonalized matrix is then (since it has zero determinant)

$$\frac{1}{4}f_3^2\begin{pmatrix} g^2+g'^2 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.45}$$

so that

$$M_Z = \frac{1}{4}f_3^2(g^2+g'^2) \tag{2.46}$$

$$M_\gamma = 0 \tag{2.47}$$

are the eigenvalues, the eigenstates being

$$Z = W_3\cos\theta_w + X\sin\theta_w \tag{2.48a}$$

$$A = -W_3\sin\theta_w + X\cos\theta_w. \tag{2.48b}$$

The mixing angle is

$$\cos^2\theta_w = \frac{g^2}{g^2+g'^2} \tag{2.49}$$

and the ρ parameter is then

$$\rho = (f_1/f_3)^2.. \tag{2.50}$$

Equation (2.50) teaches us that $\rho = 1$ is connected with the existence of an isospin-like symmetry in $\mathcal{L}_{SB}$. In particular if the global symmetry H of $\mathcal{L}_{SB}$ encompasses an $SU(2)$ under which $\vec{w}$ and J_a^μ are triplets, then it guarantees that $f_3 = f_1$ and that $\rho = 1$ to all orders in the (possibly strong) interactions of $\mathcal{L}_{SB}$. In this sense it functions as a "custodial" $SU(2)$ since it protects $\rho = 1$ against corrections from $\mathcal{L}_{SB}$.[9] Conversely, it can be shown that $\rho = 1$ implies that the low energy interactions of the Goldstone bosons $\vec{w}$ obey an effective custodial $SU(2)_{L+R}$ symmetry,[10] which need not however be an exact symmetry of $\mathcal{L}_{SB}$.

The custodial $SU(2)$ symmetry also underlies the upper bound on the top quark mass from one loop corrections to the ρ parameter[11] (or equivalently to a quantity called Δr in other renormalization schemes). The mass difference $m_t - m_b$ breaks the custodial isospin, resulting in

a correction to ρ proportional to $G_F m_t^2$ for $m_t >> M_{W'}$. Analyses[12)] of the experimentally allowed deviations from $\rho = 1$ suggest an upper bound of ~ 200 GeV for m_t.

Finally I will describe the equivalence theorem, which relates the Goldstone boson physics of $\mathcal{L}_{SB}$ to observations that can be made in the laboratory and therefore suggests an experimental strategy to study the physics of $\mathcal{L}_{SB}$. The complete electroweak lagrangian $\mathcal{L}$, eq. (2.1), is of course $SU(2)_L \times U(1)_Y$ gauge invariant, so that physics does not depend on the choice of gauge. In the U (unitary) gauge only physical degrees of freedom appear in $\mathcal{L}$ and, in particular, the Goldstone boson fields vanish, $\vec{w} = 0$. In R (renormalizable) gauges, of which there are an infinite number, the Goldstone fields $\vec{w}$ do appear in $\mathcal{L}$ and in the Feynman rules, though gauge invariance ensures that they do not appear in the physical spectrum (i.e., they never generate poles in S-Matrix elements). Since they engender the longitudinal gauge boson modes, W_L and Z_L, it is plausible that W_L and Z_L interactions reflect the dynamics of $\vec{w}$. The equivalence theorem is the precise statement of this proposition,

$$\mathcal{M}(W_L(p_1), W_L(P_2), \ldots) = \mathcal{M}(w(p_1), w(p_2), \ldots)_R + O\left(\frac{M_W}{E_i}\right). \quad (2.51)$$

As indicated the equality holds up to corrections of order M_W/E_i. In the generalized R_ξ gauge[19a)] the dependence of the gauge variant goldstone boson scattering amplitudes on the gauge parameter ξ is also of the order of the M_W/E_i corrections.

In addition to being essential to the derivation of the $W_L W_L$ low energy theorems equation (2.51) greatly simplifies perturbative calculations for heavy — and therefore strongly coupled — higgs systems. For instance, to correctly evaluate heavy higgs production and decay by WW fusion in unitary gauge require evaluation of many diagrams with "bad" high energy behavior that cancel to give the final result. But to leading order in the strong coupling $\lambda = m_h^2/2v^2$ it suffices using equation (2.51) to compute just a few simple diagrams using the interactions of the scalar potential, equation (5.9) below. The result embodies the

cancellations of many diagrams in unitary gauge and trivially has the correct high energy behavior. It is very accurate for energies above 1 TeV (of order 1 % or better).

As a simple example, consider the decay of a heavy higgs boson to a pair of longitudinally polarized gauge bosons $W_L^+ W_L^-$. In unitary gauge the $hW_L^+ W_L^-$ amplitude is

$$\mathcal{M}(H \to W_L^+ W_L^-) = g M_W \epsilon_L(p_1) \cdot \epsilon_L(p_2). \tag{2.52}$$

For $m_H >> M_W$ we neglect terms of order M_W/m_H, so that $\epsilon_L^\mu(p_i) \cong p_i/M_W$ and similarly from $m_H^2 = (p_1 + p_2)^2 \cong 2p_1 \cdot p_2$ we find

$$\mathcal{M}(H \to W_L^+ W_L^-) = g\frac{m_H^2}{2M_W} + O\left(\frac{M_W}{m_H}\right). \tag{2.53}$$

In a renormalizable gauge the corresponding amplitude can be read off (taking care with factors of 2) from the Hww vertex in the potential, equation (5.9),

$$\mathcal{M}(H \to w^+ w^-) = 2\lambda v. \tag{2.54}$$

Using equations (2.43) and (5.10) it is easy to see that equations (2.53) and (2.54) are indeed equal up to M_W/m_h corrections.

The theorem was established in tree approximation[13)] and used in a variety of calculations.[14–16)] Reference (15) sketches a proof to all orders which is not however easily extended to matrix elements with more than one external W_L. A proof to all orders in both $\mathcal{L}_{SB}$ and $\mathcal{L}_{\text{gauge}}$ is given in reference (3), and alternative treatments have been given for the portion of the proof of reference (3) that is based on the BRS identities.[17)] The suggestion has been made that the theorem may fail at higher orders, though not confirmed by an explicit calculation to one loop,[18)] or that it may fail at higher orders in $\mathcal{L}_{\text{gauge}}$.[19)] My own view is, coincidentally, that of reference (3): that the theorem is valid to all orders in all interactions when the Goldstone boson fields are appropriately renormalized.

The theorem (2.51) tells us that scattering of longitudinal gauge bosons at high energy reflects the dynamics of the underlying Goldstone

bosons. We will use this connection in the next section to learn more about the general properties of $\mathcal{L}_{SB}$.

3. SYMMETRY AND UNITARITY

In this section we continue to extract the general properties of the Higgs mechanism. We will use the general symmetry properties of $\mathcal{L}_{SB}$, eq. (2.10), and unitarity. The symmetry properties imply low energy theorems for $W_L W_L$ scattering[3,10] that correlate the unknown mass and interaction scales of $\mathcal{L}_{SB}$, (2.5) and (2.6), and allow us to estimate the scattering amplitudes if $\mathcal{L}_{SB}$ is strongly interacting. The low energy theorems are determined by the pattern of symmetry breaking and by two (known!) parameters, the vacuum expectation value and the ρ parameter. The low energy theorems together with unitarity then imply an upper limit on the energy scale at which the physics of $\mathcal{L}_{SB}$ must become visible and probably also an upper limit on the unknown mass scale M_{SB}. Experimental implications of these results will be discussed in Section 6.

Begin by considering $\mathcal{L}_{SB}$ in the absence of $\mathcal{L}_{\text{gauge}}$. The spontaneous symmetry breaking pattern $G \to H$ is sufficient to derive low energy theorems for Goldstone boson scattering in terms of the constants f_a that characterize the couplings of the Goldstone bosons to the symmetry currents. The earliest example is the Weinberg $\pi\pi$ low energy theorems.[20] Assuming the pion isotriplet to be the almost-Goldstone bosons associated with $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$ in hadron physics, Weinberg showed for example that

$$\mathcal{M}(\pi^+\pi^- \to \pi^0\pi^0) = \frac{s}{F_\pi^2} \tag{3.1}$$

where $F_\pi = 93$ MeV is the pion decay constant. Equation (3.1) neglects $O(m_\pi^2)$ corrections (which are in fact calculable) and is valid for low energy, defined as

$$s << \text{minimum}\{m_\rho^2, (4\pi F_\pi)^2\}. \tag{3.2}$$

The low energy theorems can be derived by current algebra or effective lagrangian methods. The proofs have two important features:

- they are valid to all orders in the Goldstone boson self–interactions. This is crucial since those interactions may be strong (as they are for the pion example) so perturbation theory is a non-starter.
- We needn't be able to solve the dynamics or even to know the lagrangian of the theory. In fact the $\pi\pi$ low energy theorems were derived in 1966 before QCD was discovered. (And we still don't know today how to compute $\pi\pi$ scattering directly in QCD.)

The current algebra/symmetry method was important in the path followed in the 1960's that led in the early 1970's to the discovery that $\mathcal{L}_{HADRON} = \mathcal{L}_{QCD}$. What can it teach us about $\mathcal{L}_{SB}$?

If $G = SU(2)_L \times SU(2)_R$ and $H = SU(2)_{L+R}$ as in QCD, then we can immediately conclude that[3)]

$$\mathcal{M}(w^+w^- \to zz) = \frac{s}{v^2} \tag{3.3}$$

at low energy,

$$s << \text{minimum}\{M_{SB}^2, (4\pi v)^2\}, \tag{3.4}$$

as in eq. (3.2). Here M_{SB} is the typical mass scale of $\mathcal{L}_{SB}$ and $v \simeq \frac{1}{4}$ TeV, eqs. (2.37–8). More generally, electroweak gauge invariance requires eq. (2.10) from which we can deduce the more general result[10)]

$$\mathcal{M}(w^+w^- \to zz) = \frac{1}{\rho}\frac{s}{v^2}. \tag{3.5}$$

Equation (3.5) is arguably more soundly based than (3.1) was in 1966, since (3.5) is a general consequence of gauge invariance and the Higgs mechanism while (3.1) was based on inspired guesswork as to the symmetries underlying hadron physics. The low energy theorems are proved by three different methods in reference 10: a perturbative analysis, a current algebra derivation similar to Weinberg's, and by the chiral lagrangian method.

We can next use the equivalence theorem, (2.51), to turn (3.5) into a physical statement about longitudinal gauge boson scattering. In particular we have

$$\mathcal{M}(W_L^+W_L^- \to Z_LZ_L) = \frac{1}{\rho}\frac{s}{v^2} \tag{3.6}$$

for an energy domain circumscribed by (3.4) and (2.51) as

$$M_W^2 << s << \text{minimum}\{M_{SB}^2, (4\pi v)^2\}. \tag{3.7}$$

The window (3.7) may or may not exist in nature, depending on whether $M_{SB} >> M_W$.

It is amusing that the low energy theorem (3.6) is precisely the famous "bad" high energy behavior that the Higgs mechanism is needed to cure — this emerges most clearly in the derivation of (3.6) given in reference 21. $\mathcal{L}_{SB}$ must cut off the growing amplitude in (3.6). Unitarity implies a rigorous upper bound on the energy at which this must occur. The use of unitarity here is identical to that of Lee and Yang[22)] and of Ioffe, Okun, and Rudik[22)] who used the growing behavior of fermion-fermion scattering in Fermi's four-fermion weak interaction lagrangian (also proportional to $G_F s \propto s/v^2$!) to bound the scale at which Fermi's theory must break down — essentially a bound on the mass of the W boson. In fact that bound is precisely half the value of the bound we obtain below for the scale of the symmetry breaking physics.

In particular we use partial wave unitarity. The partial wave amplitudes for the Goldstone scalars (or for the zero helicity, longitudinal gauge bosons) are

$$a_J(s) = \frac{1}{32\pi} \int d(\cos\theta) P_J(\cos\theta) \mathcal{M}(s,\theta) \tag{3.8}$$

where θ is the center of mass scattering angle. Partial wave unitarily then requires

$$|a_J(s)| \leq 1. \tag{3.9}$$

Putting $\rho = 1$, eqs. (3.6-3.9) then imply

$$a_0(W_L^+ W_L^- \to Z_L Z_L) = \frac{s}{16\pi v^2} \leq 1 \tag{3.10}$$

so that the amplitude must be damped at a scale bounded by

$$\Lambda_{\text{Cutoff}} \leq 4\sqrt{\pi} v \simeq 1.75\ TeV. \tag{3.11}$$

That is, new physics from $\mathcal{L}_{SB}$ must effect the scattering at an energy scale bounded by (3.11).

At the cutoff, $s \cong O(\Lambda)$, the $J = 0$ wave is

$$a_0(\Lambda) \cong \frac{\Lambda^2}{16\pi v^2} \tag{3.12}$$

which implies the promised correlation between the strength of the interaction and the energy scale of the new physics. If $\Lambda \lesssim \frac{1}{2}$ TeV then $a_0(\Lambda) \lesssim 1/4\pi$, well below the unitarity limit; then $\mathcal{L}_{SB}$ has a weak coupling and can be analyzed perturbatively. For $\Lambda \gtrsim 1$ TeV, we have $a_0(\Lambda) \gtrsim 1/3$, which is close to saturation; this means $\mathcal{L}_{SB}$ is a strong interaction theory requiring nonperturbative methods of analysis.

Though it is not rigorous, the most likely case is that $\Lambda_{\text{Cutoff}} = \Lambda_{SB}$ is of order the typical mass scale M_{SB} of the quanta of $\mathcal{L}_{SB}$,

$$\Lambda_{SB} \cong M_{SB}. \tag{3.13}$$

I can't prove (3.12) but can illustrate it with two examples. The first is the Weinberg–Salam model, in which s–channel Higgs exchange provides the contribution that cuts off (3.10). I assume that $m_H >> M_W$ but that m_H is small enough that perturbation theory is not too bad — say $m_H \simeq 700$ GeV so that $\lambda/4\pi^2 = m_H^2/8\pi v^2 \simeq 1/10$ (see section 5 below). Then I can calculate in tree approximation, with the result

$$a_0(s) = \frac{s}{16\pi v^2} - \frac{s}{16\pi v^2}\,\frac{s}{s - m_H^2} \tag{3.14}$$

where the first term arises from $\mathcal{L}_{\text{gauge}}$ and the second from the s-channel Higgs boson exchange given by $\mathcal{L}_{SB}$ now assumed to be the Weinberg-Salam Higgs sector (see figure (3.1)).

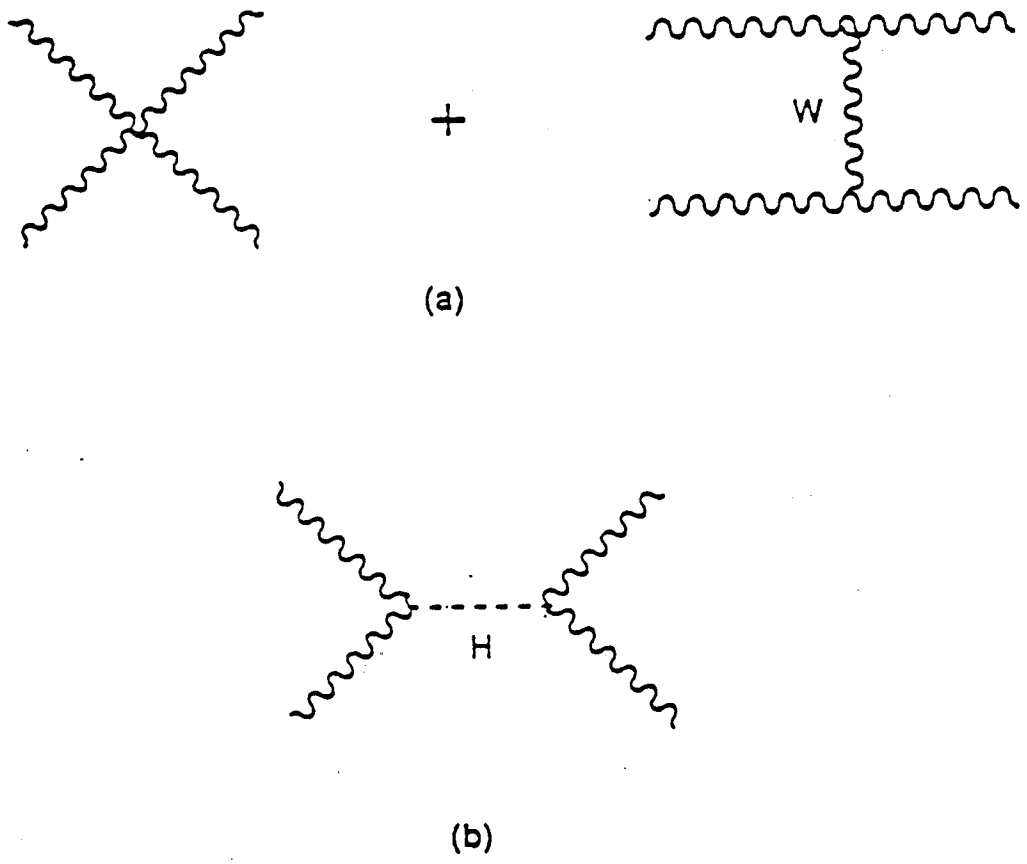

Figure 3.1: Leading diagrams for $W^+W^- \to ZZ$, including interactions from the gauge sector (a) and the s–channel Higgs boson exchange (b) — see eq. (3.14).

For $s << m_H^2$ the first term dominates, giving the low energy theorem as it must. But for $s >> m_H^2$ the two terms combine to give

$$a_0\Big|_{s >> m_H^2} = \frac{m_H^2}{16\pi v^2}. \tag{3.15}$$

Comparing (3.15) with (3.12) we see that (3.13) is indeed verified, i.e., $\Lambda \cong m_H$.

Consider next a strongly-coupled example. In this case we expect to approximately saturate the unitarity bound,

$$\Lambda_{\text{Strong}} \cong 4\sqrt{\pi} v \cong O(2) TeV. \tag{3.16}$$

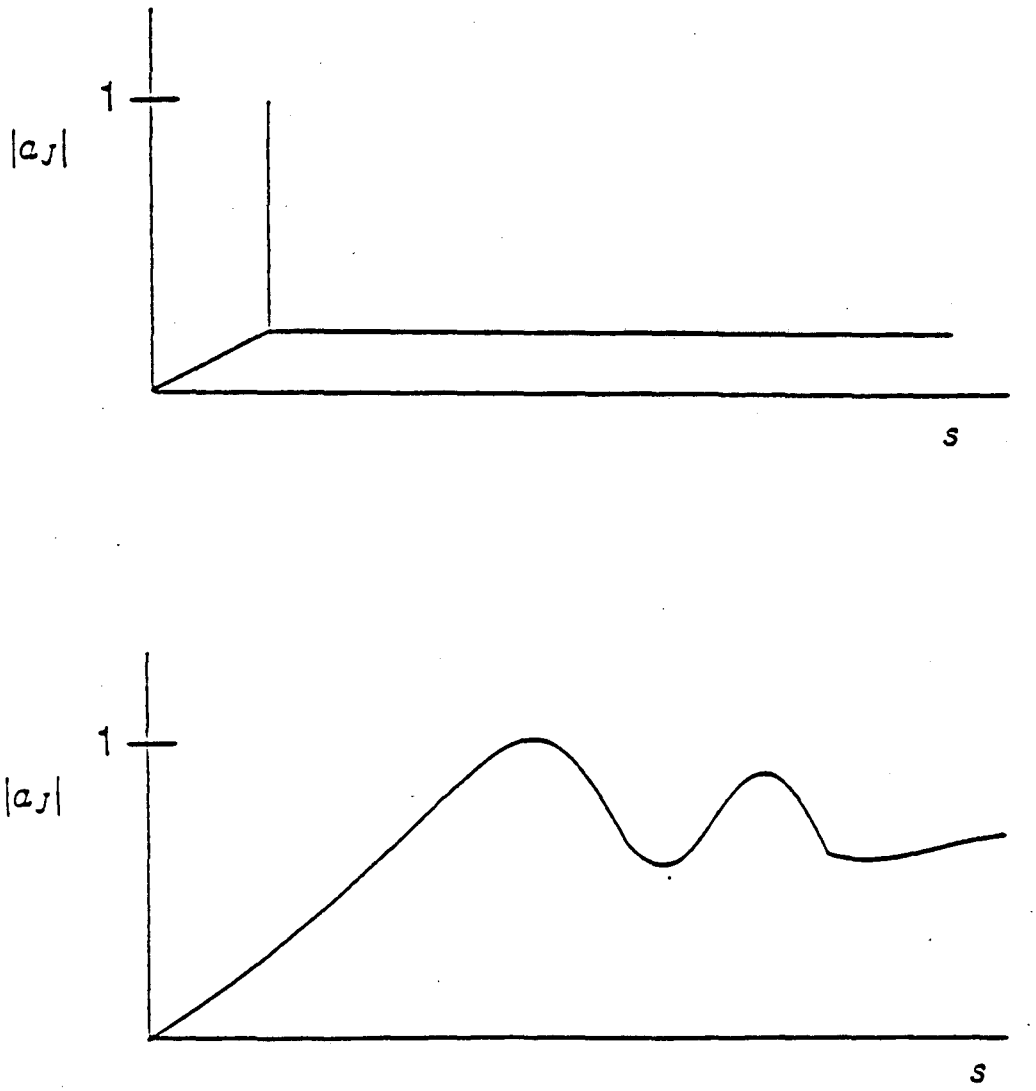

Figure 3.2: Typical behavior of partial wave amplitudes for $W_L W_L$ scattering for a weakly coupled model with narrow (Higgs) resonances (left-hand figure) or a strongly coupled model with broad resonances in the 1-2 TeV region (right-hand figure).

I can't solve for M_{SB} in this case but I can relate the problem to one that has been studied experimentally. In hadron physics the saturation scale from (3.1) would be

$$\Lambda_{\text{Hadron}} \cong 4\sqrt{\pi} f_\pi \cong 650 MeV. \tag{3.17}$$

Experimentally we know this is indeed of the order of the mass of the lightest hadrons, e.g., $m_\rho = 770$ MeV. This is not surprising: in strong coupling theories we expect resonances to form when scattering amplitudes become strong, as they do at the energy scale of the unitarity bound.

So we expect $\Lambda \cong M_{SB}$ for weak or strong $\mathcal{L}_{SB}$. The two generic cases are shown in figure (3.2). For weak $\mathcal{L}_{SB}$ we expect narrow resonances below 1 TeV — these are just the Higgs bosons. For strong $\mathcal{L}_{SB}$ we expect broad resonances in the vicinity of 1 to 2 TeV and strong $W_L W_L$ scattering, both of which can be observed at an appropriate collider.

4. THE NATURALNESS PROBLEM

In this section I will review the so–called "technical naturalness problem" that afflicts models with elementary Higgs bosons because of their quadratic divergences. I will also review two possible solutions: supersymmetry and technicolor. Both eliminate the offending quadratic divergences — supersymmetry by guaranteeing their cancellation and technicolor by doing away with elementary scalars. Both solutions also require new physics at or below the TeV scale, where it can be found at the SSC. The natural scale for technicolor is $\sim O(2)$ TeV since it is a strongly coupled theory which saturates the unitarity bound, eq. (3.11). Supersymmetry must also appear at or below the TeV scale if it is indeed the explanation of the naturalness problem, since as the SUSY breaking scale grows beyond the TeV scale the problem begins to reappear.

There are two aspects of what is called the "naturalness" or "gauge hierarchy" problem. The first is the physical origin of the very small

numbers $M_W/M_{GUT} \cong 10^{-12}$ or $M_W/M_{Planck} \cong 10^{-17}$. The second is a technical problem that is specific to Higgs boson models: even if the gauge hierarchy problem has a natural solution in lowest order, the quadratic divergences associated with scalar fields induce one loop corrections that destroy the hierarchy. In ordinary Higgs boson models these corrections require an order by order fine tuning of the subtraction constants that seems physically unnatural. In this section I will discuss this technical naturalness problem.

Consider the standard Higgs boson model, to be reviewed in Section 5. The potential V contains a wrong-sign (tachyonic) mass term for $\vec{w}$ and h, given by the coefficient of $\frac{1}{2}(\vec{w}^2 + h^2)$ in equation (5.4) , equal to $-\lambda v^2$. Because of the tachyonic sign, the state of minimum energy has a condensate v, resulting in zero mass for the triplet $\vec{w}$ and a mass $+\sqrt{2\lambda v^2}$ for h. The one loop quantum correction (figure 4.1) is quadratically divergent,

$$\delta(\lambda v^2) = \frac{9\lambda}{2} \int \frac{d^4\ell}{(2\pi)^4} \frac{1}{\ell^2 + \lambda v^2}. \tag{4.1}$$

Though expressions like equation (4.1) are shocking to novices in field theory, they lose their shock value as the student masters (i.e., is brainwashed by) the renormalization program, which shows that finite predictions can be extracted at

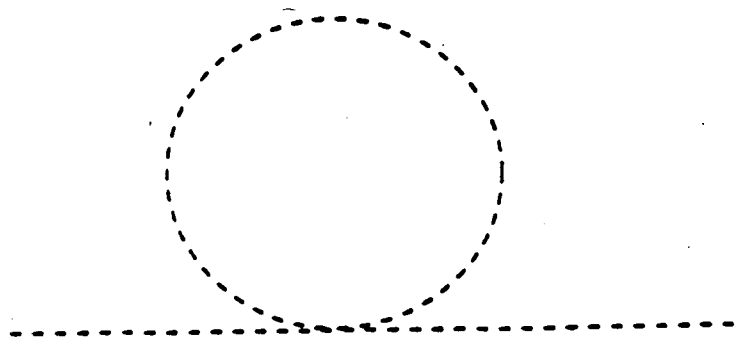

Figure 4.1: Quadratically divergent contribution to Higgs boson self–energy, as in eq. (4.1).

the cost of a small number of subtractions or redefinitions. Most no-

tably in the case of quantum electrodynamics this program has been extraordinarily successful. The divergence in equation (4.1) can be removed by introducing a counterterm that in effect shifts the initial value of λv^2 by an infinite constant cancelling the divergence generated in equation (4.1).

In the renormalization program we renounce any attempt to understand the physical origin of those parameters requiring subtraction — their values are simply fit to experiment — but we are then able to obtain finite predictions for all other physical quantities in the theory. To understand the naturalness problem it is necessary to go beyond this limited, though powerful, perspective and to ask questions about the origins of the subtracted quantities, assuming they will eventually be understood and calculable in the context of another theory formulated at a deeper level. The expectation is that the deeper theory introduces new physics at high energy that cuts off the divergent behavior of integrals like equation (4.1). Denoting the energy scale of the new physics by Λ, equation (4.1) would be replaced by

$$\delta(v^2\lambda) = C\frac{\lambda}{2\pi^2}\Lambda^2 \tag{4.2}$$

where C us a numerical constant of order unity.

Equation (4.2) tells us that the parameters of Higgs models are hypersensitive to the high energy scale of the deeper underlying theory. For example, the Higgs boson mass, given in lowest order by $m_H^2 = 2\lambda v^2$, might reasonably range from tens of MeV to perhaps the TeV scale. The scale Λ of the deeper theory might be the scale of Grand Unified Theories, $M_{GUT} = O(10^{14})$ GeV, or even the Planck scale suggested by superstring and supergravity models, $M_{\text{Planck}} = O(10^{19})$ GeV. Writing the physical mass as the sum of a bare mass plus the one loop corrections

$$m_H^2 = m_{H,\text{bare}}^2 + \frac{C\lambda}{\pi^2}\Lambda^2 \tag{4.3}$$

we see that the bare mass must be tuned with exquisite precision to make the left side much smaller than the two terms on the right side. For instance, if $m_H = 1$ TeV and $\Lambda = M_{\text{Planck}}$ then the cancellation

on the right side must work to one part in 10^{17}! Of course the renormalization program allows us to arrange the cancellation to any desired precision, but viewed from the perspective of the deeper theory such a cancellation seems extremely unnatural — one might even say, in the absence of any principle requiring or explaining such a cancellation, that it is absurdly implausible.

Though the term is also used in other ways, this is the naturalness problem that uniquely afflicts Higgs boson models. It may be thought of us as an instability of the energy scale of the theory against quantum corrections that tend naturally to drive the scale to violently larger values. The problem uniquely affects Higgs models because in 3 + 1 dimensions the only renormalizable theories with quadratic divergences are those containing scalar fields. For instance in unbroken gauge theories like QED or QCD divergences are at most given by powers of logarithms. If instead of the quadratic dependence on Λ in equation (4.3) there were a logarithmic dependence,

$$m_H^2 = m_{H,\text{bare}}^2 + \frac{C\lambda}{\pi^2} m_{H,\text{bare}}^2 \ln \frac{\Lambda}{m_{H,\text{bare}}} \tag{4.4}$$

then no fine tuning would be needed even for Λ as large as M_{Planck}.

Two strategies have been proposed to deal with the naturalness problem. One is to suppose that the symmetry breaking sector, $\mathcal{L}_{SB}$, does not contain elementary Higgs bosons. In particular, in technicolor models[23] $\mathcal{L}_{SB}$ is presumed to be a confining gauge theory like QCD at a mass scale roughly $v/F_\pi \sim 2700$ times greater than the GeV mass scale of QCD. Since QCD is known to undergo spontaneous symmetry breaking, with $SU(2)_L \times SU(2)_R$ breaking to $SU(2)_{L+R}$, giving rise to three Goldstone bosons (the pions), it is plausible that a similar theory at a higher mass scale would contain the necessary ingredients for electroweak symmetry breaking.

The second strategy is to provide a principle for the cancellation of the quadratic divergences: supersymmetry.[24] In supersymmetric theories the quadratic divergences due to scalar boson loops are precisely cancelled by fermion loop contributions. The remaining finite differ-

ence is proportional to the scale of supersymmetry breaking e.g., the mass differences of the scalar and fermion superpartners. The absence of scalars degenerate with the known leptons and quarks tells us supersymmetry cannot be exact. Naturalness then implies an upper limit on the scale of supersymmetry breaking, since the naturalness problem returns if mass differences of fermion-boson superpartners are too large. To avoid fine-tuning at less than the few percent level, superpartners cannot be heavier than a few TeV.

Supersymmetry and technicolor are discussed in the next section. It is however important to recognize that nature may have found a way to solve the naturalness problem that has not yet occurred to us.

5. MODELS

In this section I will review three specific models of $\mathcal{L}_{SB}$, concentrating on how they illustrate the general features discussed in Sections 2 and 3. The models are

- the Weinberg–Salam model
- the minimal supersymmetric extension of the standard model
- techlonicolor

5.1 The Weinberg–Salam Higgs Sector

The Weinberg–Salam model is a minimal model in that it has the smallest number of fields needed to break the gauge symmetry from $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$. Four spin zero quanta are introduced, in a complex doublet of $SU(2)_L$:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} w_1 & + & iw_2 \\ H & + & iw_3 \end{pmatrix} \tag{5.1}$$

The lagrangian is

$$\mathcal{L}_{SB} = |\mathcal{D}_\mu \Phi|^2 - V(\Phi) \tag{5.2}$$

where $\mathcal{D}$ is the gauge covariant derivative,

$$\mathcal{D}_\mu = \partial_\mu - ig\vec{T}_L \cdot \vec{W}_\mu - ig'YX. \tag{5.3}$$

The scalar self–couplings are just like the $O(3)$ model discussed in Section 2 (in fact the Weinberg–Salam model is just the extension to O(4)):

$$\begin{aligned} V &= \lambda(\Phi^+\Phi - \frac{v^2}{2})^2 \\ &= \frac{\lambda}{4}(H^2 + \vec{w}^2 - v^2)^2. \end{aligned} \tag{5.4}$$

The global symmetry group of (5.4) is

$$G = SU(2)_L \times SU(2)_R \tag{5.5}$$

(or equivalently $O(4)$). Defining $\vec{T}_L$ and $\vec{T}_R$ in terms of vector and axial–vector $SU(2)$ generators,

$$\vec{T}_{L,R} = \vec{V} \mp \vec{A} \tag{5.6}$$

the infinitesimal $SU(2)_L \times SU(2)_R$ rotations act on the fields as follows:

$$\delta_V(H, \vec{w}) = (0, \vec{\epsilon}_V \times \vec{w}) \tag{5.7a}$$

$$\delta_A(H, \vec{w}) = (\vec{\epsilon}_A \cdot \vec{w}, -\vec{\epsilon}_A H), \tag{5.7b}$$

i.e., as if $\vec{w}$ were a pseudoscalar and H a scalar.

As reviewed in Section 2 for the $O(3)$ model, the minimum energy configuration chooses a field condensate which we define to be H,

$$\langle H \rangle_0 = v. \tag{5.8}$$

Taking $H \to H + v$ the potential becomes

$$V = \frac{\lambda}{4}(H^2 + \vec{w}^2)^2 + \lambda v H(H^2 + \vec{w}^2) + \lambda v^2 H^2 \tag{5.9}$$

so that

$$m_H^2 = 2\lambda v^2 \tag{5.10}$$

$$m_{\vec{w}} = 0 \tag{5.11}$$

Inspection of (5.9) reveals that the global symmetry has broken spontaneously from

$$\begin{aligned} G = SU(2)_L \times SU(2)_R \to H_{\text{The Group}} &= SU(2)_{L+R} \\ &= SU(2)_V \end{aligned} \tag{5.12}$$

(or equivalently $O(4) \to O(3)$). There are then $6 - 3 = 3$ Goldstone bosons, the $\vec{w}$ triplet, which become the longitudinal gauge boson modes as in Section 2. The only remaining quantum in $\mathcal{L}_{SB}$ is then the scalar H.

Notice that the symmetry structure $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$ is identical to the symmetry of QCD with two massless quarks, eqs. (2.26) and (2.28). In fact $V(\Phi)$ as given in (5.4) is identical to the sigma model[24a)] with the substitutions $H \to \sigma$, $\vec{w} \to \vec{\pi}$, and $v \to F_\pi$. The sigma model was developed to model the low energy symmetries of hadron physics and played an important part in the history of the 1960's that led to the discovery and understanding of the underlying quark structure of hadrons. It is amusing that the Weinberg–Salam model could play a similar role in the effort to final $\mathcal{L}_{SB}$. In the sigma model the surviving $SU(2)_{L+R}$ symmetry is just the ordinary isospin of hadron physics. In the Weinberg–Salam model it is the custodial $SU(2)$ discussed in Section 2 that protects the ρ parameter against $O(\lambda)$ corrections.

The $|\mathcal{D}\Phi|^2$ term in (5.2) contains a contribution

$$\frac{1}{2} g v \vec{W}_\mu \cdot \partial^\mu \vec{w} \tag{5.13}$$

which is equivalent to eqs. (2.31-2.32) with $f = v$. That is, the gauge current contains a term $\frac{1}{2} g v \partial^\mu \vec{w}$. We therefore see immediately from the discussion in Section 2 that the mixing of $\vec{w}$ with $\vec{W}^\mu$ results in a gauge boson mass

$$M_W = \frac{1}{2} g v. \tag{5.14}$$

A more familiar though less general derivation is by inspection of the term quadratic in $\vec{W}^\mu$ that is contained in $|\mathcal{D}W|^2$, i.e.,

$$\frac{1}{2} \left(\frac{gv}{2}\right)^2 \vec{W}_\mu \cdot \vec{W}^\mu \tag{5.15}$$

from which (5.14) may be read directly.

Taking $\lambda/4\pi^2$ as the quantity characterizing perturbative correc-

tions, we find from (5.10) that

$$\frac{\lambda}{4\pi^2} = \frac{m_H^2}{8\pi v^2} \cong \left(\frac{m_H}{1TeV}\right)^2 \tag{5.16}$$

which shows that strong coupling sets in at roughly $m_H \geq 1$ TeV. This estimate agrees with the general analysis of section 3, as discussed following eq. (3.12), where we identify m_H with the cutoff Λ, as shown in eqs. (3.14-3.15).

The Higgs boson decay width in lowest order is

$$\begin{aligned}\Gamma(H \to WW + ZZ) &= \frac{3\sqrt{2}}{32\pi} G_F m_H^3 \\ &\cong \frac{1}{2} TeV \cdot \left(\frac{m_H}{1\ TeV}\right)^3 .\end{aligned} \tag{5.17}$$

For $m_H \gtrsim 1$ TeV the width is so big that there is no discernible resonance peak. Since the theory is strongly coupled for such values of m_H, the spectrum need not correspond in a simple way to the degrees of freedom in the lagrangian. It is in fact widely believed (the buzz word is "triviality") that the theory is inconsistent for m_H near or above 1 TeV. This conclusion was based first on a simple renormalization group analysis[25] and is supported by lattice computations.[26]

A lower bound on m_H follows from requiring the $SU(2)_L \times U(1)_Y$ broken vacuum (with $\langle H \rangle_0 = v \neq 0$) to be the lowest energy configuration in the one loop effective potential. The result is[27]

$$m_H^2 \geq \frac{3g^2}{64\pi^2}\left[2M_W^2 + \frac{1}{\cos^2\theta_w} M_Z^2 - 4m_t^2\right] \tag{5.18}$$

assuming the top quark is the only fermion as heavy as M_W. For $m_t << M_W$ the bound is $m_H \geq 7$ GeV but for $m_t > 80$ GeV the bound disappears. New bounds are obtained for $m_t > 86$ GeV from the requirement that the vacuum be stable against large Higgs field fluctuations, i.e., that the coefficient of $H^4 \ln H$ in the effective potential be positive.[28] The value of the bound depends on the value of a cutoff representing new physics beyond the Weinberg-Salam model. Consider

for instance the possibility that $m_t > 120$ GeV. Then the renormalization goup analysis of Lindner, Sher and Zaglauer[28)] gives $m_H \gtrsim 50$ GeV for $\Lambda = 10^{15}$ GeV and $m_H \gtrsim 30$ GeV for $\Lambda = 10^3$ GeV.

Fermions acquire mass from a Yukawa interaction with the Higgs boson,

$$\mathcal{L}_{\text{Yukawa}} = y_f H \overline{\psi}_f \psi_f \tag{5.19}$$

where y_f is the dimensionless coupling constant. The fermion masses are then $m_f = yv$ so that the couplings are

$$y_f = \frac{m_f}{v} = g\frac{m_f}{2M_W}. \tag{5.20}$$

Except for the top quark the y_f are extremely small, which makes Higgs boson production cross sections extremely small as well.

This is not a satisfying description since all the mysteries of the quark and lepton spectrum are hidden in the y_f which are simply introduced by hand. In fact, fermion mass generation could prove much more difficult to understand than W and Z mass generation. Fermion and gauge boson masses could be due to different condensates rather than the single condensate of the Weinberg–Salam model. Unitarity allows very different scales. Upper bounds can be obtained using partial wave unitarity as in section 3. For a fermion of mass m_f the counterpart of the 1.75 TeV bound, eq. (3.11), is[28a)]

$$\Lambda \lesssim \frac{16\pi v^2}{\zeta m_f} \tag{5.21}$$

where $\zeta = 1$ for leptons and 3 for quarks. The right hand side of (5.21) is much larger than the TeV scale, ranging from $5{\cdot}10^6$ TeV for the electron to ~ 10 TeV for a 100 GeV top quark.

5.2 Supersymmetry

The only known solution to the naturalness problem (Section 4) that allows elementary Higgs bosons is supersymmetry — that is the principal reason to believe supersymmetric partners of the known particles might be found at or below the TeV scale. In order to give mass to

quarks and leptons of weak isospin $T_{3L} = \pm\frac{1}{2}$ the constraints of supersymmetry require a minimum of two complex doublet Higgs fields. In this section I will review the Higgs sector of the minimal supersymmetric extension of the standard model,[29] which has precisely two complex Higgs doublets,

$$\Phi_a = \frac{1}{\sqrt{2}}\begin{pmatrix} w_a^1 & + & iw_a^2 \\ H_a & + & iw_a^3 \end{pmatrix} \qquad a = 1, 2. \tag{5.22}$$

The scalar potential $V(\Phi_1, \Phi_2)$ has its minimum at

$$\langle H_a \rangle = v_a \tag{5.23}$$

The W mass is

$$M_W = \frac{1}{2} g \sqrt{v_1^2 + v_2^2} \tag{5.24}$$

so that

$$v_1^2 + v_2^2 = v^2 = (\sqrt{2} G_F)^{-1}. \tag{5.25}$$

We choose H_1 to couple to $T_{3L} = +\frac{1}{2}$ and H_2 to $T_{3L} = -\frac{1}{2}$ fermions.

The two complex doublets contain eight degrees of freedom, of which three become the longitudinal $W^\pm$ and Z modes. The remaining five particles include three "pseudoscalars", $H^\pm$ and P', which are orthogonal to the "eaten" combinations of $\vec{w}_1$ and $\vec{w}_2$, and the two Higgs scalars H_1 and H_2. In general the eigenstates are mixtures with mixing angle α,

$$\begin{pmatrix} H \\ H' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}. \tag{5.26}$$

In the Weinberg–Salam model, $\lambda_{SB} = \lambda$ is a free parameter so that the Higgs boson mass, $m_H^2 = 2\lambda v^2$, is also unconstrained. In the minimal supersymmetric model, the strength of the Higgs interactions is constrained (because the scalar potential arises from a "D–term") to be

$$\lambda = g^2 + g'^2 \tag{5.27}$$

where g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge coupling constants. This means that the model is a weakly coupled $\mathcal{L}_{SB}$ in the sense of

Section 3. It also means that Higgs boson masses are not completely arbitrary, but satisfy sum rules which in lowest order are

$$m_{H^\pm}^2 = m_P^2 + M_W^2 \tag{5.28}$$

$$m_{H,H'}^2 = \frac{1}{2}(m_P^2 + M_Z)^2 \pm \frac{1}{2}\sqrt{(m_P^2 + m_Z^2) - 4m_P^2 M_Z^2 \cos^2 2\beta} \tag{5.29}$$

where β is defined by the ratio of the vevs,

$$\tan\beta \equiv v_2/v_1. \tag{5.30}$$

We then see that

$$m_{H^\pm} > M_W \tag{5.31}$$

$$m_H < M_Z \tag{5.32}$$

$$m_{H'} > M_Z. \tag{5.33}$$

Equations (5.27–5.29) are not generally true for nonmininal supersymmetric models. In particular, models containing $SU(2)_L$ singlet Higgs fields can have arbitrary couplings λ. Because they mix with the doublet Higgs fields, all Higgs boson masses are then in general arbitrary.

The one loop corrections to the minimal model sum rules have been computed, both for the charged[30] (5.28) and neutral[31] (5.29) bosons. The corrections are typically small though they can be large for certain choices of the parameters.

The search for the lighter Higgs scalar H is similar to the search for the Weinberg–Salam Higgs boson below M_Z[32]. Searches for the Weinberg–Salam Higgs boson can be used to exclude regions of the supersymmetric model's parameter space, which can be characterized by the angles α, β or, equivalently, by the masses of the scalars $m_H, m_{H'}$.

The heavy scalar H' has highly suppressed couplings to $WW + ZZ$ and is therefore probably undetectable at the SSC. However at the SSC we will be able to search directly for the superparticles, especially the

squarks and gluinos which should be observable for masses as large as 1 TeV and perhaps even beyond.[33)]

Charged Higgs bosons are of course pair-produced in e^+e^- annihilation, for $\sqrt{s} > 2\sqrt{M_W^2 + m_P^2}$. Since m_P is an arbitrary parameter, we cannot say what energy might be necessary.

5.3 Technicolor

Technicolor is the other known solution to the "technical" naturalness problem. In the context of a grand unified theory the logarithmic variation of the technicolor coupling constant might also explain[34)] the "fundamental" naturalness problem, i.e., the origin of the electroweak : GUT or Planck hierarchy. Technicolor is a good example of a strongly interacting $\mathcal{L}_{SB}$ as defined in Section 3.

The basic idea is that the Goldstone bosons w and z of $\mathcal{L}_{SB}$ are bound states of an asymptotically free gauge theory with a confined spectrum at the TeV scale. The simplest example is an unbroken $SU(N_{TC})$ gauge theory which would resemble closely the familiar dynamics of QCD. For N_F massless techniquark flavors the global symmetry group is

$$G = SU(N_F)_L \times SU(N_F)_R. \tag{5.34}$$

As in QCD we expect the ground state to have a condensate

$$\left\langle \sum_{i=1}^{N_F} \overline{q}_L^i q_R^i + \overline{q}_R^i q_L^i \right\rangle_0 \neq 0 \tag{5.35}$$

which breaks G down to the diagonal, vector–like subgroup

$$H = SU(N_F)_{L+R}. \tag{5.36}$$

For $N_F \geq 2$, H includes a custodial $SU(2)_{L+R}$ symmetry so that $\rho = 1$ is protected against large corrections from strong technicolor interactions. Since there are $N_F^2 - 1$ broken $SU(N_F)_{L-R}$ generators, there are $N_F^2 - 1$ Goldstone bosons, $w^\pm, z, \{\phi_i\}$. The ϕ_i exist if $N_F > 2$; they acquire masses from the $SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$ gauge interactions and are referred to as pseudo–Goldstone bosons. Choosing the

"technicolor pion = w, z decay constant"

$$F_\pi^{TC} = v \cong \frac{1}{4}\ TeV \tag{5.37}$$

referred to as f in eqs. (2.29–2.38), we obtain the correct value of the W mass as shown in the general discussion of Section 2.

For $N_{TC} = 3$ the theory is precisely a rescaled version of QCD and we can reliably predict (up to small corrections due to the small masses of the QCD u and d quarks) the mass and width of the techni–rho vector meson:

$$m_{\rho_T} = \frac{v}{F_\pi} m_\rho = 2.04\ TeV \tag{5.38}$$

$$\Gamma_{\rho_T} = \frac{v}{F_\pi} \Gamma_\rho = 0.40\ TeV. \tag{5.39}$$

More generally (and less reliably) in the limit of large N_{TC} and large 3 (i.e., the large N limit assumed to be valid for QCD), we have

$$m_{\rho_T} = \sqrt{\frac{3}{N_{TC}}} \cdot 2\ TeV \tag{5.40}$$

$$\Gamma_{\rho_T} = \frac{3}{N_{TC}} \cdot 0.40\ TeV. \tag{5.41}$$

The techni–rho has a spectacular though small background free signal at the SSC, as discussed in the next section.

Technicolor has potential experimental problems, from possibly light pseudo–Goldstone bosons and from flavor–changing neutral currents. However it is far from dead.[35] Possible solutions are being actively studied, including composite models[36] and models with slowly running coupling constants and elevated mass scales.[37] The potential experimental problems and the theoretical repulsiveness of specific models both result from the effort to explain quark and lepton masses. If fermion masses arise by some other still unknown mechanism, technicolor (with two flavors) is an elegant mechanism for $SU(2)_L \times U(1)_Y$ breaking, with no experimental evidence presently against it.

6. OVERVIEW OF STRONG WW SCATTERING

In Section 3 we reviewed the low energy theorems for $W_L W_L$ scattering and showed that together with unitarity they require the dynamics of $\mathcal{L}_{SB}$ to affect the scattering at an energy scale $\Lambda_{SB} \lesssim 1.75$ TeV. The most probable mechanism is the exchange of particles from $\mathcal{L}_{SB}$, so that $\Lambda_{SB} \cong M_{SB}$, as shown in two examples in Section III. In general just above the cutoff scale the $J = 0$ partial wave amplitude for scattering of the longitudinal modes $W_L^+ W_L^- \to Z_L Z_L$ is

$$a_0(W_L^+ W_L^- \to Z_L Z_L) \cong \frac{\Lambda_{SB}^2}{16\pi v^2} \tag{6.1}$$

so that the scattering is strong if $\Lambda_{SB} > 1$ TeV and weak if $\Lambda_{SB} << 1$ TeV. In fact there are three independent reaction channels, which can be chosen as $a_{IJ} = a_{00}, a_{11}, a_{20}$ where I is the index of the custodial $SU(2)$ discussed in Section 3. In addition to (6.1) the complete list of $2 \to 2$ reactions is

$$W_L^+ W_L^- \to W_L^+ W_L^- \tag{6.2}$$

$$W_L^\pm Z_L \to W_L^\pm Z_L \tag{6.3}$$

$$W_L^+ W_L^+ \to W_L^+ W_L^+ \tag{6.4a}$$

$$W_L^- W_L^- \to W_L^- W_L^-. \tag{6.4b}$$

All these channels will exhibit strong scattering for $\sqrt{s} > 1$ TeV if $\Lambda_{SB} > 1$ TeV, and some will probably have s–channel resonances with masses M_{SB} of order Λ_{SB}.

Therefore by measuring the $W_L W_L$ scattering amplitudes at high energy, $\sqrt{s} > 1$ TeV, we will learn whether $\mathcal{L}_{SB}$ is a strongly or weakly interacting theory and whether the mass scale of its quanta is at the TeV scale or below. We will probably also begin to observe the quanta directly as resonance effects in some of the $2 \to 2$ channels. A general strategy to accomplish this is based on the $W_L W_L$ fusion reaction, figure (6.1), that can be studied at a pp or e^+e^- collider. The initial state W_L's are off-mass-shell and must rescatter to appear on-shell in the final state. The contribution from rescattering by $\mathcal{L}_{SB}$ is $O(g^2 \lambda_{SB})$

where g is the $SU(2)_L$ gauge coupling constant and λ_{SB} the generic interaction strength of $\mathcal{L}_{SB}$. The dominant background from $\bar{q}q \to WW$ is $O(g^2)$. Therefore WW fusion contributes an observable increment if and only if the rescattering is strong, i.e., if and only if $\lambda_{SB}/4\pi = O(1)$ or equivalently $\Lambda_{SB} \gtrsim 1$ TeV.

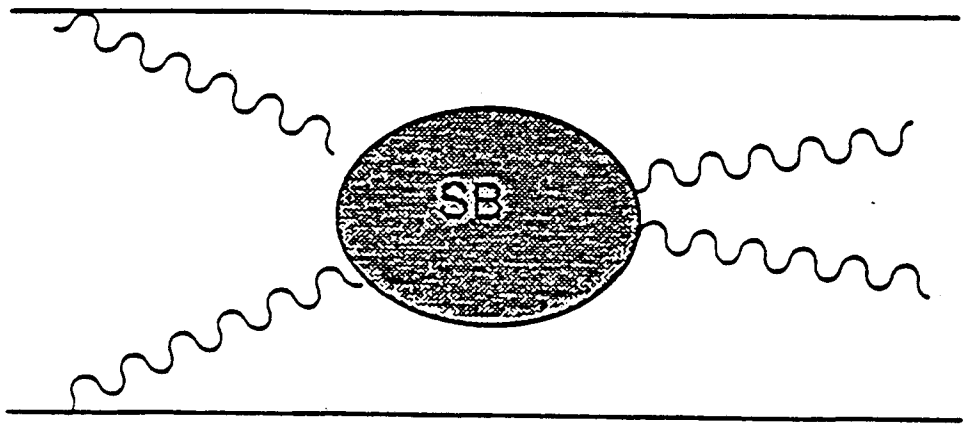

Figure 6.1: Generic $W_L W_L$ fusion via interactions of the symmetry breaking sector $\mathcal{L}_{SB}$.

Other backgrounds are $\mathcal{M}(gg \to W^+W^-, ZZ) \sim \alpha_S g^2$ via heavy quark loops[38] (e.g., top), WW bremsstrahlung with gluon exchange between the quarks,[39] $\sim \alpha_S g^2$, and WW fusion by $\mathcal{L}_{SU(2)\times U(1)}$ which is $\sim g^4$. These backgrounds are illustrated in figure (6.2). Though the backgrounds (except gg fusion) are dominated by transverse polarizations, polarization is not sufficient to separate them from the longitudinally polarized signal, though it can provide corroboration of a possible signal as discussed below.

The SSC is a minimal pp collider for this strategy. A collider of half the energy or less is not adequate, even with realistically likely higher luminosity. Because both the signal and the signal : background decrease at lower energy[40] and because the most important final states are inaccessible at high luminosity,[41] an upgrade in $\mathcal{L}$ of two to three orders of magnitude would be needed to offset a factor three loss in energy.[40]

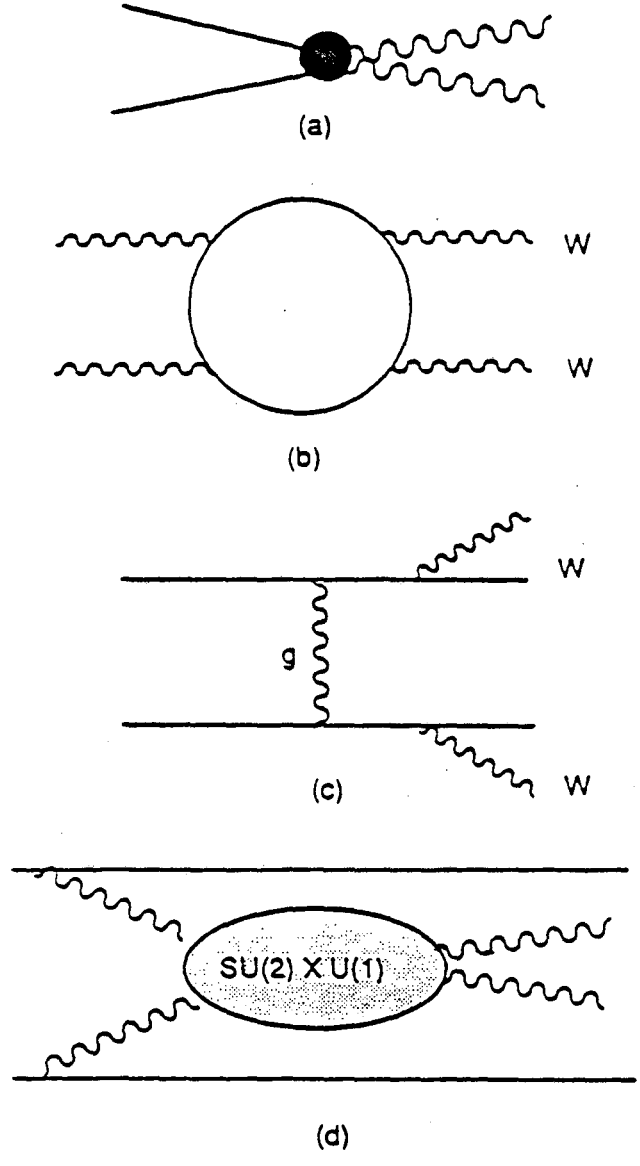

Figure 6.2: Backgrounds to $H \to WW$ signal from (a) $\overline{q}q \to WW$, (b) $gg \to WW$ via $\overline{Q}Q$ loops, (c) gluon exchange, and (d) higher order $O(g^4)$ electroweak interactions including WW fusion as shown.

An e^+e^- collider of $\sqrt{s} \cong 2-3$ TeV is probably minimal for the strong WW scattering signal,[42)] though more study is needed. See figure 6.3 for 1 TeV Higgs boson production cross sections at e^+e^- and pp colliders of various energies.[42a)]

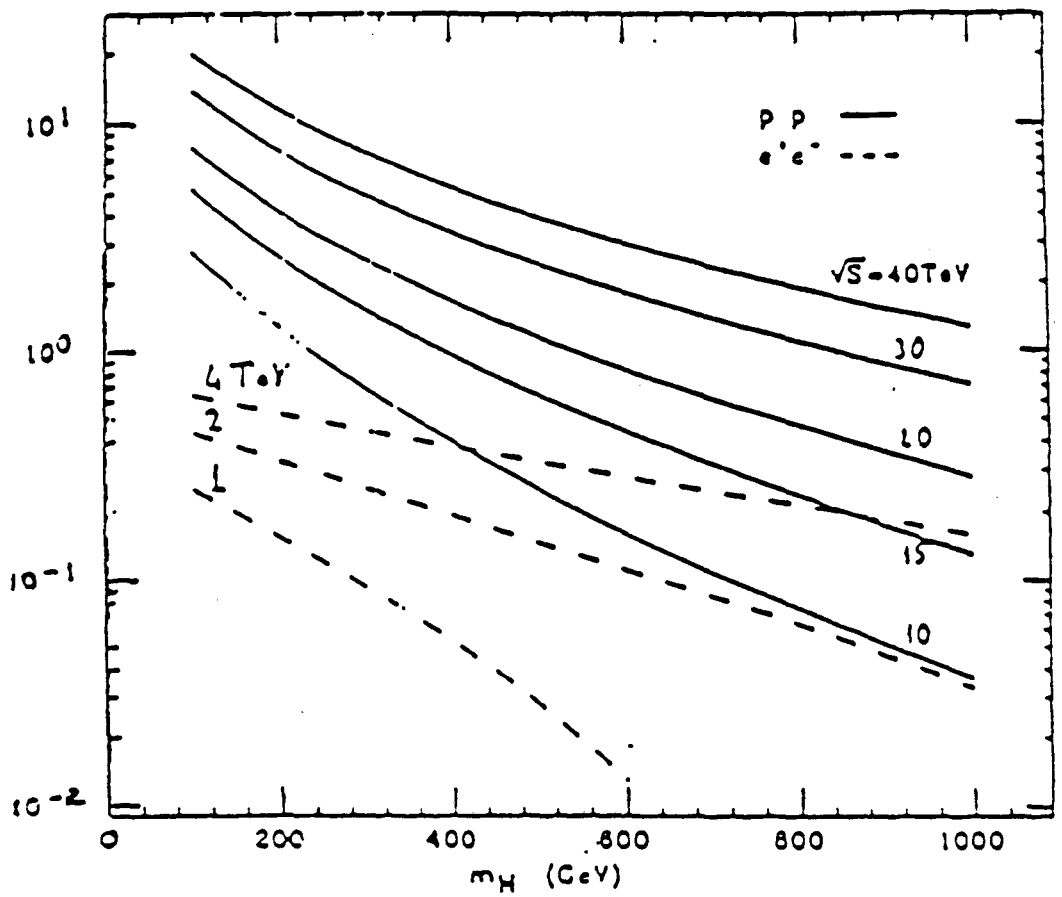

Figure 6.3: Higgs boson production cross sections in picobarns at e^+e and pp colliders with center of mass energies indicated (from ref. 42a).

In this section I consider three examples of signals for strong symmetry breaking:

1. The 1 TeV Weinberg-Salam Higgs boson
2. Strong W^+W^+ and W^-W^- scattering
3. Techni-rho production

I will consider purely leptonic final states, since they are experimentally cleanest. Larger yields will be possible if detection of $WW \to \ell v + \overline{q}q$ proves feasible.[43–45]

The method of calculation used below is the effective W approximation[52], analogous to the well known effective photon approximation of Weiszacker and Williams, which provides an effective luminosity distribution for the probablity to find colliding "beams" of longitudinally polarized gauge bosons within the colliding quark "beams" produced at the SSC. For longitudinally polarized gauge bosons V_1 and V_2 in incident fermions f_1 and f_2 the effective luminosity is

$$\left.\frac{\partial \mathcal{L}}{\partial z}\right|_{V_1V_2/f_1f_2} = \frac{\alpha^2\chi_1\chi_2}{\pi^2 sin^4\theta_W}\frac{1}{z}\left[(1+z)\ln\left(\frac{1}{z}\right) - 2 + 2z\right] \tag{6.5}$$

where $z \equiv s_{VV}/s_{ff}$ and the χ_i are the $f_i - V_i$ couplings, e.g., $\chi_W = 1/4$ for all fermions, $\chi_{Z_{u\bar{u}}} = (1 + (1 - \frac{8}{3}sin^2\theta_W)^2)/16\cos^2\theta_W$, etc ... Equation (6.5) must be convoluted with the desired V_1V_2 subprocess cross section and also with the quark distribution functions in the case of pp collisions,

$$\sigma(pp \to \cdots) = \int_\tau \left.\frac{\partial \mathcal{L}}{\partial \tau}\right|_{qq/pp} \cdot \int_x \sum_{V_L} \left.\frac{\partial \mathcal{L}}{\partial x}\right|_{V_LV_L/qq} \cdot \sigma(V_LV_L \to \cdots) \tag{6.6}$$

The effective W approximation has been compared with analytical[46] and numerical evaluations [46a] of higgs boson production. The most definitive results are probably the analytical calculations of reference 46. They show good agreement for $WW \to h$ for $m_h \geq 500$ GeV, with errors $\lesssim O(10\%)$ and decreasing with m_h and $\sqrt{s}$, while for the relatively less important process $ZZ \to h$ the errors are roughly twice as large. Above 1 TeV the errors are very small.

The signals for examples 1) and 2) are excesses of events with no discernible structure. To detect this excess reliably we must understand the background to $\pm 30\%$, a goal consistent with the level at which we can expect to understand the nucleon structure functions and perturbative QCD.[47] Realization of this goal requires an extensive program of "calibration" studies at the SSC, to measure a variety of jet, lepton, and gauge boson final states in order to tune the structure functions and confirm our understanding of the backgrounds.[48]

6.1 The 1 TeV Weinberg-Salam Higgs Boson

In the Weinberg–Salam model the generic figure 6.1 is replaced by s-channel Higgs boson exchange, figure 6.4. I consider the leptonic final state,

$$H \to ZZ \to e^+e^-/\mu^+\mu^- + e^+e^-/\mu^+\mu^-/\bar{\nu}\nu \tag{6.7}$$

for which the branching ratio is 1.1%, of which 6/7 of the events have one Z decay to $\bar{\nu}\nu$.[3,49] I require any observed Z's to be central, $|y_Z| < 1.5$, and in addition require either $m_{ZZ} > 0.9$ TeV or $(m_{ZZ})_T > 0.9$ TeV, where $(m_{ZZ})_T$ is the transverse mass, $2 \cdot \sqrt{m_Z^2 + p_T^2}$, computed from the p_T of the observed Z when the second Z decays to $\bar{\nu}\nu$.

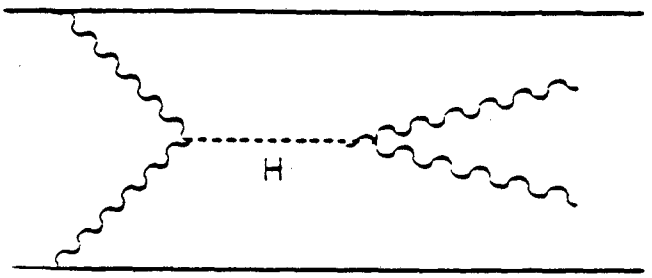

Figure 6.4: Higgs boson production via WW fusion and decay to WW.

The cuts are needed in order to see the signal above $\bar{q}q \to ZZ$ background.

An idea of the dependence of the signal on collider energy can be gotten from figure 6.5, which shows the signal alone. Figure 6.6, showing the signal over the background, illustrates the need for the cut on m_{ZZ} or equivalently on $p_T(Z)$.

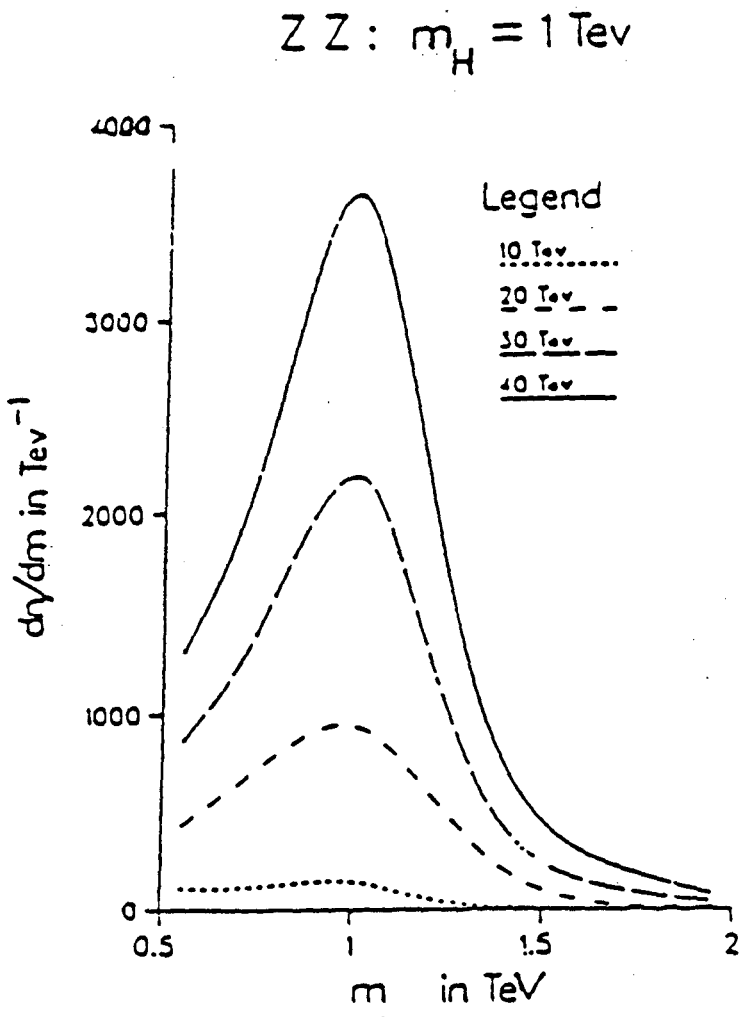

Figure 6.5: Yield dn/dm_{ZZ} in TeV^{-1} for $H \to ZZ$ at 10, 20, 30, and 40 TeV pp colliders, in events per $10^4 pb^{-1}$ with $|y_Z| < 1.5$ (from ref. 3).

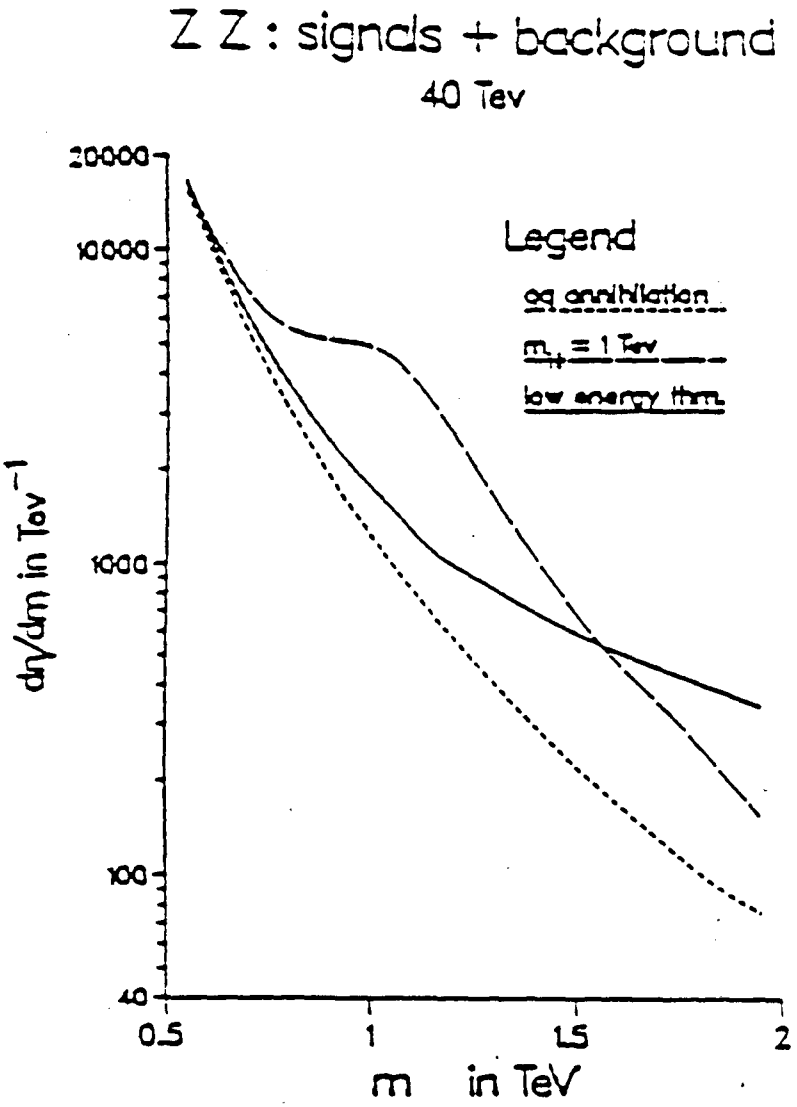

Figure 6.6: Yields defined as in figure 6.5 for a 40 TeV pp collider. The short dashed line is the $\bar{q}q \to ZZ$ background while the long dashed line is the sum of the background and the $H \to ZZ$ signal. The solid line represents the sum of signal plus background for an extrapolation of the low energy theorem as discussed in Section 6.2 (from ref. 3).

Here and elsewhere I quote yields in events per $10^4 pb^{-1}$, the integrated luminosity accumulated with $10^{33} cm^{-2} sec^{-1}$ for 10^7 sec. For $m_t = 50$ GeV the signal is 34 events over a background (from $\bar{q}q$ and $gg \to ZZ$) of 16 events (i.e., 50 events total). The situation improves with a heavier top quark due to the additional production channel $gg \to H$ via a $\bar{t}t$ loop.[50] For $m_t = 200$ GeV the signal is 100 events over a background of 22 events. The $O(\alpha_s g^2)$ gluon exchange and $O(g^4)$ $qq \to qqZZ$ backgrounds have not yet been calculated, but will not be very important after the m_{ZZ} or $(m_{ZZ})_T$ cut is applied.

Except for $gg \to ZZ$, the backgrounds are predominantly transversely polarized Z's while the signal is purely longitudinal, resulting in different angular distributions for the decays $Z \to \bar{f}f$ where f is a lepton or quark. Define θ^* as the angle in the Z center of mass system between the fermion momentum $\vec{p}_f$ and the boost axis to the laboratory frame. Then the angular distributions for longitudinal and transverse polarizations are

$$P_L(\cos\theta^*) = \frac{3}{4}\sin^2\theta^* \tag{6.8}$$

$$P_T(\cos\theta^*) = \frac{3}{8}(1+\cos^2\theta^*) \tag{6.9}$$

A strong cut against P_T throws out most of the P_L baby with the bath, and cannot be afforded given the small number of events. On the other hand, there are enough events to check that the signal is longitudinal as expected. For instance, a cut at $|\cos\theta^*| < 1/3$ reduces N_L by about 1/2 while reducing N_T by about 1/4 (see e.g. reference 51).

6.2 Strong W^+W^+ & W^-W^- Scattering

The like-charge $W_L W_L$ channel is controlled by the $I_{\text{custodial}} = 2$ low energy theorem,[3]

$$a_{02} = -\frac{s}{32\pi v^2} \tag{6.10}$$

where I have put $\rho = 1$. This is analogous to the exotic $I = 2$ channel in QCD, in which no resonance structure is observed. A simple model[3] for the continuum scattering in this channel is obtained by extrapolating

the low energy theorem (6.10) to the unitarity limit at $\sqrt{32\pi v^2} \cong 2.5$ TeV,

$$|a_{02}| = \frac{s}{32\pi v^2}\theta(32\pi v^2 - s) + 1 \cdot \theta(s - 32\pi v^2) \qquad (6.11)$$

as shown in figure 6.7. We then use the effective W approximation[51] to compute the yield from WW fusion.

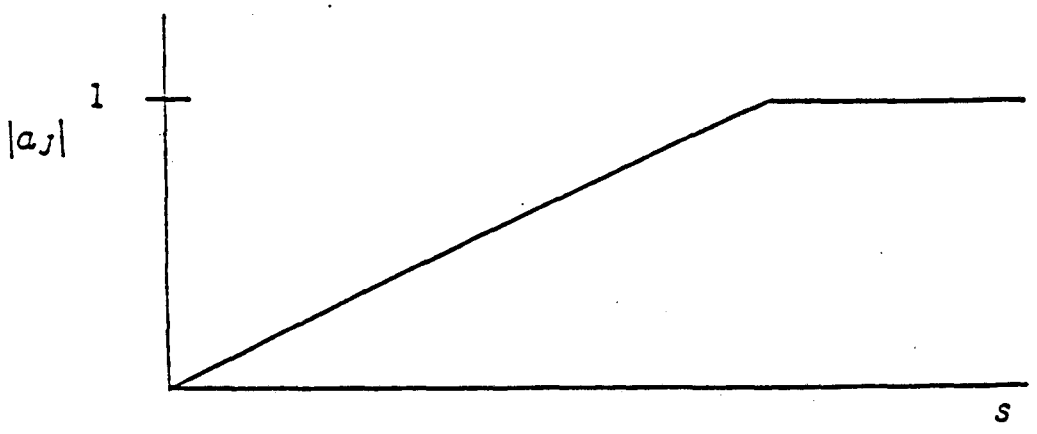

Figure 6.7: Extrapolated low energy theorem for strong W^+W^+ scattering, eq. (6.9).

The model (6.11) can be thought of as a kind of "insurance policy" against the possibility that that the mass scale M_{SB} is much larger than the unitarity limit Λ_{SB}. As discussed in Section 3 this is physically implausible though not rigorously impossible. (Ultracolor[52] with a Higgs boson above 1 TeV might provide an example.) To see how this works, compare the analogous $\pi\pi$ scattering models with experimental data. For the three channels, $(I, J) = (0,0), (1,1), (2,0)$, the models analogous to (6.11) are labeled by the curves a in figures (6.8), compared there with experimental data.[53]

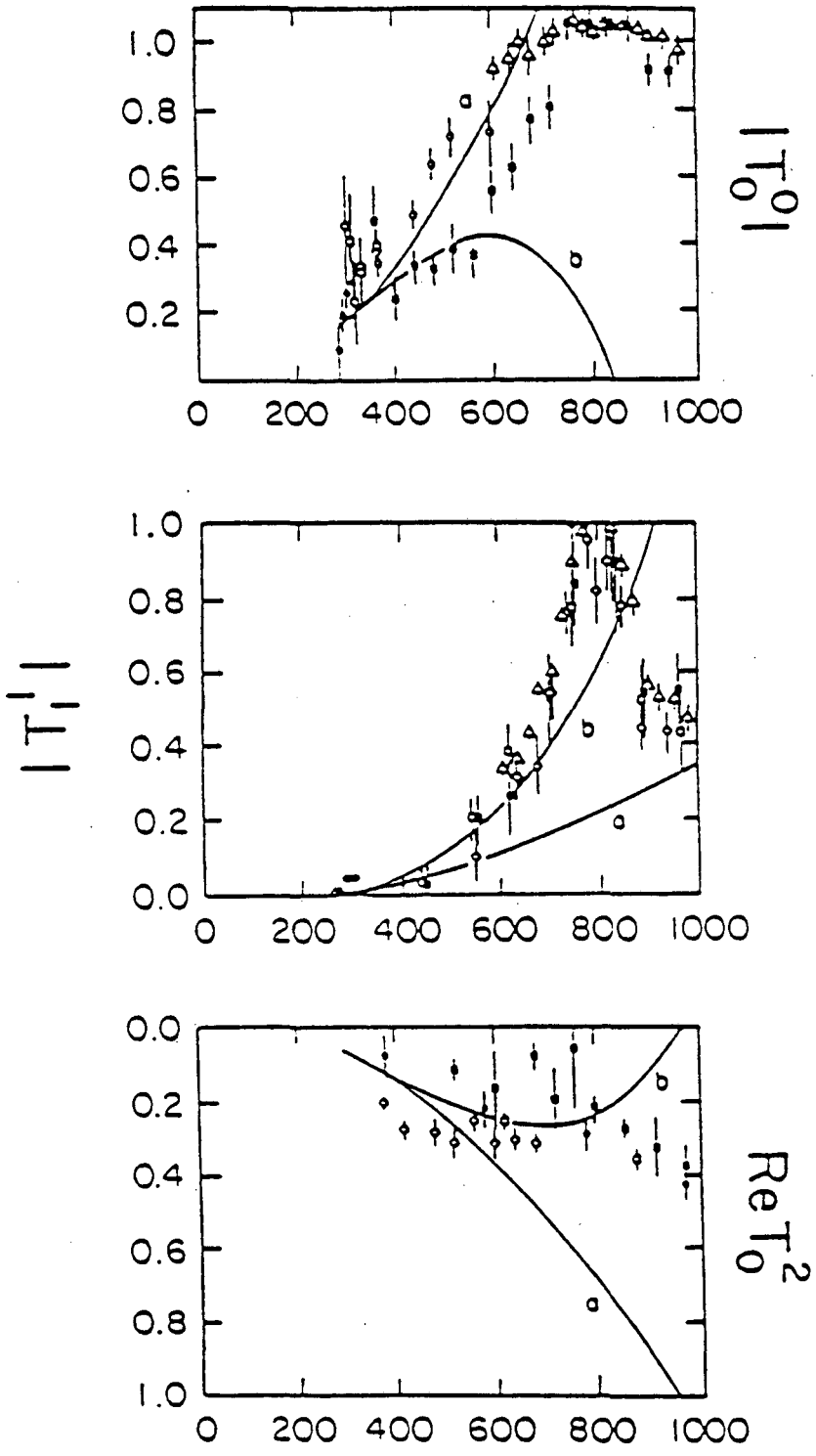

Figure 6.8: Data for $\pi\pi$ partial wave amplitudes compared with extrapolated low energy theorems (e.g., eq. (6.9)) for the three channels $I; J = (0,0), (1,1), (2,0)$. The curves labeled a correspond to the naive extrapolation as in eq. (6.9) and figure 6.6. The figures are from ref. 53.

The model for $|a_{00}|$ describes the trend of the data well. For $|a_{11}|$ it underestimates the data because it fails to account for the ρ meson peak. For $|a_{02}|$ the model overestimates the data (note that since this is an exotic channel, $Im\ a_{02} \cong 0$ and $|a_{02}| \cong |Re\ a_{02}|$ to a good approximation), because if fails to include the effects of ρ exchange in the t and u channels. The model (6.11) is then a kind of worst case scenario: it should work best in the unlikely event that the resonances are much heavier than the unitarity bound for Λ_{SB}. For instance, if the ρ were heavier, say $\geq$ 1 GeV, then curve (a) in figure (6.8) would give a better fit (to larger s) than it now does. On the other hand, if the resonances are where we naively expect, $M_{SB} \cong \Lambda_{SB}$, then at least some channels will be dramatically enhanced relative to the model. We consider a resonant (technicolor) example below. First we consider strong WW scattering with no structure as in figure (6.7).

The signal is defined by two isolated like-charge leptons,

$$W^+W^+ \to e^+\nu/\mu^+\nu + e^+\nu/\mu^+\nu. \tag{6.11}$$

(Assuming $m_t > M_W$, the branching ratio is $(2/9)^2$.) Cuts imposed are $|y_\ell| < 2$ and $p_{T\ell} > 50$ GeV where $\ell = e, \mu$. In addition a "theorist's" cut of $M_{WW} > 800$ GeV is imposed to reduce background from $qq \to qqWW$ by gluon exchange, $O(\alpha_s g^2)$, and by higher order electroweak interactions, $O(g^4)$. This is a "theorist's" cut since the two ν's prevent it from being implemented experimentally. It can eventually be replaced by a set of cuts on observables, such as the dilepton mass and the transverse mass formed from the dilepton momenta.

The corresponding signal[54)] for an SSC year (10^7 sec.) is 53 events, from both W^+W^+ and W^-W^-. The background is $\sim$ 34 events, of which 1/3 is from gluon exchange[54,55)] and 2/3 is from $O(g^4)$ processes.[56)] If instead of (6.11) we used a scaled version of the $I = 2\ \pi\pi$ data shown in figure 6.8, the signal would be decreased by about a factor 2.

6.3 Techni-rho meson

As an example of resonance production I will consider production of the techni-rho meson expected in $SU(4)$ technicolor. From eqs. (5.40-5.41)

we have

$$m_{\rho_T} \cong 1.8\ TeV \tag{6.13}$$

$$\Gamma_{\rho_T} \cong 0.3\ TeV. \tag{6.14}$$

There are two important production mechanisms: $W_L W_L \to \rho_T$ (ref. (3)) and $\bar{q}q \to \rho_T$ ref. (57)). I consider the easily observed purely leptonic final state

$$\rho_T^{\pm} \to W_L^{\pm} Z_L \to e^{\pm}\nu/\mu^{\pm}\nu + e^+e^-/\mu^+\mu^- \tag{6.15}$$

with branching ratio 0.014 (for $m_t > M_W$). With a central rapidity cut, $|y_{W,Z}| < 1.5$, and a diboson mass cut $M_{WZ} > 1.6$ TeV, I find a signal of 13 events and a background of 1.7 events. If $W \to \tau\nu$ events can also be recovered, signal and background both increase by $\sim 1\frac{1}{2}$ to 20 events over a background of 2.5.

7. CONCLUSION

The Higgs mechanism implies the existence of Higgs bosons below 1 TeV or strongly interacting particles above 1 TeV, though probably not much heavier than $\sim$ 2 TeV. With the ability to observe strong WW scattering in the 1-2 TeV region, we can decide for certain if the symmetry breaking sector is strong or not. Unlike the usual situation where a negative result leaves open the possibility that we must search at higher energy, the observed absence of strong WW scattering would imply that symmetry breaking is due to Higgs bosons below 1 TeV. The SSC is a minimal pp collider with this "no–lose" capability. A minimal e^+e^- collider probably would need $\sqrt{s} \cong 3-5$ TeV and $\mathcal{L} \geq 10^{33} cm^{-2}\,\mathrm{sec}^{-1}$.

Presently approved world facilities would leave open an "intermediate mass" window for a Higgs boson of mass 70-80 GeV $< m_H <$ 120-140 GeV. The gap could be closed by an e^+e^- collider with $\sqrt{s} \gtrsim 300$ GeV and $\mathcal{L} \gtrsim 10^{32} cm^{-2}\,\mathrm{sec}^{-1}$. Motivation for closing this window would be strengthened by the discovery of supersymmetry or by evidence that strong WW scattering does not occur.

It should be clear from the small yields quoted in Section 6 and from the not much bigger yields for lighter Higgs bosons, that discovery of the symmetry breaking sector will not be the end but the beginning of a long process of detailed studies. The handful of events that provide the initial discovery will be just the first step in our study of the new force responsible for gauge boson mass.

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