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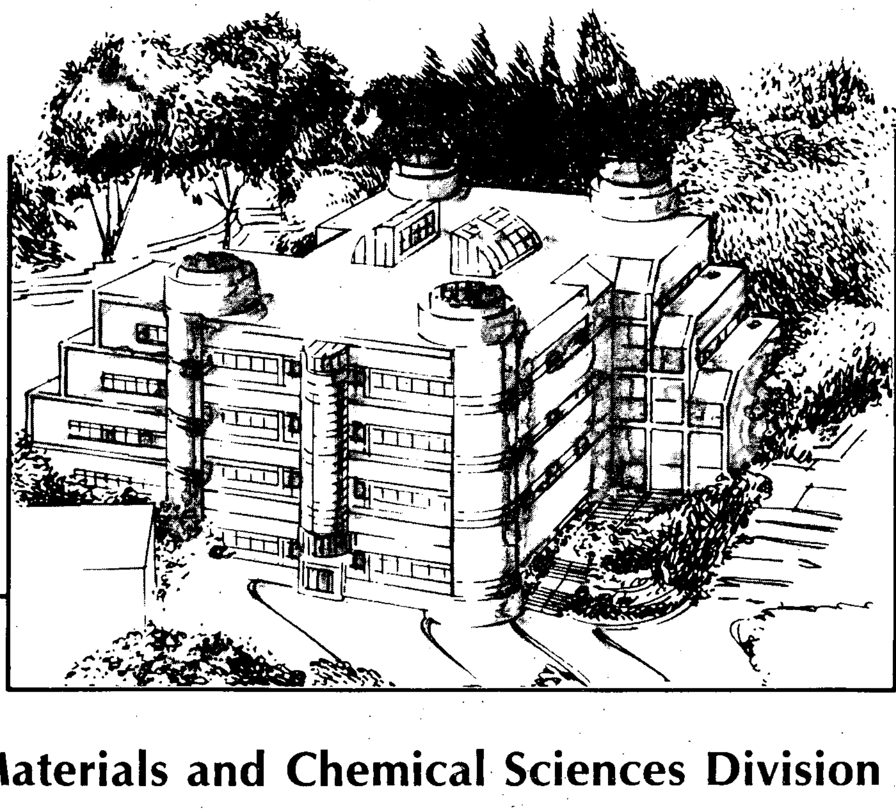

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Analysis of Transformation-Induced Crack Closure

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The deformation-induced martensitic transformation around a fatigue crack results in the phenomenon of crack closure. By approximation of the residual stress field due to the volume expansion of a slab of transformed zone as that of an edge dislocation pair, we have derived a closed-form solution of crack opening as a function of external loading.

I. INTRODUCTION

The previous studies[1-12] indicated that the deformation-induced martensitic transformation that occurs at the tip of a growing fatigue crack in metastable austenitic steels reduces the crack propagation rate in both the threshold and Paris regions. One of the explanations for that phenomenon is the residual stress field induced by the volume expansion associated with the martensitic transformation. The stress/strain concentration at a crack tip induces the martensitic transformation; as it propagates, the crack is enveloped with a transformed zone. The elastic constraint of the surrounding matrix material on the dilatant transformed zone puts the crack under compression.

It was proposed by Suresh and Ritchie[13] that the dilatant transformation around a fatigue crack should induce "crack closure". The crack closure, in the study of fatigue crack propagation, refer to the premature contact between the crack faces during the tensile portion of the fatigue cycle. It have been well documented and reviewed[13] that the crack closure may be resulted from the plastic deformation at the crack tip, oxide deposits formed within the crack, irregular crack surface morphologies, and viscous media penetrated inside the crack. The crack closure induced by the martensitic transformation has been observed as well.[8,9,12] However, the crack closure measurement data could not quantitatively explain the reduction of the crack growth rate in metastable austenitic stainless steels.[12]

It is our intention here to develop an analytic model for the crack closure induced by the martensitic transformation, in order to understand the displacement field of a crack that is under the transformation-induced residual compressive stress and exactly how the crack opens and closes under external cyclic tensile loading. The model reported here approximates the residual stress field of a slab of dilatant transformed zone with the stress field of a pair of edge dislocations, the changing of the opened crack length with increasing external tensile load is then calculated. The results from this model should also shed light on the crack closure induced by the oxide deposit, crack face irregularity, and viscous media .

II. ANALYSIS

Residual Stress

Assume that it is a two dimensional plain strain condition. As shown in Fig. 1, a through-thickness straight crack with length ($2\ a_0$) is embedded in a rectangular transformed zone of width ($2\ w$) and length $2\ (a_0 + \delta)$. The zone itself is located inside an infinitely large elastic body. The strain associated with the transformation is a pure volume expansion described by e^T ($= \Delta V / V$). The residual stress field induced by the presence of the transformed zone is modeled as the stress field of a pair of dislocations. The crack-opening stress, σ_{yy} along $y = 0$, is plotted in Fig. 1, and can be expressed as,[14]

$$\sigma^d{}_{yy} = -\frac{bu}{2\pi(1-v)}\frac{1}{a_0+\delta-x} - \frac{bu}{2\pi(1-v)}\frac{1}{a_0+\delta+x} \tag{1}$$

where the first and second terms represent the stress fields of two edge dislocations located at ($x = a_0 + \delta$) and ($x = -\ a_0 - \delta$) respectively, u is the shear elastic constant, v the Poisson's ratio. It is seen from Fig. 1 that the compressive stress reaches the maximum value at the tip of the crack and the minimum value at the center of the crack,

$$\sigma^d{}_{yy}(\max) = -\frac{bu}{2\pi(1-v)}\left(\frac{1}{\delta} + \frac{1}{2a_0+\delta}\right) \tag{2a}$$

$$\sigma^d{}_{yy}(\min) = -\frac{bu}{\pi(1-v)}\frac{1}{a_0+\delta}. \tag{2b}$$

The Burger's vector of the dislocations, **b**, should be the displacement of the transformed zone relative to the non-transformed matrix materials. The vector **b** is approximately along the y axis and with the value of ($2\ w\ e^T$), because the dimensions along x and z axis of the transformed zone are much larger than that along y axis, the displacement associated with a pure dilatant plate is mostly along the normal direction of the plate.

Loading and Crack Opening

It is supposed that this cracked elastic body is under a remotely applied tensile stress, σ_0. Let σ_0, now, increase from zero. If the stress σ_0 is small, the crack remains closed due to the residual compressive stress. As the stress σ_0 increases to a critical value such that $\sigma_0 \geq \sigma^d{}_{yy}(\min)$ of Eq (2b), the crack starts to open from the center, because the central part of the crack is under the smallest compressive stress. The opened part of the crack extends towards the crack tip as σ_0 increases further.

The determination of the opened part length of the crack as a function of the remotely applied stress σ_0 is analogical to determination of the plastic zone size at a crack tip. While the yield stress is constant everywhere inside an elastic body, the stress to open the crack varies as the absolute value of the compressive residual stress. Two methods were used in fracture mechanics to determine the crack tip plastic zone size, the Irwin plastic zone correction and the Dugdale stripe approach. Here the Dugdale approach[15] is modified to calculate the crack opening with increasing σ_0.

The basic assumption is that the stress level at the tip of the opened part of the crack can not go to infinite, the stress singularity should disappear. That means that the stress intensity K_σ due to the remotely applied σ_0 has to be compensated by the stress intensity K_p due to the compressive residual stress σ^d_{yy},

$$K_\sigma + K_p = 0. \tag{3}$$

The requirement (3) permits determination of the opened part length x_0 of the crack.

The stress intensity K_σ of a central crack of length $(2\,x_0)$ in an infinitely large elastic body under remotely applied σ_0 is

$$K_\sigma = \sigma_0 \sqrt{\pi x_0}. \tag{4}$$

The stress intensity K_p of the symmetrically distributed (with respect to $x = 0$) loads on the crack surfaces can be calculated by integration of the stress intensity solution of two pairs of concentrated splitting forces P acting on $(-x, 0)$ and $(x, 0)$ of the crack surfaces. Such solution is,[16]

$$K_G = \frac{2}{\sqrt{\pi x_0}} \frac{1}{\sqrt{1-(x/x_0)^2}} \tag{5}$$

Therefore,

$$K_p = \int_0^{x_0} \frac{2}{\sqrt{\pi x_0}} \frac{\sigma_{yy}(x)\,dx}{\sqrt{1-(x/x_0)^2}} \quad . \tag{6}$$

After replacing $\sigma_{yy}(x)$ in (6) with $\sigma^d_{yy}(x)$ of (1), (6) becomes,

$$K_p = -\frac{1}{\sqrt{\pi x_0}} \frac{bu}{\pi(1-\nu)} \left(\int_0^{x_0} \frac{1}{\sqrt{1-(x/x_0)^2}} \frac{dx}{a_0+\delta-x} + \int_0^{x_0} \frac{1}{\sqrt{1-(x/x_0)^2}} \frac{dx}{a_0+\delta+x} \right)$$

$$= -\frac{1}{\sqrt{\pi x_0}} \frac{bu}{\pi(1-\nu)} \int_{-x_0}^{x_0} \frac{1}{\sqrt{1-(x/x_0)^2}} \frac{dx}{a_0+\delta+x}$$

$$= -\frac{\sqrt{x_0}}{\sqrt{\pi}} \frac{bu}{\pi(1-\nu)} \int_{-\pi/2}^{\pi/2} \frac{d\theta}{a_0+\delta+x_0\sin\theta} . \qquad (7)$$

It is listed in reference 17 that,

$$\int \frac{dz}{a+b\sin z} = \frac{2}{\sqrt{a^2-b^2}} \arctan \frac{a\tan(\frac{z}{2})+b}{\sqrt{a^2-b^2}}, \qquad (a^2 > b^2). \qquad (8)$$

Using that result, we have,

$$K_p = -\frac{\sqrt{x_0}}{\sqrt{\pi}} \frac{bu}{1-\nu} \frac{1}{\sqrt{(a_0+\delta)^2-x_0^2}} . \qquad (9)$$

After inserting (9) and (4) into (3), (3) becomes,

$$\sigma_0 = \frac{bu}{\pi(1-\nu)} \frac{1}{\sqrt{(a_0+\delta)^2-x_0^2}} . \qquad (10a)$$

Reform (10a), express the crack opening x_0 in terms of the external load σ_0,

$$x_0 = \sqrt{(a_0+\delta)^2 - (\frac{bu}{\pi(1-\nu)\sigma_0})^2} \qquad (10b)$$

The value of σ_0 when the crack just starts to open can be calculated by replacing $x_0 = 0$ in (10a),

$$\sigma_0\,(\min) = \frac{bu}{\pi(1-\nu)} \frac{1}{a_0+\delta} \qquad (11)$$

The result is consistent with that of (2b). The value of σ_0 to fully open the crack can be evaluated by replacing x_0 in (10) with a_0, then

$$\sigma_0\,(\max) = \frac{bu}{\pi(1-\nu)} \frac{1}{\sqrt{2\delta a_0+\delta^2}} . \qquad (12)$$

The stress intensity when $\sigma_0 = \sigma_0$ (max) is,

$$K_\sigma\,(\max) = \sigma_0\,(\max)\,\sqrt{\pi a_0}$$

$$= \frac{bu}{\sqrt{\pi}(1-\nu)} \frac{\sqrt{a_0}}{\sqrt{2\delta a_0+\delta^2}} \tag{13}$$

Numerical Results

Rewrite (10b) in terms of the non-dimensional variables,

$$\sigma^*_0 = \frac{\sigma_0}{E/(1-\nu)^2}, \quad x^*_0 = \frac{x_0}{a_0}, \quad w^* = \frac{w}{a_0}, \quad \delta^* = \frac{\delta}{a_0} \tag{14}$$

where E is the Young's elastic modulus, we have,

$$x^*_0 = \sqrt{(1+\delta^*)^2 - (\frac{e^T w^*}{\pi\sigma^*_0})^2}. \tag{10c}$$

Now we use AISI 304 austenitic stainless steel as an example and input some practical values for the parameters in (10c). The volume expansion e^T associated with γ to α' martensitic transformation is about 2%; the transformation zone width w is approximately 0.5 mm; the zone size ahead of the crack δ is in the same range as w; the crack length a is about 10 mm.[12] For these conditions, the crack opening x^*_0 vs. external load σ^*_0 is plotted in Fig. (2).

III. DISCUSSION

Residual Stress

It is an approximation to represent the residual stress field due to a rectangular dilatant transformation zone with that of a pair of edge dislocations. The approximation is accurate only when the width is much smaller than the length of the rectangular zone. The exact solutions for the stress field in an infinitely large body due to a rectangular element that has an initial homogeneous strain have been solved.[18] The expression for $\sigma_{yy}(x)$ is very complicated. However, the general trend of σ_{yy} is similar to that of a edge dislocation: it is compressive inside the rectangular and tensile outside the rectangular; it has smallest compressive stress at the center, and increases to infinite at the edge.

Stress Intensity Reduction by the Residual Stress

K_σ (max) in (13) represents the maximum possible reduction of the stress intensity by the compressive residual stress field. When a cracked body is under a cyclic tensile loading, the effective cyclic stress intensity at the crack tip changes from K_{max} - K_{min} to K_{max} - K_σ(max), if K_σ(max) > K_{min}; When a cracked body is loaded monotonically, its fracture toughness increased by the amount of K_σ(max).

It has been well documented that the fracture toughness of certain ceramics can be enhanced by the martensitic transformation. The amount of enhancement were estimated theoretically with the weight function method by McMeeking and Evans.[19] Their result is that,

$$K_\sigma = 0.22\, e^T \sqrt{w}\, \frac{E}{(1-\nu)}. \tag{15}$$

It should be interesting to compare (15) and (13). Since a >> δ, (13) can be approximated as,

$$K_\sigma = \frac{e^T w E}{\sqrt{\pi}(1-\nu^2)} \frac{1}{\sqrt{2\delta}}. \tag{16}$$

δ is in the same size range with w, i.e. $\delta = \alpha$ w. For the transformation zone defined in reference 19, $\alpha = (8 / 3\sqrt{3})$. If ν is chosen as 1/3, we can rewrite (16) as,

$$K_\sigma = 0.24\, e^T \sqrt{w}\, \frac{E}{(1-\nu)}. \tag{17}$$

Crack Closure Measurement

The crack closure is usually determined by the unloading displacement vs. load curve. The slope of the curve, compliance, depends on the crack length. During unloading cycle, when the crack faces starts to contact near the crack tip, the crack length is reduced, so is the compliance, therefore the displacement vs. load straight line starts to bend. The analysis described above present a method to calculate the crack opening vs. the applied load. For specimen geometries that are different from the infinitely large plate with a central crack, the similar procedure of (3) through (13) can still applied. With the knowledge of the crack opening vs. load, plus the formula of displacement vs. crack length (usually listed in the stress analysis handbooks), the displacement vs. load curve can be estimated.

It is seen from Fig. 2 that the crack does not open until σ_0 reaches σ_0(min), immediately after that, it opens very fast ($dx_0 / d\sigma_0 = \infty$ when $\sigma_0 = \sigma_0$(min)), as the crack opens further it gets more and more difficult to open ($dx_0 / d\sigma_0$ decreases). The unloading is a reverse process of the loading. It is easy to see that the initial point where the unloading straight line starts to bend is very difficult to locate. It implies that the accuracy of determination of K_σ(max) from the compliance measurement is inherently difficult to achieve.

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REFERENCE

1. A. G. Pineau and R. M. Pelloux: *Metall. Trans. A*, 1974, vol. 5A, pp. 1103-12.

2. C. Bathias and R. M. Pelloux: *Metall. Trans. A*, 1973, vol. 4A, pp. 1265-73.

3. R. L. Tobler and R. P. Reed: *J. of Testing and Evaluation,* 1984, vol. 12, No. 6, pp. 364-70.

4. G. Schuster and C. Altstetter: *Metall. Trans. A*, 1983, vol. 14A, pp. 2077-83.

5. G. Schuster and C. Altstertter: Fatigue Mechanisms, ASTM STP 811, American Society for Testing and Materials, 1983, pp. 445-63.

6. G. R. Chanani, S. D. Antolovich, and W. W. Gerberich: *Metall. Trans. A*, 1972, vol. 3A, pp. 2661-72.

7. E. Hornbogen: *Acta Metall.*, 1978, vol. 26, pp. 147-52.

8. A. J. McEvily, W. Zagrany and J. Gonzalez: Basic Mechanisms in Fatigue of Metals, P. Lukas and J. Polak, eds., Elsevier, New York, 1988, pp. 271-79.

9. K. Katagiri, M. Tsuji, T. Okada, Kohji, R. Ogawa, G. M. Chang, and J. W. Morris, Jr., *Adv. Cryog. Eng.*, 1989, vol. 36, to be published.

10. G. M. Chang: M.S. Thesis, University of California, Berkeley, 1983.

11. Z. Mei, G. M. Chang, and J. W. Morris, Jr.: Cryogenic Materials '88, R. P. Reed, Z. S. Xing, and E. W. Collings, eds., ICMC, Boulder, Colorado, 1988, vol. 2, p. 491.

12. Z. Mei, and J. W. Morris, Jr.: *Metall. Trans. A*, submitted.

13. S. Suresh and R. O. Ritchie: Fatigue Crack Growth Threshold: Concepts, D. L. Davidson and S. Suresh, eds., TMS-AIME, Warrendale, PA, 1984, pp. 227-61.

14. J. P. Hirth and J. Lothe: Theory of Dislocations, 2nd edition, A Siley-Interscience Publication, 1982, p. 76.

15. D. S. Dugdale, *J. Mech. Phys. Sol.*, vol. 8, 1960, pp. 100-108.

16. H. Tada, P. C. Paris, and G. R. Irwin, The Stress Analysis of Cracks Handbook, 1985 edition, Paris Productions, Inc., p. 5.11.

17. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, eds, M. Abramowitz, and I. A. Stegun, National Bureau of Standards, Applied Mathematics Series 55, 1964, p.78.

18. Y. P. Chiu, *J. of Applied Mechanics*, December 1977, pp. 587-90.

19. R. M. McMeeking and A. G. Evans, *J. Am. Ceram. Soc.*, v. 65, pp. 242-6.

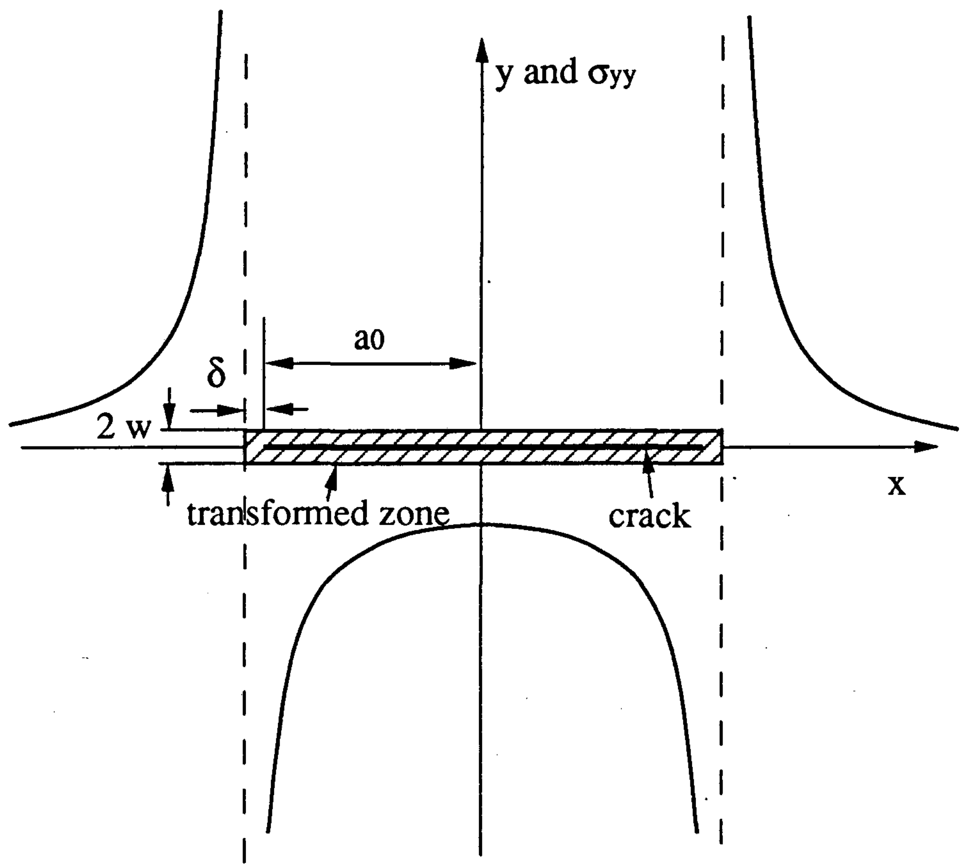

Fig. 1: A crack embedded in a rectangular transformed zone. The residual stress associated with the dilation of the transformation is close to the stress fields of a pair dislocations located at both edges of the rectangle.

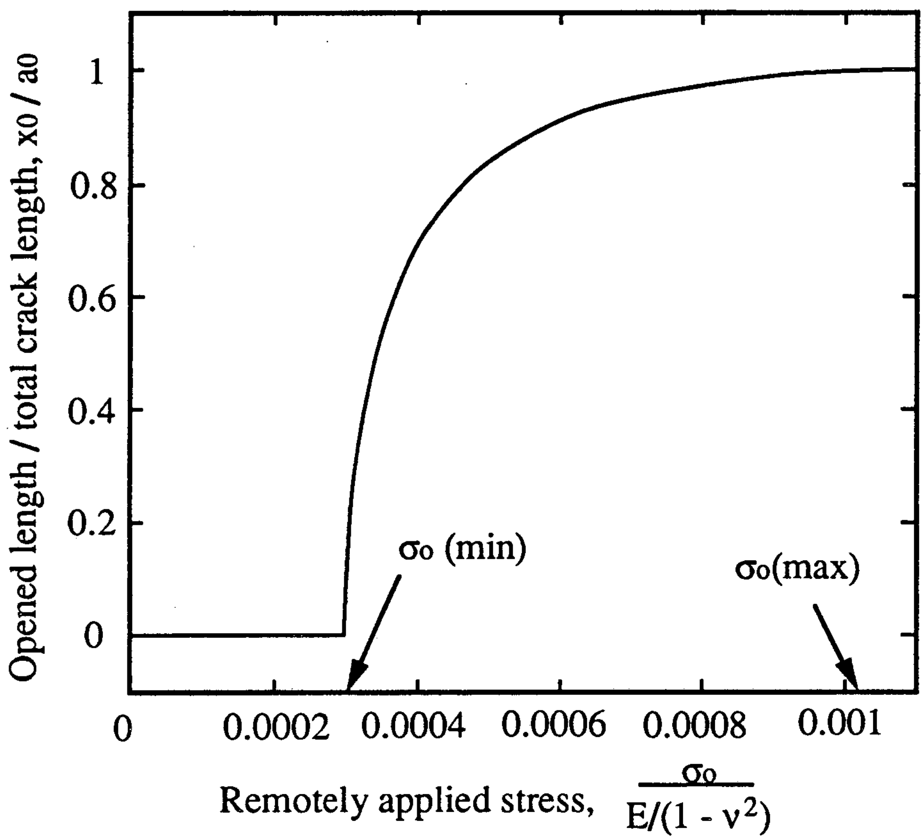

Fig. 2: Predicted crack opening vs. remotely applied tensile stress, see the text for the parameters used.

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