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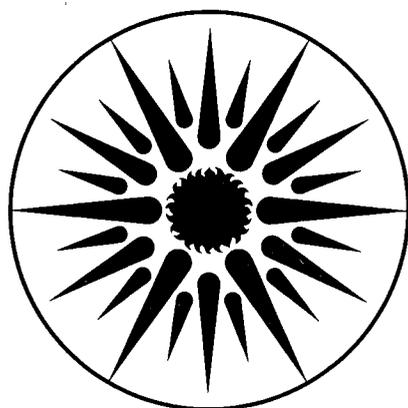
APPLIED SCIENCE DIVISION

To be presented at the 1990 Fall Meeting of the
Western States Section of the Combustion Institute,
La Jolla, CA, October 14-16, 1990, and
to be published in the Proceedings

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October 1990



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**STATISTICAL ERRORS IN THE
FRACTAL ANALYSIS OF FLAME BOUNDARIES**

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This work was supported by the Director, Office of Energy Research, Office of Basic Energy Research, Chemical Sciences Division of the U.S. Department of Energy under contract No. DE-AC-03-76SF00098.

ABSTRACT

A high speed tomographic technique is used to evaluate the effect of spatial resolution, and requirements for statistical convergence on the fractal analysis of a turbulent, premixed, stoichiometric methane/air flame at high Damkoehler number. The gas velocity at the nozzle exit is 5 m/s, the turbulence intensity is 7%, the integral length scale 3 mm and hence the turbulence Reynolds number is 70. The light source is a copper vapor laser which produces 20ns, 5 mJ pulses at a 4KHz repetition rate. Cylindrical lenses transform the 38mm circular laser beam to a sheet 50 mm high and 0.6 mm thick. A high speed Fastax camera is used to record the tomographic images formed by the scattering of light from oil droplets seeded in the reactant flow. The films are digitized and the flame front extracted from the images by a thresholding technique. Digitization noise, which appears in the fractal plots at approximately twice the pixel resolution, can obscure the inner cutoff. Simple smoothing can remove this problem if the spatial resolution is sufficient. At insufficient resolution smoothing produces plausible results are produced which in fact erroneous. If the inner cutoff is ambiguous the range over which the fractal dimension is determined will be unclear. The wide distribution of fractal dimensions obtained from the individual images indicates the necessity of ensemble averaging the fractal plots if reliable statistical results are to be obtained.

INTRODUCTION

The availability of high power lasers and the development of effective seeding techniques has made laser tomography [1,2] a very convenient diagnostics technique for investigating the effects of turbulence on premixed flames. At high Damkohler numbers, where the instantaneous flame thickness is small, the flame can be treated as a sheet and the flame sheet boundaries can be derived from the tomographic images. This data can then be analyzed to determine the spatial scales of the flame wrinkles and the turbulent burning rate [3]. A interesting new method of characterizing the geometry of flame sheets is by the use of fractal analysis [4-6] in terms of the fractal dimension, D , the inner cut-off, ϵ_i , and the outer cut-off, ϵ_o . In this paper the influence of resolution, digitization noise, the number of records used for averaging, and the method of analysis on the experimental determination of the fractal parameters of premixed, turbulent, stagnation-point flames will be investigated.

Fractal Theory for Premixed Turbulent Flames:

Fractal theory has been developed to characterize the self-similarity of non-Euclidean objects [7]. A power law relationship exists between the size of the fractal object and the measurement scale at which this size is determined. The measured length, L , of a fractal curve increases with decreasing

measurement scale, ϵ .

$$L(\epsilon) = \epsilon^{(1-D_2)} \quad (1)$$

where, D_2 , is the fractal dimension. Additional parameters exist for physical objects, such as flame boundaries, which describe the self similarity limits : ϵ_i , the inner cut-off and ϵ_o , the outer cut-off. A fractal theory for turbulent premixed flames has been developed by Gouldin [4] in which the increase in flame area per unit volume due to turbulence can be evaluated from the fractal parameters. Fractal analysis of the flame boundaries gives

$$\left(\frac{\epsilon_i}{\epsilon_o}\right)^{1-D_2} = \frac{L(\epsilon_i)}{L(\epsilon_o)} \quad (2)$$

The flame surface is not an isotropic fractal surface, and so the well known relationship which relates the fractal dimension of a cross-sectional line (D_2) to that of the surface area (D_3)

$$D_3 = D_2 + 1$$

cannot be used. The law of addition of fractal dimensions can, however, be invoked to estimate the flame area ratio. The ratio of turbulent/laminar burning rate, \bar{W} can be obtained by assuming that all cross-sections that pass through the stagnation line have the same fractal dimension. This implies that

$$\left(\frac{\epsilon_i}{\epsilon_o}\right)^{2(1-D_2)} = \frac{A(\epsilon_i)}{A(\epsilon_o)}$$

hence

$$\bar{W} = \frac{A_T}{A_L} = \left(\frac{L(\epsilon_i)}{L(\epsilon_o)}\right)^2 = \left(\frac{\epsilon_i}{\epsilon_o}\right)^{2(1-D_2)} \quad (3)$$

EXPERIMENTAL DETAILS

Since the scalar field of the premixed flames to be studied here consists essentially of burned and the unburned states separated by a thin flame sheet, the scalar properties, such as temperature or density, can be determined by measuring the intensity of light scattered from micron sized oil droplets which evaporate at the flame sheet. To obtain tomographic images of the flame zone, the oil droplets are illuminated by a laser sheet and the Mie scattering in the direction normal to the laser sheet is photo-

graphed. The flame sheet is marked as the interface between light (cold reactants with seed particles) and dark (hot products without particles) regions on the tomographic record.

A tomographic study was performed on a methane/air stagnation point premixed turbulent flame where Damköhler number based on the chemical reaction time and the integral time scale of the reactant stream is much greater than one. Figure (1) shows a schematic of the experimental setup. A uniform axisymmetric flow of premixed fuel/air mixture at 5 m/s is provided by a 50 mm diameter nozzle with a coflowing air stream at the same velocity which shields the inner flow from interaction with the room air. The reactant flow turbulence (7%), generated by a perforated plate placed 50 mm upstream of the burner nozzle, has an integral length scale of 3 mm and the turbulent Reynolds number, based on these values, is 70. The burner configuration has been described in detail elsewhere [8]. The stagnation plate was placed 100 mm downstream of the nozzle exit.

The light source was a Metalaser copper vapor laser which affords significant advantages for laser sheet imaging. It delivers 5 mJ per pulse with a 20-30 nsecs pulse width but is also capable of repetition rates up to 10 KHz. Hence it is possible, not only to resolve the instantaneous flame shape, but also to follow the evolution of the flame with time. By the use of cylindrical lenses, the 38 mm diameter laser beam is transformed to a sheet 0.6 mm thick by 50 mm high. The reactant flow is seeded with silicone oil droplets (approximately 1 micron diameter) generated by a blast atomizer which evaporate at the flame front ($\approx 500\text{K}$). The laser sheet is recorded at 4 KHz by a high speed 16 mm Fastax camera which provides a trigger pulse for the laser. Film is a convenient and economical means of recording and storing the large amount of data necessary for statistical analysis to be presented here.

Data Analysis:

The film is projected onto a screen and digitized by a video camera to give 512 X 512 pixel images with 256 gray scales of light intensity resulting in horizontal and vertical resolutions of 0.155 and 0.121 mm per pixel, respectively. A typical digitized image of the stagnation flame is shown in figure (2) where the flame boundary is clearly visible. The insert in Figure (2) shows the histogram of pixel intensity

(range of 200 grey levels) and illustrates the two state nature of the images. Digitization of film images by video camera also offers considerable flexibility in optimizing the pixel resolution for the fractal analysis. Flame boundaries as defined by an intensity threshold are generated by an edge finding algorithm which gives a contiguous flame edge (characterized by steps). The threshold is determined by inspection of the histogram of pixel intensity: the results are not sensitive to the precise value of the threshold. A fractal analysis is applied to 100 to 250 flame edges satisfying the windowing criterion and the results are ensemble averaged.

RESULTS AND DISCUSSION

Fractal Analysis of Flame Boundaries:

Many different analysis methods are available to determine the fractal parameters from the digitized flame boundaries. Among them are the stepping caliper method, the box counting method and the circle method. The stepping caliper method has been used here because the flame boundaries are continuous and it seems to be the most sensitive. With this method the flame length is determined by stepping along the boundary using a given scale ϵ_n . The flame length measured at that scale is then

$$L(\epsilon_n) = N \epsilon_n$$

where N is the number of steps necessary to cover the whole flame length. The fractal dimension can be determined from fractal plots of $\log L(\epsilon_n)$ versus $\log \epsilon_n$, figure (3a), or $\log N(\epsilon_n)$ versus $\log \epsilon_n$ figure (3b). For the same set of data the L versus ϵ plot gives a clearer indication of the fractal parameters because at scales below the inner cutoff the length of the flame is a constant and the slope of the curve 0. In the plot of the number of segments, figure (3b), however, the slope of the curve tends to -1 and at fractal dimensions characteristic of flame surfaces the position of the inner cutoff is difficult to discern.

Effects of Reduced Resolution on Fractal Parameters:

The effects of reduced spatial resolution on fractal plots is estimated by degrading the resolution of the digitized flame boundaries by summing and averaging pixels to give effective pixel resolutions down

to 1.24 mm. The effect of such averaging on a segment of a flame boundary is shown in figure (4a). Fractal plots, generated from these new data sets at reduced resolution, figure (4b), show that for all cases the impact of digitization noise starts to appear in the fractal plots when ϵ is less than approximately twice the pixel resolution. The pixel resolution is marked by crosses on figure (4b). This suggests a useful empirical criterion for estimating the resolution requirement for future work. Although changes in pixel resolution have no observable effect on the outer cut-off, ϵ_o , with reduced resolution, the inner cut-off, ϵ_i , becomes obscured and so the self-similar region over which the fractal dimension should be evaluated cannot be determined with confidence. Furthermore, when ϵ_i is not clear, the flame length ratio between the inner and outer cutoffs, given by equation (2), cannot be resolved as the maximum length plateau does not appear in the fractal plot.

Effects of Smoothing on Fractal Parameters:

The effects of digitization noise observable in figure (4b) may be removed by simple 5×7 or 5×3 unweighted averaging of the flame boundaries, figure (5a). Reanalysis of the data sets with 5×7 smoothing gives the fractal plots presented in figure (5b). The maximum length plateau is now unambiguous and the flame length ratio, the inner cutoff and the fractal dimension can be defined with confidence. For cases with insufficient resolution, however, smoothing the flame edges produces results which look reasonable but are in fact erroneous.

Convergence Criteria:

A fractal plot obtained from a single image is characterized by large scatter in the fractalized region, figure (6a) and the extraction of information from such plots, although sometimes attempted, is clearly hazardous. Ensemble averaging using 10 frames reduces the scatter but is insufficient to give the statistical mean of the fractal dimension. In this study the minimum number of images needed for convergence is twenty five. To determine the distribution of fractal dimension within the data set the fractal dimension for a single flame boundaries was calculated from the length ratio for that boundary and the cutoffs determined for the whole set. Figure (6b) shows that the fractal dimension, D has a large statistical

distribution indicating the need for statistical averaging.

CONCLUSIONS

1) The effects of spatial resolution, digitization noise, and requirements for statistical convergence on the fractal analysis of premixed turbulent flame boundaries have been identified and evaluated.

2) The determination of the fractal dimension relies on a clear indication of the fractal range i.e. the linear region of the fractal plot and its limits, the inner and outer cutoffs. For typical experimental data this can only be achieved by the statistical averaging of many realizations.

3) The inner cut-off can be obscured by digitization noise and insufficient spatial resolution. The effects of digitization noise on ϵ_i , can be removed by smoothing the flame boundaries only when the spatial resolution is sufficient.

4) The requirements necessary to resolve the outer cutoff are analagous to those for large scale turbulent fluctuations: the record size, i.e. the field of view of the tomograph, has to be larger than the largest significant scale, i.e. ϵ_o . This also implies that a consistent record size, i.e. flame boundary length, will improve the accuracy of ϵ_o .

5) In principle, the fractal parameters can be derived from fractal plots of either L versus ϵ or N versus ϵ . It is, however, more difficult to identify the fractal range on the N versus ϵ plot. Therefore, the L versus ϵ plot provides a more sensitive means of extracting the fractal parameters from experimental data.

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FIGURE CAPTIONS

- 1) Schematic of experimental apparatus
- 2) Typical image of flame edge. Inset of histogram of pixel intensity.
- 3a) Fractal plot: $\text{Log}(\epsilon)$ versus $\text{log}(L)$.
- 3b) Fractal plot: $\text{Log}(\epsilon)$ versus $\text{log}(N)$.
- 4a) The effect of reduced resolution on a flame edge.
- 4b) Fractal plots at reduced resolution.
- 5a) The effect of smoothing on a flame edge.
- 5b) Fractal plots of smoothed edges.
- 6a) The effect of ensemble averaging on fractal plots.
- 6b) The probability distribution function of fractal dimension.



HIGH-SPEED TOMOGRAPHY

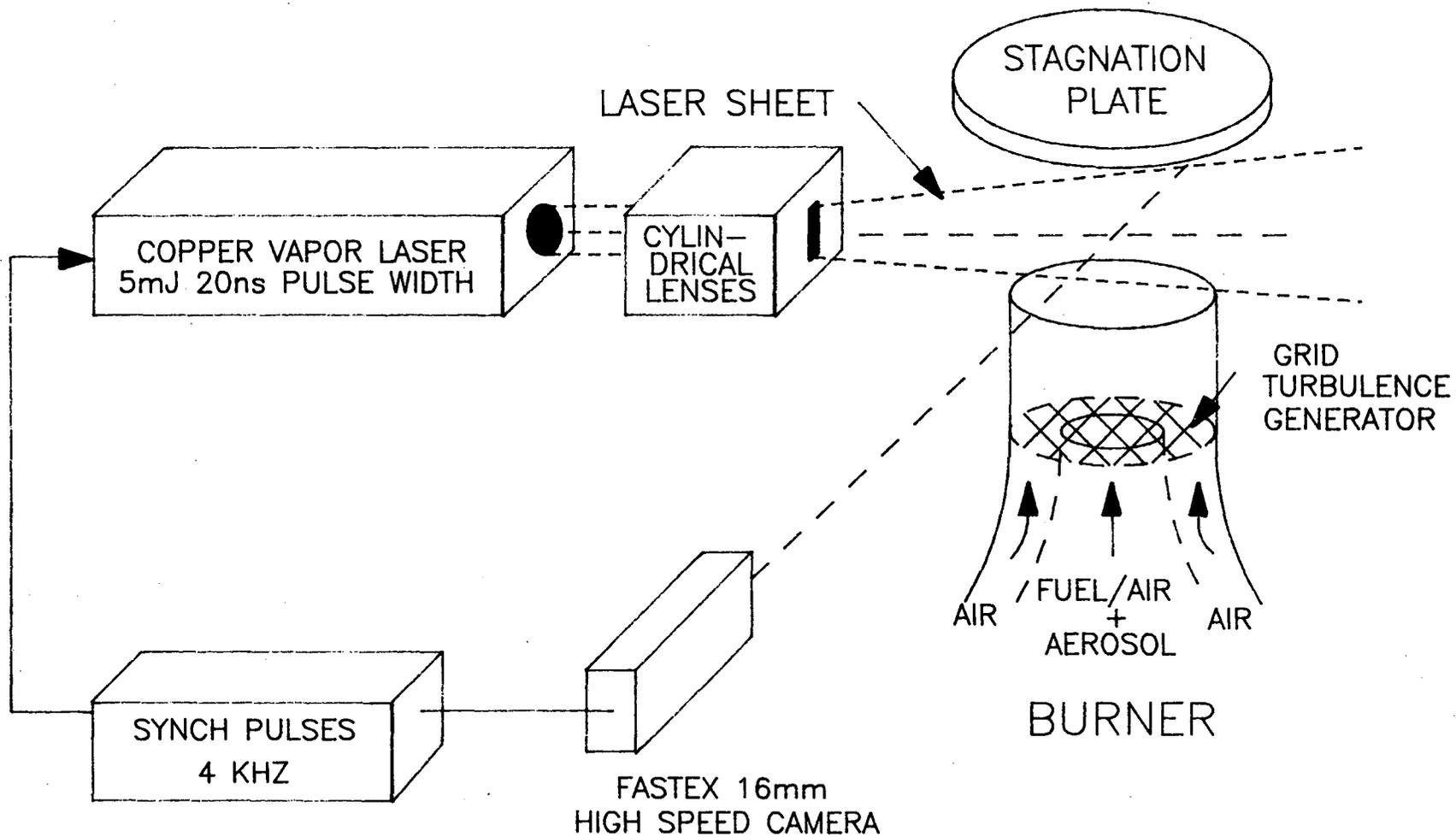


Figure 1

Instantaneous Flame Boundary

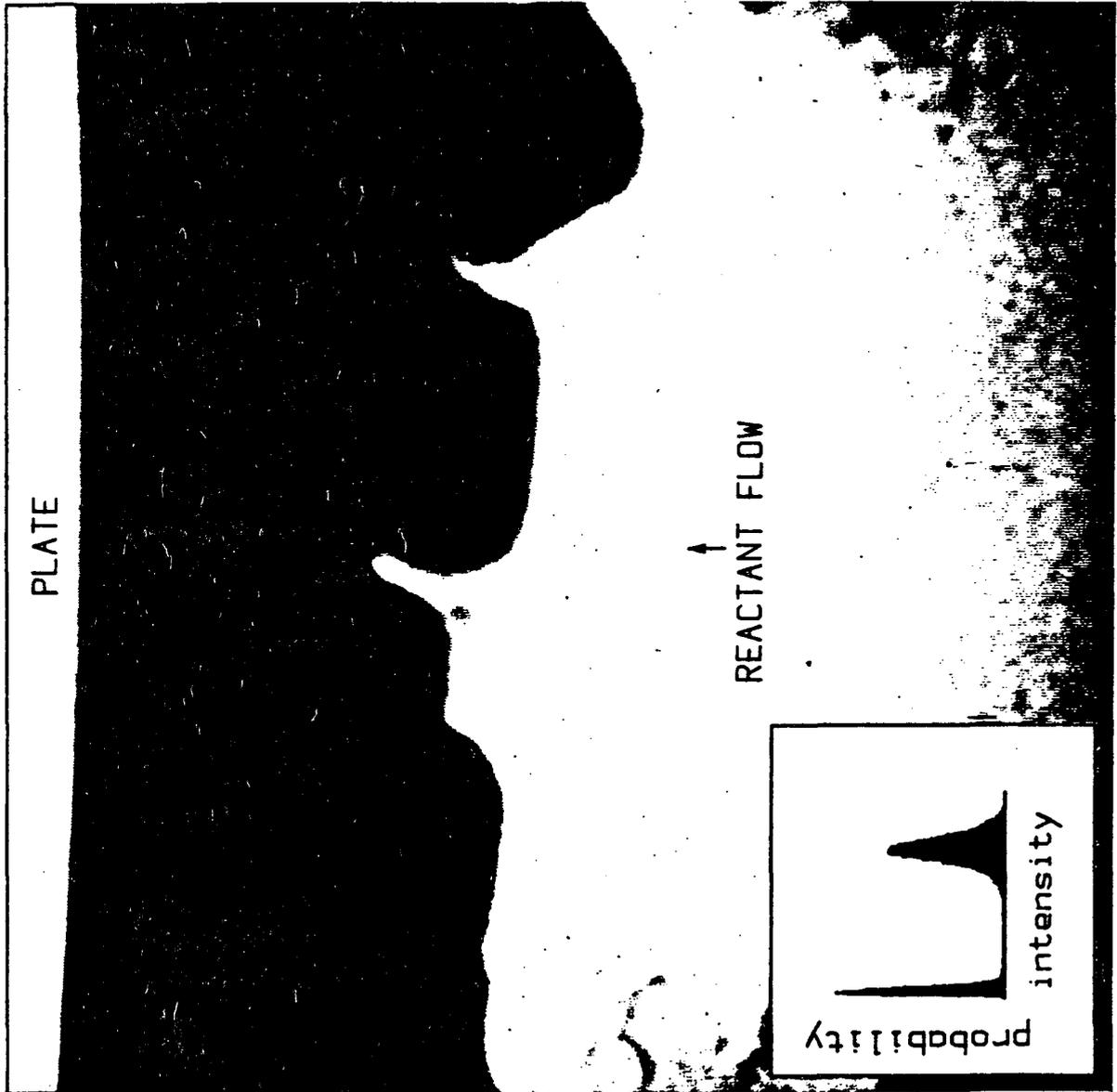


Figure 2

Fig. 2

inner cut-off=1.93 mm, outer cut-off=22.7 mm
fractal dimension = 1.16
flame length ratio = 1.48

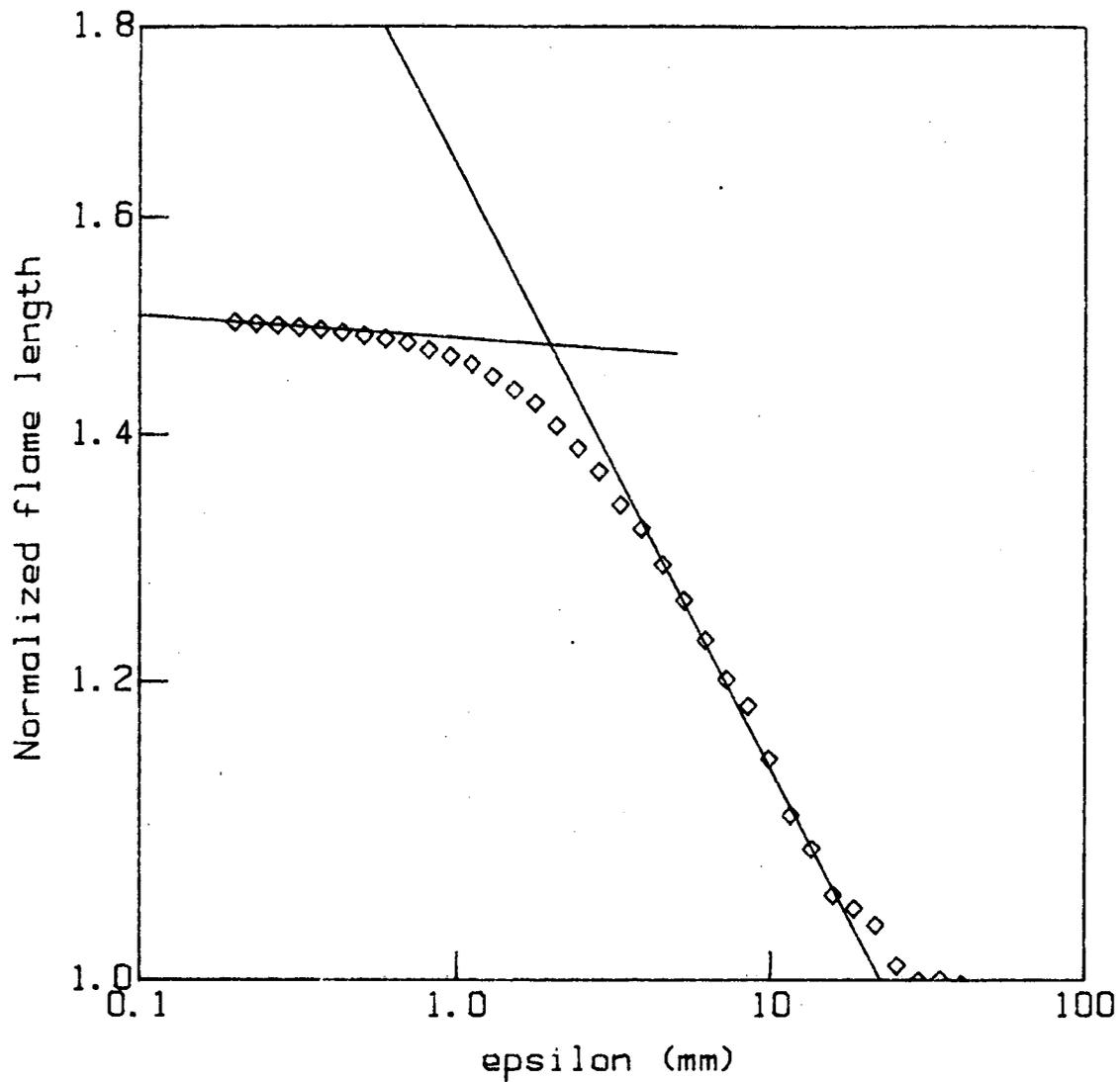


Figure 3a

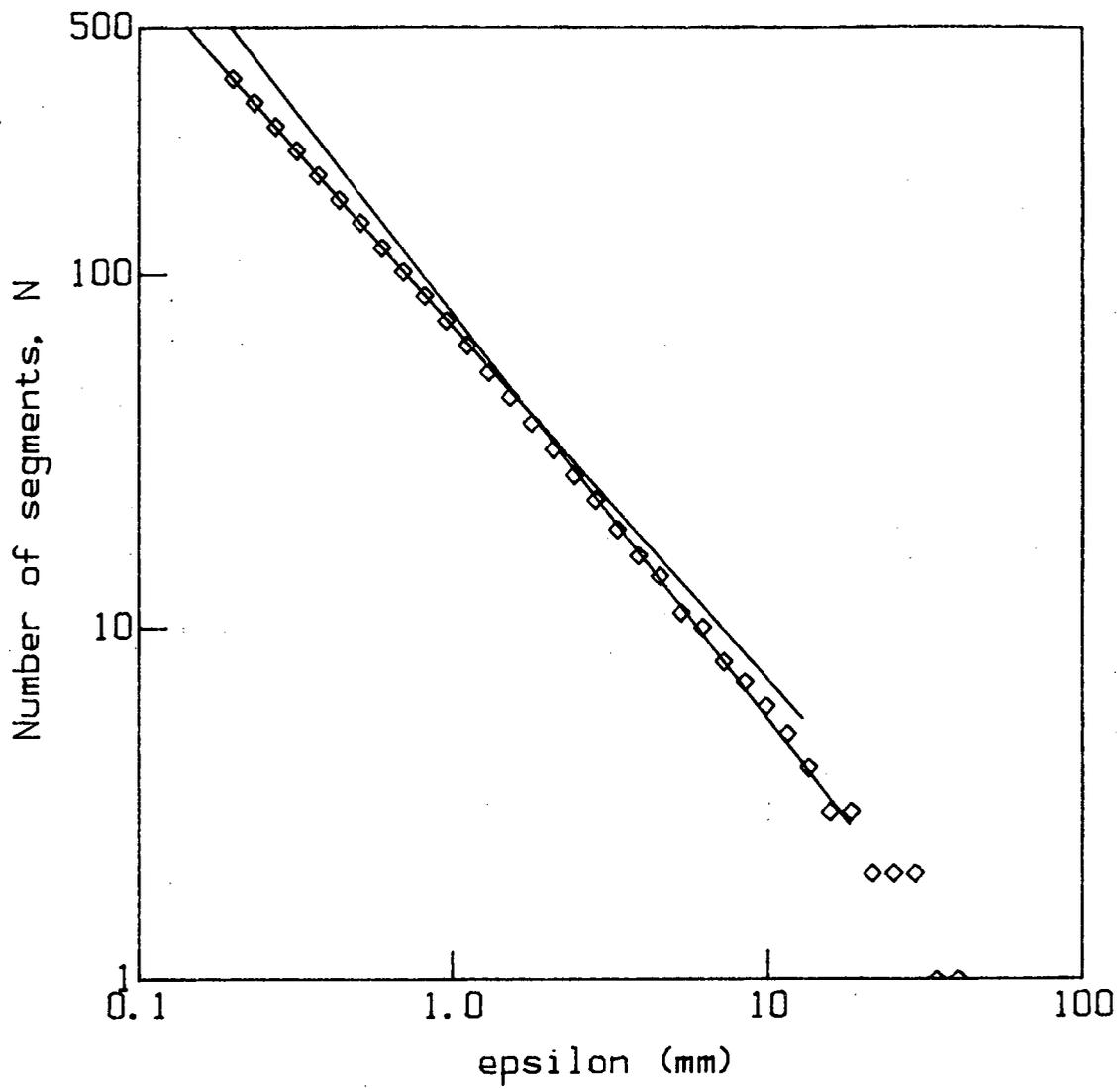


Figure 3b

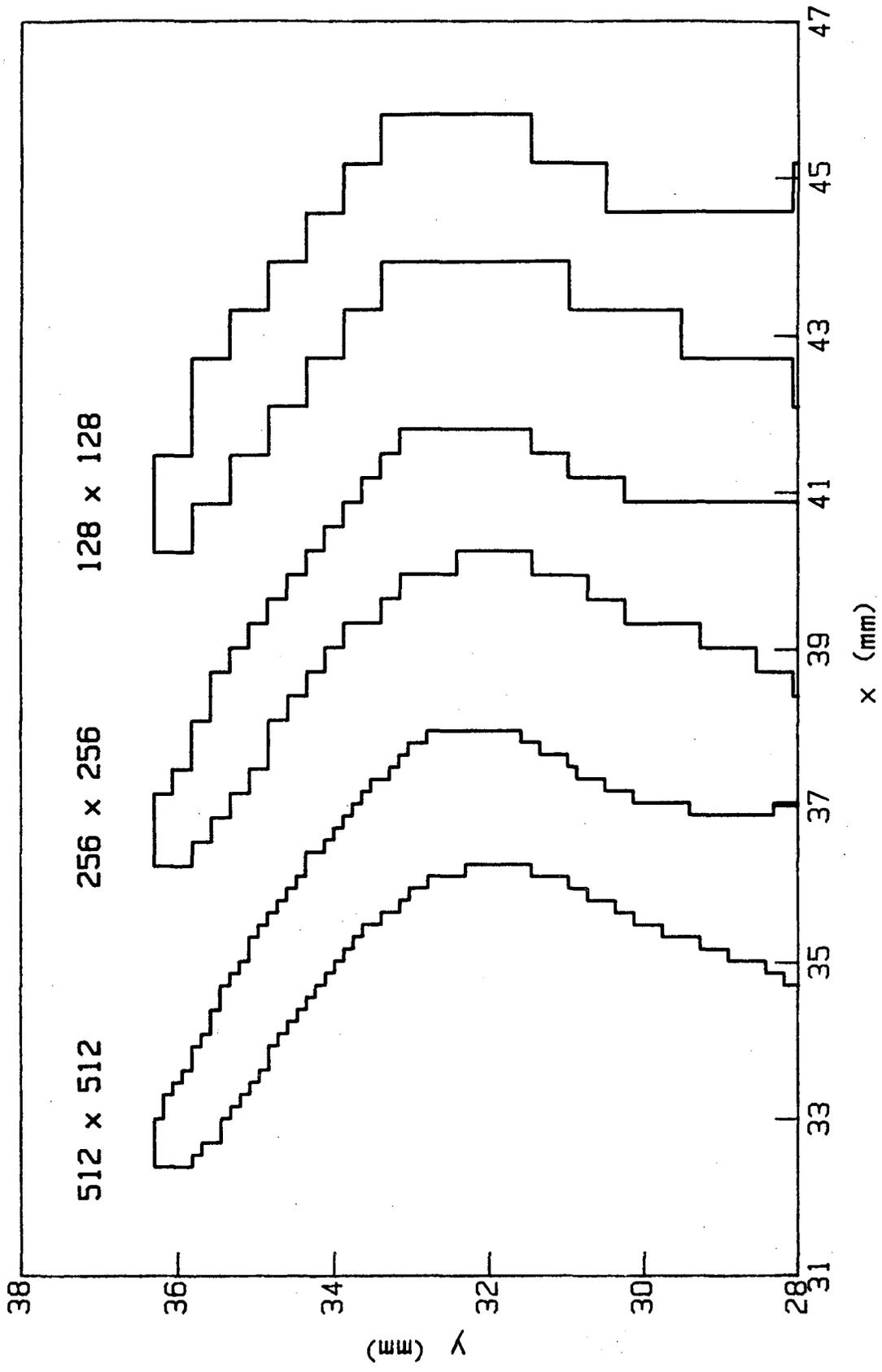


Figure 4a

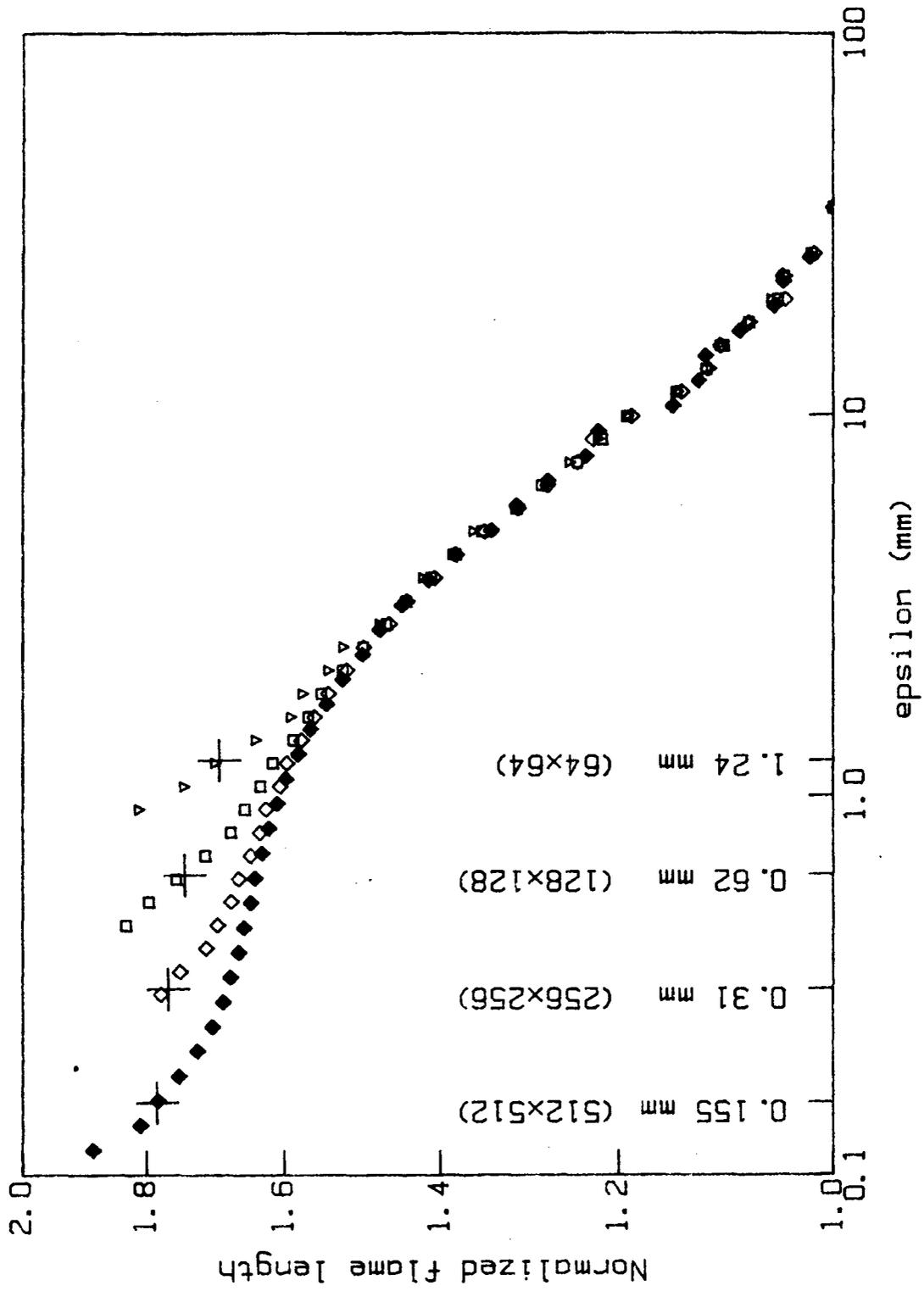


Figure 4b

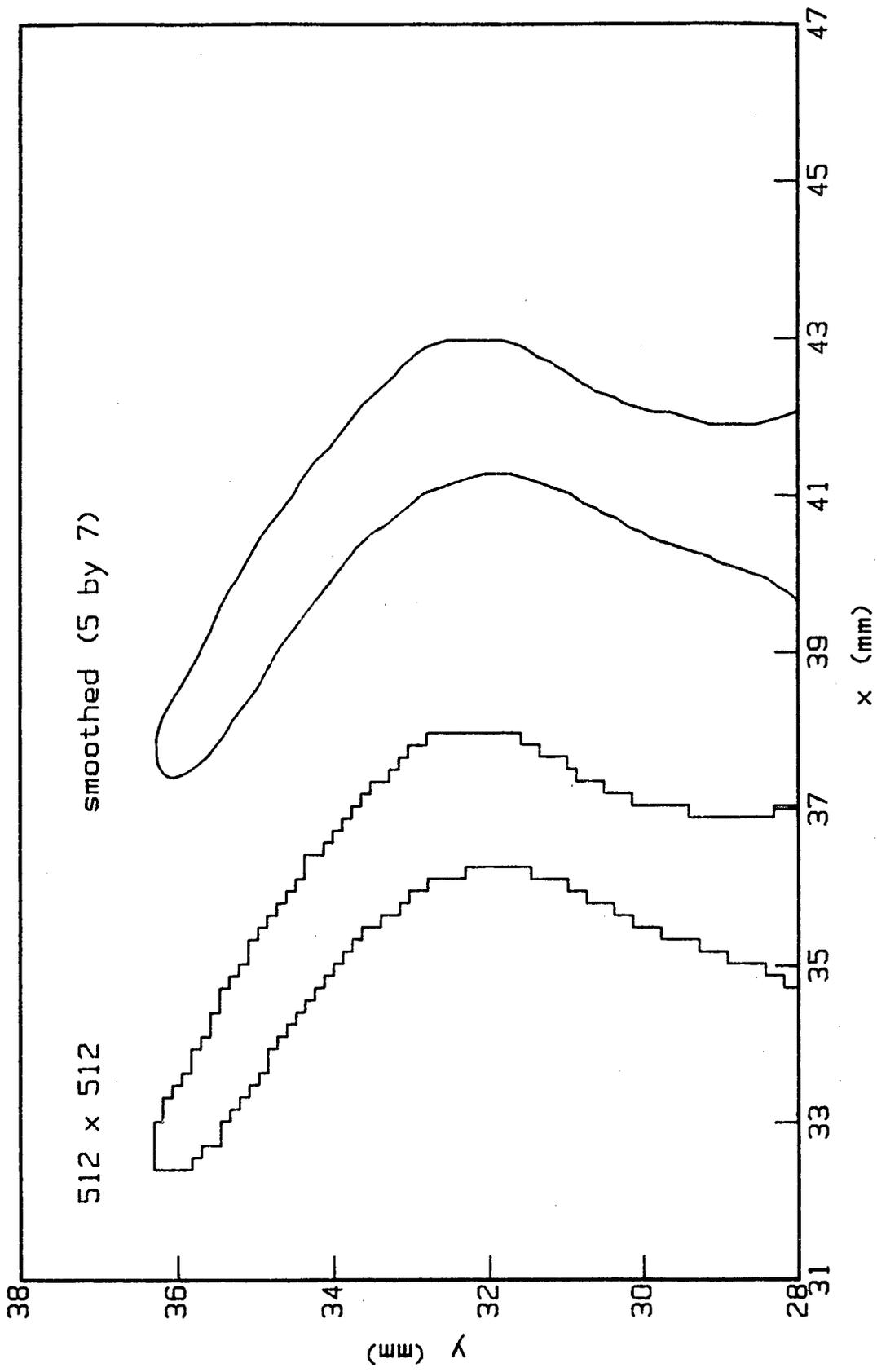


Figure 5a

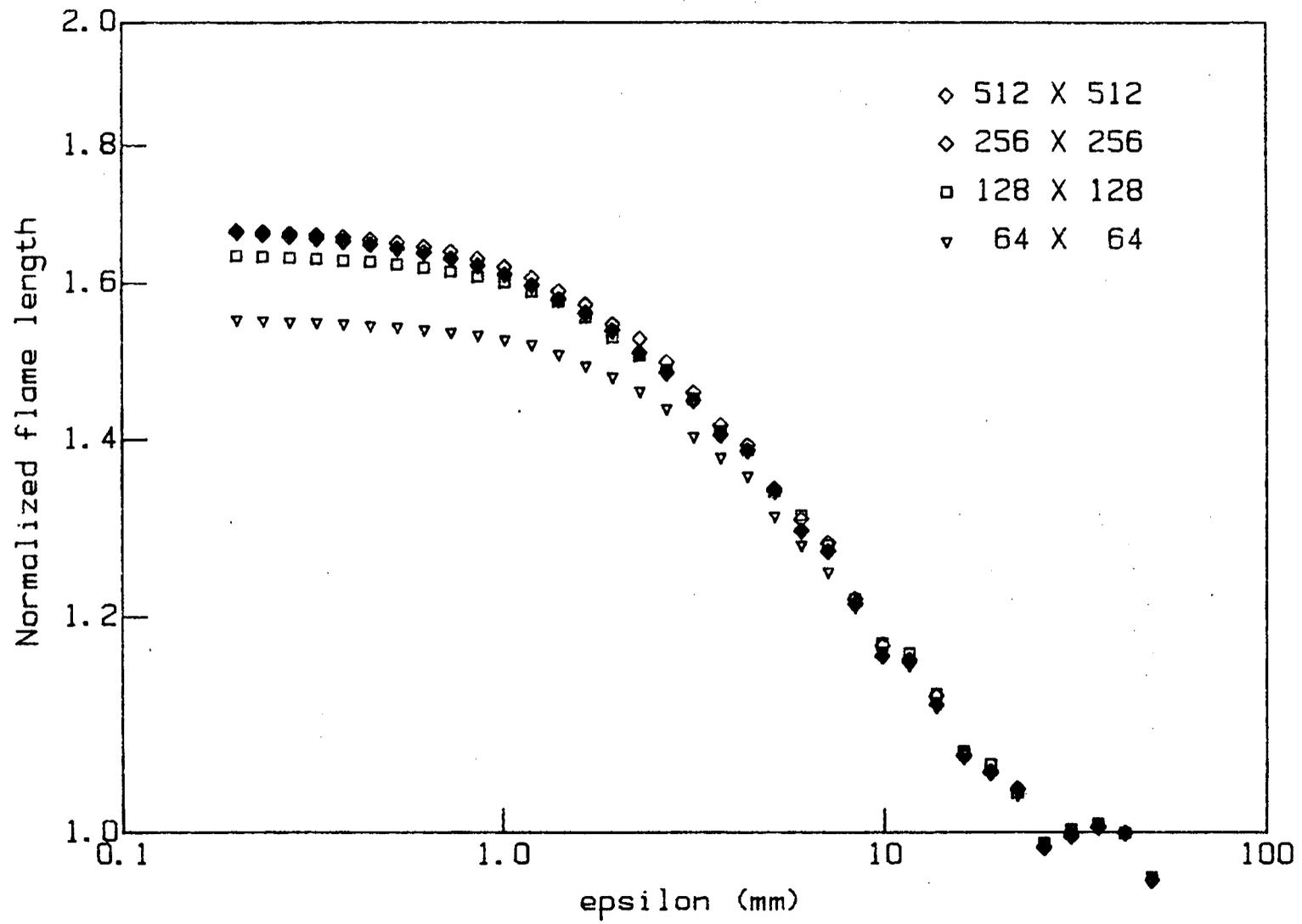


Figure 5b

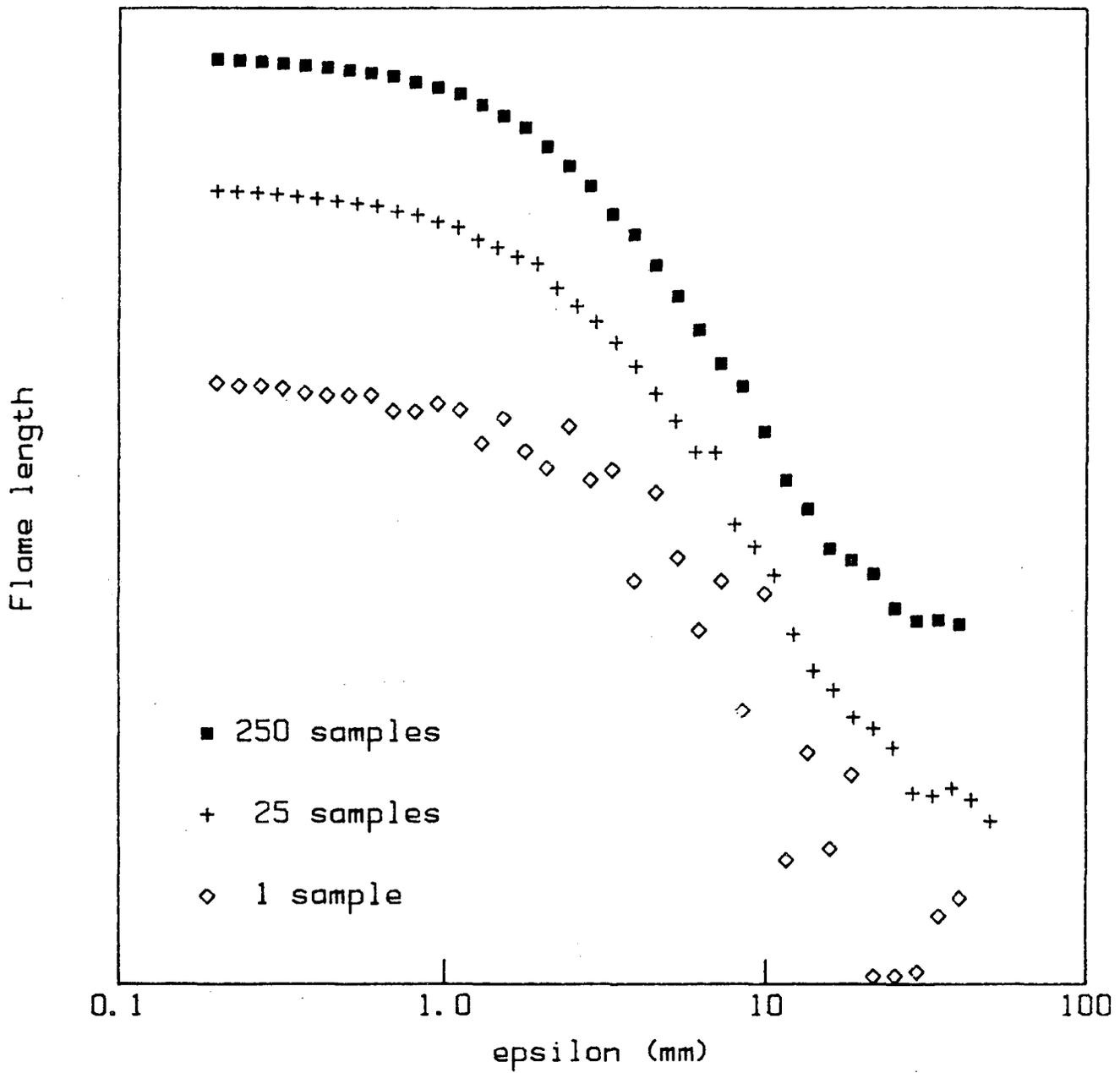


Figure 6a

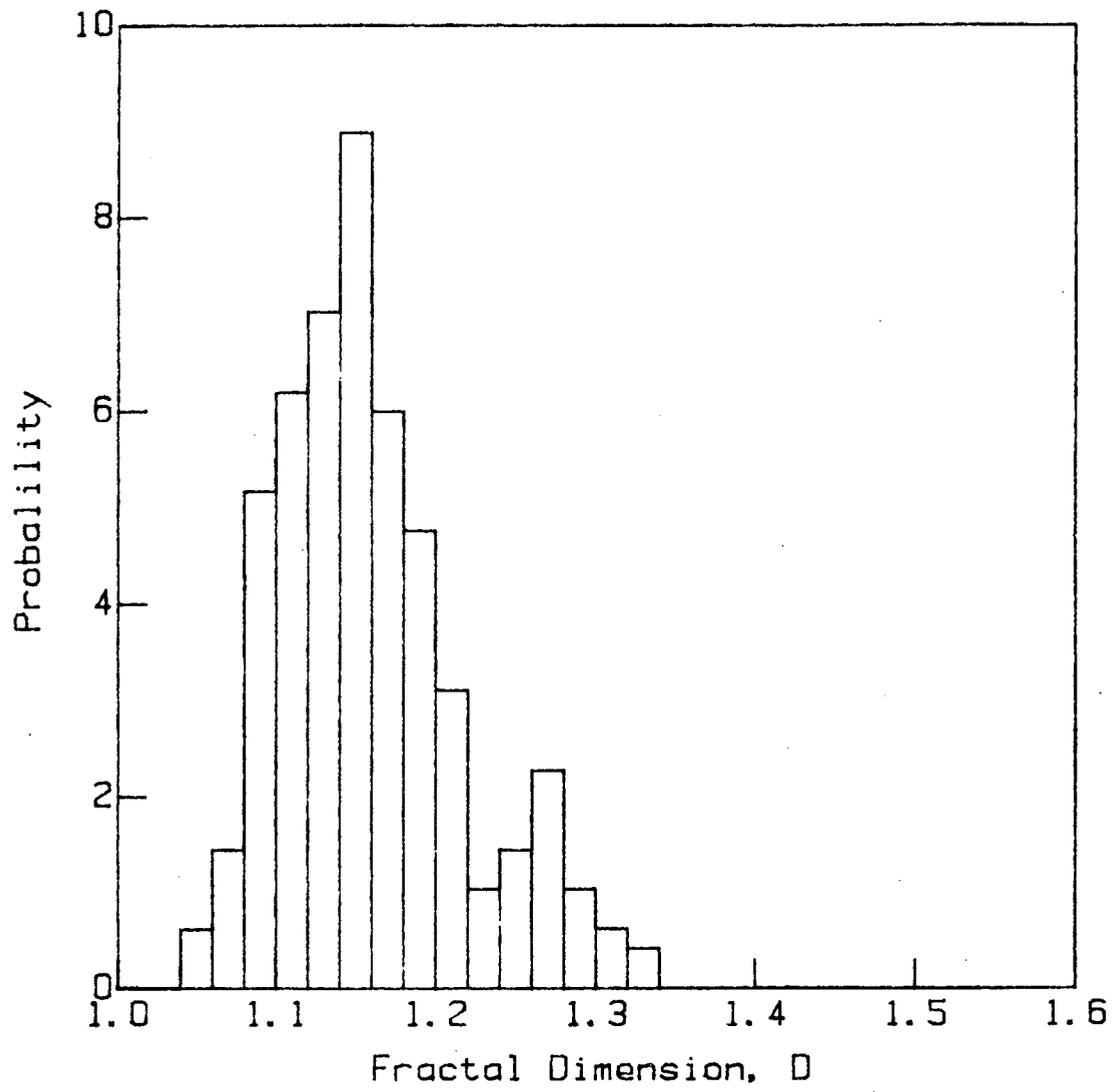


Figure 6b

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