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Abstract

Models of general relativistic rotating neutron stars, constructed from Hartle's perturbative "slow" rotation formalism of massive relativistic objects, are compared with their counterparts obtained from the exact solution of Einstein's equations. It is found that both methods, perturbative versus exact, lead to compatible results down to rotational periods $P \approx 0.5$ msec, a value which is by far smaller than the smallest yet observed pulsar period. This finding rests on the reinvestigation of Hartle's method, (1) supplementing it by a self-consistency condition inherent in the determination of the Kepler frequency, and (2) a careful analysis of sequences of star models. A collection of seventeen representative neutron matter equations of state served as an input. Because of its simple structure, Hartle's method should prove to be a practical tool for testing models of the nuclear equation of state with data on pulsar periods.

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Applicability of Hartle's Method for the Construction of General Relativistic Rotating Neutron Star Models

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1 Introduction

The discovery of the first millisecond pulsar in 1982 (Backer et al. 1982) has stimulated interest in the rotation of neutron stars especially as concerns possible constraints on their structure. The realization that globular clusters are ideal environments for the formation of binaries including neutron star binaries in which accretion from the companion spins up the compact star, promises much more data (Backer and Kulkarni 1990). Indeed half of the ten known millisecond pulsars are in globular clusters. Although even the smallest observed period does not as yet place any serious constraint on the theory of pulsar structure, this situation could easily change. In this regard it is relevant to note that pulsar surveys involve compromises that impose a bias against the detection of pulsars with small periods. The large surveys have had biases against detection below 4 ms, thus presumably distorting the statistics on pulsar periods (Tayor and Stinebring 1986).

Against this background we have reexamined Hartle's method (Hartle 1967, Hartle and Thorne 1968) of constructing rotating neutron star models. Because of their large mass densities ($\approx 10^{15}$ g/cm^3 in the cores of heavier neutron stars), the geometry of space-time deviates considerably from flat space. Therefore such objects must be treated in the framework of Einstein's theory of general relativity. However, solving Einstein's field equations for rotating objects is a very complicated and cumbersome task (see, for example, Butterworth and Ipser 1976). It has been treated to date for the case of general neutron matter equations of state only by two groups, i.e., Friedman, Ipser, and Parker (1986; 1989) and Lattimer, Prakash, Masak, and Yahil (1990). These authors studied a collection of a total of sixteen different neutron star

matter equations of state, covering realistic ones as well as equations of state which have meanwhile become outdated.

Hartle's perturbative method of treating general relativistic, rotating massive objects provides an alternative to the exact numerical treatment. However since it is perturbative with respect to deviations from spherical symmetry caused by rotation, its applicability is restricted to "slow" rotation. *The upper limit on rotation to which it is actually applicable remained to date an unanswered question.* In comparison with the exact solution of Einstein's equations, Hartle's method is much easier to implement, and from this work we believe it is an appropriate tool for the construction of models of rotating neutron stars in the framework of general relativity. In particular, the compatibility of different competing models of the nuclear equation of state with data on pulsar periods can be tested conveniently. This should help clarify the behavior of the nuclear equation of state at densities beyond normal nuclear matter density. We will find that the minimum periods so far observed of $P = 1.6$ msec in the case of pulsars PSR 1937+21 and PSR 1957+20 can easily be computed by Hartle's method, as well as those minimum periods which are likely to be achieved by *neutron stars* (Glendenning 1990).

Our aim in this article is to provide evidence concerning the applicability of Hartle's method for the construction of rotating objects in the framework of general relativity. For this purpose the investigation deals with the following four major aspects: (1) A self-consistency condition implicit in the determination of the Kepler frequency is imposed on Hartle's method (Sect. 2). This essential ingredient has only recently been discussed in connection with Hartle's method by Weber, Glendenning, and Weigel (1990*a*; 1990*b*). The Kepler frequency is calculated for a selected collection of neutron matter equations of state (Sect. 3) in the following Sect. 4. (2) The empirical formula for the Kepler frequency is discussed in Sect. 5. (3) For each of the equations of state, we construct the entire sequence of stars that are rotating at their Kepler frequencies in Sect. 6. For several of these, a detailed comparison between the results of the exact and perturbative method is performed, in particular for the Pandharipande (1971), Bethe-Johnson (1974), and Friedman-Pandharipande (1981) equations of state. (4) Finally an analysis based on the Hartle formalism is performed in Sect. 7 to motivate the empirically established formula for the Kepler frequency, which has become a practical and frequently used tool for estimating the absolute limit on rotational frequency of a neutron star model derived from a specific

equation of state. We summarize our results in Sect. 8.

2 Imposition of self-consistency on Hartle's stellar structure equations

The basic idea in Hartle's treatment is the development of a perturbation solution based on the Schwarzschild metric of a static, spherically symmetric object:

$$ds^2 = -e^{2\Phi}dt^2 + e^{2\lambda}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \tag{1}$$

Rotation distorts the star away from spherical symmetry, and its perturbed metric, expanded through second order in the star's rotational velocity, Ω, has the form (Hartle 1967; Hartle and Thorne 1968)

$$ds^2 = -e^{2\nu}dt^2 + e^{2\psi}\left(d\phi - \omega\, dt\right)^2 + e^{2\mu}\, d\theta^2 + e^{2\lambda}dr^2 + O\left(\Omega^3\right). \tag{2}$$

In this line element, ω is the angular velocity of the star's fluid in a local inertial frame and depends on the radial coordinate r. It is related to Ω, which is a constant (i.e. uniform rotation) throughout the star's fluid, and is the rotational frequency seen by a distant observer.

The metric functions in the perturbed line element of Eq. (2) have the form

$$e^{2\nu(\Omega)} = e^{2\Phi}[1 + 2\left(h_0 + h_2 P_2(\cos\theta)\right)], \tag{3}$$

$$e^{2\psi(\Omega)} = r^2\sin^2\theta[1 + 2\left(v_2 - h_2\right)P_2(\cos\theta)], \tag{4}$$

$$e^{2\mu(\Omega)} = r^2[1 + 2\left(v_2 - h_2\right)P_2(\cos\theta)], \tag{5}$$

$$e^{2\lambda(\Omega)} = \left(1 + \frac{2}{r}\,\frac{m_0 G + m_2 P_2}{1 - \frac{2mG}{r}}\right)\left(1 - \frac{2mG}{r}\right)^{-1}. \tag{6}$$

The quantity $\Phi(r)$ in Eq. (3) denotes the metric function of a spherically symmetric object and $m(r)$ the mass within r for the corresponding spherical star, and P_2 is the Legendre polynomial of order 2. The perturbation functions m_0, m_2, h_0, h_2, and v_2, all functions of r and Ω, are to be calculated from Einstein's field equations and are given as solutions of Hartle's stellar structure equations (for more details see Hartle 1967; Hartle and Thorne 1968; Datta 1988).

Next we introduce the general relativistic Kepler frequency, denoted by Ω_K. It is given as the solution, Ω, of (Friedman, Ipser, and Parker 1986)

$$\Omega = e^{\nu-\psi}\; V(\Omega) \;+\; \omega(\Omega), \quad V(\Omega) \equiv \frac{\omega'}{2\psi'}\, e^{\psi-\nu} \;+\; \sqrt{\frac{\nu'}{\psi'} + \left(\frac{\omega'}{2\psi'}\, e^{\psi-\nu}\right)^2}. \tag{7}$$

In Eq. (7) all functions are to be evaluated at the star's equator. The quantity V denotes the orbital velocity measured by an observer with zero angular momentum in the ϕ-direction. Primes refer to derivatives with respect to the radial coordinate. The dependence of ψ and ν on Ω is suppressed in Eq. 7 for the purpose of brevity.

An essential point is that in order to find the Kepler frequency defined in Eq. (7), a *self-consistency* problem must be solved, a fact apparently not recognized before in applications of Hartle's method. The reason for self-consistency lies in the dependence of Ω_K on the metric functions at the equator. These in turn can be found only as the solution to the initial value problem posed by Einstein's equations. The initial values are those of the central density (ϵ_c) and angular velocity in the local inertial frame (ω_c). Each stellar mass and the Kepler frequency for that configuration correspond to a unique value for this pair of initial values. Only one of them can be chosen arbitrarily, say the central density, because obviously at this stage one does not know the Kepler frequency, not even the mass, belonging to this choice, so it is not known how to choose the other. In other words, Eq. (7) is to be understood as a transcendental equation in the Kepler frequency, in which all quantities on the right depend on this frequency. In this work we calculate Ω_K for the sample of equations of state listed in Table 1. A detailed procedure for finding the Kepler frequency is outlined in Weber, Glendenning, and Weigel (1990*a*; 1990*b*) and will not be repeated here.

3 Collection of selected equations of state

A sample of a total of seventeen neutron matter equations of state is used for the construction of models of rotating neutron stars. These are listed in Table 1. Among those are the equations of state of Bethe and Johnson (1974), Friedman and Pandharipande (1981), and Pandharipande (1971), abbreviated by respectively BJ(I), FP(V_{14}+TNI), Pan(C), which have also been used by Friedman, Ipser and Parker for their study of rotating neutron stars in which they solve Einstein's equations exactly. These equations of state therefore allow for an immediate comparison of both methods. The equations of state labeled "1" through "11" in Table 1 have been derived in the framework of relativistic nuclear field theory, while those labeled "12" through "17" are based on nonrelativistic potential models of the nucleon-nucleon force. An inherent feature of the former is that they do not violate causality, i.e. $c_s/c = \sqrt{\partial P/\partial \epsilon} < 1$ (c_s denotes the velocity of sound), which is not the case for the potential models. Among the latter only the WFF(UV_{14}+TNI) equation of

Table 1: Equations of state (EOS) of this work

Label		EOS	Description†	Reference
1		G_{300}	R, H, K=300	Glendenning 1989*a*
2		HV	R, H, K=285	Glendenning 1985
3		G^{DCM2}_{B180}	R, Q, K=265, $B^{1/4}=180$	Glendenning, Weber, and Moszkowski 1991
4		G^{DCM2}_{265}	R, H, K=265	Glendenning,Weber, and Moszkowsi 1991
5		G^{π}_{300}	R, H, π, K=300	Glendenning 1989*a*
6		G^{π}_{200}	R, H, π, K=200	Glendenning 1986
7		Λ^{00}_{Bonn} + HV	R, H, K=186	Weber, Glendenning, and Weigel 1990*a*
8		G^{DCM1}_{225}	R, H, K=225	Glendenning, Weber, and Moszkowski 1991
9		G^{DCM1}_{B180}	R, Q, K=225, $B^{1/4}=180$	Glendenning, Weber, and Moszkowski 1991
10		HFV	R, H, Δ, K=376	Weber and Weigel 1989
11		Λ^{00}_{HEA} + HFV	R, H, Δ, K=115	Weber, Glendenning, and Weigel 1990*a*
12	C‡	BJ(I)	NR, H, Δ,	Bethe and Johnson 1974
13		WFF(UV_{14}+TNI)	NR, NP, K=261	Wiringa, Ficks, and Fabrocini 1988
14	FP‡	FP(V_{14}+TNI)	NR, N, K=240	Friedman and Pandharipande 1981
15		WFF(UV_{14}+UVII)	NR, NP, K=202	Wiringa, Ficks, and Fabrocini 1988
16		WFF(AV_{14}+UVII)	NR, NP, K=209	Wiringa, Ficks, and Fabrocini 1988
17	B‡	Pan(C)	NR, H, Δ, K=60	Pandharipande 1971

† The following abbreviations are used: R = relativistic; NR = non-relativistic; N = pure neutron; NP = n, p, leptons; π = pion condensation; H = composed of n, p, hyperons, leptons; Δ = Δ_{1232}-resonance; Q = quark hybrid composition; K = incompressibility in MeV; $B^{1/4}$ = bag constant in MeV.

‡ Notation of Friedman, Ipser, and Parker (1986;1989).

state does not violate causality. The BJ(I) and Pan(C) equations of state violate causality at ≈ 23 times normal nuclear matter density, not much above the central density of the limiting star of the sequence. The equations of state FP(V_{14}+TNI), WFF(AV_{14}+UVII), and WFF(UV_{14}+UVII) do so at considerably smaller densities of $\approx 6-7$ times normal nuclear matter density, which is less than the central densities encountered in neutron star models constructed from them. We will turn back to the discussion of this subject in Sect. 6. Four equations of state of our collection, G_{225}^{DCM1}, G_{265}^{DCM2}, G_{B180}^{DCM1}, and G_{B180}^{DCM2} have only recently been calculated (Glendenning, Weber, and Moszkowski 1991) for electrically charge neutral neutron star matter in generalized β equilibrium from the derivative coupling Lagrangian of Zimanyi and Moszkowski (1990). Those labeled DCM1 correspond to the Lagrangian of Zimanyi and Moszkowski, while those labeled DCM2 correspond to the "hybrid" coupling introduced in Glendenning, Weber, and Moszkowski (1991). The possibility of a phase transition of the dense core to quark matter is taken into account in equations of state G_{B180}^{DCM1} and G_{B180}^{DCM2}, where B180 refers to the bag constant of $B^{1/4} = 180$ MeV. Important features of all of the selected equations of state are listed in Table 1.

4 Results for the Kepler frequency

In Figure 1 we plot our results obtained for Ω_K as a function of rotational star mass, M_{rot}, calculated from the equations of state of Sect. 3. The limiting value of M_{rot} for each equation of state (labels "1" through "17" of Table 1) represents the equilibrium configuration of a star of limiting gravitational mass, rotating at its Kepler frequency. It is remarkable that most of the potential models of our collection lead, in comparison with the field-theoretic equations of state, to relatively large Kepler frequencies. In particular, with the exception of the BJ(I) equation of state (label "12"), all potential models lead to considerably larger Ω_K values than the field-theoretic ones. The softness at low and intermediate nuclear densities but rather stiff behavior at large densities of the former equations of state accounts for this behavior (Glendenning 1989*b*; Lattimer et al. 1990; Weber, Glendenning, and Weigel 1990*a*). From the relativistic models we find that the Λ_{HEA}^{00} + HFV equation of state leads to the largest Kepler frequency, i.e. $\Omega_K = 1.18 \cdot 10^4$ s^{-1}. The (Fock) exchange contribution contained in HFV, stiffening the equation of state at large densities, has been shown to be responsible for this behavior (Weber, Glendenning, and Weigel 1990*a*). However, though behaving rather stiffly at larger nuclear densities, this

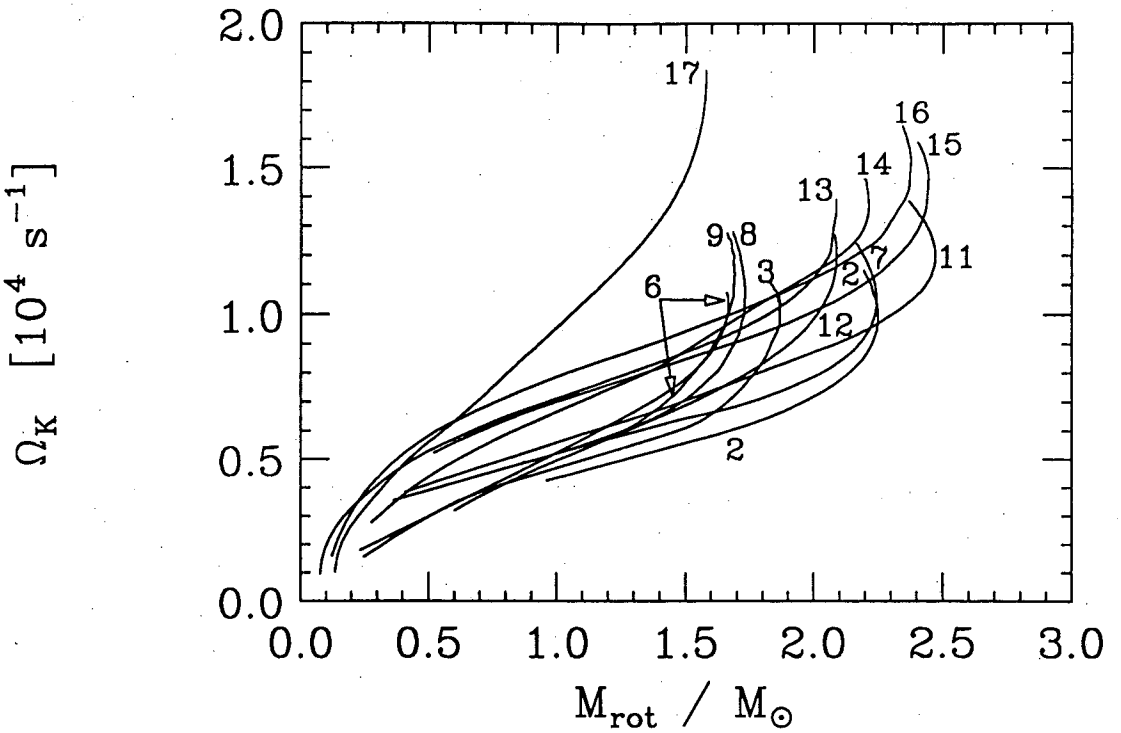

Figure 1: Kepler frequency Ω_K (solution of Eq. (7)) as a function of rotational neutron star mass, shown for a sample of the equations of state of Table 1.

equation of state stays causal in contrast to the potential model equations of state. As already mentioned among the latter only the UV_{14}+TNI equation of state of Wiringa, Ficks, and Fabrocini fulfills the condition of causality.

The rapid change of Ω_K in the vicinity of the limiting mass is apparent from Fig. 1. This is particularly so for the equations of state labeled "8", "9", "12", and "17". A careful investigation of the dependence of Ω_K on M_{rot} has been performed by Weber and Glendenning (1991) (referred to as Paper I). It was found that an uncertainty in the limiting-mass model of 1% leads to an uncertainty in Ω_K of $\pm \approx 10\%$! Such a sensitive dependence of the limiting-mass model on Ω_K is a problem for the exact method for which a compromise between numerical accuracy and radial grid spacing is to be made, implying errors in mass and radius of respectively 1% and 5% (Friedman, Ipser, and Parker 1986).

5 Empirical formula for the Kepler frequency

A useful result of the work of Friedman, Ipser and Parker (1986; 1989) is that the Kepler frequency of a neutron star rotating at its mass limit can be estimated from the mass and radius of the corresponding *nonrotating* limiting-mass star. This "empirical" relation is given by:

$$\Omega_{\mathrm{K}} \approx C \cdot \sqrt{[M_{\mathrm{s}}/M_{\odot}]/[R_{\mathrm{s}}/10\,\mathrm{km}]^3}\,. \tag{8}$$

The quantities M_{s} and R_{s} denote the gravitational mass and radius, respectively, of the spherical star of limiting mass. The quantity C in Eq. (8) is a constant for which values of $C_{\mathrm{FIP}} = 7200\ \mathrm{s}^{-1}$ (Friedman, Ipser, and Parker 1989) and $C_{\mathrm{HZ}} = 7700\ \mathrm{s}^{-1}$ (Haensel and Zdunik 1989) have been extracted. By direct numerical solution of Einstein's field equations for the same sample of neutron matter equations of state, it was found that Eq. (8) approximates the exact value of Ω_{K} to better than 10% in the case of $C = C_{\mathrm{FIP}}$ (Friedman, Ipser, and Parker 1989) and 5% for $C = C_{\mathrm{HZ}}$ (Haensel and Zdunik 1989; cf. also Lattimer, Prakash, Masak, and Yahil 1990).

A striking feature is the independence of C of the *particular* star model (and hence of the equation of state itself). To date the numerical value of C has not been derived from theory. To the best of our knowledge, only motivation for an expression as given by Eq. (8) from heuristic considerations has been given (Shapiro, Teukolsky, and Wasserman 1983). In Sect. 7 we will turn back to this topic and motivate, in the framework of Hartle's theory, an approximate analytic expression for C. (We will find that C indeed depends on the equation of state. Thus considering it as a constant applies only with reservation!) The remaining quantity in Eq. (8) is the Newtonian expression for the Kepler frequency. The importance of the simple relation of Eq. (8) lies in the fact that only the properties of the spherical neutron star are needed. These can be easily obtained by solving the Oppenheimer-Volkoff equations.

6 Bulk properties of self-consistent neutron star models

Up to now our discussion was focused on the Kepler frequency of the *limiting-mass* models. Of course, to provide strong evidence for the applicability of Hartle's method, an extensive comparison of the properties of rotating stars (like equatorial radius, equatorial velocity, dragging of local inertial frames, eccentricity, etc.) calculated from

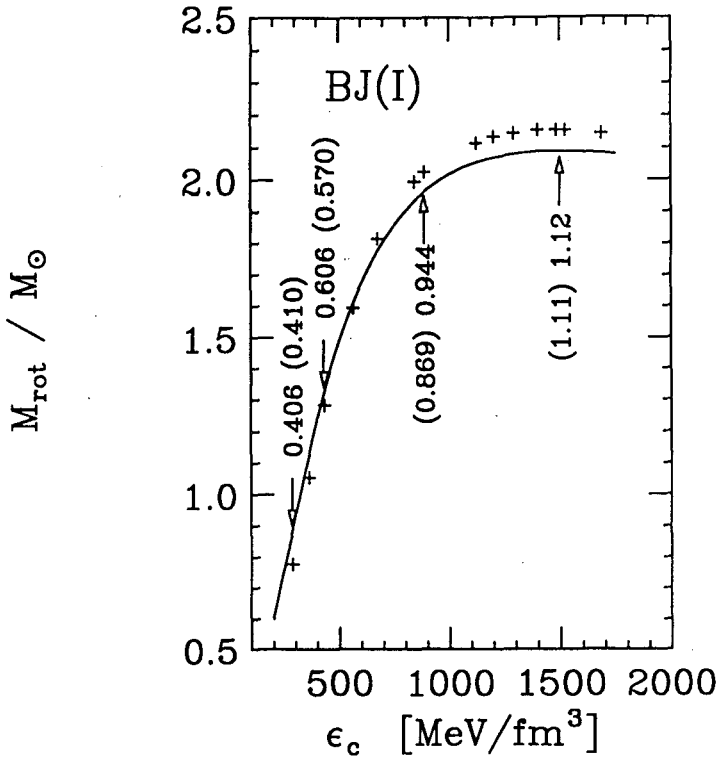

Figure 2: Comparison of rotational neutron star mass as a function of central energy density, ϵ_c, obtained from Hartle's method (solid line) with the exact results (crosses) of Friedman, Ipser, and Parker (1986). The underlying equation of state is BJ(I) (see Table 1). The numbers refer to the values of the Kepler frequency Ω_K (in units of 10^4 s^{-1}) of Eq. (7). The exact values are given in round brackets.

both the exact as well as Hartle's method is necessary. This will be performed in this section where we present sequences of models of rotating neutron stars constructed from the set of neutron matter equations of state of Table 1.

6.1 Comparison with exact results

We begin with the comparison of the properties of rotating neutron stars calculated from the equations of state of BJ(I), FP(V_{14}+TNI), and Pan(C) (labels "12", "14", and "17", respectively).

Figure 2 exhibits the rotational neutron star mass as a function of central energy density, ϵ_c, calculated for the BJ(I) equation of state (solid line). The crosses denote the mass values obtained from the exact calculation (taken from Friedman, Ipser, and Parker 1986). For a sample of ϵ_c values the Kepler frequencies Ω_K (in units of 10^4 s^{-1}) are displayed too. The numbers in round brackets refer to the exact values. One sees that the rotational star masses obtained from Hartle's method are in very good

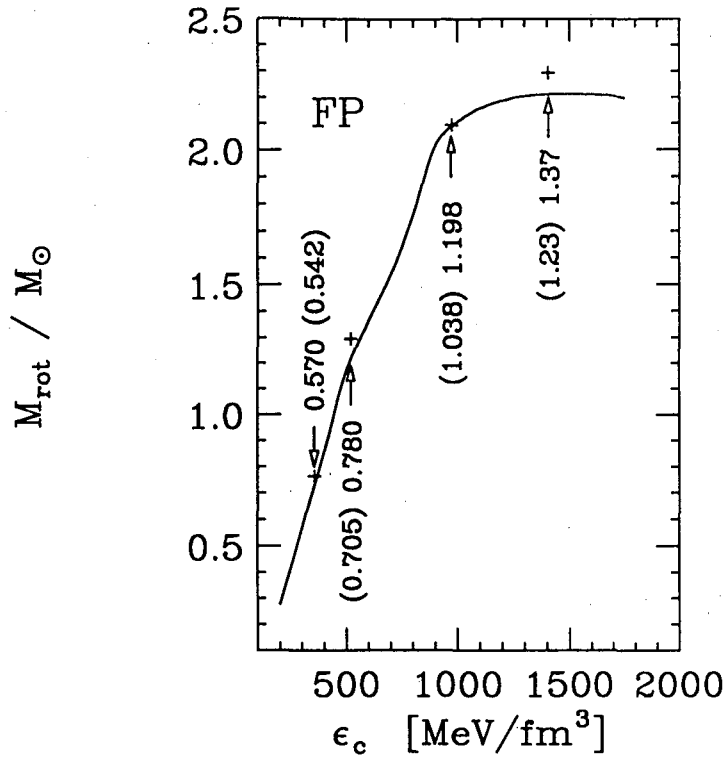

Figure 3: Same as Fig. 2, but calculated for the Friedman-Pandharipande equation of state FP(V_{14}+TNI).

agreement up to $M_{\rm rot} \approx 1.85 M_\odot$ (which corresponds to $\Omega_{\rm K} \approx 9000$ s^{-1}). Rotating star models of larger masses are characterized, in the exact treatment, by being slightly more massive. For the limiting-mass model a mass difference of $\approx 3\%$ between the two treatments is obtained. The location of the mass limit occurs, according to Fig. 2, in both treatments at more or less the same central energy density. The limiting Kepler frequency for BJ(I) has in our treatment a value of $\Omega_{\rm K} = 1.11 \cdot 10^4$ s^{-1}, in close agreement with the exact value of $1.12 \cdot 10^4$ s^{-1} (cf. Table 2).

Figure 3 is the analog of Fig. 2, but calculated for the FP(V_{14}+TNI) equation of state. The rotating neutron star masses in this case are in good agreement up to $M_{\rm rot} \lesssim 2.1 M_\odot$. The Kepler frequencies related to the latter mass value are $1.198 \cdot 10^4$ s^{-1} (Hartle) and $1.038 \cdot 10^4$ s^{-1} (exact). The limiting Hartle mass is $\approx 4\%$ smaller than the exact value. As for the BJ(I) equation of state, the mass limit occurs in both treatments at more or less the same ϵ_c value.

We emphasize once again that the self-consistency problem outlined in Sect. 2 has not been previously imposed by other authors who have employed Hartle's method (cf. Hartle and Thorne 1968; Baym, Pethick, and Sutherland 1971; Datta and Ray 1984;

Ray and Datta 1984; Datta 1988). Our results clearly demonstrate that by means of the self-consistent determination of stellar models, a rotation-induced mass increase of $\approx 15-25\%$ (depending on the equation of state, see Weber, Glendenning, and Weigel 1990*a*; 1990*b*) can be obtained. Such a mass increase is in very good agreement with the one established from the exact solution of Einstein's equations. The magnitude of this mass increase *cannot be obtained without solving the full general relativistic stellar structure equations self-consistently.* For that reason too small a mass increase ($\lesssim 8\%$, see, for example, Datta 1988) was obtained in earlier non-selfconsistent applications of Hartle's method and was viewed as an inherent weakness of the method (cf. Friedman, Ipser, and Parker 1986). The findings of this work show that this objection looses its validity when self-consistency is imposed on Hartle's equations.

In Table 2 we summarize rotating neutron star properties derived from Hartle's method, where the equations of state FP(V_{14}+TNI), BJ(I), and Pan(C) served as an input. Mass models rotating below their mass limits are labeled "a", those rotating at the mass limit are given in the rows labeled "b". The exact results, taken from Friedman, Ipser and Parker (1986, 1989), are listed in rows labeled "exact". All Hartle models shown in rows "a" have been determined self-consistently such that these possess the same rotational mass as the exact models. From the above discussion it is known that Hartle's method leads to $\approx 3-4\%$ less massive *limiting*-mass star models than the exact treatment. For that reason the limiting-mass models of rows "b" have slightly different masses. One sees from Table 2 that the central energy densities, Kepler frequencies, values of central frame dragging (ω_c/Ω_K), ratios of rotational energy to gravitational energy (T/W), equatorial velocities (V_{eq}/c), and eccentricities (e) of both methods are in good agreement. The equatorial radii (R_{eq}) of the Hartle models coincide to a less extent with the exact outcome. (It should be kept in mind that the latter have errors of $\approx 5\%$ (Friedman, Ipser, and Parker 1986).) The origin of this may lie in the neglect of the higher-order (i.e. higher than quadrupole) perturbation functions in Hartle's perturbative method.

In summary Figs. 2 and 3, and the promising agreement of the perturbative rotating neutron star models with their exact counterparts, demonstrated in Table 2, provide *strong evidence for the applicability of Hartle's method up to (at least)* $\Omega_K \approx 1.2 \cdot 10^4\ s^{-1}$, corresponding to a rotational period of $P \approx 0.5$ msec, which is a fraction of the smallest yet observed period (1.6 msec). It should be noted that this conclusion rests on the comparison of sequences of star models, which cover a large

Table 2: Comparison of the properties of rotating neutron star models calculated from Hartle's method with those of the exact method for the equations of state V_{14} + TNI, BJ(I), and Pan(C) (see Table 1). The properties listed are: $M_{\rm rot}/M_\odot$, rotational star mass in units of the solar mass; ϵ_c, central energy density; $\Omega_{\rm K}$, Kepler frequency; $R_{\rm eq}$, equatorial radius; $\omega_c/\Omega_{\rm K}$, percentage of central dragging; T/W, ratio of rotational energy to gravitational energy; $V_{\rm eq}/c$, equatorial velocity of a comoving observer; and e, eccentricity. Comparisons labeled "a" are carried out at the same mass, "b" at the limiting mass.

Equation of state		Method	$\frac{M_{\rm rot}}{M_\odot}$	ϵ_c [10^{15}g/cm^3]	$\Omega_{\rm K}$ [10^4s^{-1}]	$R_{\rm eq}$ [km]	$\frac{\omega_c}{\Omega_{\rm K}}$	$\frac{T}{W}$	$\frac{V_{\rm eq}}{c}$	e
V_{14}+TNI	a	exact	0.77	0.63	0.542	15.1	0.27	0.102	0.27	0.72
		Hartle	0.77	0.66	0.570	13.7	0.23	0.080	0.26	0.71
	a	exact	1.30	0.92	0.705	14.9	0.43	0.120	0.35	0.77
		Hartle	1.30	0.98	0.780	13.10	0.40	0.094	0.39	0.71
	b	exact	2.30	2.5	1.23	12.	0.83	0.133	0.49	0.67
		Hartle	2.21	2.7	1.37	10.7	0.77	0.117	0.62	0.68
BJ(I)	a	exact	0.78	0.51	0.410	18.1	0.25	0.071	0.25	0.71
		Hartle	0.78	0.47	0.406	17.5	0.21	0.066	0.25	0.70
	a	exact	1.29	0.77	0.570	16.9	0.40	0.093	0.38	0.74
		Hartle	1.29	0.75	0.606	15.6	0.34	0.081	0.35	0.70
	b	exact	2.16	2.7	1.11	12.9	0.79	0.110	0.47	0.68
		Hartle	2.09	2.7	1.197	11.6	0.72	0.099	0.57	0.68
Pan(C)	b	exact	1.65	5.16	1.57	9.2	–	0.107	–	0.66
		Hartle	1.58	5.16	1.81	8.0	0.76	0.108	0.61	0.67

range of different Ω_K values, i.e. $4 \cdot 10^3\ s^{-1} \lesssim \Omega_K \lesssim 1.8 \cdot 10^4\ s^{-1}$. Therefore Hartle's method should prove to be a practical tool for the construction of models of pulsars down to rotational periods of 0.5 msec.

As concerns the limiting-mass model related to the Pan(C) equation of state, given in the last row of Table 2, we find a value for Ω_K which is roughly 15% larger than the one of the exact numerical treatment ($\Omega_K = 1.57 \cdot 10^4\ s^{-1}$). However the determination of Ω_K for the Pan(C) equation of state is particularly complicated by the fact that Ω_K depends very sensitively - much more than is the case for all other equations of state of our collection - on M_{rot} (cf. Fig. 1 and also Fig. 1 of Paper I). Our precise determination of the mass limit gives $\Omega_K = 1.81 \cdot 10^4\ s^{-1}$. This is compatible with the exact method if one assumes that the mass is determined only within 2% in the latter case (Paper I). The properties of the limiting-mass models of both methods agree very well with each other (Table 2). For example the central energy density turns out to be the same in both treatments. Furthermore the instability parameter, T/W, and the eccentricity, e, are in remarkable good agreement. As before the Hartle star is slightly less massive. The difference is $\approx 4\%$.

6.2 Models of rotating neutron stars derived from the collection of equations of state

We present in the following the properties of neutron star models calculated from the *complete* sample of equations of state of Table 1 that are rotating at their absolute maximum rate. Such a systematic investigation enables one to demonstrate the impact of different models of the nuclear equation of state on the bulk properties of rotating neutron stars. Of special interest is the investigation of the compatibility of the nuclear equation of state with data on pulsar periods. Different models of the equation of state lead to different neutron star properties, and not all of them may accommodate observed data (e.g. masses and radii of magnetic X-ray burster (Fujimoto and Taam 1986), neutron star redshifts (Liang 1986)). This attains its particular interest in view of the rapid discovery pace of millisecond pulsars (Backer and Kulkarni 1990).

From the survey of both rotating as well as nonrotating neutron star properties presented in Table 3, it follows that the equations of state of our collection lead to limiting Kepler frequencies in the range of $9.28 \cdot 10^3\ s^{-1} \lesssim \Omega_K \lesssim 1.81 \cdot 10^4\ s^{-1}$. The lower and upper bounds are established by the relativistic G_{300} ("1") and nonrela-

tivistic Pan(C) ("17") equations of state, respectively. With the exception of G^{π}_{200} ("6"), G^{DCM1}_{B180} ("9"), and Pan (C), all equations of state are able to support *nonrotating* neutron star models of gravitational masses $M \geq 1.5\,M_{\odot}$. Nonrotating masses of $M \gtrsim 2M_{\odot}$ can only be derived from the relativistic equations of state "10" and "11" (because of their rather stiff behavior at large densities) as well as two of the potential model equations of state , i.e. "15" and "16". The influence of rotation on gravitational mass and equatorial radius is shown in columns 6 and 7 of this table. Because of the mass increase permitted by rotation, the equations of state "6", "9", and "17" then lead to neutron stars of rotational masses $M_{\rm rot} > 1.5M_{\odot}$, and eleven of the seventeen rotating star models possess $M_{\rm rot} > 2M_{\odot}$. The largest rotating mass value, $M_{\rm rot} = 2.47M_{\odot}$, is obtained for HFV ("10") and $\Lambda^{00}_{\rm HEA}$ + HFV ("11"). Here the latter equation of state is supplemented by two-particle correlation effects derived from a relativistic T matrix calculation, using the HEA meson-exchange potential for the nucleon-nucleon interaction as an input (Weber, Glendenning, and Weigel 1990*a*).

A characteristic feature of rotating neutron star models is the decrease of the central energy density, ϵ_c, caused by rotation. This decrease can only be obtained in the framework of a self-consistent stellar structure calculation. The ϵ_c values for both nonrotating and rotating stars are listed in columns 4 and 8, respectively. One sees that matter in the center of a rotating neutron star constructed from the Pan(C) equation of state nevertheless is compressed to $\approx$ 21 times normal nuclear matter density. (We recall that causality is violated for this equation of state at $\approx$ 23 times normal nuclear matter density.) At such extreme nuclear densities neutron star matter can certainly not be thought of as being made up of individual neutrons interacting via potential forces. In this respect the equations of state G^{DCM1}_{B180} and G^{DCM2}_{B180} are of particular interest since these account for the transition of electrically charge neutral baryon matter to quark matter (consisting of u,d,s quarks) at higher densities (Glendenning, Weber, and Moszkowski 1991). The ϵ_c values determined for the remaining equations of state of our collection are clearly smaller than in the case of Pan(C). They lie, for the potential model equations of state, in the range $11.5 \lesssim \epsilon_c/\epsilon_0 \lesssim 12.8$ for nonrotating stars, decreasing to $10 \lesssim \epsilon_c/\epsilon_0 \lesssim 10.8$ when rotation is taken into account. For the field-theoretic equations of state we find respectively $9.3 \lesssim \epsilon_c/\epsilon_0 \lesssim 13.9$ and $7.1 \lesssim \epsilon_c/\epsilon_0 \lesssim 12$. The decrease of ϵ_c due to rotation is found to be largest for the equations of state HFV and G_{300}, and is given by respectively 26% and 24%. The smallest reduction, 10%, is obtained for G^{π}_{300} and

Table 3: Calculated properties of rotating limiting-mass neutron star models, calculated from the collection of equations of state of Table 1. The properties listed are explained in table caption 2, with the exception of $M_s/M_\odot$ and R_s which denote respectively graviational mass and radius of the nonrotating limiting-mass model.

Equation of state	$\frac{M_s}{M_\odot}$	R_s [km]	ϵ_c/ϵ_0 †	Ω_K [$10^4 s^{-1}$]	$\frac{M_{rot}}{M_\odot}$	R_{eq} [km]	ϵ_c/ϵ_0	$\frac{T}{W}$	$\frac{V_{eq}}{c}$	e
1	1.792	11.12	9.37	0.93	2.04	13.60	7.16	0.10	0.50	0.70
2	1.976	11.34	9.29	0.99	2.25	13.43	7.83	0.10	0.54	0.69
3	1.605	10.48	11.36	1.01	1.87	12.42	9.43	0.09	0.51	0.69
4	1.510	10.14	11.99	1.01	1.72	12.17	10.29	0.09	0.49	0.69
5	1.808	10.97	9.57	1.02	2.06	12.87	8.67	0.10	0.54	0.69
6	1.458	10.12	12.10	1.03	1.67	11.91	10.84	0.09	0.48	0.69
7	1.969	10.97	9.29	1.04	2.24	12.97	7.96	0.11	0.55	0.69
8	1.525	9.84	12.79	1.06	1.73	11.91	10.61	0.09	0.50	0.69
9	1.494	9.48	13.93	1.13	1.69	11.31	12.07	0.09	0.51	0.68
10	2.198	10.70	9.64	1.16	2.47	12.34	7.12	0.11	0.60	0.69
11	2.195	10.63	9.64	1.18	2.47	12.24	7.66	0.12	0.61	0.68
12	1.843	9.77	12.50	1.12	2.09	11.59	10.81	0.10	0.57	0.68
13	1.848	9.24	12.79	1.29	2.09	10.97	10.02	0.11	0.59	0.69
14	1.971	9.29	12.16	1.37	2.21	10.82	10.62	0.12	0.62	0.68
15	2.197	9.56	11.54	1.45	2.44	10.70	10.07	0.13	0.66	0.64
16	2.135	9.17	12.50	1.51	2.38	10.31	10.51	0.13	0.67	0.65
17	1.412	6.90	24.07	1.81	1.58	7.98	20.73	0.11	0.61	0.67

† The quantity ϵ_0=140 MeV/fm^3 ($= 2.5 \cdot 10^{14}$ g/cm^3) denotes the density of normal nuclear matter.

G^{π}_{200}. A special feature of the latter two equations of state is the inclusion of pion condensation at supernuclear densities.

The value of the "instability" parameter, $t \equiv T/W$, obtained for all models of Table 3 takes on values $0.088 \leq t < 0.13$. Hence all rotating models reach their respective limiting-mass limit before $t = 0.14$. The latter value indicates the onset of an instability caused by a bar mode (cf. Friedman, Ipser, and Parker 1986). Indeed all models constructed from relativistic neutron matter equations of state lead to $t \leq 0.11$ with the exception of equation of state $\Lambda^{00}_{\mathrm{HEA}}$ + HFV for which $t = 0.12$. We recall that the latter leads to the largest Kepler frequency, $\Omega_K = 1.18 \cdot 10^4\ \mathrm{s}^{-1}$, among the relativistic models for the equation of state. The largest value obtained for the instability parameter, $t = 0.13$, is calculated for the potential model equations of state "15" and "16" of Wiringa, Ficks, and Fabrocini. In general all potential models have $t \geq 0.10$.

The equatorial velocity, V_{eq}, amounts at most to 67% of the velocity of light (equation of state "16"), followed by 66% in the case of "15". Above we have found that Ω_{K} (and t) are in general smaller for the relativistic equations of state. These smaller Kepler frequencies imply lower equatorial velocities than those of the potential model equations of state : we find $0.48 \leq V_{\mathrm{eq}}/c \leq 0.61$ (relativistic equations of state) and $0.57 \leq V_{\mathrm{eq}}/c \leq 0.67$ (nonrelativistic).

The eccentricities, defined by $e = \sqrt{1 - (R_{\mathrm{p}}/R_{\mathrm{eq}})^2}$ (R_{p} and R_{eq} denote respectively the polar and equatorial radius), are listed in the last column of Table 3. One sees that these do not change very much though the bulk properties of the limiting-mass models of our collection are rather different from each other. The ratio of polar radius to equatorial radius however is rather independent of the underlying equation of state. We typically find $R_{\mathrm{p}}/R_{\mathrm{eq}} \approx 3/4$.

7 Motivation of the empirical formula for Ω_{K} from Hartle's method

To date it has remained an open problem why the simple, empirically established expression for Ω_{K} of Eq. (8) for the limiting-mass star so successfully approximates the exact solution of Eq. (7). We recall that the empirical formula expresses Ω_{K} in terms of $\sqrt{M_{\mathrm{s}}G/R_{\mathrm{s}}^3}$, i.e. mass and radius of the *nonrotating* neutron star of limiting mass, times a constant C (see Eq. (8)). Obviously the rotating and nonrotating star

models are two completely different physical objects which are not related by any quantity with each other. Nevertheless the empirical formula is known to work very well (Friedman, Ipser, and Parker 1989; Lattimer et al. 1990) and one is tempted to understand this from theory. (Heuristic arguments have been given by Shapiro, Teukolsky, and Wassermann (1983).)

Our aim in this section is to provide theoretical evidence for the validity of the empirical formula. To be specific, we analyze the general relativistic expression for the Kepler frequency of Eq. (7) in the framework of Hartle's method. The investigation leads to an analytic expression for C which exhibits a weak dependence on the equation of state. The range of C values is extracted from the collection of star models of Sect. 6.2, and is found to be compatible with the empirically established range of $C \approx C_{\rm FIP} = 7200\ {\rm s}^{-1}$, $C_{\rm HZ} = 7700\ {\rm s}^{-1}$, and $C_{\rm Hartle} \approx 8500\ {\rm s}^{-1}$ (see Sect. 5).

To simplify the following discussion, we distinguish between two different cases: In the first case, (i), we will neglect the pure general relativistic effect of dragging of local inertial frames. By this a rather straighforward investigation of $\Omega_{\rm K}$ can be performed. In the second step, case (ii), the dragging effect is taken into account and its impact on the results of item (i) is demonstrated.

7.1 Dragging of local inertial frames neglected

The dragging effect of local inertial frames describes the onset of rotation of these inertial frames induced by a rotating mass (see, for example, Hönl and Soergel-Fabricius 1961; Hönl and Dehnen 1962; Brill and Cohen 1966; Misner, Thorne, and Wheeler 1973). Values of fractional dragging at the center of rotating neutron stars, ω_c/Ω, are listed in Table 2 for rotation at $\Omega = \Omega_{\rm K}$ (8th column). The dragging effect was shown to be largest at the center of rotating neutron stars with decreasing magnitude (i.e. ω considerably smaller than Ω) toward its surface (Hartle and Thorne 1968; Friedman, Ipser, and Parker 1986; Weber, Glendenning, and Weigel 1990*b*). In the limit when the rotational frequencies of the local inertial frames, $\omega(r)$, are small in comparison with the star's rotational frequency Ω, the dragging effect can be ignored. Only in this limit is the angular velocity, that determines the magnitude of the centrifugal force acting on the star's matter, equal to the star's angular frequency Ω (Hartle 1967). We restrict ourselves in the first part of the investigation to $\omega/\Omega \ll 1$! By substituting Eqs. (3) and (4) into Eq. (7), one obtains for the orbital velocity $V_{\rm s}$ of a mass element rotating at the star's equator (we assume spherical symmetry throughout this section

which is indicated by means of attaching a subscript "s" to the relevant quantities)

$$V_{\mathrm{s,\,no\ dragging}} = \Omega_c \frac{R_{\mathrm{s}}}{\sqrt{1-2M_{\mathrm{s}}G/R_{\mathrm{s}}}} \tag{9}$$

$$\longrightarrow R_{\mathrm{s}}\,\Omega_c \qquad \text{(Newtonian limit)}\,. \tag{10}$$

We recall that the relation for $V(\Omega)$ posed a self-consistency problem in Sect. 2 since knowledge of the properties of the rotating neutron star (e.g. total mass, radius), which themselves depend on the rotational frequency Ω, is necessary for the calculation of $V(\Omega_{\mathrm{K}})$. The self-consistency is avoided here by replacing the rotating neutron star configuration by its nonrotating one of limiting-mass. This approximation makes sense since the amount of dragging, in which we are interested, is determined by the mass of the star (Brill and Cohen 1966; Misner, Thorne, and Wheeler 1973). The rotation-induced mass increase however was found to be $\approx 15\%$ (compare columns two and six of Table 3), which is too small to have a crucial impact on dragging. Therefore by means of setting $M_{\mathrm{rot}} \approx M_{\mathrm{s}}$ (and $R_{\mathrm{eq}} \approx R_{\mathrm{s}}$), one can expect to account for the impact of very massive objects on the frame dragging to a reasonably good approximation. We turn back to this topic in Sect. 7.2.

The frequency Ω_c in Eqs. (9), (10) is defined by $\Omega_c \equiv \sqrt{M_{\mathrm{s}}G/R_{\mathrm{s}}^3}$. From Eq. (9) one arrives for the general relativistic Kepler frequency, with the neglect of frame dragging,

$$\Omega_{\mathrm{K}}\,|_{\mathrm{s,\,no\ dragging}} = \Omega_c\,. \tag{11}$$

One sees that the relativistic expression for the Kepler frequency Ω_{K}, with the neglect of frame dragging, coincides with the one of classical mechanics, expressing the circular movement of a massive particle in that stable orbit for which balance between centrifuge and gravity occurs (cf. Misner, Thorne, and Wheeler 1973).

7.2 Inclusion of the dragging effect

In the next step we take the rotational dragging effect of the local inertial frames into account, i.e. we investigate the case $\omega(r) \neq 0$. It is convenient to define the difference between the star's angular velocity and the rotational frequency of the local inertial

frame by introducing the function $\bar{\omega}(r) \equiv \Omega - \omega(r)$ (Hartle 1967; Hartle and Thorne 1968). Cases (i) and (ii) then differ by $\bar{\omega}(r)\,|_{\text{no dragging}} = \Omega$ and $\bar{\omega}(r)\,|_{\text{dragging}} = \Omega - \omega(r)$, respectively. After substituting Eqs. (3) and (4) into the second relation of Eq. (7), one finds for V_s, instead of Eq. (9), the more general expression ($\omega(R_s) = (2I_sG/R_s^3) \cdot \Omega$, see Hartle 1967)

$$V_{s,\,\text{dragging}}(\Omega) = -\,\alpha\,\Omega \;+\; \beta\,\Omega_c\,. \tag{12}$$

To arrive at Eq. (12), the term $(\omega'\,e^{\psi-\nu}/2\psi')^2$ has been neglected in the second of Eqs. (7), which turned out in our calculations to be two orders of magnitude smaller than ν'/ψ'. The quantities α and β in Eq. (12) are defined by

$$\alpha \;\equiv\; 3I_sG \,/\, \left(R_s^2 \sqrt{1 - 2M_sG/R_s} \right), \tag{13}$$

$$\beta \;\equiv\; \alpha\, R_s^3 \,/\, (3I_sG)\,. \tag{14}$$

The star's moment of inertia, I, occuring in Eqs. (13) and (14) is given by

$$I_s = \frac{8\,\pi}{3} \int_0^{R_s} dr\, r^4 \frac{\epsilon + P(\epsilon)}{\sqrt{1 - 2m_sG/r}}\, \frac{\bar{\omega}}{\Omega}\, e^{-\Phi}\;. \tag{15}$$

The striking feature of Eq. (12) is that in the case when dragging is taken into account, $V_{s,\,\text{dragging}}$ depends on the star's angular velocity Ω. This behavior is to be compared with Eq. (9), where the only dependence on angular velocity occurs in Ω_c. The latter frequency is determined by the bulk properties, i.e. mass M_s and radius R_s, of the *nonrotating*, sperically symmetric neutron star.

It is interesting to look at the impact of dragging on the velocity V_s. From Eqs. (9) and (12), it readily follows

$$V_{s,\,\text{no dragging}} \;-\; V_{s,\,\text{dragging}}(\Omega) \;=\alpha\,\Omega \quad > \;\; 0\;. \tag{16}$$

From Eq. (16) one sees explicitly that in the case when dragging of the local inertial frames is taken into account, the velocity of a mass element rotating at the star's equator is smaller than in the case when dragging is neglected. The actual magnitude

of the difference is proportional to the star's rotational frequency Ω and, in particular, vanishes if $\Omega \longrightarrow 0$.

In the next step we determine the Kepler frequency, Ω_K, which was given, in the general treatment, as the solution of Eq. (7). In the framework of the approximations introduced in this section, Ω_K is obtained by combining Eq. (12) with the first of Eqs. (7). The latter is given by

$$\Omega_K = \frac{1}{R_s}\sqrt{1 - 2M_sG/R_s} \cdot V_{s,\,\mathrm{dragging}}(\Omega_K) \quad + \quad 2I_sG/R_s^3 \,. \tag{17}$$

The quantities M_s and R_s are those of the nonrotating, limiting-mass neutron star (cf. discussion at the end of Sect. 7.1). The velocity $V_{s,\,\mathrm{dragging}}(\Omega_K)$ is given in Eq. (12). It can be substituted in favor of Ω_K and the functions α and β of Eqs. (13) and (14), respectively. One obtains for Ω_K after some algebraic manipulations (compare with Eq. (11))

$$\Omega_K\,|_{s,\,\mathrm{dragging}} \quad = \quad D(I_s/R_s^3) \cdot \Omega_c \,, \tag{18}$$

$$D(I_s/R_s^3) \quad \equiv \quad \left(1 + I_sG/R_s^3\right)^{-1} \qquad (\; < \; 1 \;) \,. \tag{19}$$

Because of the dragging of the local inertial frames one obtains a maximum rotation rate which deviates from Ω_c by a function D (referred to in the following as "dragging factor"). It is striking that D of Eq. (19) depends only *very weakly* on its argument I_s/R_s^3. To demonstrate this for masses and radii which are typical for static neutron stars at their mass limits, we express I_s in terms of M_s and R_s. Using $I_s \approx R_s^2 M_s$, Eq. (19) can be written as

$$D(M_s, R_s) = \left(1 + \frac{3}{2}\,\frac{M_s/M_\odot}{R_s/\mathrm{km}}\right)^{-1} \,. \tag{20}$$

To answer the question how strongly $D(M_s, R_s)$ reduces Ω_K below the classical value Ω_c, we resort to our outcome of M_s and R_s for spherical, static star models of limiting mass listed in Table 3. From these results we find $0.74 \lesssim D(M_s, R_s) \lesssim 0.81$, which implies C-values of $8500\ \mathrm{s}^{-1} \lesssim C \lesssim 9300\ \mathrm{s}^{-1}$.

Thus from the perturbation treatment we have established the approximate proportionality of Ω_K to the classical value $\sqrt{M_sG/R_s^3}$. The proportionality constant

depends only weakly on the mass and radius. The values obtained for C are rather close to those established from numerical investigations (Friedman, Iper, and Parker 198; Haensel and Zdunik 1989; Weber and Glendenning 1991). For the purpose of comparison we recall $C_{\rm FIP} = 7200\ {\rm s}^{-1}$ (Friedman, Ipser, and Parker), $C_{\rm HZ} = 7700\ {\rm s}^{-1}$ (Haensel and Zdunik), and $C_{\rm Hartle} \approx 8500\ {\rm s}^{-1}$ (our result, see Paper I).

7.3 Conclusions

In summary, the following points arise from the above investigations:

1) The reduction of the relativistic Kepler frequency compared to the classical value has its origin in the *dragging* of the local inertial frames. (The amount of dragging is given by the function D ("dragging" factor) of Eq. (20));

2) $D \equiv 1$ if dragging is neglected. This is a good approximation in the case of less massive rotating stars like white dwarfs;

3) The magnitude of the dragging factor is rather *insensitive* to variations of the bulk properties of neutron stars (i.e. masses and radii). This has been inferred from investigating the dragging factor for a variety of different limiting-mass neutron star models, constructed from a representative collection of neutron matter equations of state. We find that for neutron stars, D is less than unity and for a variety of the models studied, falls in a narrow range, which we have understood by the analysis of this section.

8 Summary

In this work we construct sequences of models of general relativistic, rotating neutron stars up to the limiting-mass model of each sequence. For this purpose Hartle's perturbative method, originally developed for "slowly" rotating massive objects, is applied. To date it remained an open question up to which rotational star frequencies, Ω, this method is applicable. We have clarified this question by means of a detailed comparison of models of rotating neutron stars constructed from Hartle's method with their exact counterparts. For this purpose the equations of state of Pandharipande (1971), Bethe and Johnson (1974), and Friedman and Pandharipande (1981) (cf. Table 2) served as input. Furthermore a collection of a total of seventeen neutron matter equations of state has been applied for investigating the impact of the nuclear equation of state (i.e. the functional dependence of pressure on energy density) on the structure of rotating as well as nonrotating neutron star models. These equations of state cover nonrelativistic potential models as well as field-theoretic ones.

In essence, our results confirm the applicability of Hartle's perturbative method up to (at least) $\Omega_K \approx 1.2 \cdot 10^4$ s^{-1}. (The application of the method up to $\Omega_K = 1.81 \cdot 10^4$ s^{-1} has been performed, and we find good agreement even for such extreme frequencies.) The above Kepler frequency corresponds to a rotational period of $P \approx 0.5$ msec which is a fraction of the smallest yet observed periods (PSR 1937+21 and PSR 1957+20, $P = 1.6$ msec.) Therefore Hartle's method should prove to be a practical tool for testing competing models of the nuclear equation of state on data of millisecond pulsars.

Finally we have performed an analytic investigation concerning the empirical formula for the general relativistic Kepler frequency. We arrived at an expression for Ω_K which exhibits two essential features: firstly it relates the masses and radii of spherically symmetric, limiting-mass stars with the Kepler frequency, i.e. $\Omega_K = C \cdot \sqrt{[M_s/M_\odot]/[R_s/10 \text{ km}]^3}$, and secondly an expression for the "constant" C could be derived, which indicates that considering C as a constant applies only with restriction. In the framework of the approximations introduced in Sect. 7, we find that C changes at most by $\simeq 800$ s^{-1} for all the equations of state treated in this work. Hence only by ignoring a dependence of C on the properties of a particular star model (and hence on the equation of state itself) up to such amounts, can it be viewed as a constant.

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