LBL-30295
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Submitted to Physical Review D

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March 1991

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Phase Transitions with More than One Conserved Charge: Consequences for Neutron Stars†

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March 17, 1991

†This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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ABSTRACT

We extend the theory of first order phase transitions from simple bodies having one conserved charge to complex bodies having more than one. The properties of the transition are quite different in the two cases. Perhaps most importantly the pressure varies continuously with the concentration of phases in equilibrium, and is not a constant in the mixed phase as in the example of the gas-liquid transition. An external field such as gravity separates the phases in the second case but not in the first. As an example, the consequences are developed for the structure of a neutron star in which the transition to quark matter in the core occurs. It is also pointed out that the general result pertains to relativistic nuclear collisions in the so-called stopping or baryon rich domain where there are three conserved charges (baryon, electric and strangeness), and impacts the expected phase transition from confined hadronic matter to quark matter as regards signals that are supposedly driven by pressure.

PACS

05.70.Fh, 12.38.Mh, 64.60-i, 97.60.Jd

Phase Transitions with More than One Conserved Charge: Consequences for Neutron Stars

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1 Introduction

The most notable features of a system that possesses a single conserved quantity, and that is in a state consisting of two phases in equilibrium, sometimes referred to as the mixed or coexistence phase, is the constancy of the pressure and of the densities of each of the two phases and the constant density discontinuity between them that hold at all proportions of the phases. The transition from one homogeneous phase to the other through the mixed phase is referred to as a first order phase transition. The gas-liquid transition, in which the number of H_2O molecules is conserved, is the text-book example [1]. This phase transition is used as the paradigm for many others. However these well known properties of first order phase transitions in simple bodies are all modified for systems possessing more than one conserved charge (meaning additive attribute of the constituents or in some contexts the number of molecules of an independent component). For brevity, we shall refer to a body with more than one conserved charge as complex.

The essentially different character of a first order phase transition in a simple and a complex body originates in the possibility that in the latter the conserved charges can be shared by the two phases in equilibrium in different proportions than the actual proportions of the phases themselves. For example in the sub-saturation nuclear liquid-vapor transition, the number of neutrons and protons is certainly conserved by the body at all times, but their ratio can be different in the vapor and liquid regions of the mixed phase. It is only their sum in all regions of the body that is conserved. This degree of freedom is not available to a one component system. We can go further: It is apparent that when the body is almost all liquid, or all vapor, the added degrees of freedom are of little importance; the conservation laws impose almost as stringent conditions as in either pure phase. So clearly the degree to which the body can exploit the added freedom when in the mixed phase is a function of the concentrations. Consequently the internal pressure is a function of concentration in the mixed phase and does not remain constant while the body converts from one pure phase to the other. Of course at *each* concentration the pressure is common in each phase in equilibrium. As a corollary we may conclude also that all other properties

in the mixed phase are non-linear functions of the concentration. In the presence of gravity, or other external field, the differences are accentuated between first order phase transitions in systems that possess only one conserved quantity and those that have more than one.

Since many treatments of phase transitions are modeled after the text-book example involving a single chemical potential corresponding to a single component, our discussion may be relevant to many astrophysical problems, including phase transitions in the early universe, and the condensation of galaxies and other structures. It is also relevant to the nuclear gas-liquid transition where the neutron and proton number are conserved, and to the expected phase transition from confined hadronic to quark matter in laboratory collisions at relativisitic energies between heavy nuclei, where the conserved charges are baryon, electric and strangeness. There are possibly many other examples in other branches of physics and chemistry. It certainly applies to neutron stars for which the ground state is in beta equilibrium, involving several species of baryons, and at the same time is charge neutral. The conserved quantities are of course baryon number and electric charge but not strangeness, since this quantum number is not conserved on macroscopic time scales (ie. longer than the weak interaction time)

In previous studies of the transition between the confined and deconfined phases in neutron stars, most have treated the star as automatically charge neutral by ignoring beta equilibrium and assuming that the confined phase is purely neutron [2]. In this case the transition is described in terms of only the neutron chemical potential. Several other authors have allowed for beta equilibrium, but have approximated the phase transition by assuming that both phases in equilibrium are separately charge neutral and imposing this on the solution [3]. Both of the approximations have prevented the star from exploiting degrees of freedom that are available to a system with more than one chemical potential. Most importantly for the structure of the star, in both approximations the pressure is constant at all densities in the mixed phase. In the presence of a gravitational field, this means that the two phases will be separated by gravity, the denser sinking to the center. As a consequence there can be no region of mixed phase and there is a discontinuity in the density of the star at the phase boundary which occurs at a definite radius, the discontinuity being that between the two phases in equilibrium. This is the situation so far treated in the literature. It is a spurious result of the approximation in both cases.

In this paper we derive some general thermodynamic properties that are characteristic of a body that possesses components carrying more than one conserved charge and that undergoes a first order phase transition, and show how they differ from those of the paradigm provided by the gas-liquid transition. Some of the results on phase transitions developed in section 3 were anticipated in refs. [4, 5, 6, 7, 8] but not in the completeness nor generality derived here.

The plan of the paper is as follows. In section 2 we briefly review the properties of a first order phase transition in a simple body, but do so without reference to a figure of pressure as a function of volume as is usually done; rather we introduce the notion of internal variables which are the solutions of the equations of motion presumed to describe the microphysics of the state of the body, and define what a first order phase

transition means in terms of these and the thermodynamic variables. This is a useful preparation for a discussion of the general case of a body possessing several conserved charges, called a complex body, which we take up in section 3. We briefly discuss the possible geometric structure that the mixed or coexistence phase can have in section 4. We illustrate the general results by considering the transition in equilibrium from confined hadronic to quark matter that may take place in neutron stars if the central pressure is high enough. To make the contrast more vivid, we exhibit the phase transition and star structure in the approximation of a purely neutron star whose core undergoes a transition to the quark matter phase in section 2. This follows the familiar paradigm. The mixed phase must be absent because of the constancy of the pressure. Then in section 5 we show that for a neutron star in beta equilibrium, the mixed phase *will* be present and have a finite radial extension if the Gibbs phase equilibrium conditions are satisfied within the star. We illustrate the behavior of the thermodynamic variables through the phase transition, and show the composition of the star, including its mixed phase core.

2 Phase Transitions with One Conserved Charge

Here we recall the features of a first order phase transition in a system that carries one conserved charge, as for example the number of H_2O molecules in the case of the gas-liquid transition. Gibbs condition for phase equilibrium is that the system be in chemical, thermal and mechanical equilibrium, meaning that for the two phases, we must have

$$\begin{aligned} &\mu_1 = \mu_2 = \mu, \qquad T_1 = T_2 = T, \\ &p_1 \equiv p(\{\phi_1\}, \mu, T) = p(\{\phi_2\}, \mu, T) \equiv p_2 \end{aligned} \tag{1}$$

where the subscript $1, 2$ denotes the two phases respectively and $\{\phi_i\}$ denotes the internal variables that characterize a solution to the equations of motion of the system, exclusive of the chemical potential, in phase i at temperature T. If there are m internal variables

$$\{\phi\} = \phi_1, \cdots \phi_m, \tag{2}$$

they satisfy the equations of motion denoted by

$$\mathcal{D}_j(\{\phi\}, \mu, T) = 0, \qquad j = 1, \cdots m. \tag{3}$$

If and only if a solution, μ, to eq. (1) exists for $\{\phi_1\} \neq \{\phi_2\}$ the system possesses a first order phase transition. The density of the charge whose chemical potential is μ is different in the two phases since the function $\rho(\{\phi\}, \mu)$ is a different function of μ because the internal variables $\{\phi\}$ have different values in the two phases.

$$\rho_1 = \rho(\{\phi_1\}, \mu, T), \qquad \rho_2 = \rho(\{\phi_2\}, \mu, T). \tag{4}$$

This is true of other functions of μ, such as the energy density. The only properties that are necessarily common to the two phases that are in equilibrium are those

indicated in Gibbs relation (1) and these are constant for all densities or in other words all concentrations in the 'mixed phase' that lies in the interval $\rho_1 \leq \rho \leq \rho_2$. The pure (ie homogeneous) phases lie above and below this range respectively. The mixed phase is some mixture of the two phases that occupy different volumes of space and the average density in the mixed phase varies linearly with their concentration according to

$$\rho = (1 - \chi)\rho_1 + \chi\rho_2, \tag{5}$$

where $0 \leq \chi \leq 1$. The precise structure of the mixed phase is not specified by the physics considered so far. It is likely to resemble the mixed phase of the gas-liquid transition in sub-saturation nuclear matter [9, 10]. The regions occupied by the denser phase near the low density end of the mixed phase is likely to be spheres surrounded by the less dense phase. Their size is specified by surface energy, and Coulomb energy. The spheres give way to rods and then slabs at higher concentration, with the roles of the two phases reversing for $\chi > 1/2$. However, since the pressure is independent of the concentration of phases in a simple body, they will will be separated by any external field that distinguishes between them, such as gravity, which distinguishes their different densities.

We now illustrate with a concrete example, the transition from neutron matter to quark matter in a compact star, and the consequences for the structure of compact stars that stem from treating the transition as if it occurred in a simple body. This is the mold into which all discussions of this phase transition have been fitted heretofore, apparently unconsciously, usually by the seemingly innocuous neglect of beta equilibrium, (ie by assuming a purely neutron star) [2]. For the purpose of illustration, we adopt for the description of the confined hadronic phase, the Lagrangian of Zimanyi and Moszkowski [11] solved in the mean field approximation, and for the quark matter phase the bag model [12]. Of course none of the general properties outlined above depend on these choices. To correspond with the earlier discussions of quarks in neutron stars [2], we assume that the only baryons present are neutrons. For the quarks we assume that the u,d,s quarks are massless. Then both phases are automatically charge neutral and there is only one chemical potential, that for baryon number. For the quarks, under the assumption of equal masses, the quark phase *is* in equilibrium. (Alternately one could impose the constraint that $n_d = 2n_u$ which again is charge neutral and ignore the strange quarks as has been done by Baym and Chin [2]. This choice is extremely beta unstable. It has the consequence that the computed energy per added baryon, $\mu = \partial\epsilon/\partial\rho$ is very large compared to an equilibrium model, so that the predicted phase transition is at a correspondingly high baryon density. Said another way, the neutron chemical potential at which the pressure in the quark phase vanishes is a factor $((1 + 2^{4/3})/3)^{3/4}$ larger than in the equilibrium model of massless u,d,s quarks. The chemical potential at which the pressure curve crosses that of the hadronic phase is correspondingly higher. This case is also shown in Fig. 1 by the dashed lines. The number density at which conversion of hadronic matter begins is a very high density of 1.03 fm^{-3} in this case as compared to 0.306 fm^{-3} in the equilibrium case of u,d,s quark matter. We thus see how a seemingly innocuous assumption lead to a major overestimate of the transition density in the

model adopted to explore the question.) However all such choices are arbitrary since beta equilibrium is ignored in the initial premise that the hadronic part of the star is purely neutron. This has been the case in the above cited studies, and our purpose here is to contrast the implications of such treatments with the structure that emerges when beta equilibrium is allowed to occur.

The pressure and neutron chemical potential at phase equilibrium are given by the crossing of the curves in Fig. 1. The dashed line in this particular instance shows schematically the way the pressure evolves as a function of chemical potential when there is a first order phase transition in a body for which one has a complete theory for the internal variables describing the state of the body. (In our example this is not so. We cannot solve QCD through the confined-deconfined transition and so use separate models for each phase. Hence in this instance the dotted line is schematic.) Fig. 2 shows the constancy of the pressure and chemical potential and the linear dependance on density of the energy density in the mixed phase. The constant pressure of the mixed phase in the presence of a gravitational field has the consequence that gravity separates the two phases, and one is left with only the pure phases. Said another way, the mixed phase at one density cannot support the mixed phase at a higher one since the pressure exerted by each is the same. Consequently there are no stable stars with central densities falling in the range of the mixed phase, and for those stars with central densities that lie above that range, the distribution of matter as a function of the radial coordinate suffers a discontinuity, falling from ρ_2 to ρ_1 at the radius where the pressure equals that of the phase equilibrium. These aspects are illustrated in Fig. 3 and 4 by the dashed lines. The solid lines correspond to stars in beta equilibrium with a first order phase transition in which baryon number and electric charge are conserved, the case that we discuss next.

3 Phase Transitions with More than One Conserved Charge

Chemical potentials are used to express the conditions of equilibrium of a multi-component system in which transformations among the components are possible. The transformations define relations among the chemical potentials. Usually there are conservation laws involving additive attributes or 'charges' of the components. We denote these by Q_α, $(\alpha = a, b, \cdots, n)$. Then in chemical equilibrium the chemical potentials of all components can be written as a linear combination of as many independent chemical potentials as there are conservation laws. We call these $\mu_a, \mu_b, \cdots$. The coefficients in the linear combination refer to the number of the conserved quantities that the component possesses. The numbers need not be integers. The baryon and electric charge possessed by quarks are fractional.

A body has two phases and can undergo a first order phase transition between them if, and only if, there exist two distinct simultaneous solutions, $\{\phi_1\} \neq \{\phi_2\}$, to the equations of motion for its internal structure.

$$\mathcal{D}_j(\{\phi_1\}, \mu_a, \mu_b, \cdots, T) \;=\; 0,$$

$$\mathcal{D}_j(\{\phi_2\}, \mu_a, \mu_b, \cdots, T) \;=\; 0, \quad j = 1, \cdots, m \tag{6}$$

subject in both cases to the relevant conservation laws and such that the Gibbs condition can be satisfied. Note that (when a comprehensive theory of the internal structure of the body is available) the equations of motion and all other functions of the internal and thermodynamic variables are the same functions in the two phases, the distinction in phases being that the internal variables $\{\phi\}$ describing the body occupy different regions of the internal space. The Gibbs condition for phase equilibrium at fixed T when there are several conserved charges, say n, is

$$\begin{aligned} &\mu_{1,a} = \mu_{2,a} = \mu_a, \cdots, \qquad T_1 = T_2 \\ &p(\{\phi_1\}, \mu_a, \mu_b, \cdots, T) = p(\{\phi_2\}, \mu_a, \mu_b, \cdots, T). \end{aligned} \tag{7}$$

The pressure is the same function on the two sides of the equation but evaluated in different regions of the space of internal variables. Clearly this equation, unlike the example of one chemical potential, does not suffice to specify the pressure and chemical potentials at phase equilibrium. If there is a first order phase transition, the pressure as a function of one of the chemical potentials behaves in the general manner of Fig. 1. Let this one be μ_a. Generally one thinks of varying the volume, V, of the body by external means at a constant temperature. This of course corresponds to varying the densities, Q_α/V, of conserved charges, and hence their chemical potentials, among them μ_a. We use it as an independent variable. The remaining $n-1$ chemical potentials are determined by the conservation laws (eg. charge neutrality for a star). The independent one in the case of a star would be the baryon chemical potential, and corresponding to a chosen value there is a particular baryon density or densities. The conservation conditions are different functions of the chemical potentials in the two phases, because they correspond to different regions of the internal variables,$\{\phi\}$. Let us write the n conservation laws for the charge densities as

$$q_\alpha(\{\phi_i\}, \mu_a, \mu_b, \cdots, T) = Q_\alpha/V, \quad (i = 1, 2) \;\; (\alpha = a, b, \cdots, n), \tag{8}$$

where i denotes one of the pure phases, 1 or 2, and $\alpha = a, b, \cdots$ refers to one of the n conserved quantities. One could think of solving $n-1$ conditions in each phase, $i = 1$ and 2, for $n-1$ chemical potentials as functions of the remaining one, μ_a:

$$\mu_{i,\beta} = \mu_{i,\beta}(\mu_a, \{\phi_i\}, T), \quad (i = 1, 2) \;\; (\beta = b, c, \cdots, n). \tag{9}$$

The equality of the chemical potentials at phase equilibrium requires a careful examination of how phase equilibrium is to be found. We study first the end points of the mixed phase where the concentration of one of the phases is very small. Then we turn to the intermediate region. Imagine that we begin with the body in the phase 1, and increase the pressure by decreasing the volume and hence increasing the non-zero charge densities. At some pressure as yet unknown, parts of the body will begin to convert to the other phase. The body is said to be in the mixed phase. At pressures below and to the point where this conversion *begins* the conservation conditions are expressed by the $n-1$ functions $\mu_{1,\beta}(\mu_a)$ (where now for brevity we suppress mention

of the other dependences expressed in eq. (9)). At this point, Gibbs conditions for equilibrium expressed in eq. (7) takes the specific form

$$p(\{\phi_1\}, \mu_a, \mu_{1,b}(\mu_a), \cdots, T) = p(\{\phi_2\}, \mu_a, \mu_{1,b}(\mu_a), \cdots, T), \tag{10}$$

which can be solved for μ_a and all the other chemical potentials are then also calculable in terms of this one. Similarly if the mixed phase is approached from the pure phase 2, Gibbs conditions are expressible as the solution for μ_a of

$$p(\{\phi_1\}, \mu_a, \mu_{2,b}(\mu_a), \cdots, T) = p(\{\phi_2\}, \mu_a, \mu_{2,b}(\mu_a), \cdots, T). \tag{11}$$

We have now two different equations for the boundaries of the mixed phase, one at each pure phase and the solutions will be different in general. They are marked on Fig. 5 as points '0' and '1' respectively, in a figure that shows pressure and two chemical potentials. Therefore, unlike a simple body having one conserved charge, in a complex body *the pressure and all the chemical potentials are different at the two ends of the mixed phase.* They are guaranteed by these two equations to be equal within the two phases in equilibrium.

Note that it would be incorrect to replace the two equations (10,11) by the single equation

$$p(\{\phi_1\}, \mu_a, \mu_{1,b}(\mu_a), \cdots, T) = p(\{\phi_2\}, \mu_a, \mu_{2,b}(\mu_a), \cdots, T),$$

because then the $n-1$ chemical potentials would be unequal in the two phases in contact, $\mu_{1,b}(\mu_a) \neq \mu_{2,b}(\mu_a)$, etc. This equation requires that both phases in equilibrium individually satisfy all of the last $n-1$ conservation laws, eq. (8). *Such a requirement is seen to be incompatible with phase equilibrium.*

It remains to describe the mixed phase at arbitrary concentration of the two pure phases. So far we have described it only at infinitesimal concentration of the one phase or the other, and in doing so have established that a first order phase transition in a complex body (having more than one conserved charge) is markedly different than in a simple body. Let $1-\chi$ denote the fraction of the volume occupied by phase 1 and χ by phase 2. The conservation laws written above in eqs. (8) for the pure phases now must be written as a *single* equation for each charge that involves *both* phases,

$$(1-\chi)q_\alpha(\{\phi_1\}, \mu_a, \mu_b, \cdots, T) + \chi q_\alpha(\{\phi_2\}, \mu_a, \mu_b, \cdots, T) = Q_\alpha/V, \quad (\alpha = a, b, \cdots). \tag{12}$$

These reduce to the separate laws for the pure phases for $\chi = 0$ or 1. As noted above we must allow the phases in equilibrium the freedom to satisfy these less restrictive conditions and not impose conditions separately on each phase in equilibrium, else Gibbs' condition on the equality of chemical potentials would be violated. The equations governing the body in the mixed phase are the equations of motion, eq. (6), of which there are $2m$, namely m equations for the solution of the m internal variables $\{\phi_i\}$ in each phase, $i = 1, 2$, the n conservation laws, eqs. (12) and the Gibbs condition, eq. (7) for the n chemical potentials and the concentration, χ. There are therefore $2m + n + 1$ simultaneous equations that describe the state of the body in the mixed phase at each concentration. If a solution exists at the chosen density of

charges, Q_α/V, with $0 < \chi < 1$ and $\{\phi_1\} \neq \{\phi_2\}$ it represents a state of the body in the mixed phase.

It is perhaps instructive to have a picture in mind to illustrate the above. In the case of one conserved charge (like the number of H_2O molecules), the pressure as a function of chemical potential for this charge has the familiar form of a curve that turns in direction, crossing itself at some pressure and chemical potential as in Fig. 1. The crossing *point* corresponds to the equilibrium values of pressure and chemical potential at the given temperature. Now imagine a third axis corresponding to an additional chemical potential as in Fig. 5. Pull the curve as an elastic sheet out into this new direction. Of course it need not be by parallel displacement and indeed the surface so created may have convolutions. The *point* of intersection, A, of the pressure in the two pure phases has become a *curve* labeled AA'; the two phases at each point on the curve have equal chemical potentials and pressure, as required for equilibrium, *but* in general they vary along the curve. However the entire curve does not correspond to physically accessible states of the body, given the conservation laws. To understand this, consider the pure homogeneous phases. The conservation law, eq. (8), for charge 'b' in the pure phases can be used to express μ_b in terms of μ_a as in eqs. (9); these describe *two* curves in the plane $p = 0$, one for each pure phase because each phase corresponds to a different region of the space of internal coordinates of the system. The projection of each of these curves onto the part of the pressure surface corresponding to the same phase describes a curve on each surface. Segments of these in the maximum pressure phases are labeled a0 and 1b. On these curves the body is in the pure phase 1 and 2 respectively and satisfies the conservation laws. Phase equilibrium is possible only when each of these curves crosses the curve of pressure intersection AA'. The points of crossing will not generally be coincident, as we proved above, and they define the extremes of the mixed phase. The concentration of phase 2 at the point '0' is zero and at '1' it is unity. The concentration varies between these values along the curve 01. This is the *curve* of phase equilibrium. In general the pressure, and both chemical potentials vary along it. Of course at each point on the curve, which corresponds to some definite concentration of phases, χ, all of these quantities are equal throughout the body, as required by the conditions of equilibrium. The construction as described serves to illustrate why a first order phase transition with more than one conserved charge is so different from a transition with exactly one. When there is more than one conserved charge, phase equilibrium is possible when the pressure as a function of the independent chemical potentials has the topology of the generic form illustrated.

Once the $2m+n+1$ equations (6,7,12) are solved simultaneously, all other additive quantities can be computed from the rule, expressed here for the charge density corresponding to the conservation law labeled 'a',

$$q_a = (1-\chi)q_a(\{\phi_1\}, \mu_a, \mu_b, \cdots, T) + \chi q_a(\{\phi_2\}, \mu_a, \mu_b, \cdots, T), \tag{13}$$

We have seen that the pressure and all chemical potentials are different at the extremes of concentration in the mixed phase. We now see that they vary continuously as a function of concentration in the mixed phase, as do all other additive properties such as densities and energy density, but they do not necessarily vary linearly with

concentration χ. For example, compare eq. (5) and (13). All of these aspects of a first order phase transition in a body with more than one conserved charge are at variance with those corresponding to a body with only one. The feature that is common in both cases is the discontinuity in the densities across the interface between the two phases in equilibrium; this is what distinguishes a first from a second order phase transition. However the magnitude of the discontinuity in the general case varies with the concentration.

The variation of pressure with concentration is perhaps the most consequential difference and leads to structurally different compact stars in which the confined hadronic to quark matter phase transition occurs, than would be computed in approximations that treat the star as a simple body having a single conserved charge (baryon number) rather than a complex one. The variation of pressure also impacts expectations that have been developed for signals of the phase transition in relativistic nuclear collisions in the 'stopping' or 'baryon rich domain' that have been built around the notion of a 'plateau' in pressure and hence in observables that are driven by pressure.

4 Structure in the Mixed Phase

So far we have discussed the mixed phase in very general terms using the phrases 'equations of motion' and 'internal variables' to denote their solution. For any particular system this phrase may embrace more or less detail. For example, the mixed phase of sub-saturation nuclear matter is believed to possess geometric structure, depending on the concentration, ranging from a small concentration of nuclei in a background of electrons and a very dilute gas of nucleons (actually approximated as negligible in density) to extended rods and then slabs of nuclear matter (spaghetti and lasagna) as the parameter χ reaches $1/2$, with the role of nuclear matter and gas reversing at higher concentrations of the dense phase [9, 10]. In any particular application this degree of detail could be included, at least in principle, in the framework of our general discussion by adding additional equations that describe the energy and interaction peculiar to the geometric structure, or it might be added as a perturbation afterward, as in the above references. In either case it is by no means a trivial problem. In any case the range in density between the pure homogeneous phases, referred to as the mixed phase, may be punctuated by phase transitions corresponding to the changing geometry.

5 Hybrid Neutron-Quark-Matter Stars

To illustrate the theory developed above for a first order phase transition with more than one conserved charge, we consider a compact star in general beta equilibrium. It has two conserved charges, baryon and electric. The overall charge of the star is zero. Charge neutral matter in equilibrium we call neutron star matter. We contrast the structure of a star made from it with the fictional case of a pure neutron star. Pure neutron matter is not the ground state of dense charge neutral matter, but it

is the model most often studied in connection with the possible phase transition to quark matter in the core of a neutron star [2]. It has only one conserved charge, the baryon number, since charge neutrality is automatic. Being so, a degree of freedom is lost that is associated with the possibility of achieving charge neutrality through a population of baryons and fractionally charged quarks that are not separately neutral.

The general features discussed do not depend on the particular models adopted to describe the confined hadronic and the quark matter phases. We adopt as a model of the confined phase a generalization of the Zimanyi-Moszkowski [11] nuclear field theory Lagrangian. With the addition of the rho meson which introduces an isospin symmetry force, the model describes the bulk properties of symmetric matter reasonably well although the compression modulus is probably too small:

$$\rho_0 = 0.16 \text{ fm}^{-3}, \quad B/A = 16 \text{ MeV}, \quad a_{sym} = 32.5 \text{ MeV}, \\ \text{K} = 225 \text{ MeV}, \quad m^{\star}_{sat}/m = 0.855 \ . \tag{14}$$

The coupling constants are

$$(g_\sigma/m_\sigma)^2 = 7.487, \quad (g_\omega/m_\omega)^2 = 2.615, \quad (g_\rho/m_\rho)^2 = 4.774 \ \text{ fm}^2. \tag{15}$$

The generalized Lagrangian is

$$\begin{aligned} \mathcal{L} \ = \ & \sum_B \Big\{ (1 + g_{\sigma B}\sigma/m_B)\overline{\psi}_B[i\gamma_\mu\partial^\mu - g_{\omega B}\gamma_\mu\omega^\mu - \tfrac{1}{2}g_{\rho B}\gamma_\mu \boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu]\psi_B - \overline{\psi}_B m_B \psi_B \Big\} \\ & + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & - \tfrac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \tfrac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu + \sum_\lambda \overline{\psi}_\lambda(i\gamma_\mu\partial^\mu - m_\lambda)\psi_\lambda. \end{aligned} \tag{16}$$

Here the sum on B is over the baryon species neutrons, protons and all charge states of hyperons to convergence. The sum on λ is over e^- and μ^-. The mesons are the scalar, vector and isovector,$\sigma, \omega, \boldsymbol{\rho}$. How this can be solved in the mean field approximation is discussed in detail elsewhere [11, 13]. The chemical potentials for the baryons can be written in terms of the chemical potentials for the two conserved charges, μ_n, μ_e according to their baryon and (electron) charge content, $\mu_B = \mu_n - q_B\mu_e$. These enter the problem through the equations that define the Fermi momenta of the particles

$$\mu_B = e_B(k_B), \qquad \mu_\lambda = (k_\lambda^2 + m_\lambda^2)^{1/2} \tag{17}$$

where $e_B(k_B)$ denotes and energy eigenvalue for baryon of charge and type B found as a solution to the mean field equations of the Lagrangian. In this description of the confined hadronic phase the internal variables are the mean values of the meson fields, σ, ω, $\boldsymbol{\rho}$. Note from eq. (17) that the Fermi momenta, k_B depend on these and on the chemical potentials. Therefore the expressions for the baryon and charge density in this phase,

$$\begin{aligned} \rho &= \sum_B \frac{2J_B + 1}{6\pi^2} k_B^3 \\ q &= \sum_B \frac{2J_B + 1}{6\pi^2} q_B k_B^3, \end{aligned} \tag{18}$$

are in accord with the dependences quoted in eq. (8).

We use a simple version of the bag model for quark matter for which the pressure, energy density and baryon number and charge density at $T = 0$ are given by

$$
\begin{aligned}
p &= -B + \sum_f \frac{1}{4\pi^2}\Big[\mu_f(\mu_f^2 - m_f^2)^{1/2}(\mu_f^2 - \frac{5}{2}m_f^2) + \frac{3}{2}m_f^4 \ln\Big(\frac{\mu_f + (\mu_f^2 - m_f^2)^{1/2}}{m_f}\Big)\Big] \\
\epsilon &= B + \sum_f \frac{3}{4\pi^2}\Big[\mu_f(\mu_f^2 - m_f^2)^{1/2}(\mu_f^2 - \frac{1}{2}m_f^2) - \frac{1}{2}m_f^4 \ln\Big(\frac{\mu_f + (\mu_f^2 - m_f^2)^{1/2}}{m_f}\Big)\Big] \\
\rho &= \sum_f \frac{(\mu_f^2 - m_f^2)^{3/2}}{3\pi^2} \\
q &= \sum_f q_f \frac{(\mu_f^2 - m_f^2)^{3/2}}{3\pi^2}
\end{aligned}
\tag{19}
$$

where the sum, f, is over flavors. In this simple model of quark matter there are no internal variables since the quarks are assumed to form a free Fermi gas. Pressure, densities, etc. are specified simply by the chemical potentials. We take for the masses, $m_u = m_d = 0$, $m_s = 150$ MeV, $m_c = 1500$ MeV. Because of the long time scale, strangeness is not conserved in a star. The quark chemical potentials are therefore related to those for baryon number and electron by

$$
\mu_u = \mu_c = \frac{1}{3}(\mu_n - 2\mu_e), \qquad \mu_d = \mu_s = \frac{1}{3}(\mu_n + \mu_e). \tag{20}
$$

In Fig. 6 the behavior of the pressure, energy density, and chemical potentials is shown. The monotonic increase of pressure in the mixed phase is in sharp contrast with the behavior in the description involving a single chemical potential which was shown in Fig. 2. It is interesting also to compare the density of hadronic and quark matter in the mixed phase in the two cases. In the present instance this is shown in Fig. 7. The densities are different in the two phases in equilibrium, but only by a few percent except at the low density end of the mixed phase, and they vary with the concentration. In contrast, when only one charge is conserved, as in Fig. 2, the density of each phase is a constant for all concentrations and the density of the quark phase is about a factor two larger than the confined hadronic phase. In a word, a first order phase transition in a system with more than one conserved charge is *smooth* compared to one in which there is a single charge. The populations in the star at the mass limit are shown as a function of Schwarzschild radius in Fig. 8. The entire core of the star out to 7.5 km is in the mixed phase. Populations of quarks are very high within 5 km. The quark matter phase is seen to be negatively charged, while the hadronic matter phase is positively charged. Their charges are almost equal and opposite through most of the star, the lepton populations which complete the charge neutrality being very small.

The masses of stars in the two cases was compared in Fig. 3. Unlike the situation where the star is artificially treated as having a single conserved charge with a consequent constant pressure in the mixed phase and therefore an absence of stable

stars with central densities falling in the range of the mixed phase, in the realistic case with two conserved charges, as corresponds to a star in beta equilibrium, there is no unstable range until the normal Oppenheimer mass limit is reached. The density profile of the two cases was compared in Fig. 4.

We have solved both models in the limit of infinite matter. Thus we assume that all significant regions of space occupied by either phase are large in the sense that the volume energy is large compared to the surface energy. We have also neglected the Coulomb rearrangement energy associated with the non-uniform distribution of charge in the mixed phase. In all likelihood geometrical structures of the two phases in equilibrium similar to that discussed for sub-saturation nuclear matter will occur so as to minimize the sum of all these energies. We have neglected the rearrangement energy associated with this structure, though it is clearly an interesting area of investigation. Note that the volume energy against which its importance is to be assessed can be read from Fig. 7 and is seen to be ~ 1 GeV/fm^3. We also note that the difference in the volume energies of the two phases of the mixed phase are nearly the same, so we expect the surface energy to be small. Further, it is a general result that the Coulomb rearrangement energy is twice the surface energy and is therefore also small [9]. Therefore although the geometric texture of the mixed phase is likely to be present, the energy associated with creating it is likely to be small.

6 Summary

We extended the discussion of first order phase transitions to the general case that there is more than one conserved charge in the system. Compared to the well known case of a single conserved charge, the differences are dramatic. The constancy of the pressure at all concentrations of phases in equilibrium in the latter case is peculiar to there being but one conserved 'charge'. When there is more than one, the mixed phase has degrees of freedom that are otherwise not available: The charges are conserved over all, but not necessarily separately or in proportion to the concentration of the phases in equilibrium. In fact it is incorrect to insist that the phases in equilibrium satisfy the conservation laws individually; Gibbs conditions would be violated. The condition that the phases in equilibrium fulfill the conservation laws not individually but overall amounts to a weaker constraint. How much weaker obviously *depends on the concentration* of the phases. (At the extremes of the mixed phase the constraint is as strong as in the neighboring pure phase, while it is weaker in between.) The energy will therefore vary in a *non-linear* fashion with respect to the concentration (or density). Therefore the pressure will *vary* as the concentration and also the chemical potentials.

Finding phase equilibrium in the general case is far from trivial. It requires solving simultaneously the equations of motion of the internal structure of the body, eqs. (6), for *two* solutions, $\{\phi_1\}, \{\phi_2\}$, that are coupled by the conservation laws, eqs. (12) and that satisfy Gibbs conditions summarized in eq. (7) for a number of concentrations χ in the mixed phase.

Many systems in physics and astrophysics that undergo phase transitions are

complex in the sense of containing more than one conserved charge. In all such cases the phase equilibrium should be treated according to the general findings of this paper. There are many cases in the literature where a complex system has been fitted into the mold of a simple one by approximation. Consequences in the case that we have examined are drastic. For example, to assess whether neutron stars are likely to have quark cores, many authors approximated both phases as the equivalent of pure neutron matter. This turns a complex system into a simple one and has unexpected and unintended consequences, both for the estimate of the density at which the transition is expected to occur, as well as for the structure of the resulting stars if the transition does occur. First, neutron matter is beta unstable and the corresponding quark phase especially so. This latter fact leads to an estimate of the density at which neutron matter converts to quark matter that is $\sim 3-4$ times too high in comparison with a calculation that takes account of the equilibrium of the system and the conservation of the two charges of the resulting problem. Thus an estimate that the transition density is at $\sim 10\rho_0$ and therefore beyond the density found in neutron stars becomes $\sim 3\rho_0$, a density that *is* expected in neutron stars. Furthermore, there are structural differences in the star model that result from the treatment of the phase transition as if it occurred in a simple body. The mixed phase *in a simple body* cannot exist in the presence of gravity, because the pressure in that phase is constant. As noted this causes a discontinuity in the density distribution in the star occurring at the radius where Gibbs criteria are satisfied. When beta equilibrium is properly treated, involving as it does both the conserved baryon and electric charge, the transition region is smoothed out; the density distribution is continuous; only its derivative is discontinuous at the boundaries of the mixed phase with the pure phases. The mixed phase occupies a finite region of the star. In our particular example, it occupied the inner sphere of seven kilometers radius.

The question naturally arises whether the differences in star structure attributable to a correct handling of the conservation laws compared to an approximate one would cause the physics of the star to be different in an observable way. One cannot directly measure the density distribution for example. What our finding does imply in this respect is that hybrid stars do not form a separate class from neutron stars as far as their ability to sustain fast rotation is concerned. Another way in which we expect differences to occur that are connected to observables is in viscosity which influences stability to fast rotation [14] and transport properties which of course influence the cooling rate and the electrical conductivity of the core, which in turn influences the decay rate of the magnetic field on which the observation of pulsed radiation depends. These expectations are grounded on the finite width of the mixed phase and its expected varying geometrical structure (spheres, rods, slabs) with depth on the one hand, and the different transport properties of the two phases on the other [15].

The two most noteworthy qualitative differences in first order phase transitions in systems that are simple or complex. Pressure is a constant in the mixed phase in a simple system and this has the consequence in a gravitational field that the phases will always be separated (as water and vapor are on earth). In a complex system where the pressure is a function of the concentration in the mixed phase, gravity does not separate them. In addition to the quark phase transition in compact stars that

was used as an example in this paper, this has potential relevance to astrophysical problems related to phase transitions in the early universe as well possibly to phase transitions associated with the development of late time structure. The other qualitative difference has to do with the discontinuity of densities across the interface of phases in equilibrium. In complex systems, at least in our example, and it is plausibly a general characteristic based on the fact that pressure and all chemical potentials vary throughout the mixed phase, the phase transition is smoother than in a simple body.

Acknowledgements: I appreciate conversations on this work with M. Gyulassy, J. Randrup and W. Swiatecki. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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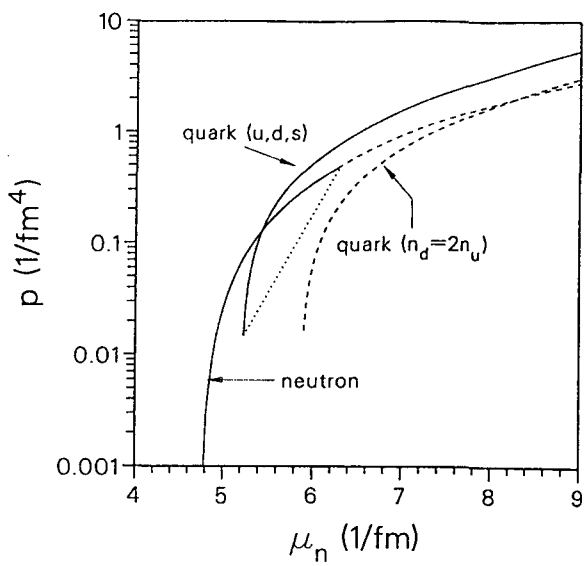

Figure 1: Pressure as function of chemical potential for first order phase transition (solid lines). Crossing is point of phase equilibrium. Dotted line is schematic. See text for description of dashed lines.

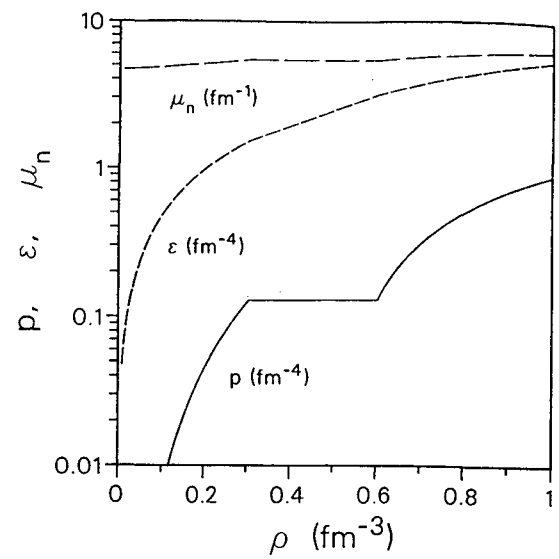

Figure 2: Pressure, energy density and chemical potential when there is only one conserved charge. Flat region in p and μ is mixed phase.

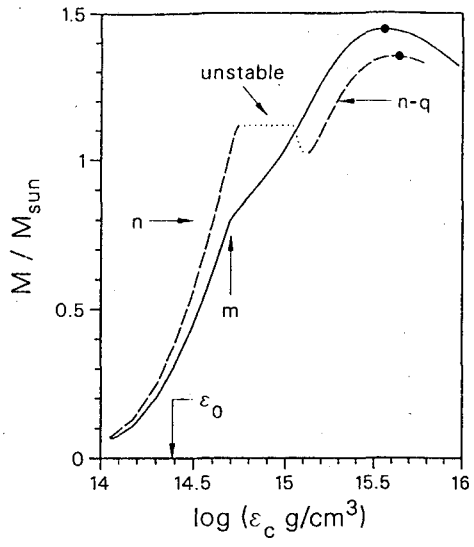

Figure 3: As a function of central density the dashed line interrupted by dots is the family of stars for idealized case having one chemical potential. Regions are; n - pure neutron stars; n-q - quark core and neutron matter mantle stars. Solid curve is family of compact stars in beta equilibrium (two chemical potentials); stars below 'm' are made of neutron star matter; those above have mixed phase cores.

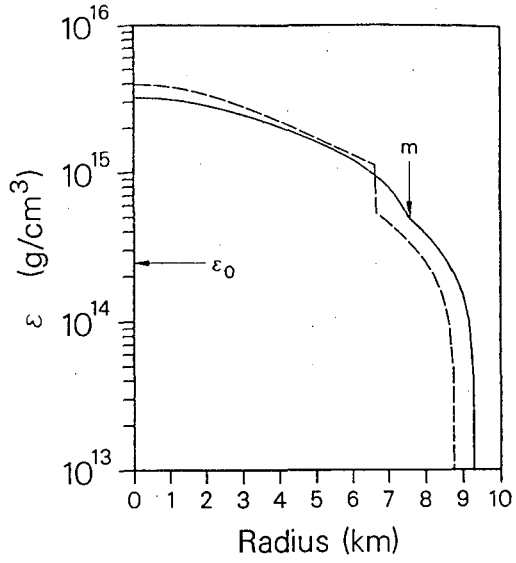

Figure 4: Dashed line is profile of an idealized star with one chemical potential at mass limit of previous figure. Core is pure quark matter and exterior is pure neutron matter. Solid curve is star at mass limit which is in beta equilibrium. The core is in the mixed phase of hadronic and quark matter. The exterior is neutron star matter. Dividing point marked by 'm'.

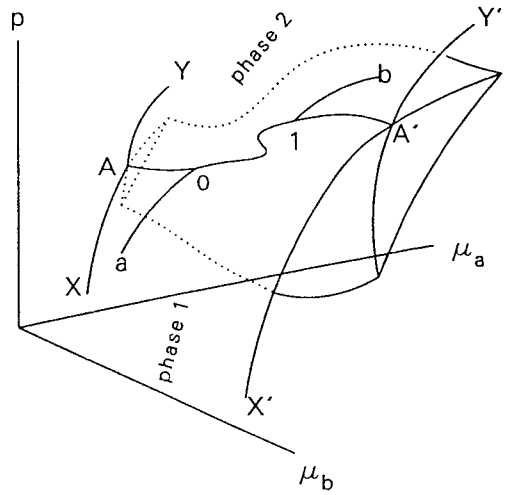

Figure 5: Schematic of pressure as a function of μ_a, μ_b, in the more general case of two conserved charge. The physical pressure is the curve a0 (pure phase 1), 01 (mixed phase with concentration χ between 0 and 1), and 1b (pure phase 2). Dotted curves lie below foreground pressure surfaces.

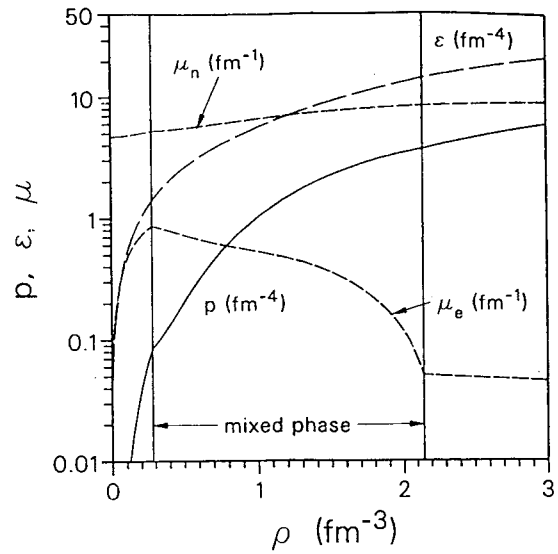

Figure 6: Pressure, energy density and chemical potentials as function of baryon density when there is more than one conserved charge.

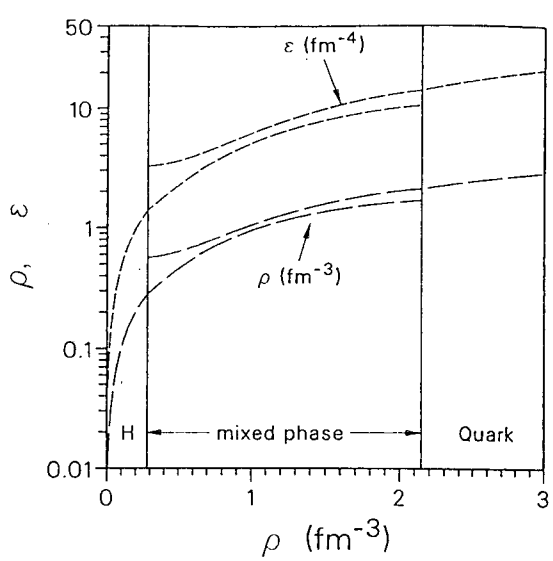

Figure 7: Baryon density and energy density of each phase as a function of average baryon density. The concentration of the phases varies from pure hadronic at the low end of the mixed phase to pure quark matter at the high end.

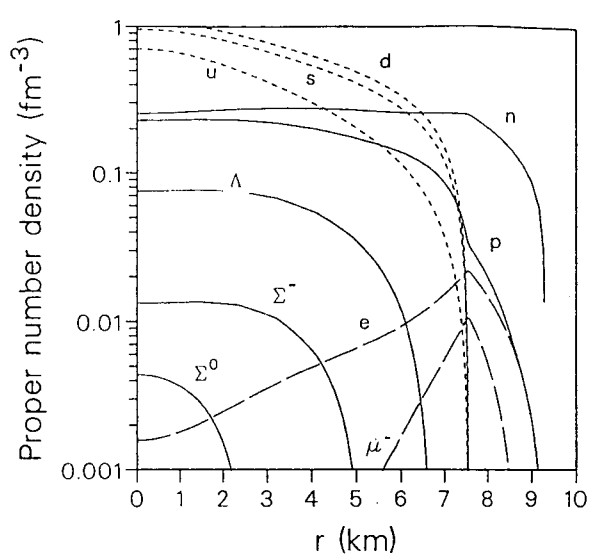

Figure 8: Composition of a hybrid star at the mass limit. Exterior to about 7.6 km is neutron star matter. Interior to this is a mixed phase of confined hadronic matter and quark matter which is overall charge neutral.

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