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B. Cohen, M. Mostrom, D. Nicholson,
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SIMULATION OF LASER BEAT HEATING OF A PLASMA

B. Cohen, M. Mostrom, D. Nicholson, C. Max, A. Kaufman,

Department of Physics and Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

and

A. B. Langdon

Lawrence Livermore Laboratory
University of California
Livermore, California 94550

June 10, 1974

ABSTRACT

We describe a new relativistic electromagnetic computer simulation code, with one spatial dimension, which explicitly follows right- and left-going electromagnetic waves by integrating along the characteristics of Maxwell's equations. The code is suited to simulating laser-plasma interactions. As an example, we discuss simulations of the heating of plasma by two opposed lasers whose beat frequency drives a local plasma oscillation. Excellent agreement is obtained with the analytic theory, in the linear-response regime.

I. INTRODUCTION

This paper presents computer simulations of laser-plasma interactions, introducing a new code for performing relativistic particle simulations with fully electromagnetic interaction. The application studied is the heating of plasma by two lasers (of frequencies ω_0, ω_1) whose beat frequency $(\Omega \equiv \omega_0 - \omega_1)$ is near the plasma frequency.

The nonlinear interaction may be considered as an induced decay ($\omega_0 \rightarrow \omega_1 + \Omega$), in which the fraction R of the incident power at ω_0 is converted to ω_1 and Ω, with the fraction $R\Omega/\omega_0$ appearing as longitudinal plasma oscillation and, because of damping, ultimately as heat. It is the aim of the theory and simulations to determine the dependence of this efficiency parameter R on the parameters of the problem: laser intensities, density scale length, temperature.

In the present paper we discuss the general theory of the interaction, with specific attention to the regime of linear longitudinal response of the nonuniform plasma. This theory is then tested by the simulations, with excellent agreement obtained. Particular interest attaches also to the regime of nonlinear response;[1] however, its analysis is still in progress, and will be reported in a later publication.

It should be kept in mind that the process studied here, involving three electron waves (two transverse and one longitudinal), with no ambient magnetic field, is illustrative of the more general three-wave process, possibly involving ions, and in a magnetic field. Thus the principle of electron heating, by the damping of a resonant excitation from the beat of two high-frequency waves, can be extended to the analogous heating of ions in a magnetically confined plasma.

II. THE CODE

There is a considerable literature concerning electromagnetic codes.[2] Most algorithms for solution of Maxwell's equations require solving a current driven wave equation for the vector potential. In this code, we solve for the electromagnetic fields explicitly by

integrating Maxwell's equations along their characteristics. Dawson and Langdon[3] first used this method in 1966.

Charged particles are represented by clouds of infinite cross-sectional area in the plane transverse to the grid. In the one dimension in which spatial variations are followed and particle positions are assigned, particles have finite size. Charge densities are calculated by linear interpolation according to the PIC-CIC model.[4] In this same dimension, designated "longitudinal", there are components of particle velocity and electric field, and all wave propagation occurs. The electromagnetic waves are linearly polarized in the direction of the single transverse velocity component (see Figure 1). The self-consistent and external magnetic fields lie in the transverse plane and are perpendicular to the polarization direction. The equations of motion are relativistic. There are versions of the code for which the plasma is assumed periodic or, alternatively, finite.

For the particular geometry we have described (Figure 1), the two Maxwell curl equations take the following form:

$$-\partial B_z/\partial x - c^{-1}\,\partial E_y/\partial t = 4\pi J_y/c$$

$$\partial E_y/\partial x + c^{-1}\,\partial B_z/\partial t = 0 \quad .$$

By adding and subtracting these equations, we obtain

$$(\partial/\partial x)\left[E_y \pm B_z\right] \pm c^{-1}(\partial/\partial t)\left[E_y \pm B_z\right] = \mp 4\pi J_y/c \quad . \qquad (1)$$

If we define the right- and left-going electromagnetic field quantities respectively as $F_{\pm} \equiv E_y \pm B_z$, the two Maxwell equations become

$$\left[(\partial/\partial x) \pm c^{-1}(\partial/\partial t)\right] F_{\pm} = \mp 4\pi J_y/c \quad . \tag{2}$$

Given the particle positions and velocities, from which we obtain the current J_y, Eq. (2) is integrated along the vacuum characteristics $x \mp ct = \text{const}$. Gridpoints in the space-time mesh are linked by the vacuum characteristics. Then $\Delta x/\Delta t \equiv c$, and there is no Courant condition in the usual sense.[5] By solving for the fields explicitly and by calculating a new current J_y with some smoothing at the half time-step, spurious numerical dispersion is minimized. Consequently, if we do the mechanics of the particle motion relativistically there should be no numerical Cerenkov instability. Furthermore, the parameters for which light waves in a drifting plasma can become unstable, due to finite differencing, are unphysical and can be easily avoided.[6]

The differential equations which the code solves can be summarized as follows: the equations for the fields, given the sources, i.e., charge density and current, are Eq. (2) and the Poisson equation

$$-\partial^2\phi/\partial x^2 = 4\pi en \quad . \tag{3}$$

Electrons have charge e. We assume a single species here (with fixed neutralizing background) but the code deals with two in general. The equations for the particle and current densities (before linear interpolation) are

$$\begin{aligned} n(x) &= \sum_m \delta(x - x^m) \\ J_y(x) &= \sum_m e v_y^m \, \delta(x - x^m) \quad . \end{aligned} \tag{4}$$

The equation of motion for the particles is

$$(d/dt)\left[m\underset{\sim}{v}(1 - v^2/c^2)^{-\frac{1}{2}}\right] = e(\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B}/c) \quad . \tag{5}$$

The closed set of equations yields to standard space-time centering and leap-frog techniques (Fig. 2). The equation of motion (5) is integrated forward in time using a hybrid, fast half-acceleration and rotation method.[7] Because we are interested in the Fourier transform of the electrostatic potential, we solve Poisson's equation by means of fast Fourier transforms although faster techniques exist. The differences between the bounded and periodic versions of the code appear in the boundary conditions on the potential ϕ, the particles, and the electrostatic and electromagnetic fields at the system walls. Our simulation of a finite plasma assumes that the walls are radiation transparent and particle reflecting. In the bounded version, the longitudinal field E_x vanishes at the system walls. The magnetostatic, vacuum field contribution to B_z is an arbitrary constant value throughout, in either version of the

code. We have found the code quite inexpensive to use; typical computer experiments with 4000 particles have required 0.25 sec CPU per time-step on the C.D.C. 7600 at the Lawrence Berkeley Laboratory (this includes all operations: field solving, particle pushing, and diagnostics).

III. BEAT HEATING

The theory of beat heating, of interest for confined plasmas, has been discussed by Cohen, Kaufman, and Watson[8-10] and by Rosenbluth and Liu.[11] Two linearly polarized transverse waves are oppositely incident on a finite, inhomogeneous, underdense plasma (Fig. 3). There can be a resonant interaction with a longitudinal normal mode of the plasma if the electrostatic disturbance, driven by the ponderomotive force at the beat frequency and wavenumber ($\Omega \equiv \omega_0 - \omega_1 \ll \omega_0, \omega_1$; $\underset{\sim}{\kappa} \equiv \underset{\sim}{k}_0 - \underset{\sim}{k}_1$), approximates the Bohm-Gross dispersion relation somewhere in the plasma. Because of the plasma inhomogeneity, the three-wave interaction is resonant only in a finite region around the position of exact matching shown in Fig. 3. The dissipation of the electron plasma oscillation introduces irreversibility into the three-wave process, and is the mechanism for the eventual thermalization of part of the energy provided by the electromagnetic waves. The dissipation may be due to collisions, Landau damping, convective loss, or nonlinear mode coupling processes. Inasmuch as our present studies encompass both the linear and nonlinear regimes for the beat wave, we shall reformulate and extend the main ideas of Ref. 9, pointing out those results which remain valid for a nonlinear beat wave.

From the wave equation for the total vector potential (of the two transverse waves), we obtain

$$\left[(\partial^2/\partial t^2) + \omega_p^{\,2}(x) - c^2(\partial^2/\partial x^2)\right] u_y(x,t) = -\omega_p^{\,2}\left[\delta n(x,t)/n_0\right]u_y \ , \tag{6}$$

where $u_y(x,t) \equiv -eA_y(x,t)/mc$ is the transverse oscillation velocity. (Corrections to Eq. (6) are of relative order $u_y^{\,2}/c^2$, $v_{th}^{\,2}/c^2 \ll 1$, as shown by a Vlasov analysis.) We use a WKB representation for the vector potential, or transverse velocity:

$$u_y(x,t) = u_0(x,t)\exp\left[-i\omega_0 t + i\int^x k_0(x')dx'\right] + \text{c.c.}$$
$$+ \ u_1(x,t)\exp\left[-i\omega_1 t - i\int^x k_1(x')dx'\right] + \text{c.c.} \ , \tag{7}$$

in terms of the slowly varying complex amplitudes u_0, u_1 of the two opposed transverse waves; the wave numbers satisfy local dispersion relations: $k_\ell^{\,2}(x) \equiv \left[\omega_\ell^{\,2} - \omega_p^{\,2}(x)\right]c^{-2}$.

For the density perturbation (not assumed small), we use a beat representation:

$$\delta n(x,t) = \tilde{n}(x,t)\exp\left[-i\Omega t + i\int^x \kappa(x')dx'\right] + \text{c.c.} \ , \tag{8}$$

where $\Omega \equiv \omega_0 - \omega_1$ is the beat frequency, and $\kappa \equiv k_0 + k_1$ is the local beat wavenumber. We can ignore the density perturbation at the sum frequencies ($\omega_0 + \omega_1$, $2\omega_0$, and $2\omega_1$), for the following reason. Since they represent high frequency, high phase velocity nonresonant perturbations, they can be only collisionally damped and are not normal modes. But if we consistently ignore collisional loss in high frequency perturbations the density perturbations at these sum frequencies simply

couple back into the electromagnetic waves, to produce nonlinear frequency shifts.[12] (The nonlinear frequency shifts for the transverse waves are of order $(\omega_p^2/\omega_0)(|u_y|^2/c^2)$, and are, in this paper, less than 10% of the frequency mismatches, which are at least $0.01\,\omega_p$, when the magnitude of the mismatch is averaged over the resonance zone.)

We further assume that $\tilde{n}(x,t)$ is slowly varying in time (on the Ω-scale) and in space (on the κ-scale). We can then obtain, from Eqs. (6)-(8),

$$\left[(\partial/\partial t) + c_0(\partial/\partial x) + c_0 d\ell n k_0^{\frac{1}{2}}/dx\right]u_0(x,t) = -(i/2)(\omega_p^2/\omega_0)(\tilde{n}/n_0)u_1 ,$$

$$\left[(\partial/\partial t) - c_1(\partial/\partial x) - c_1 d\ell n k_1^{\frac{1}{2}}/dx\right]u_1(x,t) = -(i/2)(\omega_p^2/\omega_1)(\tilde{n}^*/n_0)u_0 , \tag{9}$$

where only slow temporal variations are kept in the nonlinear coupling terms. The transverse group velocities are $c_\ell \equiv k_\ell c^2/\omega_\ell$.

The energy density of each wave is proportional to $\omega_\ell^2|u_\ell|^2$ (Ref. 8). Multiplying the two equations of (9) by u_0^* and u_1^*, and adding their complex conjugates, we obtain the conservation law for transverse action:

$$(\partial/\partial t)(\omega_0|u_0|^2 + \omega_1|u_1|^2) + (\partial/\partial x)(c_0\omega_0|u_0|^2 - c_1\omega_1|u_1|^2) = 0 . \tag{10}$$

This law (Manley-Rowe, or photon conservation) is valid for uniform or nonuniform plasma, and for linear or nonlinear density petrubation; but it is violated if our assumption of slowly varying amplitudes breaks down. The conservation of action implies that transverse energy is not conserved: as action is transferred from the higher

frequency wave (ω_0) to the lower frequency one (ω_1), the energy difference, of relative size Ω/ω_0, is deposited in the plasma, as a coherent oscillation or as heat. In the latter case, the process is irreversible, and only the $\omega_0 \to \omega_1$ transition can occur. In the former case, the deposit can be withdrawn, and energy transferred from ω_1 back to ω_0. (This is observed when the beat wave traps electrons.[1])

The rate of action transfer is, from Eqs. (9), given by

$$(\partial/\partial t)(\omega_0|u_0|^2) + (\partial/\partial x)(c_0\omega_0|u_0|^2) = -\omega_p^2 \, \mathrm{Im}(u_0 u_1^* \tilde{n}^*/n_0) \quad . \tag{11}$$

On using the Poisson equation for the density and scalar potential amplitudes, $\kappa^2\tilde{\phi} = 4\pi\tilde{n}e$, the right side of (11) becomes

$$-\kappa^2 \mathrm{Im}\left[u_0 u_1^* (e\tilde{\phi}^*/m)\right] \quad . \tag{12}$$

The potential ϕ is the longitudinal response to the ponderomotive potential energy[13] $\Psi(x,t) \equiv \left\langle \frac{1}{2} mu^2 \right\rangle (x,t)$ of the electrons; $\langle\ \rangle$ represents an average over the rapid temporal variation at ω_0, ω_1, yielding a beat variation $\Psi(x,t) = \tilde{\Psi}(x,t)\exp\left[-i\Omega t + i\int \kappa dx\right]$ + c.c., with $\tilde{\Psi}(x,t) = mu_0 u_1^*$. If the longitudinal response is linear, we have

$$e\tilde{\phi}(x,t) = (\varepsilon^{-1} - 1)\tilde{\Psi} = (\varepsilon^{-1} - 1)mu_0 u_1^* \quad , \tag{13}$$

where ε is the electron dielectric function, evaluated at Ω, κ. (If the space-time variation of (u_0, u_1) is not sufficiently slow, we should instead use $\Omega + i\partial/\partial t$ and $\kappa - i\partial/\partial x$ as the arguments of ε; in that case, the present formulation is not the most expedient,

and a three-wave analysis is preferable.) When the response is nonlinear,[1] certain of the nonlinear aspects may be incorporated by modifying the form of ε, so that ε depends on ϕ implicitly. We now use (12) and (13) to express the right side of (11) as

$$\kappa^2|u_0|^2|u_1|^2 \operatorname{Im} \varepsilon^{-1}(\Omega,\kappa) \quad . \tag{14}$$

For a uniform medium, the nonlinear equations for $|u_0|^2(x,t)$, $|u_1|^2(x,t)$ can be solved analytically, as discussed in Ref. 10. For a nonuniform medium, we limit our analytic study to the steady state $(\partial u_0/\partial t = \partial u_1/\partial t = 0)$, whence (11) and (10) become

$$(d/dx)(k_0|u_0|^2) = (d/dx)(k_1|u_1|^2) = (\kappa^2/c^2)|u_0|^2|u_1|^2 \operatorname{Im} \varepsilon^{-1} \tag{15}$$

where $\varepsilon(\Omega,\kappa;\ x)$ has an explicit x-variation through its parameters: density, temperature, possibly non-Maxwellian electron distribution. In order to use the same notation as in Ref. 9, the (absolute) action density <u>fluxes</u> are expressed as $(mc^2/e)^2 J_\ell$, with $J_\ell \equiv (k_\ell/2\pi)|u_\ell/c|^2$, so that (15) reads as in Eq. (3) of Ref. 9: $dJ_0/dx = dJ_1/dx = \bar{8}\ J_0 J_1 \operatorname{Im} \varepsilon^{-1}(x)$, with $\bar{8} \equiv 2\kappa^2/k_0 k_1 \approx 8$ for $\Omega \ll \omega_0$. Upon integrating over x, we found the solution

$$\Delta\ \ell n(J_0/J_1) = \bar{8}\pi\ \tilde{J}\int_a^b dx \operatorname{Im} \varepsilon^{-1}(x) \quad , \tag{16}$$

where $\Delta f \equiv f(x = a) - f(x = b)$, a and b are any two x-planes (such as the "edges" of the plasma), and $\tilde{J} \equiv J_0 - J_1$ is the constant (signed) action density flux.

In the limit of weak damping ($\mathrm{Im}\,\varepsilon \equiv \varepsilon'' \ll 1$), the x-integral can be carried out exactly.[9] We write $\mathrm{Im}\,\varepsilon^{-1}(x) = -\pi\delta\left[\varepsilon'(x)\right]$, where $\varepsilon' \equiv \mathrm{Re}\,\varepsilon$; the integral is then

$$\int_a^b dx\ \mathrm{Im}\,\varepsilon^{-1}(x) = -\pi\left|\partial\varepsilon'/\partial x\right|^{-1}_{\varepsilon'=0} \equiv -\pi L_n \quad , \tag{17}$$

defining the effective density scale length L_n. In this limit, the action transfer of Eq. (15) takes place over the infinitesimal region where $\varepsilon'(\Omega,\kappa;x) = 0$, i.e., at the position x where the Bohm-Gross frequency at the beat wavenumber, $\omega(\kappa;\ x)$, matches the beat-frequency Ω. More realistically with finite ε'', we have

$$\mathrm{Im}\,\varepsilon^{-1} = -\varepsilon''/(|\varepsilon'(x)|^2 + |\varepsilon''|^2) \quad ,$$

and it can be shown that $\mathrm{Im}\,\varepsilon^{-1}$ has a half-width of order $\varepsilon'' L_n = 2(\nu/\omega_p)L_n$, where ν is the total damping rate of a Langmuir oscillation. Equation (17), however, remains unaltered in the limit that the half-width is small compared to the plasma length. In order that our WKB representation be valid, we must require that the transfer zone width $(\nu/\omega_p)L_n$ exceed the wavelengths, i.e., $(\nu/\omega_p) \gg (k_\ell L_n)^{-1}$. (Typical parameters for a θ-pinch, $n_0 \sim 10^{17}/\mathrm{cm}^3$, $T_e \sim 100$ eV, $\omega_p/\omega_0 \sim 0.1$, and $L_n \sim 10$ cm, satisfy this inequality, since $\nu/\omega_p \gtrsim 10^{-2}$, while $(k_0 L_n)^{-1} \sim 10^{-4}$. For our simulations the resonance zone was of order 10 wavelengths long.) If the damping is <u>not</u> weak ($\nu \sim \omega_p$), so that $\varepsilon(x)$ does not become $\ll 1$, the integration can still be done, for known $\varepsilon(x)$. Since strong damping implies $\mathrm{Im}\,\varepsilon^{-1} = -\mathcal{O}(1)$, we obtain in place of (17)

$$\int dx \, \mathrm{Im}\, \epsilon^{-1}(x) = -\mathcal{O}(L) \quad , \tag{18}$$

where L is the length of the plasma. In a real plasma, when $L \sim \mathcal{O}(L_n)$, we have the important result that the action transfer is, in order of magnitude, the <u>same</u> for strong as for weak damping of the longitudinal response. Thus, for given κ, the dependence on $\kappa\lambda_D$ is weak; and for $\kappa\lambda_D \ll 1$, the dependence vanishes, since the integral is truly independent of ν for the model of a linear gradient.

In a simulation model, for reasons of economy the slab thickness L may be smaller than L_n, and even smaller than the resonance width $(\nu/\omega_p)L_n$. In that case appropriate corrections must be made in comparing theory and simulation. A typical simulation for beat heating when the density perturbations are linear is shown in Fig. 4.

Inserting (17) into (16), we have the result (Eq. (5) of Ref. 9):

$$\frac{1}{2}\, \bar{8}\pi\, k_0 L_n |u_0/c|^2_{in} = (1 - R - \rho)^{-1} \ln\left[(1 - R)(\rho + R)/\rho\right] , \tag{19}$$

an implicit equation for the relative action transfer $R \equiv \Delta J/J_0^{in}$, in terms of the input ratio $\rho \equiv J_1^{in}/J_0^{in}$ and the input amplitude $|u_0|_{in}$. (See Fig. 2 of Ref. 9 for a plot, also Fig. 5 here.) This result is remarkable not only in its independence of the damping rate ν (and thus of the temperature, the collision rate, and the damping mechanism), but also in that its dependence on the power parameter $|u_0|^2_{in}$ and the scale length L_n is only through their product.

The relation (19) was tested by simulation for the case of equal input actions ($\rho = 1$), corresponding roughly to maximum transfer for given total power input. The excellent agreement, in the linear regime, is shown in Fig. 5.

To test the dependence on $|u_0|^2_{in}$ and L_n through their product only, three runs were made, varying each but holding the product fixed. The agreement is shown also in Fig. 5.

The damping rate ν in these simulations was due to Landau damping of the beat wave. It was chosen to be in the range $10^{-2}\,\omega_p$ to $10^{-1}\,\omega_p$, corresponding to $\kappa\lambda_D$ between 0.30 and 0.45.

IV. CONCLUSION

We have described a fully electromagnetic, relativistic, finite-sized particle simulation code. The code is free from beam-Cerenkov numerical instability. The region of parameter space over which two light waves can interact with the grid and a plasma drift to give numerical instability is limited to unphysically short wavelengths and large time-steps. We found relativity to be important when nonlinear effects are included because individual particles can attain very large velocities. Of course, for some astrophysical applications, relativity is essential. Finally, the code was found to be economical to use.

We have used the code to study beat heating of a plasma in the linear and nonlinear regime of the driven density disturbance. Steady-state theory was found to be useful in understanding the action transfer and plasma heating for small amplitude electron waves. There was good quantitative agreement between simulation and theory.

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FIGURE CAPTIONS

Fig. 1. The $1\frac{1}{2}$-dimensions (x, v_x, v_y) of the code are pictured schematically. Wave propagation and density variation occur parallel to x. Transverse waves are linearly polarized in the y-direction. Magnetic fields are parallel to z. The three-wave interaction is diagrammed.

Fig. 2. The equations describing transverse waves and particle dynamics are integrated forward in time using a time-centered, leap-frog technique. Currents are calculated from charge locations measured over consecutive time-steps and from velocities at the half time-steps $\left[J_y = (J_y^+ + J_y^-)/2\right]$.

Fig. 3. Beat heating in an inhomogeneous medium. Because of the resonance condition, there arises a resonance region h. The density gradient, described by the scale length $L_n \equiv (d\ell n n_0/dz)^{-1}$, is parallel to the propagation direction of waves.

Fig. 4. Beat heating in a finite, inhomogeneous medium: (a) the right- and left-going electromagnetic waves before onset of beating; (b) (x, v_x) phase space after a fairly large amplitude electron plasma wave has been established.

Fig. 5. Relative energy or action depletion ($R \equiv \Delta W/W$) of the high frequency wave vs dimensionless parameter (scale length × pump strength) $4\pi k_0 L_n |u_0|^2/c^2$ for beat heating in an inhomogeneous medium. The driven electron plasma waves are small in amplitude. An input ratio $J_1^{in}/J_0^{in} = 1$ has been selected. The data points for $4\pi k_0 L_n |u_0|^2/c^2 = 0.5$ represent three parameter choices: ∇ : $4|u_0/c|^2 = 0.008$ and $k_0 L_n = 18.3$; $\square$: $4|u_0/c|^2 = 0.010$ and $k_0 L_n = 15.2$; and Δ : $4|u_0/c|^2 = 0.012$ and $k_0 L_n = 13.7$. We conclude that it is the combination of scale length $k_0 L_n$ multiplied by pump strength $|u_0/c|^2$ that determines action transfer.

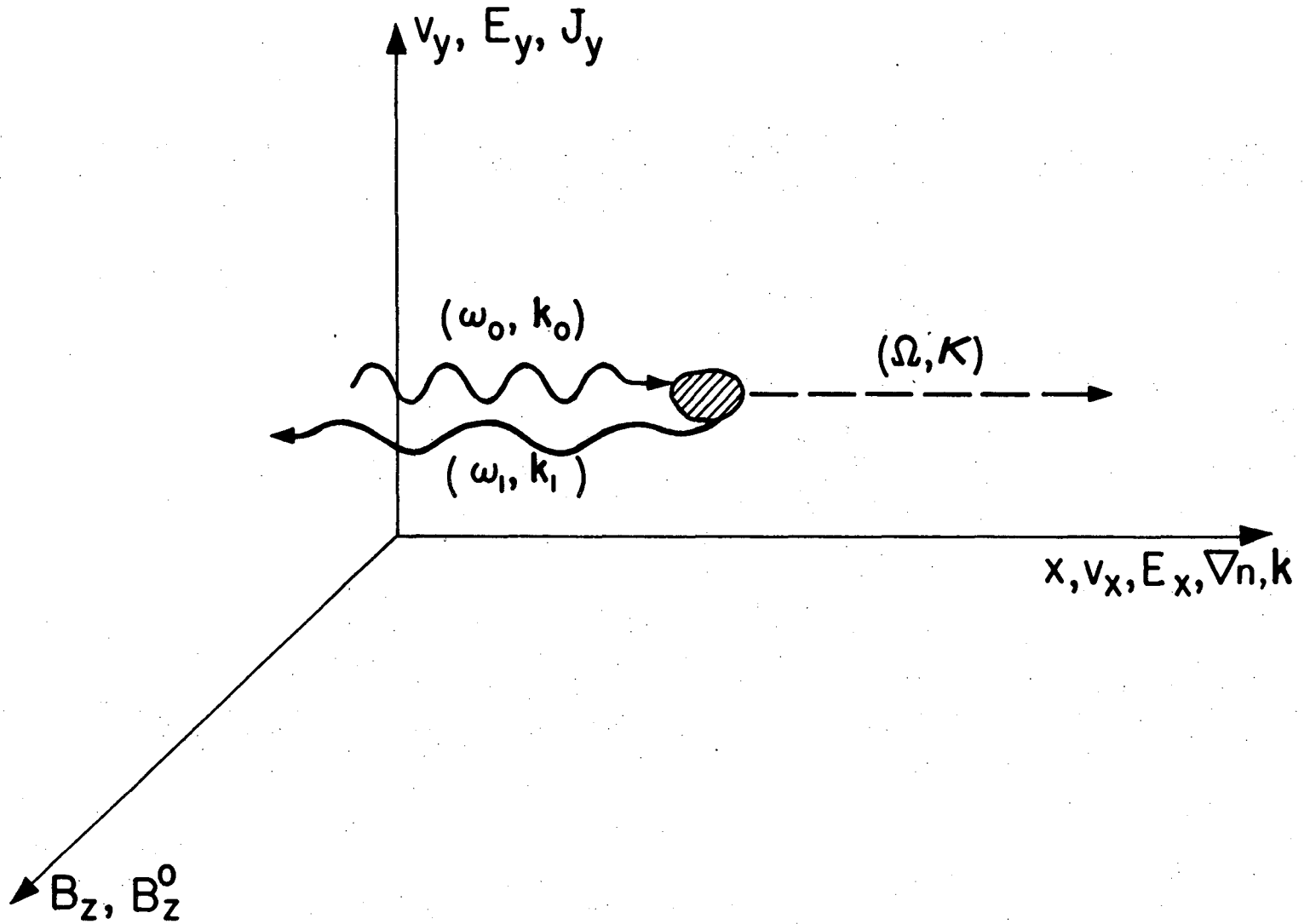

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Fig. 1.

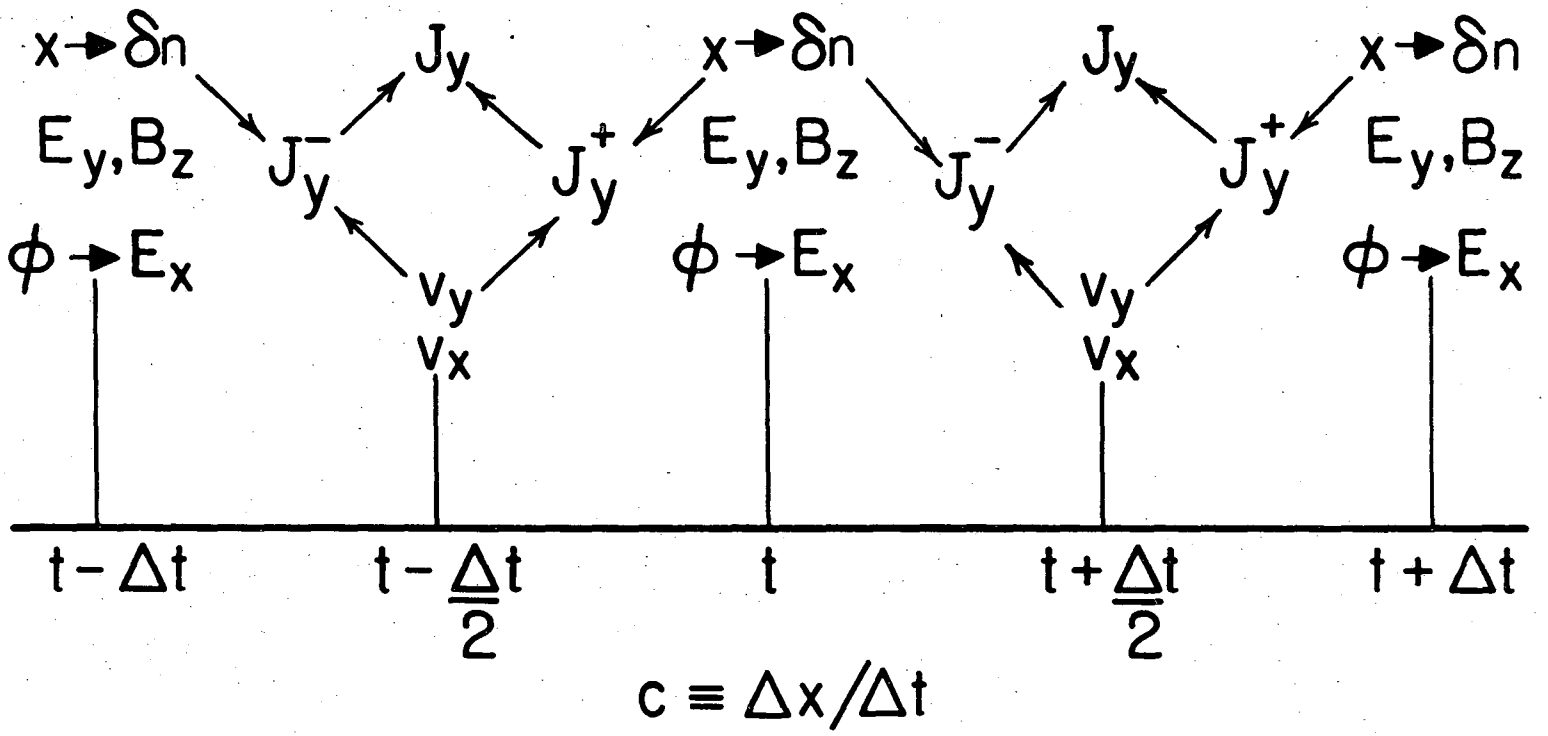

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Fig. 2.

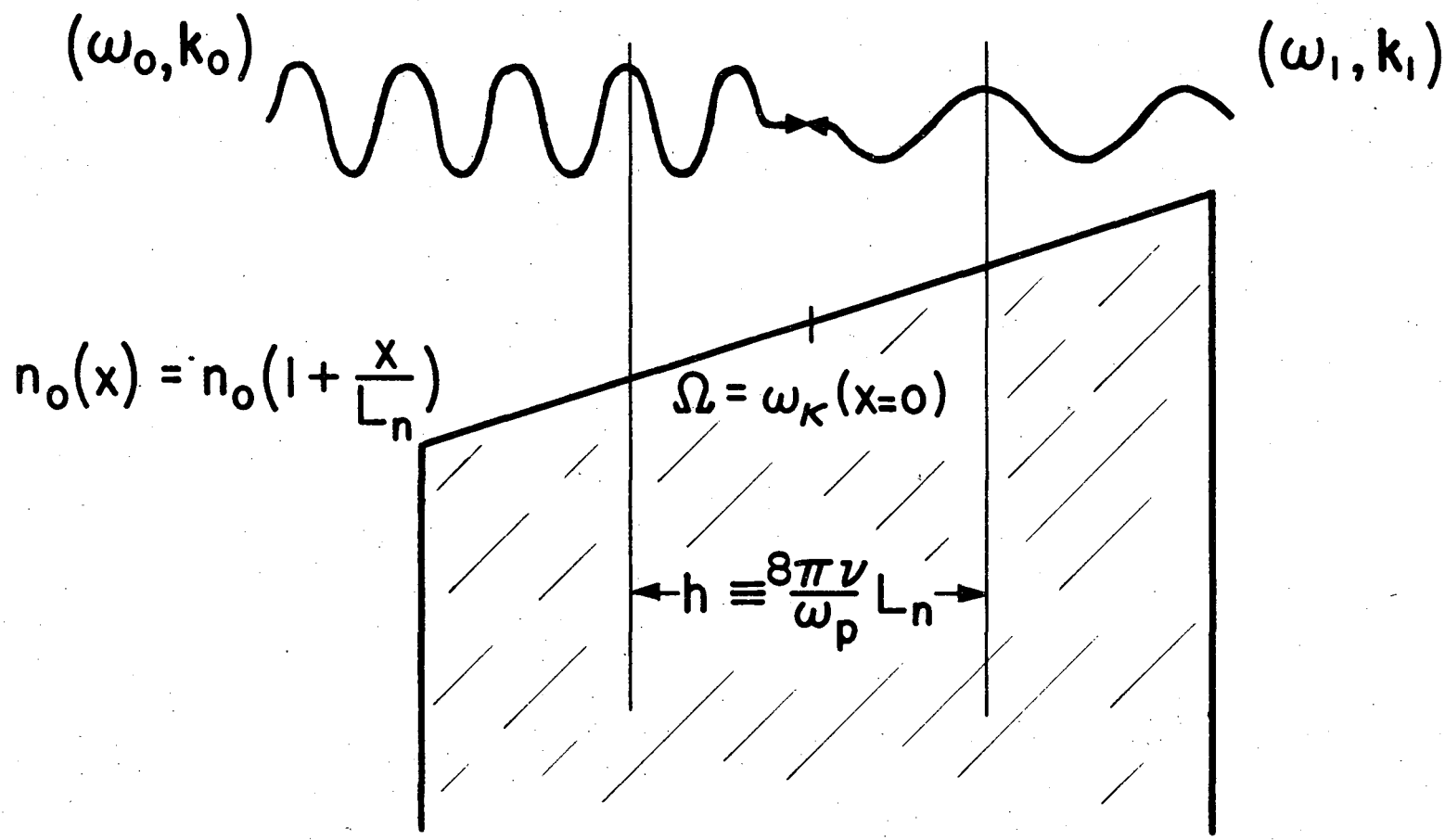

Fig. 3.

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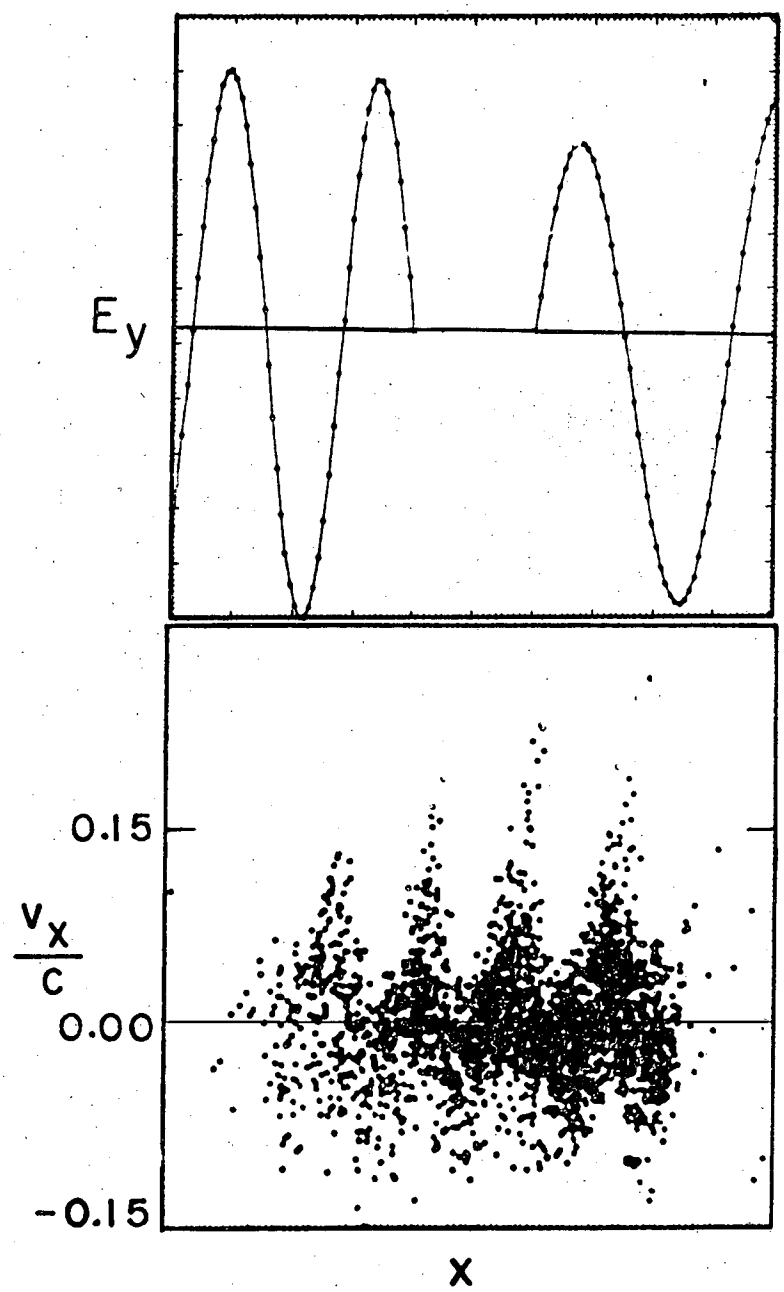

XBL745-3023

Fig. 4.

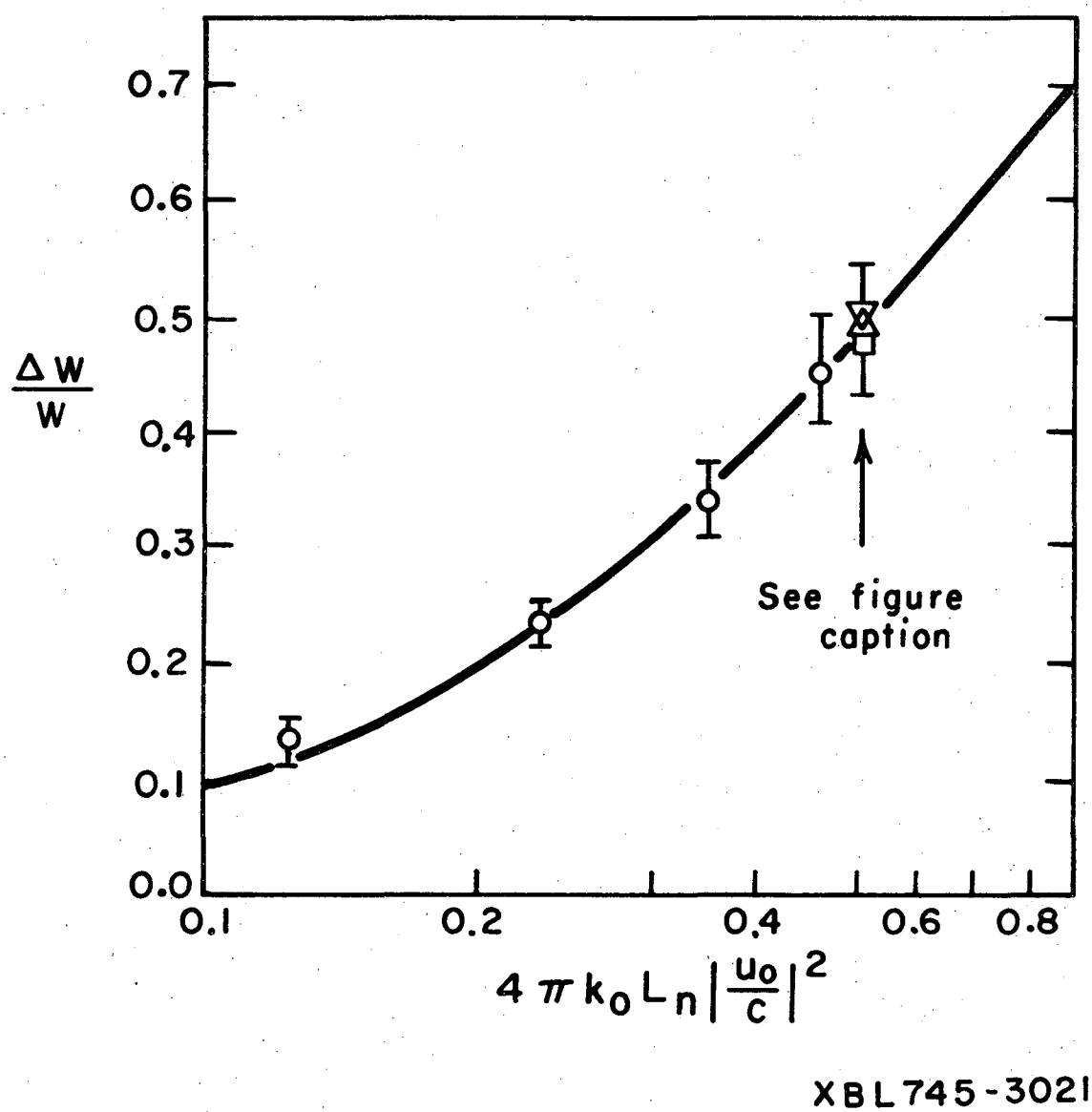
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
ΔW/W
See figure caption
0.1
0.2
0.4
0.6
0.8
4 π k₀ Lₙ |u₀/c|²
XBL745-3021

Fig. 5.

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TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720