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Multiple Particle Production Processes in the "Light" of Quantum Optics

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MULTIPLE PARTICLE PRODUCTION PROCESSES IN THE 'LIGHT' OF QUANTUM OPTICS

by

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1. A bit of history

Is there an obsession with cascade processes ?

Ever since the observation [1] that high-energy "nuclear active" cosmic-ray particles — today we would say hadrons — create bunches of penetrating particles upon hitting targets, a controversy has raged about whether these secondaries are created in a "single act" (a sort of "mini-Big Bang", probably first termed by Heisenberg [2] as "Multiple Production") or whether — in analogy to the succession of ("particle-poor") bremsstrahlung-pair production processes in electromagnetic cascades — many hadrons are just the result of an intra-nuclear cascade, yielding one meson in every step (this used to be termed "Plural Production" [3]).

I cannot escape the impression that: a) the latter kind of model appeals naturally as a consequence of an innate bio-(or should one say anthropo- ?) morphism in our way of thinking (multiplication by cell division, rarity of twins and multiplets. . .), and that b) in one guise or another it has tenaciously survived to this day, also for hadron-hadron collisions, via multi-peripheral models to the modern parton shower approach.

Do "true" multiple processes occur in nature ?

The above remarks are not meant to imply that the cascade-type of approach to multiparticle production processes is the wrong one, but rather as a reminder of the fact that — in spite of its many successes — it is not the only one. Indeed, from the very beginning of theoretical consideration of multiparticle production [4] [5],the possibility of many particles arising from a single "hot" system ("fireball" ?) has been explored, with many fruitful results, not the least of which are the $s^{1/4}$ dependence of the mean produced particle multiplicity and the "thermal" shape of the p_T spectra.

An important consequence of the thermodynamical-hydrodynamical models (hereafter THM's) is that particle emission is treated in analogy to black-body radiation, implying for the (mostly bosonic!) secondaries a set of specific Quantum-Statistical (hereafter QS) properties, very similar to those observed in quantum optics.

From here on I shall try to review a number of implications and applications of this QS analogy in the study of multiplicity distributions of the produced secondaries (MD's).

I will touch only in passing another very important topic of this class, viz. the Bose-Einstein two-particle correlations; it will be amply covered by other speakers at this meeting and in particular by Professor Gerson Goldhaber.

2. Facts and Fits

Approximate Negative Binomial Behavior of MD's

For a long time people have paid relatively little attention to the shape of the multiplicity distributions; they rather concentrated on the mean multiplicity <n> . It should have been clear from the very beginning, however, that P(n) contained a lot of "dynamical" information. Thus e.g., if the THM's are indeed the adequate framework of true multiparticle production, the QS analogy implies that for a single emitting source the P(n) should have the single-cell Bose-Einstein form

$$P_{B.E.}(n)=\frac{1}{1+m}\left(\frac{m}{1+m}\right)^n \qquad (1)$$

where m=<n>; for the general case of k cells in phase space the k-fold convolution of such distributions is the well-known Planck-Polya distribution

$$P_{NB}(n|m,k)=\frac{(n+k-1)!}{n!\,(k-1)!}\left(\frac{k}{m+k}\right)^k\left(\frac{m}{m+k}\right)^n \qquad (2)$$

known in mathematical terms as the negative binomial (NB).

If, on the other hand, particles are independently emitted from the fireball (Eq.(2) implies a lot of correlations !; see below) then n would be distributed according to the Poisson law (PD)

$$P_{Poiss}(n|m)=\frac{e^{-m}m^n}{n!} \qquad (3)$$

It should be noted here that both Eqs. (2) and (3) could be arrived at starting from very different viewpoints and models; thus e.g. the PD could be simply obtained as the limit of the NB if the number of cells increases indefinitely; conversely, an intrinsically Poisson distributed multiplicity n for which the expectation value m is a random variable obeying a gamma distribution (e.g. due to inelasticity fluctuations, to be discussed in Appendix A), n will end up by following an overall NB distribution.

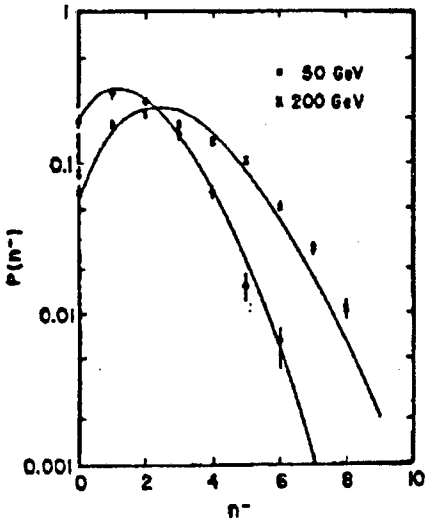


Fig. 1

The earliest reliable observations of P(n) were obtained in bubble chamber experiments at the CERN PS at $\sqrt{s}$ = 9 GeV and indicated a Poissonian shape. Later observations at higher energies ([6] and [7]) showed more and more marked departures from the PD, as can be seen from Fig. 1 (full lines=PD).

A good measure for the departure of an MD from the PD are the factorial cumulants fq (also known as Mueller functions), to be considered in more detail below, which should be zero for the PD in all orders q>1. The best known of these, f_2 is given by:

$$f_2=\langle n^2\rangle-\langle n\rangle^2-\langle n\rangle \qquad (4)$$

Experimental values of f_2 obtained with hydrogen bubble chambers up to the highest energies covered in early FNAL experiments are plotted in Fig. 2 against <n>.

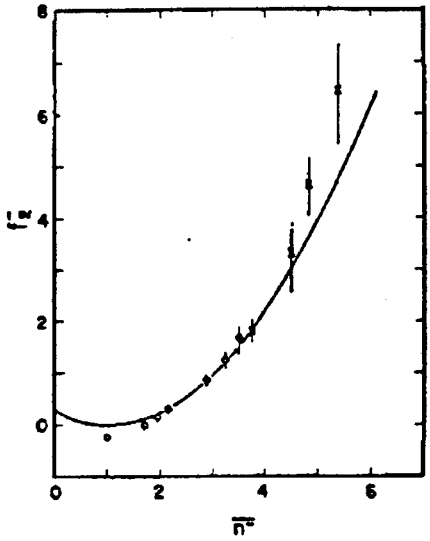
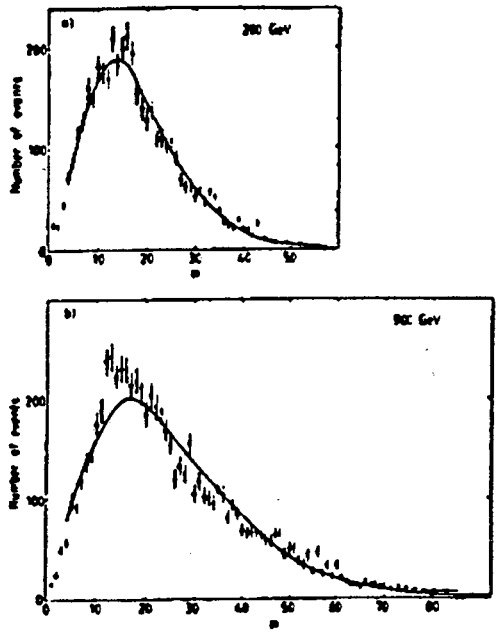


Fig. 2 ↑

It is obvious that with increasing energy, f_2 increases to large positive values, i.e. the distributions become broader than the PD. When strong departures from Feynmann scaling and hence from its asymptotic consequence known as KNO scaling were established at considerably higher cms energies, the NB was suddenly "re-discovered" [8] and it was shown that it provides indeed a reasonable description of the MD's over a large range of s-values. That this description is far from perfect is shown in Fig. 3 obtained when more accurate measurements at the CERN SPPS [9] noticed systematic departures from an NB.

Fig. 3 →



Centered Rapidity Windows

If the MD of "complete" events, in which produced particles from all regions of phase space were counted, have been an object of study for a relatively long time, it took quite a while to recognize [10] the fact that significant "dynamical" information could be revealed if restricted regions of phase space — more precisely, in rapidity — were treated in each event as a "sub-event" and then the MD of such sub-events were investigated. This concept was also re-discovered at the SPPS [11], and then exploited by many experimental groups. Most large-scale investigations concerned rapidity windows centered about zero cms rapidity (usually called "cuts" or "symmetrical windows" going from -Yc to Yc). The main facts established in this field (e.g., [12]) are the good fit yielded by the NB in practically all experiments and a systematic lowering of the width parameter k (i.e. a widening of the MD) with decreasing window size Yc. (Fig. 4, below).

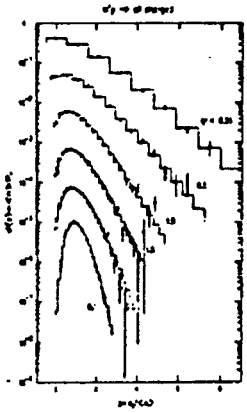


Fig. 4

Shifted Rapidity Windows

The behavior of sub-events collected from windows shifted away from cms rapidity ("shifts" or "asymmetrical windows") (10[], [13]) reveals that along with the widening of the MD due to decreasing window size (as seen already in the cuts), the width of the MD decreases as the center of the accepted window shifts away from y=0 (ref. 10 and 13). The quality of NB-fits also deteriorates.

Forward-Backward Correlations

A special case of an asymmetrical window is one starting at y=0 and extending up to some Y_c; if Yc extends to infinity (or rather to its allowed kinematical limit) a whole hemisphere in rapidity, containing nF particles is covered; in studying correlations with the number n_B of particles observed in the mirror reflection of the accepted interval, one then speaks of "Forward-Backward" correlations. These would be non-existent (i.e. the slope of the $\langle n_F \rangle = a + bn_B$ regression line would be zero) if the particles were Poisson distributed and the sampling done by opening one hemisphere only would be simply binomial. In fact, significant slopes b are observed, which furthermore increase with $\sqrt{s}$.

Chaotic and Coherent Sources

Bose - Einstein Correlations: The Unmistakable Signal

The fact that any source emitting hadrons (mainly bosons) should obey the laws of QS is obvious; *how* this occurs in detail is a much more complex problem, which has not been completely explored to this day. The unmistakable signal that QS is at work is the observation of strong (like-sign) two- pion correlations [14] with the characteristic signature of Bose-Einstein (BE) statistics, namely the strong enhancement of the correlation function at low 4-momentum differences [15][16]. Along with this signal, all experiments [17] have also shown that the correlation never reaches its maximum allowed degree (a value of 2 at zero intercept); the most natural interpretation (though by far not the only one!) [18] for this effect is that the source is not 100% chaotic, as would be expected from a purely thermalized system, but that a certain degree of coherence is present in the boson radiation. Details (both theoretical and experimental) of these investigations will fill at least one of the sessions of our workshop. Except for one (essential) connection, to be mentioned further on in my talk, I shall from now on leave this subject to my other colleagues and concentrate on some consequences of the "co-existence" of chaoticity and coherence in the source, for the multiplicity distribution of the emitted particles.

Pure and Partially Coherent Sources

The simplest assumption about the origin of the created secondaries is that they are emitted from a single source; if the field has a chaotic (π_{ch}) and a coherent (π_c) component[19]:

$$\pi(x) = \pi_c(x) + \pi_{ch}(x) \tag{5}$$

such that

$$\langle n_{ch} \rangle = \langle | \pi_{ch}^2 | \rangle \ , \quad \langle n_c \rangle = \langle | \pi_c^2 | \rangle \tag{6}$$

the mean number of particles emitted will be:

$$\langle n \rangle = \langle | \pi^2 | \rangle = \langle n_c \rangle + \langle n_{ch} \rangle \tag{7}$$

and one may define a coefficient of chaoticity (for short "chaoticity") p:

$$p = \frac{\langle n_{ch} \rangle}{\langle n \rangle} \ , \quad [\, 0 \ \leq p \leq 1 \,] \tag{8}$$

We are interested in the shape (or , lacking that, at least some numerical characteristics) of the resulting multiplicity distributions (MD's).

The case p=0 corresponds to purely coherent sources. For a single source the predicted MD coincides, oddly, with the prediction for a set of independently emitted particles , i.e. with the Poisson distribution (PD, Eq.3). As was already noted [20]), a superposition of many coherent sources leads to a generalization of the PD known as the hyper-Poisson (HP) distribution,

$$P_{HP}(n|\alpha,\beta)=\frac{\beta!\,\alpha^n}{{}_1F_1(1;\beta+1;\alpha)\,(\beta+n)!} \qquad (9)$$

where ${}_1F_1$ is the confluent hypergeometric function. The HP obviously reduces to the PD if $\beta \to 0$. An example of how well the HP describes MD' s, which are definitely wider than the PD ($f_2 > 0$), is shown in Fig.5 [21]. It would be tempting to assume that all reactions are intrinsically coherent, and to assign all the widening of MD's with increasing $\sqrt{s}$ to the change of the additional parameter β. However, at very large $\sqrt{s}$ the fits deteriorate [21] and it becomes obvious that some chaoticity has to be present, as was already indicated by the BE correlations.

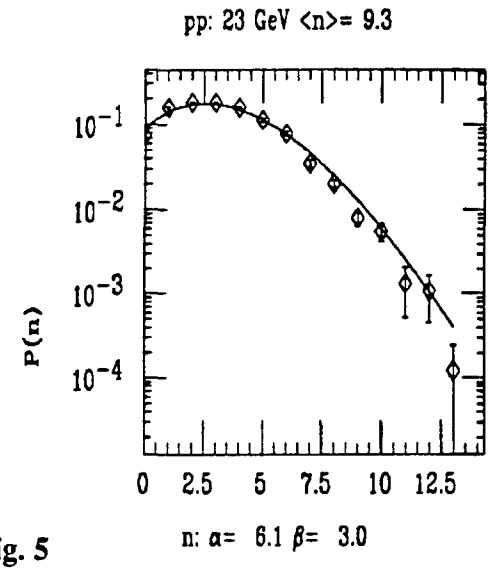


Fig. 5

The general case of a superposition of coherent and chaotic fields (described by Eqs.(5) - (8)) is easier to consider if the variable x is identified with a kinematic variable which – in analogy to time in optics – has the property of stationarity. Such a variable, in the case of multiparticle production, appears to be (at least for the bulk of the created secondaries) the longitudinal rapidity y. (Its conjugate is then the boost variable.) The stationarity is revealed in the two-particle correlation

$$\langle \pi_{ch}(y)^+ \pi_{ch}(y')\rangle = e^{-\frac{|y-y'|}{\xi}} \qquad (10)$$

for the chaotic component , which is a function of $|y-y'|$ only. The quantity ξ is the rapidity correlation length of the chaotic component (and appears as such in the BE correlations, when measured in terms of y -). If $\xi \to \infty$ (as one would expect, e.g. in the vicinity of a phase transition) then P(n) is known to obey the so-called Partially Coherent Laser Distribution (PCLD, also known as Glauber-Lachs.) :

$$P_{PCLD}(n\,|\,m,p,k)=\left(\frac{k}{k+mp}\right)^k\left(\frac{mp}{k+mp}\right)^n \exp\left(-\frac{mk(1-p)}{k+mp}\right) L_n^{k-1}\left(-\frac{k^2(1-p)}{p(k+mp)}\right) \qquad (11)$$

the L_n^{k-1} are generalized Laguerre polynomials, k is the number of "cells" in phase space, and m the mean multiplicity. It is easy to see that the PCLD has as limiting cases the PD if $p \to 0$ and the NB if $p \to 1$. It appears to give [22] a qualitative description of the widening of MD's with $\sqrt{s}$, provided p increases with $\sqrt{s}$ by about an order of magnitude in the energy range covered by the CERN ISR and SPPS colliders. This is illustrated in Fig.6, which shows the ratios R_q between the computed and the observed normalized moments

$$C_q \equiv \frac{\nu_q}{\nu_1^q}, \quad \nu_q=\langle n^q\rangle \qquad (12)$$

along with the $\sqrt{s}$ – dependence of the estimated p–values.

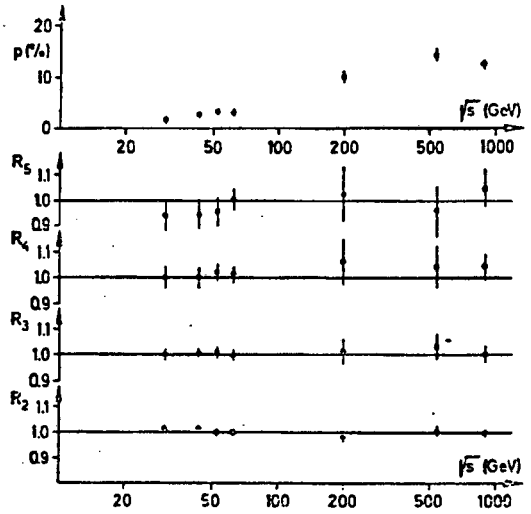


Fig. 6

Forward cone in p-A collisions: Self-Induced Transparency?

The relative width of the PCLD can be measured by its normalized factorial moments of order q,

$$\Phi_q \equiv \frac{F_q}{F_1^q}, \quad F_q = \langle n(n-1)\ldots(n-q+1)\rangle$$

which are given by [22] :

$$\Phi_q = q!\left(\frac{p}{q}\right)^q L_q^{k-1}\left(-k\frac{1-p}{p}\right) \; ; \text{PD: } \Phi_q = 1 \; ; \text{ NB: } \Phi_q = \frac{(q+k-1)!}{k^q(k-1)!} \quad (14)$$

they thus decrease when $p \to 0$, i.e. with increasing coherence of the source. One is tempted to connect the narrowing of the MD's for "shifted" rapidity windows (as the center of the "shift" moves away from y_{cms} towards the kinematic limit Y_c) with a decrease of the chaoticity p.

On the other hand, it is known [23] that in p-A collisions the particle density near Y_c is insensitive to the target thickness (measured by $A^{1/3}$); in other words, the target appears to be transparent to the fastest (high $-y$) bosons. This brings to mind an effect well known in Quantum Optics [24], namely the so-called self-induced transparency when a coherent beam traverses a medium with at least one metastable state. Fig.7 [10] compares for proton-Emulsion collisions at 200 GeV four Y windows (covering four quarters of the kinematically accepted y range (going from $-Y_c$ to Y_c):

a) with respect to "nuclear transparency", as evidenced by the variation (or lack thereof) of the particle density in the given window with the multiplicity N_h of slow target fragments (a good measure of target thickness) and

b) the shape of the multiplicity distribution in the same window. The straight lines are one-cell BE MD's while the curves are the best fitting PD. It is apparent that as one approaches the projectile fragmentation region, "coherence" (low p) and "transparency" are well correlated. Although each of these effects, taken separately, is susceptible to a few other interpretations , their simultaneous occurrence is very suggestive of the quantum-optical analogy.

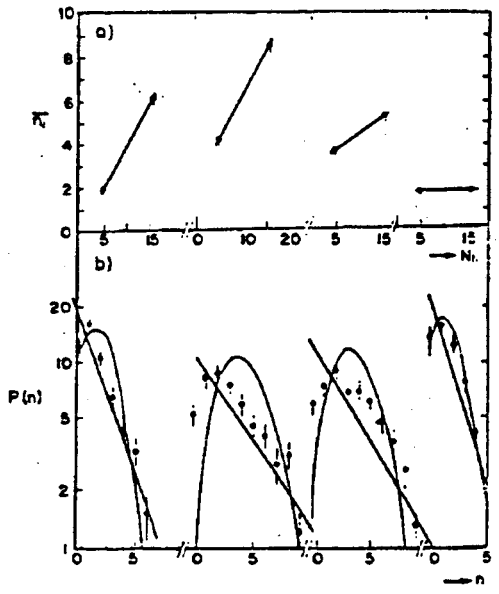


Fig. 7

Multi-Component Convolutions

A reasonable interpretation of the decrease of the apparent degree of chaoticity when y increases towards Y_c can be found [25] by assuming that in each collisions two sources are at play, one of which is purely chaotic (thermal fireball due e.g. to gluon-gluon interactions), while the other behaves coherently (as would be expected, e.g., from valence quark bremsstrahlung). The resulting MD of the total number n of produced secondaries would then be given by a convolution,

$$P_{2\ comp.}(n|m_1,m_2,k) = \sum_{n_1+n_2=n} P_{NB}(n_1|m_1,k)\ .\ P_{Poiss}(n_2|m_2) \qquad (26)$$

where the subscripts 1 and 2 refer to the actual and the mean multiplicities of the chaotic and the coherent sources, respectively. Since the coherent source does not contribute to the factorial cumulants (Eq.(14)) it is easy to estimate m_1 and m_2 from the observed moments of the overall MD. Estimated values for m_1 and m_2 are shown in Fig..8.1 for $\sqrt{s}$ ranging from about 20 to 900 GeV. The chaotic component is well described by a $s^{1/4}$ – law, as expected from a thermal source [4] (QGP ?) . The chaotic component varies rather like log(s) consistent with a bremsstrahlung origin. Fig.8.2.a shows the estimated rapidity distribution of the two sources at $\sqrt{s}$=546 GeV. The chaotic one is concentrated at low cms rapidities while the coherent one is more or less flatly spread out towards high y. Fig.8. b then shows the estimated apparent chaoticity

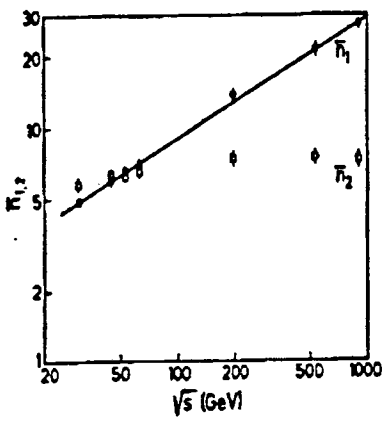


Fig. 8.1

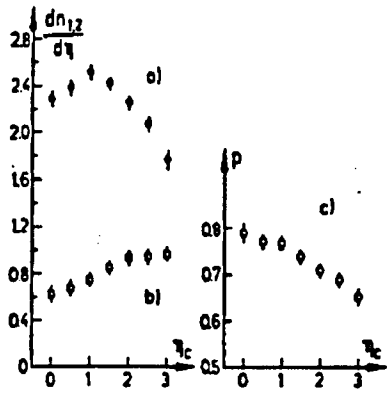


Fig. 8.2

$$p^* = \frac{m_1}{m_1+m_2} \qquad (16)$$

as a function of rapidity; p^* decreases from ≈ 80% to ≈ 60% over the observed rapidity range.

Partially Coherent Sources with Finite Correlation Lengths

When the coherence length ξ of the chaotic component is finite, two things happen: the bad news is that it is not possible to get a closed expression for P(n); the good news, however, is that it becomes possible to predict the behavior of finite rapidity windows (at least the "cuts") from information deduced from the global distribution P(n). We shall now address these two problems in more detail.

Though no closed formula exists for P(n), numerical approximations have been worked out recently [26] , [27]. Even for low <n> the computational effort is enormous and for the large r <n> occurring at large existing or projected colliders the logistics become prohibitive. What is available is a set of closed expressions for the normalized factorial cumulants μ_q of P(n):

$$\mu_q \equiv \frac{f_q}{f_1^q}\ , \qquad f_q = \left.\frac{d^q \log(G_F(z))}{dz^q}\right|_{z=1} \qquad (17)$$

where $G_f(z)$ is the factorial moment generating function:

$$G_F(z) = \sum_{n=0}^{\infty} z^n P(n) . \tag{18}$$

They are given [28][19] by :

$$\mu_q = (q-1)! \left(\frac{p}{k}\right)^{q-1} \left[p B_q(\beta) + q(1-p) \overline{B}_q(\beta) \right] \tag{19}$$

The B's are a set of elementary (but, for larger values of q, increasingly nasty !) functions of the ratio $\beta = \Delta Y/\xi$ only (where ΔY is the width of the rapidity interval counted); the first three pairs are :

$$B_1(\beta) = \overline{B}_1(\beta) = 1 , \tag{20}$$

$$B_2(\beta) = \left(\frac{1}{2}\right)\left(e^{-2\beta} + 2\beta - 1\right) \beta^{-2} , \ \overline{B}_2(\beta) = 2 \left(e^{-\beta} + \beta - 1\right) \beta^{-2} , \tag{21}$$

and

$$B_3(\beta) = \left(\frac{3}{2}\right)\left((\beta+1)e^{-2\beta} + \beta - 1\right) \beta^{-3} , \ \overline{B}_3(\beta) = \left(-e^{-2\beta} + 2(\beta+4)e^{-\beta} + 4\beta - 7\right) \beta^{-3} . \tag{22}$$

The normalized factorial cumulants μ_q can be easily connected with experimentally measured quantities, such as the normalized moments C_q or the equivalent k of a fitted negative binomial. For q=2 the simple relationships are:

$$\mu_2 = \Phi_2 - 1 = C_2 - 1 - \frac{1}{\langle n \rangle} \rightarrow \frac{1}{\tilde{k}} \tag{23}$$

The fact that for a given p the μ_q are functions of the ratio β only, allows one to scale cumulants (in a more rudimentary analysis, equivalent k - values) from one window width ΔY (e.g. the full Y range) to a more restricted one, as long as ξ remains unchanged. This property, which we call beta-scaling explains qualitatively why the MD's of symmetric windows ("cuts") become systematically wider (in "normalized" parlance, i.e. in terms of μ_q or k) as ΔY decreases [30] . The same formalism can be applied to explain the increase with $\sqrt{s}$ of the Forward-Backward multiparticle correlations [29] , because the width of the MD in, say, the forward cms hemisphere is determined by that of the global MD except for the change $\beta \rightarrow \beta/2$ in Eq.(19) .

By combining the results of measurements of the moments (ergo also the cumulants) of the MD's with the information from Forward-Backward Correlation, it becomes possible to estimate simultaneously both p and ξ.

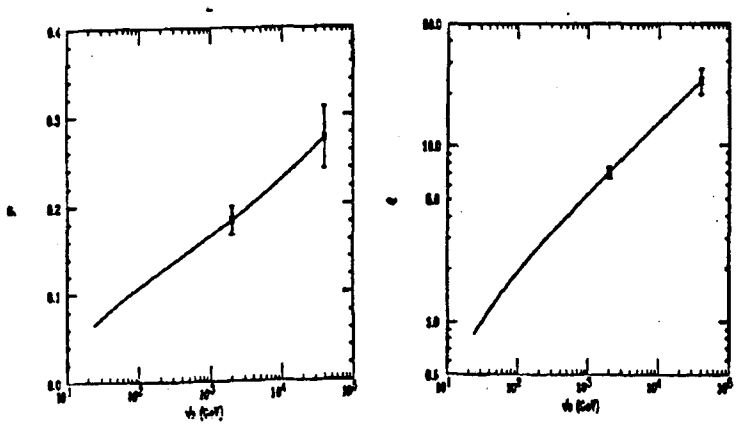


Fig. 9

It could be shown [29], [31], that both parameters increase with $\sqrt{s}$. These increases are shown in Fig.9 a) and b) for the ISR – SPPS energy range along with projection to TEVATRON and SSC energies (the latter with error bars).

Consequences for BE – Correlation (HBT) Experiments

As mentioned earlier, the fact that the two-particle correlation function $C_{(2)}$ recorded in BE-correlation experiments never reaches its maximum allowed value of 2, can be most easily interpreted in terms of partial coherence ($p < 1$) of the source. As has been shown by Weiner [32], mixed (i.e. chaotic + coherent) fields lead to a correlation function

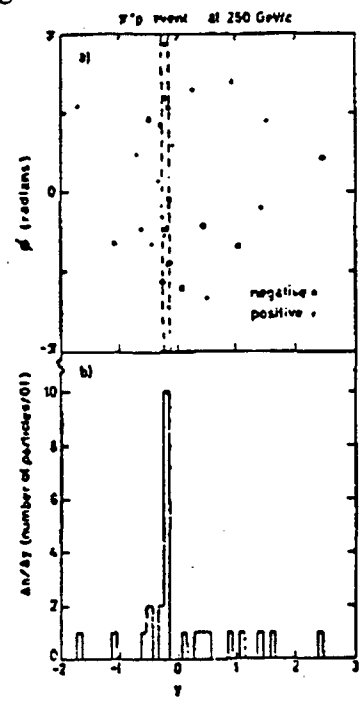
$$C_{(2)}=1+2p(1-p)\, e^{-|x_1-x_2|/\xi}+p^2\, e^{-2|x_1-x_2|/\xi} \qquad (24)$$

in which even a slight admixture of coherence substantially decreases the intercept (at $|x_1 - x_2|=0$.

Very small rapidity windows: Intermittency?

It has been noticed for some time ([33] – [36]) that with decreasing size Δy of the rapidity bite considered there occur strong local fluctuations in rapidity density ("spikes). A typical example observed at $\sqrt{s}$=22 GeV by the NA22 Collaboration is shown in Fig.10. The uniform distribution in azimuth illustrated in the upper half of the figure excludes the interpretation of the "spike" in y seen in the lower half as a ("hard" parton-parton) jet.

Fig. 10



In order to deal quantitatively with this phenomenon, dubbed in analogy to hydrodynamics as intermittency, it has been proposed [37] to study the behavior of normalized factorial moments Φ_q with Δy. Certain models predict a power law behavior, whereas experimental data seem to flatten off at very small Δy. It has been shown [38] that the overall behavior of the Φ_q can be obtained directly from Eq. (19) as a consequence of the decrease of β with Δy. Two examples of this kind of analysis are shown in Figs11 and 12 for $\sqrt{s}$ of 22 and 546 GeV, respectively. It should be noted that most of the "noisiness" noticeable for q=4 and q=5 in Fig.11 is due to the influence of the "spiky" event of Fig10 and possibly that of a few similar but less prominent extreme events ; their origin (see e.g., Appendix B) may very well lie outside the scope of QS, which otherwise

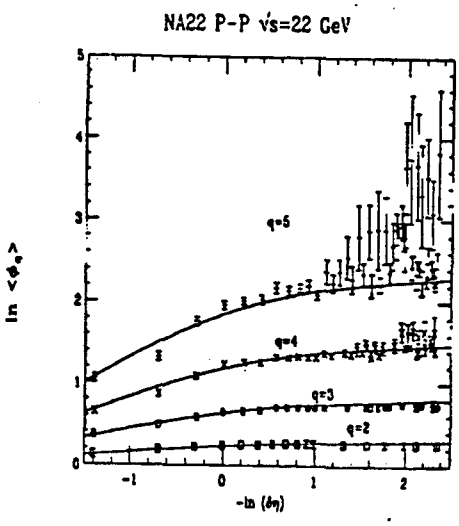
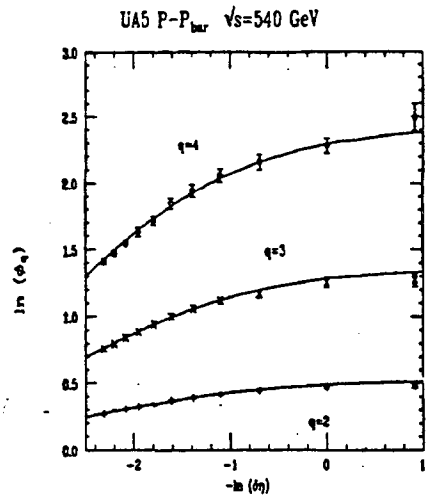


Fig. 11



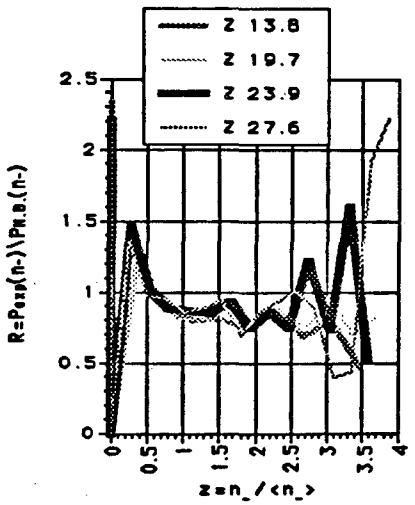
predicts the overall behavior of the Φ_q rather well (It must be stressed that in Figs.11 and 12 the parameters p and ξ were fitted just to the second factorial moments and the good description of the higher moments followed automatically!).

Fig. 12

How well is P(n) *really* described by the QS formalism?

As we have seen until now, both the very fashionable negative binomial distribution and the different refinements introduced by a more detailed consideration of QS phenomena manage to give a good overall description of the shape of P(n). A legitimate question is whether "fine structures" exist which would be drowned out in the ensemble averaging reflected in the moments or be hidden in the (also very fashionable) logarithmic presentations of P(n). A suspicion that such fine structures could be present arose [39] from the consideration of possible "squeezed" quantum states, which would lead (in NB jargon) to MD's with an apparent k < 1. Given adequate values of the parameters of such a model one might see an oscillatory behavior superimposed on the "global" shape of the P(n).

Fig. 13 →



In order to "put the P(n) under a magnifying glass" we plot in Figs. 13-15 the ratios R between the experimental P(n) and the "smooth" prediction of the best fitted NB. Similar analyses have been done using as reference (instead of the NB) the PCLD or the two-component convolution (Eq.26), with essentially similar results. For better clarity the data have been grouped into "low" (√s < 28 GeV), "medium" (√s=30 – 60 GeV , CERN ISR), and "high" (√s >200 GeV, CERN SPPS) energy groups. The R - values are plotted against the normalized multiplicity z=n/<n> (of KNO fame) so as to try to detect systematic deviations from a smooth distribution. Although the mid-range, i.e. z=1 - 2 data points agree roughly with the NB prediction (best in the ISR case) it is apparent that correlated structures appear around z=0.5 and z=2.5. Although statistics deteriorate towards the extremities of the z- scale (to illustrate this, I present in Fig. 16 the 200 GeV UA5 data alone, complete with error bars which would have irremediably obscured the preceding graphs), the correlated trends of the different distributions appear too similar to be due to chance fluctuations only and deserve further, quantitative, investigation. Besides squeezed quantum states one might think of other possible explanations of these deviations, such as mixtures of events obeying quite different shapes of P(n). If such a mixture could be detected by tagging the different kinds of events by means of characteristics extraneous to the MD's it might make the "good fit" by the NB distribution to appear as an accident of numbers (just as the analysis by the UA5 Collaboration had shown the apparent KNO-behavior in the ISR range be accidental).

Fig. 14

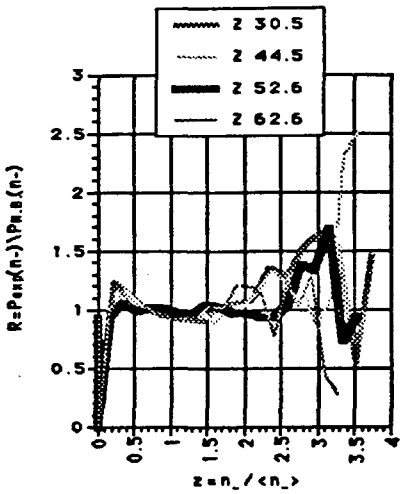


Fig. 15

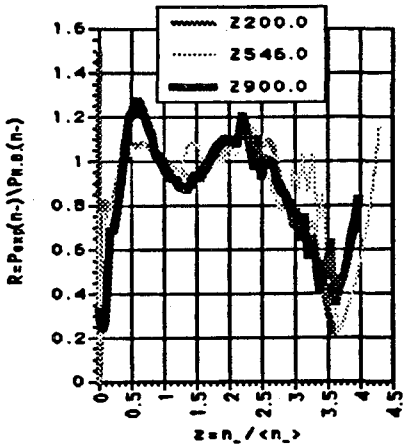
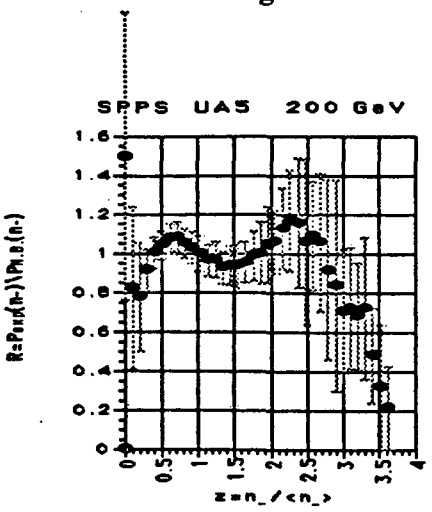


Fig. 16



Concluding Remarks

It goes without saying that many if not all the physical characteristics of multiplicity distributions, *taken individually*, could be explained by a host of other models (clusters, parton cascading, minijets, to name just a few). On the other hand – from quite general principles – the processes leading to multiparticle production just *have* to obey the laws of Quantum Statistics; with this in mind it is amazing how easily QS leads to a quite consistent picture of most if not all the observed phenomena. Maybe this is why it deserves further careful consideration in future investigations.

On the theoretical side these must include the effects of non-stationarity in Y, simplifications in the assumptions about the boost variable, and a better understanding of the effects of inelasticity. (See Appendix A.)

On the experimental side considerably more accurate determinations of both P(n) and the BE-correlations, with careful filtering out of systematic effects appear very desirable. Whether or not this will remain just a pious wish will depend to a large extent on the concrete programs of future accelerators as the good reliable ones (like the ISR and the SPPS) succumb to the looming budget crunch.

The ideas and results presented in this talk came about through the efforts and collaboration of the following colleagues, [grouped and re-grouped in almost all n!/k! (n-k)! combinations..!] all of which have contributed very much to whatever understanding I have of the subject and none of which bears any responsibility for my presentation thereof : M.Biyajima, P.Carruthers, G.N.Fowler, C.X.He, F.S.Navarra, U.Ornik, M.Pluemer, F.W.Pottag, C.C.Shih, I.Stern, A.Vourdas, R.M.Weiner, J.M.Wheeler and G.Wilk.

APPENDIX A.

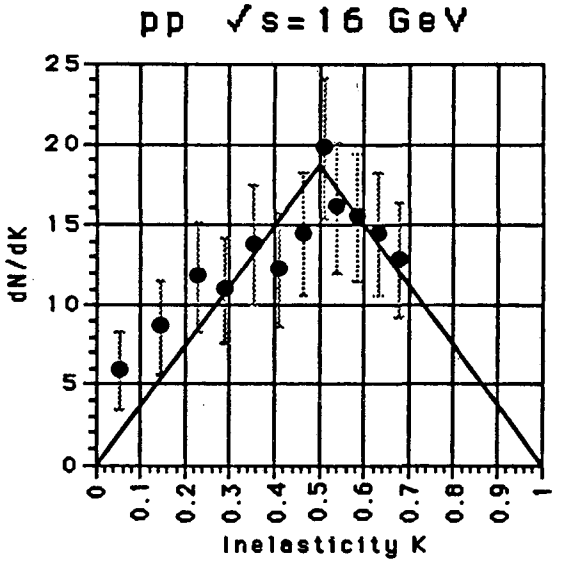
All Collisions are Equal, but some are More Equal than others. . .

An essential ingredient in any single-source model of particle production is the recognition of the fact that the incident particles deposit only a fraction K of their total energy into the "source" so as to make available only an "effective" energy $W=K\sqrt{s}$ for particle production. In the treatment of multi-particle production W is the only relevant energy variable. The quantity K called the coefficient of inelasticity (or, for short, the inelasticity) fluctuates from event to event ($0 < K < 1$) so that events produced by beam-target combinations producing different $\sqrt{s}$ may end up with the same physical initial conditions.

As a consequence, if observations are made at a fixed cms energy $\sqrt{s}$ and some model predicts a multiplicity distribution $P(n \mid W(K))$ then the observed multiplicity distribution $P'(n)$ will be given by

$$P'(n \mid \sqrt{s}) = \int_{K=0}^{1} P(n \mid W(K))\, \chi(K)\, dK \quad \text{(A!)}$$

Fig. A1



where $\chi(K)$ is the Inelasticity Distribution.

This probability distribution (which can be intuitively, if not necessarily adequately, connected with an impact parameter representation) is hardly known from experiment. It was recognized as early as the fifties (again from cosmic-ray measurements) that $\langle K \rangle$ is of the order of 1/2, i.e. that leading particles (usually the surviving baryons) carry away about half the initial energy. The only direct measurement of $\chi(K)$ [40] from collisions of an accelerator beam at $\sqrt{s}$ =16 GeV, yielded a rather broad distribution centered about K=0.5.

What is better known is the Feynmann x-spectrum of individual leading protons, which is close to a flat distribution, almost from x=0 to 1. However, the concrete spectrum of K depends on the total energy loss of both incident particles and hence on the degree of correlation of the fractional energy contents x1 and x2 of the leading secondaries. It can be easily shown that if x1 and x2 are totally correlated, then K is uniformly distributed between 0 and 1, whereas if cov(x1,x2)=0 the distribution of K should be triangular and centered about $\langle K \rangle$=1/2 (this is more or less what is seen in Fig. A.1).

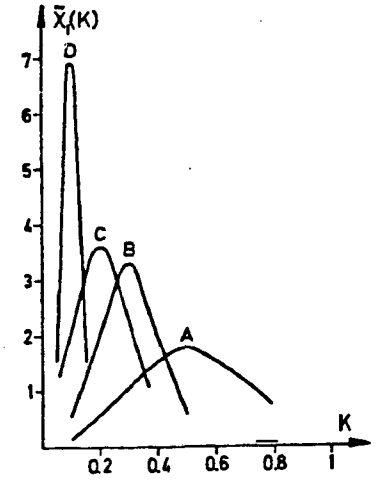


Fig. A2

It has been noted [41] that the relative widths of the rapidity distributions at $\sqrt{s}$=53 and 546 GeV, together with the relatively weak change in transverse momentum spectra at the two energies, imply a decrease of $\langle K \rangle$ with $\sqrt{s}$. Within the framework of QCD it has been shown [41] that assuming gluon-gluon interaction as the vehicle for energy deposition in the "source" leads to K-distributions similar to those observed hitherto and predicts a shrinking of c(K) with $\sqrt{s}$ as depicted in Fig. A.2 (Here A, B, C and D refer to $\sqrt{s}\approx$50, 500, 2000 and 40000 GeV, respectively). With this kind of information at hand, it becomes possible to correct for the effects of $\chi(K)$ in P(n) by means of Eq.(A1) or similar procedures [22],[30].

APPENDIX B

Fractal substratum of rapidity spikes? A toy model

It s well known that for certain critical values, of their parameters many non-linear dynamical systems with a very deterministic behavior turn chaotic. The most trivial example is the logistic map in which a variable x defined recursively via

$$x' = \lambda\, x\ (1-x) \qquad \text{(B1)}$$

turns chaotic (in other words with a quasi - stochastic behavior) near λ=4. It seemed amusing to see what the apparent probability distribution of x - values produced by such a pseudo-random number generator is like. A histogram of a set of 200 x-values produced in this way is shown in Fig. B.1. This distribution shows an uncanny similarity to a $\cos^n \theta$ angular distribution once x is re-mapped to the "cosΘ" interval -1 to +1 .

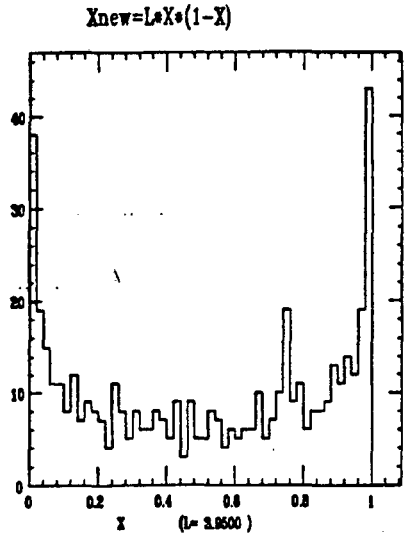


Fig. B1

A simple re-mapping to

$$\eta \equiv \tan h^{-1}(2x-1) \qquad \text{(B2)}$$

turns this into a "pseudo-pseudo-rapidity distribution" shown in Fig. B.2. It displays a typical spike quite similar to those observed in(e.g. heavy ion-)experiments. One such spike turns up in a few hundreds of simulated "events" of comparable "multiplicity" . No comment !

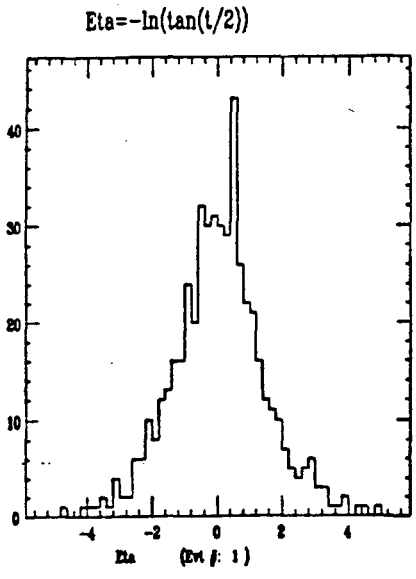


Fig. B2

REFERENCES

[1] L.Janossy and G.D.Rochester, Proc.Roy.Soc. **A183**, 183 (1944)
[2] W.Heisenberg Kosmische Strahlung, Springer,Berlin (1953), p.148 ff.
[3] W.Heitler and L.Janossy, Proc.Phys.Soc. **62**,374 (1949)
[4] E.Fermi, Progr.Theor.Phys. **5**, 270 (1950)
[5] L.Landau, Izv.A.N.SSSR, s.fiz., **17**, 1 (1953)
[6] V.Ammossov *et al.*, Phys.Rev.Lett.,**42B**, 519 (1972)
[7] A.Firestone *et al.*, Phys.Rev. **D10**, 2080 (1974)
[8] G.J.Alner *et al.*, Phys.Lett. **160B**, 199 (1985)
[9] R.E.Ansorge *et al.*, Zs.f.Phys.C– Particles and Fields, **43**,357 (1989)
[10] G.N.Fowler,E.M.Friedlander,R.M.Weiner, Phys.Lett. **104B**, 239 (1981)
[11] J.Alner *et al.*, Nucl.Phys. **B291**, 445 (1987)
[12] M.Adamus *et al.*, Zs.f.Phys. **C37**, 215 (1988)
[13] G.J.Alner *et al.*, Phys.Rep. **154**, 247 (1987)
[14] G.Goldhaber *et al.*, Phys.Rev. **120**, 840 (1960)
[15] G.J.Kopylov and M.I Podgoretsky, Yad.Fiz. **18**, 656 (1973)
[16] G.Cocconi, Phys.Lett. **49B**, 459 (1974)
[17] M.Deutschmann *et al.*, Nucl.Phys. **103B**, 198 (1976)
[18] M.Gyulassy ,S.Kaufmann and L.Wilson, Phys.Rev. **C20**, 2267 (1979)
[19] G.N.Fowler and R.M.Weiner, Phys.Rev. **D17**, 3118 (1978)
[20] T.F.Hoang , Phys.Rev. **D10**,1043 (1974)
[21] E.M.Friedlander and I.Stern, to be published
[22] G.N.Fowler *et al.*, Phys.Rev.Lett. **56**, 14 (1985)
[23] J.Cohen *et al.*, Lett.Nuovo Cim., **9**, 337 (1974)
[24] S.L.McCall and E.L.Hahn, Phys.Rev.Lett. **18**, 908 (1967)
[25] G.N.Fowler *et al.*, ibid., **17**, 2119 (1986)
[26] C.C. Shih, Phys.Rev. **A40**, 1456 (1989)
[27] E.M.Friedlander and M.Pluemer, LBL Report 27082 (1989)
[28] A.K.Jaiswal and C.L.Mehta, Phys.Rev.**A 2**,168 (1970)
[29] G.N.Fowler *et al.*, Phys.Rev.**D37**, 3127 (1988)
[30] G.J.Alner *et al.*, Phys.Lett. **B160**, 199 (1985)
[31] G.N.Fowler *et al.*, LBL-28956,April 1990 and Journ. of Phys., under press
[32] R.M.Weiner, Proc. Hadron Matter In Collision, Tucson (1988) p.187
[33] T.H.Burnett *et al.*, Phys.Rev.Lett., **50**, 2062 (1983)
[34] E.M.Friedlander, Proc. LESIP I Conference Bad Honnef, 1984, p.105
[35] W.Kittel Proc.XVIII Int.Symp. on Multiparticle Dynamics, Tashkent (1987) p.3
[36] E.M.Friedlander *et al.*, Proc. Hadron Matter In Collision, Tucson (1988) p.210
[37] A.Bialas and R.Peschanski, Nucl.Phys. **B273**, 703 (1986)
[38] P.Carruthers *et al.*, Phys.Lett., **B222**, 487 (1989)
[39] A.Vourdas and R.M.Weiner , Phys.Rev. **D38**, 37 (1988)
[40] D.Brick *et al.*, Phys.Lett. **103B**, 242 (1981)
[41] G.N.Fowler *et al.*, Phys.Rev. **D37**, 3127 (1988)

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