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COMMENTS ON QUANTUM THEORY WITH SHADOW STATES*

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ABSTRACT

It is shown that in shadow-state theories the particle mediating a one-shadow-particle-exchange process is propagated via a principal-value propagator. This leads to striking acausal effects that in principle are observable. It is also noted that the pole-factorization and cluster properties hold in shadow-state theories, contrary to recent claims.

In an attempt to resolve some of the difficulties with quantum field theory Professor Sudarshan has introduced the idea of shadow states. According to this idea certain particles are identified as shadow particles. States that contain shadow particles are called shadow states, and they are propagated via the time symmetric principal-value propagator, rather than the usual causal propagator.[1]

The use of this noncausal propagator would naturally be expected to lead to difficulties with the causality properties of the theory. However, proponents of shadow-state theories claim that there are no real difficulties because shadow particles are not observable; they merely provide an unobservable background. On the other hand, a recent analysis[2] of the effects of the unusual analyticity properties of shadow-state theories has led to the conclusion that shadow particles can be detected by their effects on ordinary particles, and

that they should, if present, produce acausal effects that are strikingly different from what has been observed in nature. Moreover, these acausal effects apparently lead to a breakdown of the usual general interpretational principles of quantum theory.[2]

It has been suggested[1] that these results of the analysis of ref.2 arise from an improper application of the principal-value propagation rule to the individual shadow particle, rather than to the entire shadow state, as required by the basic rule of shadow-state theory. It has also been asserted[1] that the pole-factorization property fails to hold in shadow-state theory. The purpose of this note is first to show that the analysis of ref.2 is based on a correct application of the basic principal-value rule of shadow-state theory, and second to point out that the pole-factorization property, and also the cluster decomposition property, do in fact hold in shadow-state theory.

In ref.1 the work of Richard[3] is cited as basis of the treatment of the shadow-state rule of propagation. Richard gives a detailed treatment of an example that is essentially the same as the one discussed in ref.2. Thus the quickest way to proceed is simply to describe the trivial changes in several of Richard's equations needed to obtain the result used in Ref.2.

The reaction discussed in Ref. 2 is exhibited in Fig. 1. The

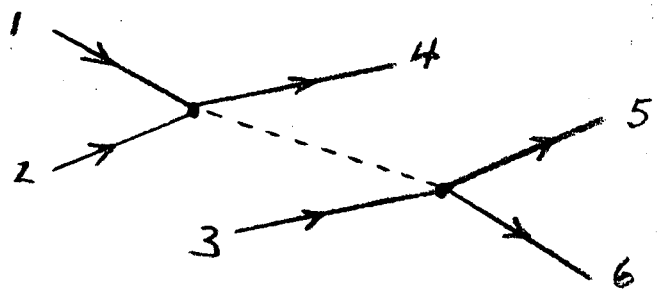

Fig. 1. The reaction under consideration.

six solid lines represent physical particles, and the dotted line represents a shadow particle. The corresponding interaction Lagrangian, which replaces Richard's interaction term (58), is

$$L_I(x) = :\varphi^\dagger(x)\ \psi^\dagger_4(x)\ \psi_2(x)\ \psi_1(x): + :\psi^\dagger_6(x)\ \psi^\dagger_5(x)\ \psi_3(x)\varphi(x): + \text{H.C.}, \tag{1}$$

where the $\psi_i(x)$ are the (scalar) fields associated with the physical particles i, and $\varphi(x)$ is the (scalar) field associated with the intermediate shadow particle. The second-order contribution to the scattering operator is, according to Richard's equation (44),

$$T^P_{(2)} = -\frac{i}{2}\int dx_1 dx_2\, \mathcal{T}\left[(L_I(x_1)\ L_I(x_2)\right] + \frac{i}{2}\int dx_1\ dx_2\ L_I(x_1)\ \Pi^S\ L_I(x_2)\ , \tag{2}$$

where $\mathcal{T}$ is the time-ordering operator and Π^S is the projector on shadow states. The operator $\Pi^S\ L_I(x_2)$ acting on physical states is $L_I(x_2)$, since $L_I(x_2)$ creates a shadow particle. [This statement corresponds to Richard's (65).] Then Wick's expansion[4] gives

$$T^{P}_{(2)} = -i \int dx_1\, dx_2 \Big(\varphi^{\cdot}(x_1)\, \varphi^{\cdot\dagger}(x_2) - \tfrac{1}{2}\, \varphi_{\cdot}(x_1)\, \varphi_{\cdot}^{\dagger}(x_2) - \tfrac{1}{2}\, \varphi_{\cdot}^{\dagger}(x_2)\, \varphi_{\cdot}(x_1) \Big)$$

$$\times\, :\psi_6^{\dagger}(x_1)\, \psi_5^{\dagger}(x_1)\, \psi_3(x_1)\, \psi_4^{\dagger}(x_2)\, \psi_2(x_2)\, \psi_1(x_2):$$

$$+ \text{ noncontributing terms,}$$

where the following definitions have been introduced:[5]

$$\varphi^{\cdot}(x_1)\, \varphi^{\cdot\dagger}(x_2) = \langle \mathcal{T} \varphi(x_1)\, \varphi^{\dagger}(x_2) \rangle_0$$

$$= \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x_1-x_2)}\, \frac{i}{k^2 - M_S^{\,2} + i\epsilon}$$

$$\varphi_{\cdot}(x_1)\, \varphi_{\cdot}^{\dagger}(x_2) = \langle \varphi(x_1)\, \varphi^{\dagger}(x_2) \rangle_0$$

$$= \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x_1-x_2)}\, 2\pi\, \theta(k^0)\, \delta(k^2 - M_S^{\,2})$$

$$\varphi_{\cdot}^{\dagger}(x_2)\, \varphi_{\cdot}(x_1) = \langle \varphi^{\dagger}(x_2)\, \varphi(x_1) \rangle_0$$

$$= \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x_2-x_1)}\, 2\pi\, \theta(k^0)\, \delta(k^2 - M_S^{\,2}).$$

(The signs in the above equations correspond to a positive-metric shadow particle. There would be an extra overall minus sign in the last line of each of the three above equations if the intermediate shadow particle were an indefinite-metric particle, as it is in Richard's example.)

Introducing $p = p_1 + p_2 - p_4 = p_5 + p_6 - p_3$ one obtains the momentum-space matrix element

$$T_{(2)}(p_1, p_2, p_3 : p_4, p_5, p_6) = N\left[\frac{1}{p^2 - M_S^2 + i\epsilon} + i\pi\,\delta(p^2 - M_S^2)\right]$$

$$= N\left[\text{P.V.}\ \frac{1}{p^2 - M_S^2}\right], \qquad (3)$$

where N is a normalization factor, and P.V. signifies the principal-value resolution of the pole singularity.

Equation (3) shows that the intermediate shadow particle is propagated by the principal-value propagator, as stated in Ref. 2. Sudarshan criticized that work by asserting that the shadow-theory propagator refers to the entire system (i.e. the whole state), <u>not</u> to the individual particles. However, the above calculation shows that, for the case under consideration, the shadow-theory rule for the propagation of states entails that the single exchanged shadow particle is also propagated by the principle-value propagator.

This result--that the principal-value rule for the propagation of shadow states implies the principle-value rule for the propagation of the single exchanged shadow particle--was mentioned explicitly in Ref. 2. The result can be derived also directly from the second-order term $V\,G\,V \equiv V(G_R\,\Pi^P + G_S\,\Pi^S)V$ in the expansion of T described by Sudarshan.[1]

Later on in Ref. 1 it was asserted that shadow particles do <u>not</u> have a pole-factorization property of the kind presupposed in

Ref. 2. The pole factorization property is not used to obtain the results about shadow-state theories described in Ref.2. The lowest-order pole contribution by itself is sufficient to produce the effects described. But in any case the ordinary pole-factorization property does in fact hold in shadow theory. This is not evident from a superficial examination of the shadow-theory formulas, but it nevertheless follows from those formulas. The point is that the various terms that appear to violate the pole-factorization property cancel out.

To obtain these cancellations one must sum over the terms of $T = V + VGV + \cdots$ corresponding to all of the different orders of the vertices of one of the two parts of the single-particle-exchange-diagram relative to those of the other part. For example, one must sum over all the terms of $T = V + VGV + \cdots$ that correspond to the diagram of Fig. 2, keeping the four vertices of the upper subdiagram

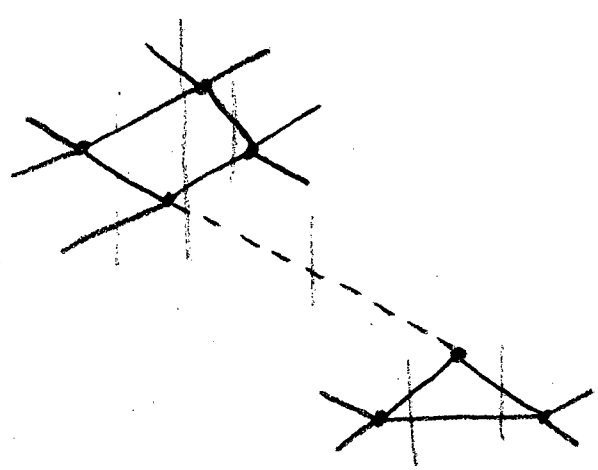

Fig. 2. Diagrams contributing the pole term.

in a fixed order relative to each other, and keeping the three vertices of the lower subdiagram in a fixed order relative to each other, but summing over all orders of the upper four vertices relative to the

lower three vertices. This summation yields a factorized form. One factor is just the pole corresponding to the single exchanged particle. The residue of this pole is a product of two factors, one of which is exactly the contribution to the $T = V + VGV + \cdots$ of the upper subprocess (considered alone) that corresponds to the specified fixed order of the vertices of upper subdiagram. The other is similarly related to the lower subdiagram. The shadow-theory rule for the propagation of states is imposed by averaging over a positive and negative imaginary increment to the energy of each of the shadow states. These increments are allowed to go to zero in any order <u>after</u> taking the limit of the Feynman $i\epsilon$'s for the ordinary (i.e., nonshadow) particles. This averaging causes the pole associated with the exchanged particle to become a principle-value pole, just as in (3), provided this particle is a shadow particle. A final sum over all orders of the vertices of each of the two individual subgraphs yields the usual pole-factorization property.

An expeditious way to perform the sum over all orders is first to fix the order of all vertices but the right-most vertex of the upper subdiagram. Then a summation over the possible positions of this vertex yields the factorization of the denominator associated with the lines connected to it. Next one sums over the possible positions of the next right-most vertex of this upper subdiagram, and so on, until one comes to the vertex V^u_{ex} connected to the exchanged line. Then one switches to the left-most vertex of the upper subdiagram and works to the right. Finally one sums over all positions of V^u_{ex}. This yields the complete factorization described above. This construction for the usual causal case is undoubtedly well known. This same

procedure, applied carefully, also allows one to show that the usual cluster decomposition property holds in shadow theory, contrary to recent claims.[6]

The singularity structure exhibited in Eq.(3) has important experimental consequences.[2,7] It ensures that shadow particles can be produced and detected by interactions with ordinary particles, and that the propagation of shadow particles through space and time, as revealed by experiments designed to produce shadow particles in one space-time region and detect them in another space-time region, is the same as the propagation of ordinary particles, except that half of the amplitude is propagated backward in time, instead of forward. This means that shadow particles can be detected in space-time regions that lie earlier than the space-time regions in which they are produced.

FOOTNOTES AND REFERENCES

* This work was supported by the U. S. Atomic Energy Commission.

1. E. C. G. Sudarshan, in Causality and Physical Theories, AIP Conference Proceedings No. 16, edited by William B. Rolnick. At the conference this report preceded the one of Ref. 2, and contained none of the comments on Ref. 2 that are the subject of the present note.

2. Henry P. Stapp, in Causality and Physical Theories, ibid.

3. J. L. Richard, Phys. Rev. D7, 3617 (1973).

4. See, e.g., J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields, McGraw-Hill, New York (1965), p. 181.

5. Ref. 4, Appendix C.

6. C. C. Chiang, Bogoliubov Boundary Condition and the Theory of Shadow States in Quantum Theory (to be published in Physical Review D).

7. H. P. Stapp, Phys. Rev. 139B, 257 (1965); D. Iagolnitzer, in Lectures in Theoretical Physics, vol. XID, ed. K. T. Mahanthappa and W. E. Brittin (Gordon and Breach, New York, 1969), p. 250.

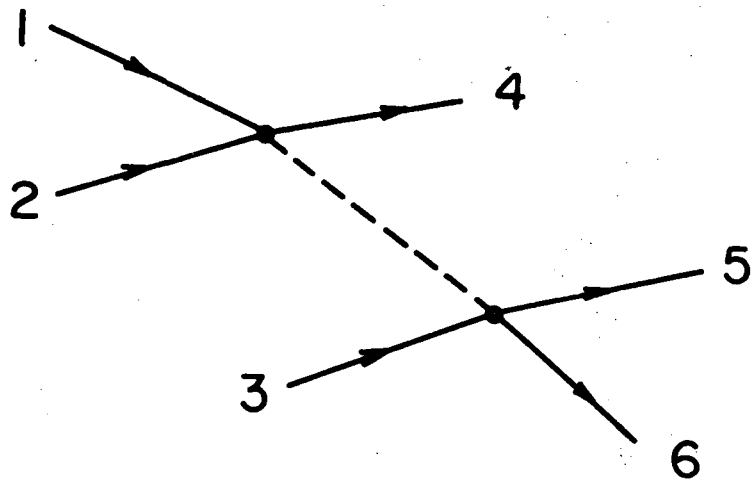

Figure 1. The reaction under consideration.

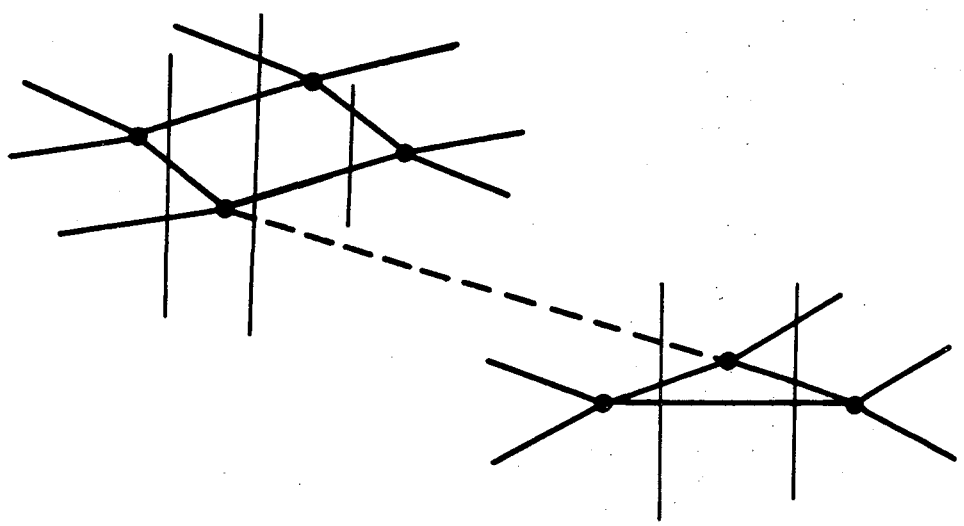

Figure 2. Diagrams contributing the pole term.

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