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COMPARISON OF HEAVY CHARGED PARTICLES AND X-RAYS FOR AXIAL TOMOGRAPIC SCANNING

R. H. Huesman, A. H. Rosenfeld, and F. T. Solmitz

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COMPARISON OF HEAVY CHARGED PARTICLES AND X-RAYS FOR AXIAL TOMOGRAPHIC SCANNING

R. H. Huesman, A. H. Rosenfeld, and F. T. Solmitz

ABSTRACT

A comparison is made between x-rays of various energies and heavy charged particles for their effectiveness in imaging of three-dimensional distributions of biological samples. It is shown that low-z heavy charged particles give lower radiation doses than x-rays for imaging the human head.

Dose versus resolution calculations for imaging with heavy charged particles include nuclear scattering as well as multiple Coulomb scattering. Calculations for x-rays neglect the skin dose which is large compared to the average dose sustained by the patient.

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ERRATA

Please make the following corrections on subject report:

1) Cover and title page reads: ... Axial Tomograpic Scanning,

should read: ... Axial Tomographic Scanning.

2) Abstract page is missing, please insert attached Abstract.

3) Page 6, (3rd line of text) reads: ... relative dose which dose not ...

should read: ... relative dose which does not ...

4) Page 6 (Equation 1) reads: $= \frac{E_\gamma \exp(\mu_o L)}{\mu_o^2}$

should read: $= \frac{\varepsilon_\gamma \exp(\mu_o L)}{\mu_o^2}$

5) Page 12 (Equation 17) reads: $= \frac{nEd^3(\delta\rho)^2}{DM_j^{-1}}$

should read: $= \frac{nEd^3(\delta\rho)^2}{1.6D}$

6) Page 13 (top line of text) reads: Tables 2 and 3

should read: Tables 1 and 2

7) Page 16, Table 3, heading of third column reads: $(1/p\beta)_{av}$

should read: $(A/p\beta)_{av}$

Comparison of Heavy Charged Particles and X-Rays for Axial Tomograpic Scanning

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I. INTRODUCTION

Axial tomographic scans using x-rays have been used for some time as input for 3-dimensional reconstruction of electron density distributions of biological samples. The attenuation of an x-ray beam is measured over many coplanar paths through a "slice" of the object to be reconstructed. Each of these measurements yield the integral of the attenuation coefficient (for the particular x-ray energy used) over a line in the plane. The attenuation coefficient is roughly proportional to electron density.

Integrated electron densities can also be determined by measuring the slowing down of heavy charged particles after having passed through a biological sample. In this report we shall compare the resolution and relative dose required by heavy charged particles and x-rays.

We shall assume that a water bath surrounds the object to be reconstructed so that the measurement of integrated electron density shall be over a fixed path length L. What is important then is how accurately one can determine the difference between the integral with the object in the bath and the integral over a length L of water. It is also assumed that electron density of the biological sample is close to that of water.

From the slowing down of a heavy charged particle, what is actually measured is an integral of the linear stopping power of the medium. This is directly proportional to electron density but also depends weakly on the mean exitation energy of the atoms of the medium. Also, in the x-ray case the linear attenuation coefficient is only directly proportional to the electron density when the attenuation is due only to Compton scattering. For the purpose of this comparison we shall normalize to the stopping power and attenuation coefficient of water and assume that deviations due to the presence of the biological sample are due to changes in the electron density.

II. ATTENUATION OF X-RAYS

We assume that a monoenergetic beam of N_γ x-rays of energy ϵ_γ is incident on the water box with the biological sample present. The number of x-rays emerging from the opposite side of the water box is given by,

$$N = N_\gamma \exp\left[-\int_0^L \mu(x)dx \right] \tag{1}$$

where $\mu(x)$ is the linear attenuation coefficient of the water and the sample. We define,

$$I_\gamma = \frac{1}{\mu_o} \left[\int_0^L \mu(x)dx - \mu_o L \right] = \frac{1}{\mu_o} \int_0^L \mu(x)dx - L = \frac{1}{\mu_o} \ln\frac{N_\gamma}{N} - L \tag{2}$$

where μ_o is the attenuation coefficient for water. By normalizing to water, we have defined I_γ to be the integral of the relative electron density difference from water when the x-ray attenuation is only due to Compton scattering (i.e., directly proportional to electron density).

The accuracy of I_γ depends on N_γ in the following way:

$$\delta I_\gamma = \left|\frac{\partial I_\gamma}{\partial N}\right| \delta N = \frac{\delta N}{\mu_o N} = \frac{\sqrt{N}}{\mu_o N} = \frac{1}{\mu_o \sqrt{N}} \tag{3}$$

But,

$$N \approx N_\gamma \exp(-\mu_o L) \tag{4}$$

if the attenuation coefficient of the sample does not differ very much from that of water, so that

$$\delta I_\gamma = \frac{1}{\mu_o \sqrt{N_\gamma \exp(-\mu_o L)}} \tag{5}$$

We now calculate the energy deposited as a function of δI_γ. Of the N_γ incident x-rays, the energy of $(N_\gamma - N)$ of them is absorbed so that the total energy deposited is given by,

$$E_\gamma = \epsilon_\gamma N_\gamma \left[1 - \exp(-\mu_o L) \right] \tag{6}$$

From eqn (5),

$$N_\gamma = \frac{\exp(\mu_o L)}{(\mu_o \delta I_\gamma)^2} \tag{7}$$

so that,

$$E_\gamma = \frac{\epsilon_\gamma [\exp(\mu_o L) - 1]}{(\mu_o \delta I_\gamma)^2} \tag{8}$$

For all practical cases $\exp(\mu_o L) >> 1$, so that

$$E_\gamma = \frac{\epsilon_\gamma \exp(\mu_o L)}{(\mu_o \delta I_\gamma)^2} \tag{9}$$

III. SLOWING DOWN OF HEAVY CHARGED PARTICLES

Heavy charged particles (large mass compared to an electron) slow down when passing through matter because of interactions with electrons. Due to the statistical nature of this process, two identical particles passing through the same material will not slow down exactly the same amount. This well known phenomenon is called range straggling.

We assume that a monoenergetic beam of N_p heavy charged particles of atomic number A and kinetic energy ϵ_p is incident on the water box with the biological sample present. After emerging form the opposite side of the box, the particles enter a second homogeneous medium of linear stopping power λ_s (relative to water) and come to a stop, and the average depth of penetration R is measured. If $\lambda(x)$ is the relative linear stopping power of the water and sample then,

$$\lambda_s R + \int_0^L \lambda(x) dx = \lambda_s R_o + L \tag{1}$$

where R_o is the depth of penetration when the sample is not present and a thickness L of water is traversed. We define

$$l_p = \int_0^L \lambda(x)dx - L = \lambda_s(R_o - R) \tag{2}$$

By normalizing to water, we have defined l_p to be the integral of the relative electron density difference from water apart from the weak dependence of the stopping power on mean exitation energy.

The accuracy of l_p depends of N_p in the following way:

$$\delta l_p = \left|\frac{\partial l_p}{\partial R}\right| \delta R = \lambda_s \delta R \tag{3}$$

But σ, the uncertainty in the longitudinal stopping point of each particle, is approximately proportional to the total range divided by the square root of the mass, so that

$$\sigma = \frac{S}{\sqrt{A}}\left(R_o + \frac{L}{\lambda_s}\right) = \frac{SL_o}{\lambda_s\sqrt{A}} \tag{4}$$

where L_o is the range in water and S is the constant of proportionality which depends on the initial velocity but varies slowly and is about 0.01. By taking the average stopping point of the N_p particles, the uncertainty in R is reduced to,

$$\delta R = \frac{\sigma}{\sqrt{N_p}} = \frac{SL_o}{\lambda_s\sqrt{AN_p}} \tag{5}$$

so that,

$$\delta l_p = \frac{SL_o}{\sqrt{AN_p}} \tag{6}$$

In addition to range straggling, the particles are deflected by interactions with the nuclei of the medium they are passing through. This phenomenon is called multiple Coulomb scattering and makes it impossible to know the path of each particle exactly. In Section V we shall show that by measuring the entrance and exit positions and angles of each particle, the uncertainty of the path of the particles is greatly reduced. For the purpose of relative dose calculations in the next section, we neglect this effect.

We now calculate the energy deposited as a function of δl_p. The N_p particles deposit some fraction, f, of their energy into the water bath and biological sample they pass through. Then the energy deposited is given by $f\epsilon_p N_p$.

Another type of interaction which the particle can undergo is a non-elastic nuclear collision. Particles undergoing such collisions are not useful to us, but increase the number of incident particles needed to have N_p useful ones. Let g be the fraction which suffer nuclear collisions and assume that they also deposit a fraction f of their energy. Then the total energy deposited is,

$$E_p = \frac{f\epsilon_p N_p}{1 - g} \tag{7}$$

From eqn. (6),

$$N_p = \frac{1}{A}\left(\frac{SL_o}{\delta I_p}\right)^2 \tag{8}$$

so that,

$$E_p = \frac{f\epsilon_p}{A(1 - g)}\left(\frac{SL_o}{\delta I_p}\right)^2 \tag{9}$$

IV. RELATIVE AVERAGE DOSE

The relative dose between x-rays and heavy charged particles is determined by the average energy deposited per gram of the biological sample. What we calculate here is the average relative dose which dose not take into account two effects:

a) The energy deposited by a beam of x-rays is much larger near the entrance to the water box than near the exit.

b) The energy deposited by a beam of heavy charged particles is slightly larger near the exit than it is near the entrance to the water box.

In order to make a comparison, we shall calculate the quantities $E_\gamma(\delta I_\gamma)^2$ for x-rays and $E_p(\delta I_p)^2$ for heavy charged particles. These numbers will be directly proportional to the dose needed to obtain a measurement of I_γ (or I_p) with uncertainty δI_γ ($=\delta I_p$).

For the purpose of comparison we shall assume a water bath thickness L = 25 cm and heavy charged particles of range L_o = 32 cm of water. These parameters are suitable for imaging the human head.

From eqn. (II-9),

$$E_\gamma(\delta I_\gamma)^2 = \frac{E_\gamma \exp(\mu_o L)}{\mu_o^2} \tag{1}$$

where E_γ, δI_γ and μ_o have the same meaning as in Section II. We have obtained values of μ_o for various x-ray energies from ref. 1. Table 1 gives values of $E_\gamma(\delta I_\gamma)^2$ for various values of E_γ assuming L = 25 cm. For the convenience of the reader we also show the fraction of attenuation which is due to the photoelectric effect.

From eqn. (III-9),

$$E_p(\delta I_p)^2 = \frac{f\epsilon_p(SL_o)^2}{A(1 - g)} \tag{2}$$

where E_p, δI_p, f, ϵ_p, S, A and g have the same meaning as in Section III. We have obtained values for ϵ_p/A and f from ref. 2, assuming an initial range of 32 cm of water and a residual range of 7 cm of water after leaving the water box. Values for g have been calculated using the approximate formula for cross sections of two complex nuclei of atomic weights A_1 and A_2 given by,

$$\sigma = \pi(1.26)^2(A_1^{1/3} + A_2^{1/3} - .8)^2 \quad (\text{fermi}^2) \tag{3}$$

Values for S have been obtained from ref. 3 where we have used values for protons in berellium (whose mean exitation energy is close to that of water) for the appropriate value of ϵ_p/A. Table 2 gives values of $E_p(\delta l_p)^2$ for various heavy charged particles.

We again remind the reader that for heavy charged particles we have neglected the effect of multiple Coulomb scattering. By the methods of Section V we shall be able to know the path of the particle to within a small fraction of the deviation from a straight line which is due to multiple scattering. For our worst case (protons of range 32 cm of water passing through 25 cm of water) we know the projected path (halfway through the water box) to a precision of about ±0.8 mm.

Within the above framework, relative doses between x-rays of various energies and various heavy charged particles can be compared directly by inspection of the last columns of Tables 1 and 2.

V. MULTIPLE COULOMB SCATTERING

When a heavy charged particle passes through matter its path is deflected by elastic scattering with the nuclei of the medium. This phenomenon has been studied extensively and is called multiple Coulomb scattering. The distribution of projected angles of deflection after passing through a small thickness x of matter is known to be approximately Gaussian, and from ref. 4 we get the relation:

$$\langle\theta^2(x)\rangle = \frac{1}{2}\left(\frac{zE_s}{p\beta}\right)^2 \frac{x}{L_{rad}} \tag{1}$$

where $\langle\theta^2(x)\rangle$ is the mean square projected angle of deflection; z, p and β are the charge, momentum and velocity of the particle respectively; L_{rad} is the radiation length of the material and

$$E_s = m_e\sqrt{4\pi\times 137} = 21.2 \text{ MeV} \tag{2}$$

where m_e is the mass of the electron. We rewrite eq. (1) as,

$$\langle\theta^2(x)\rangle = \alpha x \tag{3}$$

where

$$\alpha = \left(z\frac{15 \text{ MeV}}{p\beta}\right)^2 \frac{1}{L_{rad}} \tag{4}$$

If one divides the length x into n slabs each of thickness d, so that x = nd, then we can write down the correlation between the angular changes between the ith and j^{th} slab as,

$$\langle\Delta\theta_i\Delta\theta_j\rangle = \delta_{ij}\alpha d \tag{5}$$

where δ_{ij} is the Kronecker delta and the angular changes in different slabs are

uncorrelated. Taken to the infinitesimal limit this becomes,

$$\left\langle \frac{d\theta}{dx}(x')\frac{d\theta}{dx}(x'') \right\rangle = \alpha\delta(x' - x'') \tag{6}$$

where $\delta(x' - x'')$ is the Dirac delta function. We then have the relation,

$$\theta(x) = \int_0^x dx'\frac{d\theta}{dx}(x') \tag{7}$$

To obtain the lateral displacement of the particle downstream a distance x, we must remember that an angular deflection $d\theta$ at a distance x' will be projected as a lateral displacement $(x - x')d\theta$ at a distance x. Therefore, the displacement at a distance x is given by,

$$y(x) = \int_0^x dx'(x - x')\frac{d\theta}{dx}(x') \tag{8}$$

In both (7) and (8) above we have assumed the initial conditions,

$$\theta(0) = y(0) = 0 \tag{9}$$

In what follows we shall assume that a heavy charged particle is incident on a homogeneous medium of thickness L. We shall make the approximation that $1/(p\beta)$ of the particle is constant over the path, and in practice we use the average value. We shall estimate the lateral deviation of the particle (from a straight line path) by a function $y^*(x)$, which depends on the exit position and angle ($y(L)$ and $\theta(L)$ respectively which we can measure).

We will now evaluate several expressions which we shall need below.

$$\begin{aligned}\langle\theta(x_1)\theta(x_2)\rangle &= \left\langle \int_0^{x_1} dx'\frac{d\theta}{dx}(x') \int_0^{x_2} dx''\frac{d\theta}{dx}(x'') \right\rangle \\ &= \int_0^{x_1} dx' \int_0^{x_2} dx'' \left\langle \frac{d\theta}{dx}(x')\frac{d\theta}{dx}(x'') \right\rangle \\ &= \alpha \int_0^{x_1} dx' \int_0^{x_2} dx''\, \delta(x' - x'') = \alpha x_<\end{aligned} \tag{10}$$

where $x_<$ is the smaller of x_1 and x_2. After similar integrations we find,

$$\langle\theta(x_1)y(x_2)\rangle = \frac{1}{2}\alpha x_<(2x_1 - x_<) \tag{11}$$

and

$$\langle y(x_1)y(x_2)\rangle = \frac{1}{6}\alpha x_<^2(3x_> - x_<) \tag{12}$$

where $x_>$ is the larger of x_1 and x_2.

We now estimate $y(x)$ by the function,

$$y^*(x) = av(L) + b\theta(L) \tag{13}$$

and to determine a and b, we minimize the mean square deviation of $y^*(x)$ from the true path $y(x)$. Let Q be this expression, then

$$Q = \langle[y^*(x) - y(x)]^2\rangle = \langle[ay(L) + b\theta(L) - y(x)]^2\rangle \tag{14}$$

After setting the partial derivatives with respect to a and b to zero and doing some algebra we get,

$$\begin{aligned} a\langle y^2(L)\rangle + b\langle y(L)\theta(L)\rangle &= \langle y(x)y(L)\rangle \\ a\langle y(L)\theta(L)\rangle + b\langle\theta^2(L)\rangle &= \langle y(x)\theta(L)\rangle \end{aligned} \tag{15}$$

After solving for a and b, substituting from eqs. (10), (11) and (12) we get (after much more algerba),

$$a = \frac{x^2}{L^3}(3L - x)$$

$$b = -\frac{x^2}{L^2}(L - x) \tag{16}$$

and we have for the estimated path,

$$y^*(x) = \frac{x^2}{L^3}(3L - 2x)y(L) - \frac{x^2}{L^2}(L - x)\theta(L) \tag{17}$$

Finally we get for the mean square deviation (after yet more algebra),

$$Q(x) = \frac{\alpha x^3}{3L^3}(L - x)^3 \tag{18}$$

which is largest at $x = .5L$ where

$$Q(.5L) = \frac{1}{192}\alpha L^3 = (5.21\times10^{-3})\alpha L^3 \tag{19}$$

In Table 3 we give values for the maximum r.m.s. deviation of the estimated path from the true path for various heavy charged particles (i.e. $\sqrt{Q(.5L)}$). For the purpose of this comparison we have used a water bath of thickness $L = 25$ cm and particles of range $L_o = 32$ cm of water. $(1/p\beta)_{av}$ is the average value of $1/p\beta$ over the

25 cm of water, and we have used L_{rad} = 36.4 cm.

For other estimates of the path of the particle than that given by eqn. (17), all the values of $\sqrt{Q_{max}}$ on Table 3 should be multiplied by a factor. We give this factor, without proof, for two other path estimates:

a) For the parabola described by

$$y^*(x) = \frac{x^2}{L^2}\, y(L) \tag{20}$$

the maximum occurs at x = .6L, and the values of $\sqrt{Q_{max}}$ on Table 3 should be multiplied by 1.49.

b) For the straight line described by

$$y^*(x) = \frac{x}{L}\, y(L) \tag{21}$$

the maximum occurs at x = .5L, and the values of $\sqrt{Q_{max}}$ on Table 3 should be multiplied by 2.

VI. THE RECONSTRUCTION

Looking toward the reconstruction of the distribution of electron density over the 2-dimensional slice, we now define the back projection. We assume that the data, I_i, consist of a collection of integrated electron densities over n coplanar paths through the slice. We take a square area to be reconstructed which has dimensions D×D, and we subdivide it into small square cells of dimensions d×d. Then the area to be reconstructed consists of $(D/d)^2$ cells, each of which is assumed to contain uniform electron density.

We define the back projection, B_k, for the kth cell to be the sum over the n paths of the integrated electron density times the line length through the kth cell,

$$B_k = \frac{D}{nd} \sum_i \frac{I_i}{d} \frac{\ell_{ik}}{d} = \frac{D}{nd^3} \sum_i I_i \ell_{ik} \tag{1}$$

where ℓ_{ik} is the line length of the ith path through the kth cell, and I_i is the integrated relative electron density difference from water over the ith path as defined in Sections II and III above. The normalization factor, $D/(nd^3)$ is chosen such that if the kth cell has unit density difference and all other cells have zero density difference from water, then B_k is approximately equal to unity.

If ρ_k is the relative electron density difference from water, then I_i is simply given by,

$$I_i = \sum_j \ell_{ij} \rho_j \tag{2}$$

and we can write the backprojection in terms of the density as,

$$B_k = \frac{D}{nd^3} \sum_i I_i \ell_{ik} = \frac{D}{nd^3} \sum_{ij} \ell_{ik} \ell_{ij} \rho_j \tag{3}$$

Defining the matrix M by the expression,

$$M_{kj} = \frac{D}{nd^3} \sum_i \ell_{ik} \ell_{ij} \tag{4}$$

and substituting into eq. (3) we see that,

$$B_k = \sum_j M_{kj} \rho_j \tag{5}$$

A diagonal element of the matrix M is given by,

$$M_{jj} = \frac{D}{nd^3} \sum_i \ell_{ij}^2 \tag{6}$$

but ℓ_{ij} is non-zero about d/D of the time, and when non-zero it is about equal to d, so that

$$M_{jj} \simeq \frac{D}{nd^3} \frac{nd}{D} d^2 = 1 \tag{7}$$

which justifies the normalization factor stated above.

From eq. (5) we see that the backprojection is just a matrix multiplication with the density vector, and if the problem is well posed, the density vector can be obtained after a matrix inversion. Since the matrix M may be very large, it is usually not practical to invert it. We shall put this problem aside for the moment and return to it later.

Investigating the uncertainty in the reconstruction, we express the density vector in terms of the inverse of the matrix M as,

$$\rho_j = \sum_k M_{jk}^{-1} B_k = \sum_k M_{jk}^{-1} \frac{D}{nd^3} \sum_i I_i \ell_{ik} = \frac{D}{nd^3} \sum_i I_i \sum_k M_{jk}^{-1} \ell_{ik} \tag{8}$$

The uncertainty of ρ_j due to the uncertainty in the I_i is easily calculated. Assuming that the uncertainty of all I_i are equal to δI we have,

$$(\delta\rho_j)^2 = \left(\frac{D\delta I}{nd^3}\right)^2 \sum_i \left(\sum_k M_{jk}^{-1} \ell_{ik}\right)^2 = \left(\frac{D\delta I}{nd^3}\right)^2 \sum_{km} M_{jk}^{-1} M_{jm}^{-1} \sum_i \ell_{ik} \ell_{im} \tag{9}$$

Substitution from eq. (4) gives,

$$(\delta\rho_j)^2 = (\delta I)^2 \frac{D}{nd^3} \sum_{km} M_{jk}^{-1} M_{jm}^{-1} M_{km} = (\delta I)^2 \frac{D}{nd^3} M_{jj}^{-1} \tag{10}$$

It is well known that the operation of backprojection is simply a convolution with the function 1/r (see ref. 5). Therefore the diagonal elements of the matrix M are equal and the off-diagonal elements decrease proportional to the reciprocal of the distance between cells. That is, M_{jk} is proportional to the reciprocal of the distance between the kth and j^{th} cells. Since it is usually impractical to invert the matrix M, we have attempted to find an approximation to M^{-1} which is also a convolution and which is limited in extent. We have set M^{-1}_{jk} equal to zero when the jth and k^{th} cells are greater than a specified distance apart and have solved for the remaining M^{-1}_{jk} which best satisfy the relationship,

$$\sum_i M^{-1}_{ji} M_{ik} = \delta_{jk} \tag{11}$$

in the least squares sense, where δ_{jk} is the Kronecker delta. For all ranges of non-zero M^{-1}_{jk} tried, the diagonal element (central element of the convolution) has remained stable and is equal to 1.6.

A second approach to finding the convolution M^{-1} was also tried. In this approach we used the 2-dimensional Fourier convolution theorm which states in our case,

$$F_2(B) = F_2(M*\rho) = F_2(M)F_2(\rho) \tag{12}$$

where F_2 indicates 2-dimensional Fourier transformation and * indicates convolution. Solving for $F_2(\rho)$ we get,

$$F_2(\rho) = F_2(B)/F_2(M) = F_2(B*M^{-1}) = F_2(B)F_2(M^{-1}) \tag{13}$$

so that,

$$M^{-1} = F_2^{-1}[1/F_2(M)] \tag{14}$$

where F_2^{-1} indicates inverse 2-dimensional Fourier transformation. With this approach the central element of the convolution M^{-1} was also found to be equal to 1.6. From eq. (10) we therefore have the result,

$$(\delta\rho)^2 = (\delta I)^2 \frac{1.6D}{nd^3} \tag{15}$$

and solving for $(\delta I)^2$ we get,

$$(\delta I)^2 = (\delta\rho)^2 \frac{nd^3}{1.6D} \tag{16}$$

so that,

$$E(\delta I)^2 = \frac{nEd^3(\delta\rho)^2}{DM^{-1}_{jj}} \tag{17}$$

which has been tabulated in Tables 2 and 3 for x-rays of various energies and various heavy charged particles, respectively.

The energy deposited in the slice is given by,

$$nE = \frac{1.6D[E(\delta l)^2]}{d^3(\delta\rho)^2} \tag{18}$$

and the dose is given by (assuming a mass density of 1 g/cm^2 within the slice),

$$\begin{aligned} \text{dose} &= \frac{nE}{tD^2} \quad \text{MeV/g} \\ &= \frac{nE}{tD^2} \times (1.6\times10^{-5}) \quad \text{mrad} \\ &= \frac{1.6[E(\delta l)^2]}{tDd^3(\delta\rho)^2} \times (1.6\times10^{-5}) \quad \text{mrad} \end{aligned} \tag{19}$$

where t is the thicknes; of the slice.

Table 4 gives dose calculations for varying cell size and a ±1% uncertainty in the reconstruction. Table 4 assumes a 25 cm water bath and gives values for both He ions of range 32 cm of water and 80 keV x-rays.

VII. CONCLUSIONS

We have shown that heavy charged particles are applicable to the problem of 3-dimensional reconstruction of electron density distributions of biological samples. Inspection of the right hand columns of Tables 1 and 2 shows that the use of low z heavy charged particles gives about an order of magnitude advantage in dose over x-rays when the resolution of the reconstruction is large compared to the multiple scattering and when the photoelectric absorbtion of the x-rays is negligible.

We have shown that the transverse uncertainty in the path of a heavy charged particle due to multiple scattering can be reduced by measureing the entrance and exit positions and angles of the particle. Table 3 gives this uncertainty half way through the water bath, where it is largest.

We have compared patient doses for He ions and 80 keV x-rays under conditions suitable for imaging the human head. Table 4 gives these doses for various reconstructed cell sizes.

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TABLE 1. ENERGY DEPOSITED BY X-RAYS TIMES THE VARIANCE OF THE RELATIVE INTEGRATED ELECTRON DENSITY MEASURED*

ϵ_γ (MeV)	μ_o (cm^{-1})	% photo	$E_\gamma(\delta I_\gamma)^2$ (MeV-cm^2)
.03	.370	37.3	2280
.04	.267	20.5	445
.05	.227	11.8	283
.06	.206	7.3	244
.08	.184	3.2	237
.10	.171	1.7	246
.15	.151	0.5	287
.20	.137	0.2	327
.30	.119	0.07	415
.40	.106	0.04	504
.50	.097	0.02	600

* The x-rays are assumed to be attenuated by 25 cm of water.

TABLE 2. ENERGY DEPOSITED BY HEAVY CHARGED PARTICLES TIMES THE VARIANCE OF THE RELATIVE INTEGRATED ELECTRON DENSITY MEASURED*

	A	z	ϵ_p/A (MeV)	f	1 - g	S ($\times 10^{-2}$)	$E_p(\delta I_p)^2$ (MeV-cm^2)
H	1	1	228.3	.581	.64	1.00	21
D	2	1	152.8	.575	.53	1.04	18
T	3	1	121.3	.573	.47	1.06	17
He	4	2	228.3	.581	.43	1.00	32
C	12	6	445.3	.599	.27	0.94	89
O	16	8	535.2	.606	.20	0.92	141
Ne	20	10	619.5	.611	.16	0.91	201

* The particles are assumed to have a range of 32 cm of water and pass through 25 cm of water.

TABLE 3. MAXIMUM UNCERTAINTY OF TRANSVERSE POSITION OF THE PATH OF A HEAVY CHARGED PARTICLE

	A	z	$(1/p\beta)_{av}$ $(MeV^{-1}\times10^{-3})$	α $(cm^{-1}\times10^{-6})$	$\sqrt{Q_{max}}$ (cm)
H	1	1	3.55	78.1	.080
D	2	1	5.17	41.3	.058
T	3	1	6.77	31.5	.051
He	4	2	3.55	19.5	.040
C	12	6	1.96	5.9	.022
O	16	8	1.67	4.3	.019
Ne	20	10	1.48	3.4	.017

* The particles are assumed to have a range of 32 cm of water and pass through 25 cm of water.

TABLE 4. DOSE AS A FUNCTION OF CELL SIZE FOR ALPHA PARTICLES AND 80 keV X-RAYS

1.	Cell size (mm)	6	4	2	1
2.	Cells along one side, n [a]	42	63	125	250
3.	Dose to patient for He (mrad) [b,c]	1.5	5.1	37	328
4.	Dose to patient for 80 keV x-rays	11	38	304	2430

(a) For a total area of 25 cm × 25 cm.
(b) For reconstruction good to ±1%. If only ±2% is desired, then divide by 4, etc.
(c) For a 1 cm thick slice. If the slice thickness is only 5mm, multiply by 2, etc.

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