Submitted to Physics Letters B;
Presented at the XVIIth International
Conference on High-Energy Physics,
London, England, July 1-10, 1974

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June 18, 1974

Prepared for the U. S. Atomic Energy Commission
under Contract W-7405-ENG-48

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LBL-3002

MULTIPLICITY BEHAVIOR OF $\pi^- p \to pX$: A MULTIPERIPHERAL MODEL DESCRIPTION*

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Abstract.—For the inclusive reaction $\pi^- p \to pX$, the average charged multiplicity $\langle n \rangle$ of the system X as a function of missing mass M^2 and momentum transfer t is studied in terms of a multiperipheral model. For $M^2 \gtrsim 20\ \mathrm{GeV}^2$ (above the low-mass pion diffraction region) and for $|t| \lesssim 1\ \mathrm{GeV}^2$, the model is found to be in good agreement with data at 205 GeV/c, which show $\langle n(M^2,t) \rangle \sim \ln M^2$ for fixed t, and $\langle n(M^2,t) \rangle$ only weakly dependent on t for fixed M^2. Further experimental and theoretical investigation is suggested.

Introduction.—The multiperipheral model [1] has had considerable success in describing many features of multiparticle production at high energy. Here we apply a simple version of the model to the inclusive inelastic reaction

$$\pi^- p \to pX \quad . \tag{1}$$

We study the behavior of the average charged multiplicity $\langle n \rangle$ of the produced system X as a function of M^2 and t, where M^2 is the mass-squared of X and t is the four-momentum transfer squared to the recoil proton. The model is shown to provide a good description of data at 205 GeV/c [2], which, for $M^2 \gtrsim 20\ \mathrm{GeV}^2$ and $|t| \lesssim 1\ \mathrm{GeV}^2$, show $\langle n(M^2,t) \rangle \sim \ln M^2$ at fixed t, and $\langle n(M^2,t) \rangle$ approximately independent of t for fixed M^2.

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*Work supported by the U. S. Atomic Energy Commission.

We first discuss some theoretical details and then compare model predictions with average multiplicity data for reaction (1). Although we specifically treat reaction (1), the model is applicable to the general class of reactions of the form $a + p \to pX$, as well as to reactions involving charge exchange at the nucleon vertex, such as $a + p \to nX$ and $a + p \to \Delta^{++}X$.

Multiperipheral Model.—To obtain $d\sigma/dM^2dt$ for reaction (1) we make two basic assumptions:

(i) The M^2 dependence of $d\sigma/dM^2dt$ can be described by a single Pomeron-pole term for sufficiently large values of M^2.

(ii) The system X is produced mainly by a non-Pomeron exchange mechanism for $M^2 \gtrsim 20\ \mathrm{GeV}^2$, as suggested by the observed inclusive M^2 distribution for reaction (1) at 205 GeV/c [2].

A detailed multiperipheral model embodying these assumptions has been described in ref. [3]. For the present, we employ a simplified version of this model which assumes that the system X is produced by pion exchange (ignoring G-parity restrictions). Although other exchanges are clearly possible, this simplified model predicts essentially the same M^2 and t behavior for the average charged multiplicity of X as the more realistic model, and has the advantage of being calculationally considerably easier to use.

The inclusive differential cross section for reaction (1) for $20\ \mathrm{GeV}^2 \lesssim M^2 \lesssim s$ is then

$$\frac{d\sigma}{dM^2dt} = \frac{g^2}{16\pi^2 s}\frac{1}{(t-m_\pi^2)^2}\,\bar{\beta}_\alpha\left(\frac{s_o}{s_o-t}\right)^{\alpha+1}\left(\frac{M^2}{s_o}\right)^{\alpha} . \qquad (2)$$

Here g^2 is the effective coupling at the nucleon vertex, s is the center-of-mass energy squared, and α is the intercept of the Pomeron pole. The last three factors represent the high-energy off-shell $\pi\pi$ scattering amplitude; this form for the $\pi\pi$ amplitude is suggested by an approximate analytic solution

to the multiperipheral integral equation [4]. The factor $\overline{\beta}_\alpha$ depends only on α, and s_o is related to the squared masses of the prominent $\pi\pi$ resonances (thus $s_o \approx 1\ \mathrm{GeV}^2$).

We now calculate $\langle n \rangle$, the average charged multiplicity of X, as a function of M^2 and t. Assuming $\langle n \rangle$ is a constant fraction of the overall charged plus neutral multiplicity of X, $\langle n \rangle$ can be obtained from the inclusive differential cross section using [5,3]

$$\langle n(M^2,t) \rangle = C \frac{\partial}{\partial\alpha} D(M^2,t)/D(M^2,t) \quad ,$$

where C is a constant related to the $\pi\pi$ coupling strength and $D(M^2,t) = d\sigma/dM^2dt$. Inserting (2) we find

$$\langle n(M^2,t) \rangle = C[\ln M^2 + \ln (\frac{s_o}{s_o-t})] + d \quad , \tag{3}$$

with $d = C[\partial(\ln \overline{\beta}_\alpha)/\partial\alpha - \ln s_o]$. Thus, for positive C, <u>$\langle n(M^2,t) \rangle$ is predicted to increase logarithmically with M^2 at fixed t, and to decrease very slowly with t for fixed M^2</u>. Furthermore, $\langle n(M^2,t) \rangle$ is predicted to be independent of s. Although both constants C and d in (3) are calculable from the multiperipheral model, they depend strongly on the details of the model and we therefore choose to obtain them by fitting the data, as described below.

We now proceed, by integrating (3) with respect to t or M^2, to calculate two other quantities of interest. The average charged multiplicity at mass M^2 is

$$\begin{aligned} \langle n(M^2) \rangle &= \int \langle n(M^2,t) \rangle D(M^2,t)dt / \int D(M^2,t)dt \\ &= C[\ln M^2 + \varphi(u_m)] + d \quad , \end{aligned} \tag{4}$$

where u_m is the minimum allowed $|t|$ value for given M^2 and s, and

$$\varphi(u_m) = \ln [(\frac{s_o}{s_o+u_m})(\frac{u_m}{s_o+u_m})^{u_m/s_o}] \quad .$$

The function $u_m(M^2)$ represents the Chew-Low boundary: for values of M^2 not too close to s, $u_m \approx (M^2/s)^2 m_p^2 (1-M^2/s)^{-1}$, where m_p is the proton mass. When $M^2/s \ll 1$, u_m is close to zero and the term $\varphi(u_m)$ in (4) has negligible contribution. However, as M^2, and therefore u_m, increases, $\varphi(u_m)$ gives a small negative contribution, so that for $s/2 \lesssim M^2, \lesssim s$, $\langle n(M^2)\rangle$ is predicted to deviate downward slightly from a simple $\ln M^2$ dependence.

The average charged multiplicity of X as a function of t, for $M^2 > 20$ GeV^2, is similarly obtained by integrating over M^2:

$$\langle n(t)\rangle = \int_{M^2_{MIN}}^{M^2_{MAX}} \langle n(M^2,t)\rangle D(M^2,t)dM^2 \Big/ \int_{M^2_{MIN}}^{M^2_{MAX}} D(M^2,t)dM^2 \quad .$$

Here $M^2_{MIN} = 20$ GeV^2 and $M^2_{MAX} \approx (s/2m_p^2)(-t)^{1/2}[(4m_p^2 - t)^{1/2} - (-t)^{1/2}]$, $\approx (s/m_p)(-t)^{1/2}$ for $-t \ll 4m_p^2$. Using $\langle n(M^2,t)\rangle$ as given by Eq. (3) we find

$$\langle n(t)\rangle = C\left[\frac{(M^2_{MAX})^{\alpha+1} \ln M^2_{MAX} - (M^2_{MIN})^{\alpha+1} \ln M^2_{MIN}}{(M^2_{MAX})^{\alpha+1} - (M^2_{MIN})^{\alpha+1}} + \ln\left(\frac{s_o}{s_o - t}\right) - \frac{1}{\alpha+1}\right] + d \quad . \qquad (5)$$

For $-t \gtrsim 0.1$ GeV^2, the M^2_{MIN} terms in (5) can be neglected and we obtain, using $M^2_{MAX} \approx (s/m_p)(-t)^{1/2}$,

$$\langle n(t)\rangle \approx C\left[\ln \frac{(-t)^{\frac{1}{2}} s_o}{m_p(s_o - t)} + \ln s - \frac{1}{\alpha+1}\right] + d \quad . \qquad (6)$$

Thus, for fixed s, $\langle n(t)\rangle$ is predicted to rise with $-t$ from $t \approx 0$, reaching a maximum at $-t = s_o \approx 1$ GeV^2.

Comparison with Data.—The data to be considered consist of 1566 inelastic events of reaction (1) at 205 GeV/c ($s = 385$ GeV^2), observed in the Fermilab 30-inch hydrogen bubble chamber. The outgoing proton, identified by ionization, has momentum ≤ 1.4 GeV/c. The quantities M^2 and t were calculated from measurements of the beam and recoil proton. Further experimental details are given in ref. [2].

In order to apply the present version of the multiperipheral model, we assume that the produced system X consists entirely of charged and neutral pions. We assume, furthermore, that the charged/neutral pion ratio is constant, so that the average charged multiplicity of X is a constant fraction of the average overall multiplicity of X. These assumptions are supported by a study of $\pi^- p \to K_s^o$ + anything and $\pi^- p \to \pi^o$ + anything at 205 GeV/c [6] which suggests that the kaonic component of X is $\lesssim 10\%$, and that π^+, π^-, and π^o are produced in approximately equal proportions.

Figure 1 shows $d\sigma/dM^2dt$ as a two-dimensional scatter plot of M^2 vs t. (As discussed in ref. [2], the observed exponential falloff with t of $d\sigma/dtdM^2$ is such that biases introduced by the 1.4 GeV/c cutoff in proton momentum ($-t \leq 1.4$ GeV^2) are negligible up to $M^2 \approx 180$ GeV^2.) For fixed t, events extend in M^2 up to the Chew-Low boundary, $M^2_{MAX} \approx (s/m_p)(-t)^{1/2}$. The low-t cluster of events which peaks at $M^2 \approx 2$ GeV^2 is produced by diffraction dissociation of the incoming pion [2]. In the following, pion diffraction dissociation (i.e., production of the system X in reaction (1) via Pomeron exchange) is assumed to be unimportant for M^2 above 20 GeV^2.

<u>M^2-Dependence</u>.—Figure 2 shows the average charged multiplicity, $\langle n(M^2)\rangle$, of the system X for all t values combined (solid circles). In agreement with the model, $\langle n(M^2)\rangle \sim \ln M^2$ for $M^2 \gtrsim 10$ GeV^2. Fitting $\langle n(M^2)\rangle$ over the region $20 < M^2 < 180$ GeV^2 using Eq. (4) without the small $\varphi(u_m)$ term, we find $\langle n(M^2)\rangle = C \ln M^2 + d$ with $C = 1.3 \pm 0.1$ and $d = 0.3 \pm 0.3$, as shown by the straight line in Fig. 2.

Figure 2 also gives $\langle n(M^2,t)\rangle$ vs M^2 for $-t = 0.0$-0.1, 0.1-0.3, and 0.3-0.7 GeV^2. The behavior of $\langle n(M^2,t)\rangle$ for each of these t-intervals is similar, indicating at most a weak t-dependence.

<u>t-Dependence</u>.—Figure 3 shows $\langle n\rangle$ as a function of t for $M^2 < 20$ GeV^2 (pion diffractive region) and for $M^2 > 20$ GeV^2. We observe that $\langle n(t)\rangle$

for the $M^2 < 20$ GeV^2 region is nearly constant for $-t \lesssim 0.4$ GeV^2 and may be rising at higher t. On the other hand, for $M^2 > 20$ GeV^2, $\langle n(t) \rangle$ rises rapidly for $-t \lesssim 0.1$ GeV^2, and increases slowly for $0.1 \lesssim -t \lesssim 1$ GeV^2; the depression of $\langle n(t) \rangle$ at small t is a Chew-Low boundary effect.

The solid curve in Fig. 3 is the prediction for $M^2 > 20$ GeV^2 using Eq. (6) with $s_o = 1$ GeV^2 and $\alpha = 1$, and with $C = 1.3$, $d = 0.3$ (as determined from the above fit to $\langle n(M^2) \rangle$). Good agreement with the data is observed. The prediction is not sensitive to the precise value of α. Using an effective Pomeron [7] with an intercept of 0.85, for example, would shift the entire curve downward by only 0.05 units.

To investigate the t-dependence of $\langle n(M^2,t) \rangle$ at <u>fixed</u> M^2, we show in Fig. 4 $\langle n \rangle$ vs t for several representative small intervals in M^2 above the pion diffractive region. In general, we find that <u>$\langle n \rangle$ at fixed M^2 is nearly independent of t for $-t \lesssim 1$ GeV^2</u>. The solid curves in Fig. 4 are the predictions of Eq. (3) with $s_o = 1$ GeV^2, again using the previously determined values of C and d. Reasonable agreement with the data is observed.

<u>Conclusions</u>.—We have shown that the average charged multiplicity data for the reaction $\pi^- p \to pX$ at 205 GeV/c can be reasonably understood with a simple version of the multiperipheral model for $20 \lesssim M^2 \lesssim 200$ GeV^2 and $|t| \lesssim 1$ GeV^2. It would clearly be of interest to extend the present multiperipheral analysis to higher values of s, M^2, and $|t|$; to the higher multiplicity moments of X, such as f_2 and f_3; and to other reactions of the form $a + p \to pX$.

We would like to thank the members of the Fermilab-LBL-UCB 205 GeV/c $\pi^- p$ collaboration for making available their data and for helpful comments.

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FIGURE CAPTIONS

Fig. 1. Two-dimensional plot of missing mass (M^2) vs momentum transfer (t) for 1566 inelastic events of the reaction $\pi^- p \rightarrow pX$ at 205 GeV/c. A cutoff of 1.4 GeV/c in the recoil proton momentum limits $-t$ to ≤ 1.4 GeV^2.

Fig. 2. Average charged multiplicity $\langle n \rangle$ as a function of M^2 for $-t < 1.4$ GeV^2 (solid circles). The straight line is the functional form $\langle n \rangle = C \ln M^2 + d$ predicted by the multiperipheral model, with fitted parameters $C = 1.3 \pm 0.1$ and $d = 0.3 \pm 0.3$ for $20 \leq M^2 \leq 180$ GeV^2. Also shown is $\langle n \rangle$ vs M^2 for three smaller intervals of momentum transfer: $0 < -t < 0.1$ GeV^2 (open circles), $0.1 < -t < 0.3$ GeV^2 (squares), and $0.3 < -t < 0.7$ (triangles).

Fig. 3. Average charged multiplicity $\langle n \rangle$ vs t for $M^2 < 20$ GeV^2 (open circles) and for $M^2 > 20$ GeV^2 (solid circles). The curve is the prediction of the multiperipheral model for $M^2 > 20$ GeV^2.

Fig. 4. Average charged multiplicity $\langle n \rangle$ vs t for $M^2 = 20$-40, 60-80, 100-120, and 140-160 GeV^2. The curves are the predictions of the multiperipheral model.

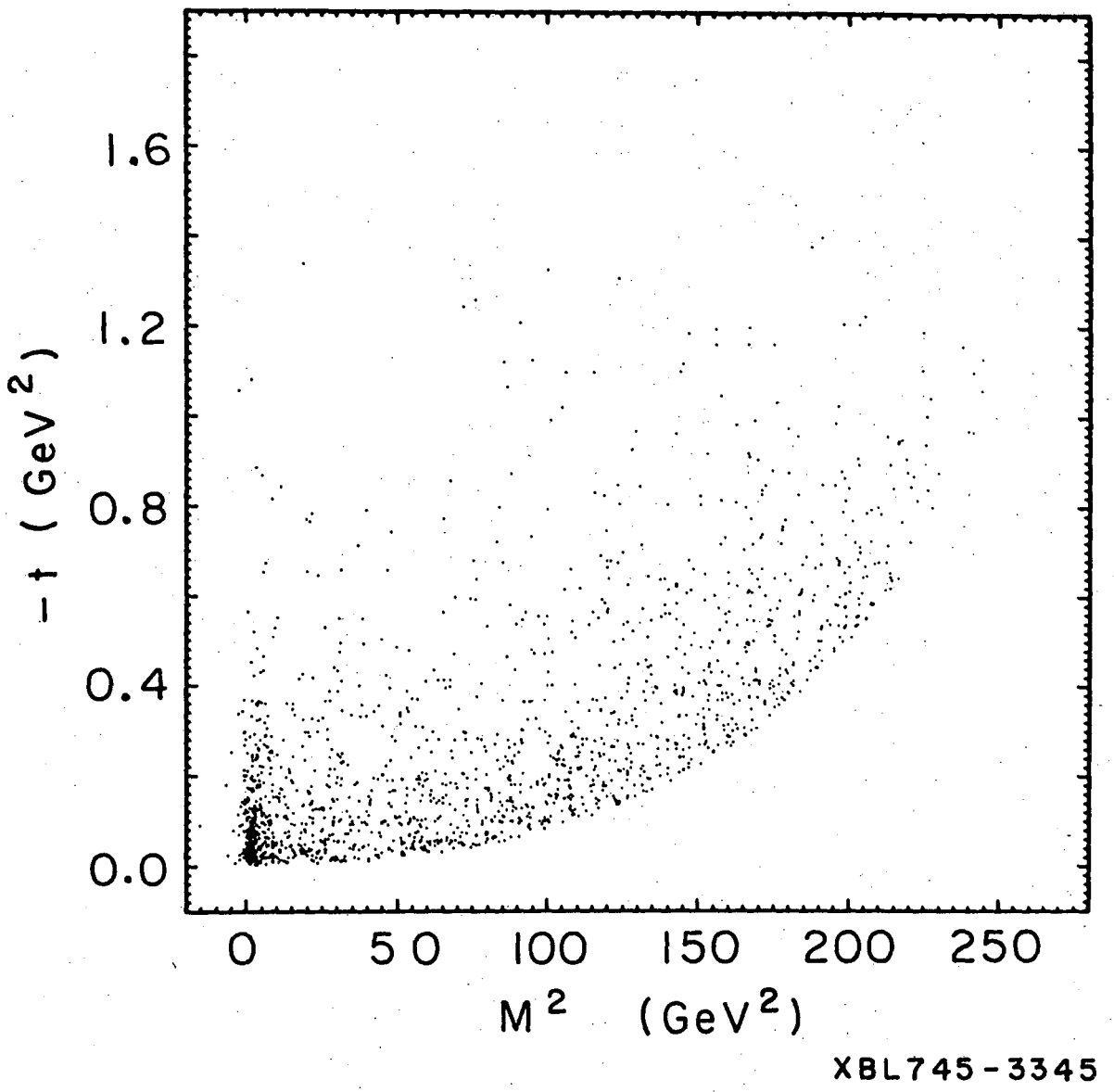

Fig. 1

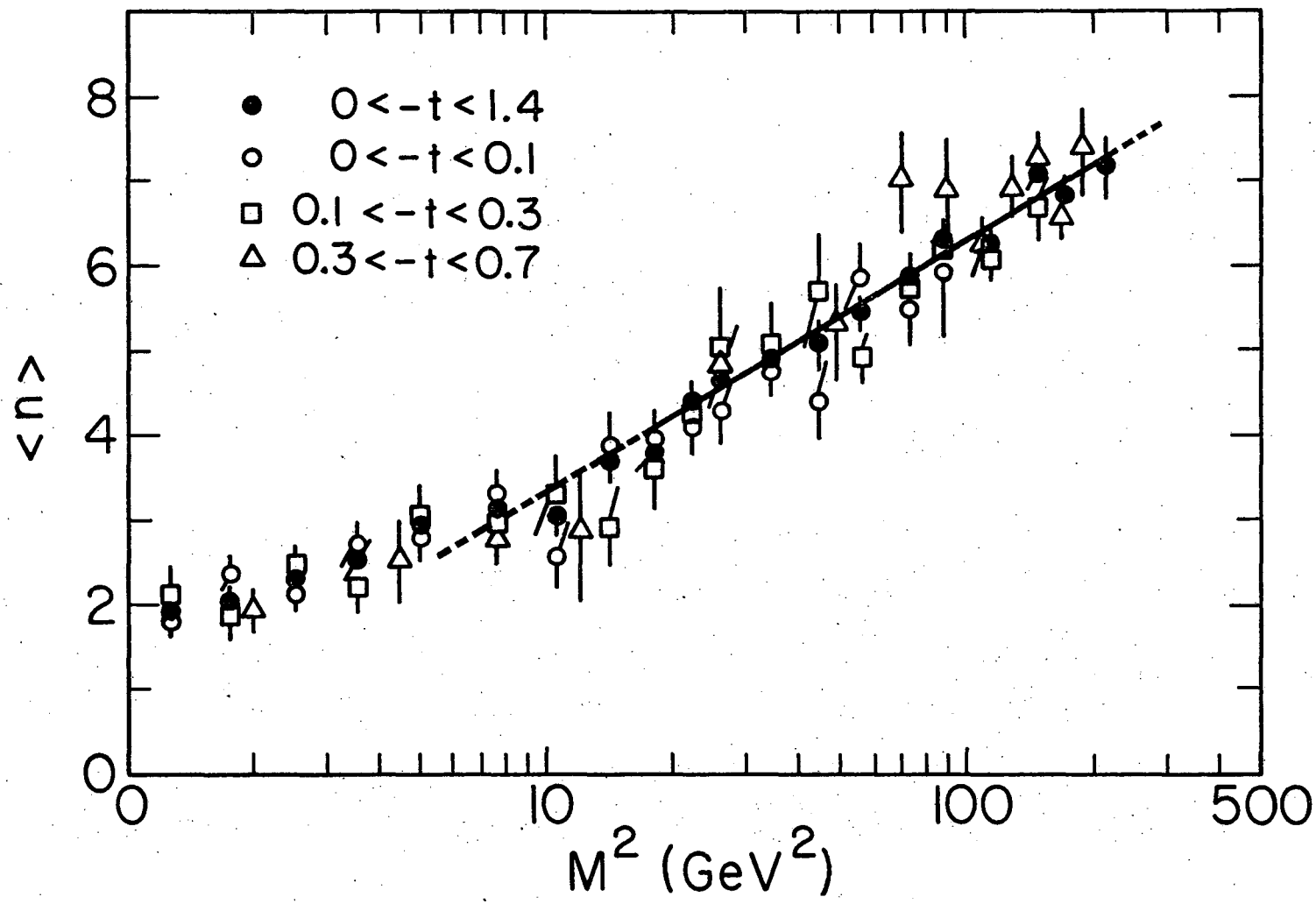

Fig. 2

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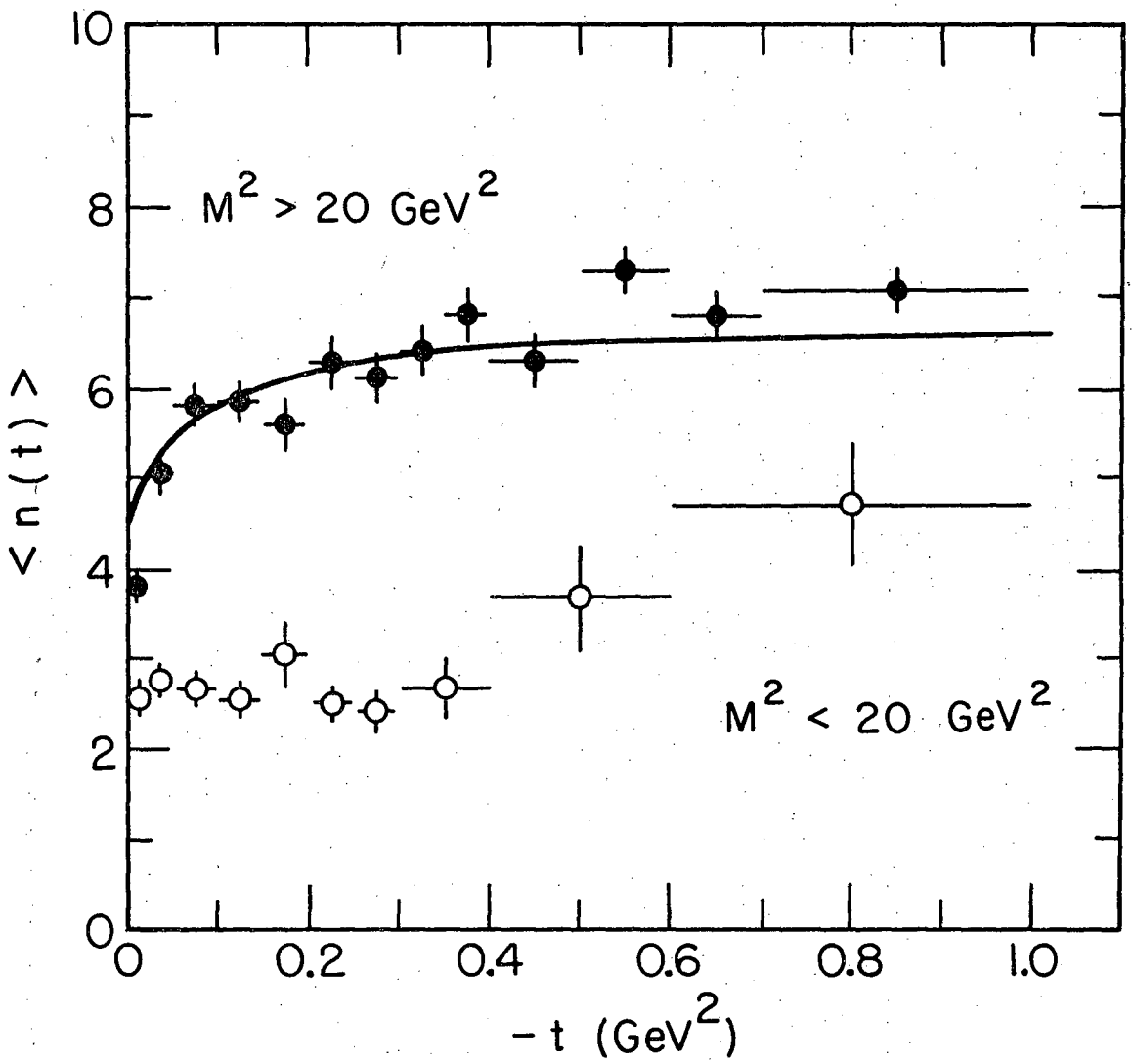

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Fig. 3

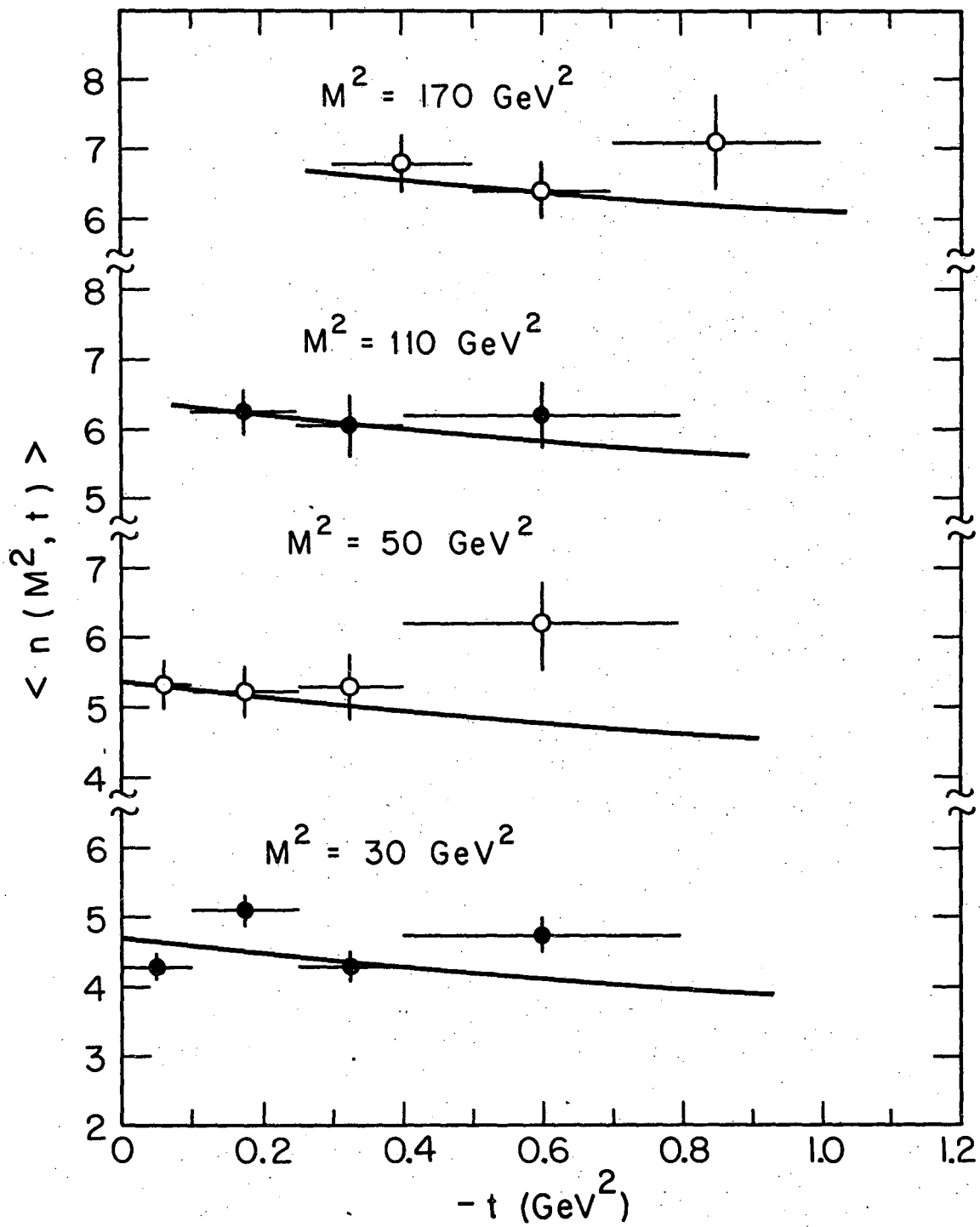

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Fig. 4

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