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ϵ'/ϵ: Review and Recent Progress

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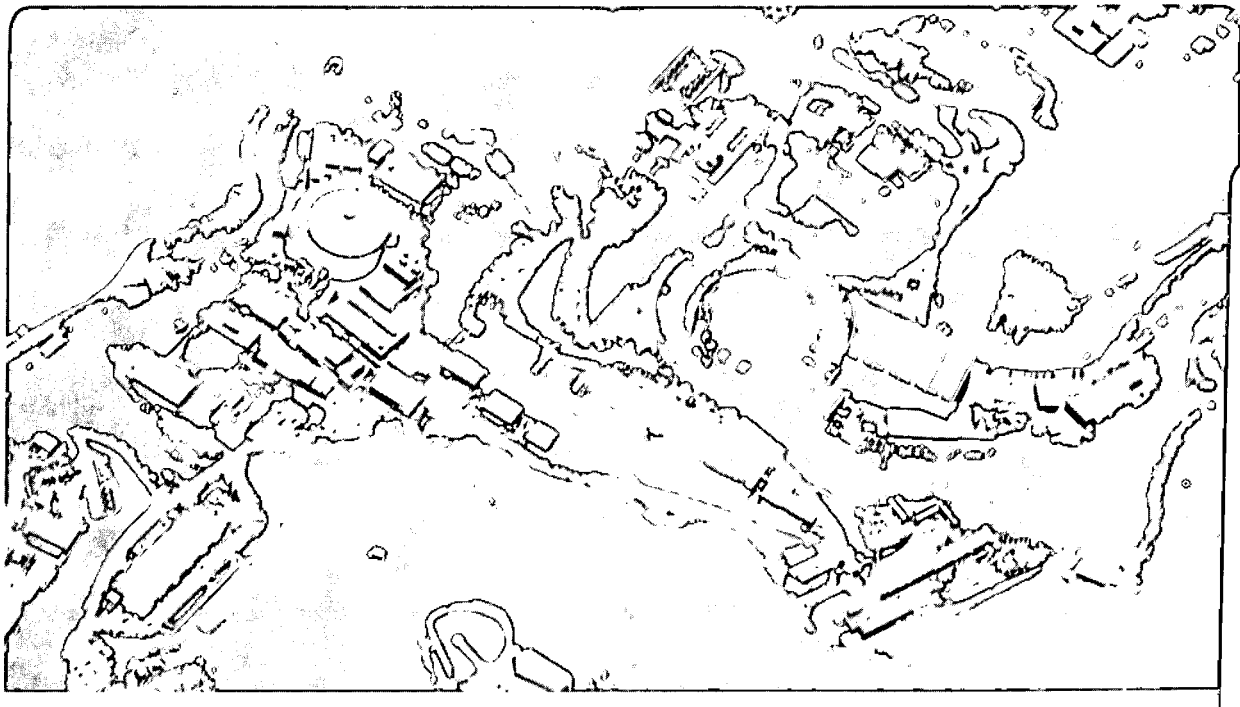

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ϵ'/ϵ: REVIEW AND RECENT PROGRESS*

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Abstract

The evolution of the theoretical perspective on ϵ'/ϵ is reviewed. The introduction of the Z^0 penguin and the effects of high m_t are discussed, in particular the possibility for ϵ'/ϵ to be identically zero. Recent calculations of ϵ'/ϵ based on current estimates and bounds on the input parameters are presented.

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1. Introduction

Essentially since the discovery[1] of CP violation in 1964 the search has been on for direct CP violation, CP violation in the decay of kaons rather than due to the CP impurity of K_L and K_S, i.e., the mixing of K_1 and K_2. In 1964 as well, Wolfenstein[2] proposed his superweak scenario, where CP violation is present in some new interaction contributing at lowest order to the $\Delta S = 2$ mass matrix. Its contribution to direct CP violation would therefore be essentially zero. The prediction for the magnitude of direct CP violation in the Standard Model has of course never been nearly as well-determined, although 'non-zero' has been accepted for quite some time.

Since the progress of experimental results over the last couple of decades is fairly well known (and since at any rate this is a theoretical talk) we confine ourself to the usual summary graph, and from there move on to establishing notation. The main part of this talk covers the evolution of the Standard Model perspective on ϵ'/ϵ, the measure of direct CP violation, over the last two decades, culminating in the recent introduction of the Z^0 penguin contribution,[3] which leads to the possibility of ϵ'/ϵ crossing zero at large m_t ($m_t \approx 200$ GeV). We present the phenomenology of ϵ'/ϵ based on the latest calculations and the current estimates and bounds on input parameters, and conclude with a brief look at predictions for ϵ'/ϵ in various non-Standard models.

2. Experimental Microreview

Fig. 1 shows the evolution of the measured value of ϵ'/ϵ over the last two decades. Two decades is a convenient cutoff, since many of the results before 1970 tend to be based on measurements of η_{00} and η_{+-} (see next section for definitions) by separate experiments. The results shown, in chronological order, are: 1972, Banner *et al.*,[4] Brookhaven/Princeton; 1972, Holder *et al.*,[5] CERN; 1979, Christenson *et al.*,[6] Brookhaven/NYU; 1985, Black *et al.*,[7] Brookhaven/Yale; 1985, Bernstein *et al.*,[8] Fermilab/U. Chicago/Saclay (E731); 1988, Woods *et al.*,[9] E731; 1988, Burkhardt *et al.*,[10] CERN (NA31); 1990, Patterson *et al.*,[11] E731; 1990, latest NA31 result.[12] Note that not all measurements from the same experiment are independent, e.g., the latest NA31 number quoted above includes the 1988 number averaged in.

The latest measurements (both from 1990) are those of E731[11] and NA31[12]

$$-0.4 \pm 1.4 \pm 0.6 \times 10^{-3} \quad \text{(E731)} \qquad\qquad 2.7 \pm 0.9 \times 10^{-3} \quad \text{(NA31)}. \tag{2.1}$$

The hope for the future is to get errors down to $\mathcal{O}(10^{-4})$ in the next five to ten years.[13]

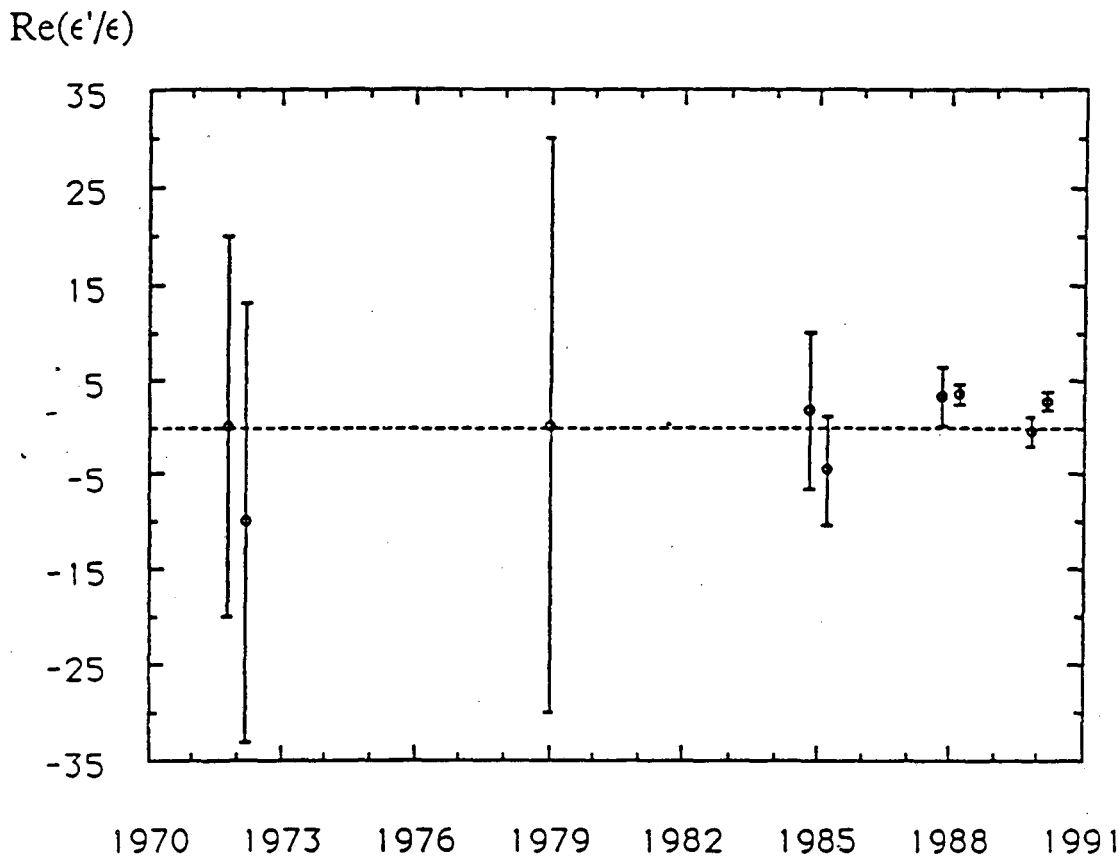

Figure 1. Experimental measurements of Re$\frac{\epsilon'}{\epsilon}$ (in units of 10^{-3}) from the last two decades.

3. Conventions, Notations and Beginnings

We begin by stating the conventions that we shall use, and relating them to some of the other ones in common usage. In the limit where CP is conserved, we have

$$CP\,|K^0\rangle = -\,|\bar{K}^0\rangle \qquad\qquad CP\,|\bar{K}^0\rangle = -\,|K^0\rangle\,, \tag{3.1}$$

giving eigenstates

$$|K_{1,2}\rangle = \frac{|K^0\rangle \mp |\bar{K}^0\rangle}{\sqrt{2}}. \tag{3.2}$$

Since K_1 is CP even it decays quickly to two pions; the CP odd state K_2 decays slowly to three pions.

Introducing CP violation as a small off-diagonal element in the mass matrix, the new eigenstates are given by

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} |K_{L,S}\rangle = e_{L,S}\,|K_{L,S}\rangle \tag{3.3}$$

giving

$$|K_{L,S}\rangle = \left[(1+\tilde{\epsilon})\,|K^0\rangle \pm (1-\tilde{\epsilon})\,|\bar{K}^0\rangle\right] \Big/ \sqrt{2(1+|\tilde{\epsilon}|^2)} \tag{3.4}$$

where $\tilde{\epsilon}$ is given by the expression

$$\frac{1+\tilde{\epsilon}}{1-\tilde{\epsilon}} = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}}\,. \tag{3.5}$$

(The $\tilde{\epsilon}$ defined here is distinguished with a tilde because, as we shall see, it is only equal to the observable ϵ in certain phase conventions.) Solving for the eigenvalues $e_{L,S}$ and taking their difference gives

$$\Delta M - \frac{i}{2}\Delta\Gamma = 2\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}. \tag{3.6}$$

We choose to *define* ϵ and ϵ' by the expressions:

$$\epsilon \equiv \frac{\langle \pi\pi\ I=0|\,H\,|K_L\rangle}{\langle \pi\pi\ I=0|\,H\,|K_S\rangle} \tag{3.7}$$

$$\epsilon' \equiv \frac{1}{\sqrt{2}}\left[\frac{\langle \pi\pi\ I=2|\,H\,|K_L\rangle}{\langle \pi\pi\ I=0|\,H\,|K_S\rangle} - \epsilon\,\frac{\langle \pi\pi\ I=2|\,H\,|K_S\rangle}{\langle \pi\pi\ I=0|\,H\,|K_S\rangle}\right] \tag{3.8}$$

where $\langle \pi\pi\ I=0,2|$ denotes a two pion state ($\pi^+\pi^-$ or $\pi^0\pi^0$) with a definite isospin, 0 or 2. The expression for ϵ' can be rewritten as proportional to the difference between ϵ and a quantity like ϵ for the $I=2$ amplitudes, thus showing that ϵ' is a measure of the difference in the relative amounts of CP violation in the $I=0$ and $I=2$ amplitudes.

Using these expressions, the familiar forms

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^-|\,H\,|K_L\rangle}{\langle \pi^+\pi^-|\,H\,|K_S\rangle} \approx \epsilon + \epsilon' \tag{3.9}$$

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0|\,H\,|K_L\rangle}{\langle \pi^0\pi^0|\,H\,|K_S\rangle} \approx \epsilon - 2\epsilon' \tag{3.10}$$

follow trivially from the Clebsch-Gordan expansions of the charged and neutral two pion states into states of isospin zero and two. Here the approximate sign refers to dropping terms of order $\epsilon(\epsilon'/\epsilon)^2$.

With the definitions

$$\langle \pi\pi\ I=0|\, H\, |K^0\rangle \equiv A_0 e^{i\delta_0} \qquad \left(\langle \pi\pi\ I=0|\, H\, |\bar{K}^0\rangle = -A_0^* e^{i\delta_0}\right) \tag{3.11}$$

$$\langle \pi\pi\ I=2|\, H\, |K^0\rangle \equiv A_2 e^{i\delta_2} \qquad \left(\langle \pi\pi\ I=2|\, H\, |\bar{K}^0\rangle = -A_2^* e^{i\delta_2}\right) \tag{3.12}$$

we get

$$\epsilon = \tilde{\epsilon} + \frac{e^{i\pi/4}\mathrm{Im}A_0}{\sqrt{2}\mathrm{Re}A_0} \tag{3.13}$$

$$\epsilon' = \frac{e^{i(\delta_2-\delta_0+\pi/2)}}{\sqrt{2}}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]. \tag{3.14}$$

In the CP conservation limit $A_{0,2} = A_{0,2}^*$. A difference in phase between A_0 and A_2 means CP violation in the decay as well as in the mass matrix. Thus, the following three statements are equivalent:

- $\epsilon' \neq 0$
- A_2 is complex in the basis in which A_0 is real
- There is CP violation in the decay.

The usual expression for $\tilde{\epsilon}$ follows from a series of approximations:

1. CP violation small $\rightarrow \tilde{\epsilon} << 1$ and $\Delta M,\ \Delta\Gamma \approx 2\mathrm{Re}M_{12},\ 2\mathrm{Re}\Gamma_{12}$
2. the observed coincidence $\Delta M \approx -\Delta\Gamma/2$
3. and the expected inequality $\mathrm{Im}\Gamma_{12} << \mathrm{Im}M_{12}$ giving

$$\tilde{\epsilon} \approx e^{i\pi/4}\mathrm{Im}M_{12}/(\sqrt{2}\Delta M). \tag{3.15}$$

Since ϵ' was isolated before these approximations were made, it is unaffected (to leading order) by these approximations, which could in fact be large compared to ϵ'.

For ϵ and ϵ' we thus have the expressions

$$\epsilon = \frac{e^{i\pi/4}}{\sqrt{2}}\left(\frac{\mathrm{Im}M_{12}}{\Delta M} + \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right) \tag{3.16}$$

$$\epsilon' = \frac{e^{i(\delta_2-\delta_0+\pi/2)}}{\sqrt{2}}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]. \tag{3.17}$$

The second term in each expression is a phase convention dependent term that can, if we pick an appropriate phase convention, be defined to be zero. (The rest of each expression

is also of course phase convention dependent, in the opposite direction, as ϵ and ϵ' are the phase convention independent observables.) Since there exists a phase convention choice in which ϵ is given by the first term only (A_0 real, a popular choice in the literature), ϵ can be said to be given by the mass-mixing CP violation (the standard $K\bar{K}$ box diagram) *alone*, and have no contribution from decay CP violation. Normally, however, we calculate in the "quark basis", where all phases have been removed from the $\Delta S = 1$ interactions involving only the light quarks (as is done in the usual CKM parametrization), and ϵ has a $\mathrm{Im}A_0$, or, as we shall see, "penguin" contribution.

Let us then introduce onstage our first penguin. For the moment we shall stick to old-fashioned penguins, the gluonic penguin, and introduce photon and Z_0 penguins later.

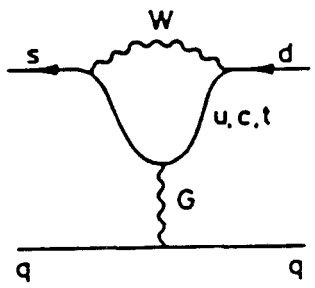

Figure 2. The gluonic penguin.

The gluonic penguin is $\Delta I = 1/2$ only, and therefore only contributes to A_0 and not A_2. Its amplitude is complex if the CKM phase δ is non-zero. Thus (in the quark basis) ϵ gets a $e^{i\pi/4}\xi/\sqrt{2}$ contribution where

$$\xi = \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \tag{3.18}$$

and ϵ' is proportional to ξ:

$$|\epsilon'| \approx 0.032|\xi| \qquad \text{or} \qquad \left|\frac{\epsilon'}{\epsilon}\right| = 14|\xi|. \tag{3.19}$$

The numbers in eq. (3.19) come from using the experimentally well measured[14]

$$|\epsilon| = (2.258 \pm 0.018) \times 10^{-3} \qquad \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} = 0.045 \tag{3.20}$$

and have been well determined for quite some time now.

It is a reassuring exercise to check that we get the same result in the A_0 real phase convention, which is arrived at by redefining the K^0 phases:

$$A_0 e^{i\xi} \to A_0 \qquad |K^0\rangle \to e^{-i\xi}|K^0\rangle \qquad |\bar{K}^0\rangle \to e^{i\xi}|\bar{K}^0\rangle . \tag{3.21}$$

Then

$$\begin{aligned} \mathrm{Im}\langle K^0|M|\bar{K}^0\rangle &\to \mathrm{Im}\left\{e^{2i\xi}\langle K^0|M|\bar{K}^0\rangle\right\} \\ &\approx \mathrm{Im}\langle K^0|M|\bar{K}^0\rangle + 2\xi\mathrm{Re}\langle K^0|M|\bar{K}^0\rangle \end{aligned} \tag{3.22}$$

giving

$$\frac{\mathrm{Im}M_{12}}{\Delta M} \to \frac{\mathrm{Im}M_{12}}{\Delta M} + \xi, \tag{3.23}$$

in agreement with above, using $\Delta M \approx 2\mathrm{Re}M_{12}$.

In the Standard Model, the imaginary parts of both the box and penguin diagrams are proportional to $\sin\delta$, the non-trivial phase in the CKM matrix. Thus the Standard Model accommodates CP violation but does not predict it, since $\sin\delta$ is arbitrary. Moreover, if the mass matrix CP violation (ϵ) is of Standard Model origin, i.e. is due to a CKM matrix with a non-zero phase, then (barring "accidental" cancellations), $\frac{\epsilon'}{\epsilon}$ is non-zero, i.e., there is direct CP violation as well. The task at hand in determining $\frac{\epsilon'}{\epsilon}$ consists of

1. overcoming calculational difficulties
2. determining (or at least constraining) $\sin\delta$ from other experimental and theoretical inputs.

4. Evolution of the Theoretical Perspective

Before moving on to the most recent theoretical developments, namely the realization of the importance of the photon and Z_0 penguins, we would like to establish a context by reviewing the evolution of the theoretical outlook and expectations for ϵ'/ϵ over the last two decades. We must emphasize that this is only a partial and somewhat arbitrary survey, intended to give the flavor of progress and a general timetable rather than an encyclopedic summary.

1973 – We start our timetable with Kobayashi and Maskawa,[15] who showed that four quarks (and minimal gauge bosons, Higgses, etc.) were insufficient to produce CP violation, i.e., that extra fields are needed. They suggested, among other scenarios, a six quark model. They made no prediction for ϵ'/ϵ.

1974 – The charm quark (or rather the J/ψ) is discovered.[16]

1976 – Weinberg[17] proposes a model with extra Higgses that could account for CP violation (see 1981) solely in the Higgs sector, with no phases in the quark mass matrix. Particularly interesting before six quarks were known to exist, since if there are six quarks there is no particular reason for the mass matrix phase to be zero.

1976 – Ellis, Gaillard and Nanopoulos[18] have penguins (not by name) but estimate them to have magnitude similar to the W loop diagram that results if one neglects the gluon in the penguin diagram. The resultant $1/M_W^2$ suppression led them to estimate $\left|\frac{\epsilon'}{\epsilon}\right| \lesssim \frac{1}{450}$.

76 – 9 – Penguins get named; bottom (or rather the Upsilon) gets discovered.[19]

1979 – First Gilman and Wise[20] paper — calculates the ratio of the imaginary and real parts of the penguin amplitude to lowest order. The calculation depends on: $\theta_2 = \tan^{-1}\frac{V_{\mu d}}{V_{cd}}$ (these days expected to be in the range $1-5°$); μ (a light hadron mass scale) and m_t. The phase $\sin\delta$ is fixed by using the measured value of ϵ. They estimated $\left|\frac{\epsilon'}{\epsilon}\right| \approx \frac{1}{13}$ to $\frac{1}{100}$. The first number comes from taking $m_t = 15$ GeV, $\mu = 0.2$ GeV, and $\theta_2 = 15°$. $\left|\frac{\epsilon'}{\epsilon}\right|$ decreases as m_t, m_c/μ and θ_2 increase. Recalling from eq. (3.19) that ξ, the penguin contribution to ϵ in the quark basis, is about thirty times ϵ', we see that with these estimates the penguin contribution to ϵ itself could be quite significant.

1979 – Second Gilman and Wise[21] paper — calculates the ratio of the imaginary and real parts of the penguin amplitude by doing an all orders leading logarithm calculation using successively W boson very heavy; top quark very heavy; bottom quark very heavy; charm quark very heavy. The parameters are θ_2 and m_t again, and the QCD scale parameter Λ in

$$\alpha(Q^2) = \frac{12\pi}{33 - 2n_f}\frac{1}{\ln(Q^2/\Lambda^2)}. \tag{4.1}$$

In addition α evaluated at the scale of light hadrons was varied between 0.75 and 1.25. With $\Lambda^2 = 0.1$ GeV^2 $\left|\frac{\epsilon'}{\epsilon}\right| \approx \frac{1}{50}$ to $\frac{1}{150}$ for $m_t = 15-30$ GeV, $\theta_2 = 15°$; with $\Lambda^2 = 0.01$ GeV^2 $\left|\frac{\epsilon'}{\epsilon}\right| \approx \frac{1}{200}$ to $\frac{1}{350}$ for $m_t = 15-30$ GeV, $\theta_2 = 15°$.

1981 – Deshpande;[22] Sanda[23] rule out Weinberg's CP-violation model by calculating penguins to get $\left|\frac{\epsilon'}{\epsilon}\right| \approx 0.045$ (this number might be modified by the inclusion of non-gluonic penguin diagrams).

After this period of establishing how to calculate penguins, a somewhat more phenomenological era was entered, from about 1983 to 1987. During this time period changes

in the theoretical estimation of $\left|\frac{\epsilon'}{\epsilon}\right|$ came more from the incorporation of bounds or improved values for input parameters in the calculation than any fundamental changes in the way the penguins were calculated.

1983 – Gilman and Hagelin[24] used bounds from $K_L \to \mu\mu$, as well as the experimentally measured value of ϵ, to come up with the bound

$$\left|\frac{\epsilon'}{\epsilon}\right| \gtrsim 2 \times 10^{-3} \frac{0.33}{B_K} \times \text{penguiny uncertainties.} \tag{4.2}$$

1983 – Gilman and Hagelin,[25] and Buras *et al.*,[26] used measurements of the b lifetime along with bounds on $\Gamma(b \to u)/\Gamma(b \to c)$ to get bounds such as

$$\left|\frac{\epsilon'}{\epsilon}\right| \gtrsim 0.005 \text{ to } 0.01. \tag{4.3}$$

These bounds, however, are only good for m_t in the then expected range 30 to 50 GeV, and drop sharply for larger m_t.

We conclude the discussion of this phenomenological era by showing the situation in 1987. With gluonic penguins recalculated for large m_t and $B\bar{B}$ mixing constraints taken into account,

$$10^{-3} \lesssim |\epsilon'/\epsilon| \lesssim 7 \times 10^{-3} \tag{4.4}$$

was considered[27] representative.

Fig. 3 shows the predicted ϵ'/ϵ versus m_t from Altarelli and Franzini[28] in 1987 for what were then considered central expectations for the relevant parameters.

In terms of a parameterization of the CKM matrix based on those of Maiani[29] and Wolfenstein:[30]

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\rho e^{i\phi} \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho e^{-i\phi}) & -A\lambda^2 & 1 \end{pmatrix}, \tag{4.5}$$

the values used are $\rho = 0.6$ (corresponding to $\Gamma(b \to u)/\Gamma(b \to c) = 0.04$), $A = 1.05$ and $B_K = 1$. All 'penguiny' uncertainties (including an estimate of photon and $\pi_0\eta\eta'$ effects) are included in the parameter P, which is expected to be in the range 0.5 to 5. The constraint of $B_d\bar{B}_d$ mixing (as first measured by ARGUS,[31] $x_d \equiv (\Delta M/\Gamma)_{B_d} = 0.73 \pm 0.18$) is imposed, and gives the upper (low m_t) boundary (minimal mixing) and the lower (high m_t) boundary (maximal mixing). The constraint of the experimental value of ϵ is not imposed here to constrain the phase in the CKM matrix; $\cos\phi = +1$ gives the upper right boundary and $\cos\phi = -1$ gives the lower left boundary. Decreasing the value of ρ used would bring these contours inwards. The vertical lines indicate the variations of some of the contour points when P is varied as indicated and also $A = 0.88$ for $P > 2.5$ and $A = 1.22$ for $P < 2.5$.

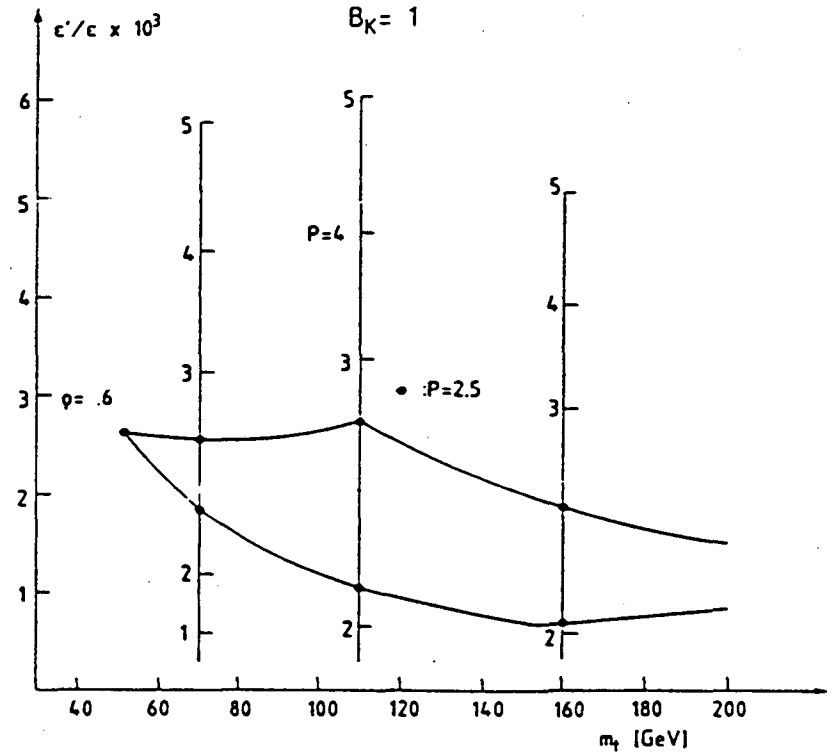

Figure 3. Calculated values of ϵ'/ϵ versus m_t for $A = 1.05$, $\rho = 0.6$, $B_K = 1$, $P = 2.5$. The vertical lines indicate variations due to the values of P shown, and, for $P > 2.5$, $A = 0.88$; for $P < 2.5$, $A = 1.22$.

5. The Era of the Electroweak Penguin

The main differences between 1981 and 1991 are:

1) More/different information on input parameters:

m_t expected to be high

more information from $\Gamma(b \to u)/\Gamma(b \to c)$ limits, and from the measurement of $B\bar{B}$ mixing.

As a result, the Standard Model expectations for ϵ'/ϵ have moved from a range of $\frac{1}{50}$ to $\frac{1}{200}$ to a few $\times 10^{-3}$, probably $\gtrsim 1 \times 10^{-3}$.

2) Electroweak penguins:

As a consequence of m_t being large, photon, and most importantly, Z^0 penguins, *are not negligible*, as previously was assumed. And the Z^0 contribution tends towards cancelling the gluonic contribution. While the photon and Z^0 contributions are $\frac{\alpha}{\alpha_s}$ suppressed, they are $\frac{A_0}{A_2}$ enhanced, since they can contribute to the $\Delta I = \frac{3}{2}$ amplitude as well as to the $\Delta I = \frac{1}{2}$ amplitude, unlike the gluonic contribution.

The Standard Model expectations for ϵ'/ϵ then move to a range of -0.3 to 2×10^{-3} depending on m_t. This is particularly notable in that ϵ'/ϵ being identically zero is not excluded in the Standard Model, contrary to the beliefs held for many years now.

In 1989, Flynn and Randall[3] calculated the effects of the photon and Z^0 penguins. The photon penguin *increases* ϵ'/ϵ, and for this reason was generally more or less ignored in past calculations, since it tends to cancel the effects due to isospin breaking corrections from π^0 mixing with η and η', which are estimated (see below) to decrease ϵ'/ϵ by about 25 to 45%. But the dominant effect is the decrease due to the Z^0 contribution, for m_t greater than about 100 GeV. This is illustrated in Figure 4.

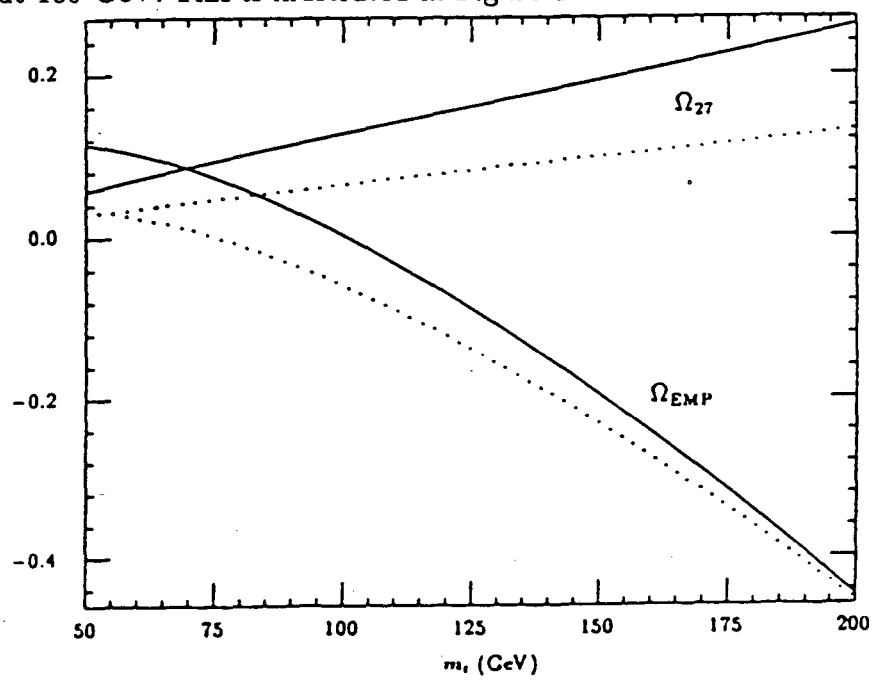

Figure 4. Terms contributing to ϵ'/ϵ from electroweak penguins, from Ref. 3. The dotted and solid lines show the variation due to varying the 4 flavor QCD scale parameter Λ, from 100 MeV (solid) to 300 MeV (dotted). These calculations are done in the vacuum insertion approximation. The Ω's are defined in the text.

The quantities Ω are defined so that

$$\frac{\epsilon'}{\epsilon} = \left(\frac{\epsilon'}{\epsilon}\right)_{gluon} (1 + \Omega_{EMP} + \Omega_{27} + \Omega_{\eta+\eta'} + \cdots) . \tag{5.1}$$

Ω_{EMP} (electromagnetic penguin) is the chirally enhanced contribution from the photon and Z_0 penguins that transforms as $(8_L, 8_R)$. Ω_{27} is another piece transforming as $(27_L, 1_R)$. $\Omega_{\eta+\eta'}$ is the isospin breaking correction term mentioned above.

A similar calculation was done shortly afterwards by Buchalla *et al.*,[32] including some smaller terms and calculating using the $1/N$ method. Their results are in good agreement with those of Flynn and Randall, and we present some of their phenomenological results here. A related paper has also been published by Paschos, Schneider and Wu.[33]

First in Fig. 5 we present the analogous figure to Fig. 4, from Buchalla *et al.* Their Ω is defined with the opposite sign. $\bar{\Omega}$ is the sum of all the terms shown, plus a constant value of 0.3 for $\Omega_{\pi^0\eta+\eta'}$, based on a 1987 $1/N$ calculation of Buras and Gerard[34] giving 0.27, and a 1986 chiral perturbation theory calculation of Donoghue *et al.*[35] giving 0.40 ± 0.06. Ω_{EWP} (electroweak penguin) is (with opposite sign) the same as the Ω_{EMP} of Ref. 3. Ω_{octet} and Ω_P are additional terms previously estimated by Flynn and Randall to be small. $\bar{\Omega}$ reaches 1, and therefore ϵ'/ϵ crosses zero, for m_t around 200 GeV.

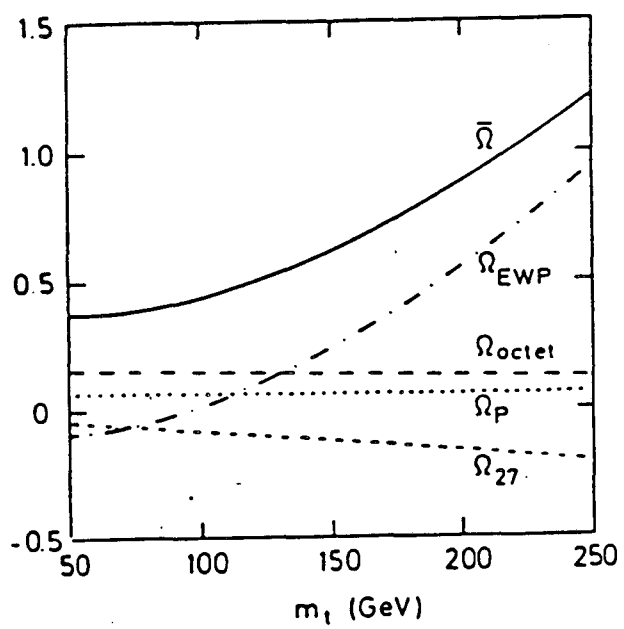

Figure 5. Terms contributing to ϵ'/ϵ from electroweak penguins, from Ref. 32, for $\Lambda = 0.2$ GeV and $m_s(1\text{ GeV}) = 175$ MeV. These calculations are done using the $1/N$ method. The Ω's are defined in the text.

Figure 6 summarizes the effect of adding Z_0 penguins on ϵ'/ϵ. A central set of values is used: $B_K = 0.75$, $\bar{R} = \Gamma(b \to ue^-\bar{\nu})/\Gamma(b \to ce^-\bar{\nu}) = 0.02$, $s_{23} \approx V_{cb} = 0.05$, $\Lambda = 0.2$ GeV, and $m_s(1\text{ GeV}) = 175$ MeV. Curve 1) is the pure QCD case, the inclusion of gluonic penguins alone, or setting $\alpha_{QED} = 0$. Curve 2) shows the result of including the $\pi^0\eta\eta'$ effects and the photonic penguins, without the Z^0 penguins, showing that these diagrams do indeed cancel to good approximation. Curve 3) is the full analysis of Ref. 32, including Z^0 penguins, W box diagrams, and using the $1/N$ approach to estimate matrix elements. Curve 4), for comparison, is the same calculation using the vacuum insertion approximation to estimate matrix elements. Figure 7 shows the value of m_t at which ϵ'/ϵ crosses zero for varying $m_s(1\text{ GeV})$ and Λ.

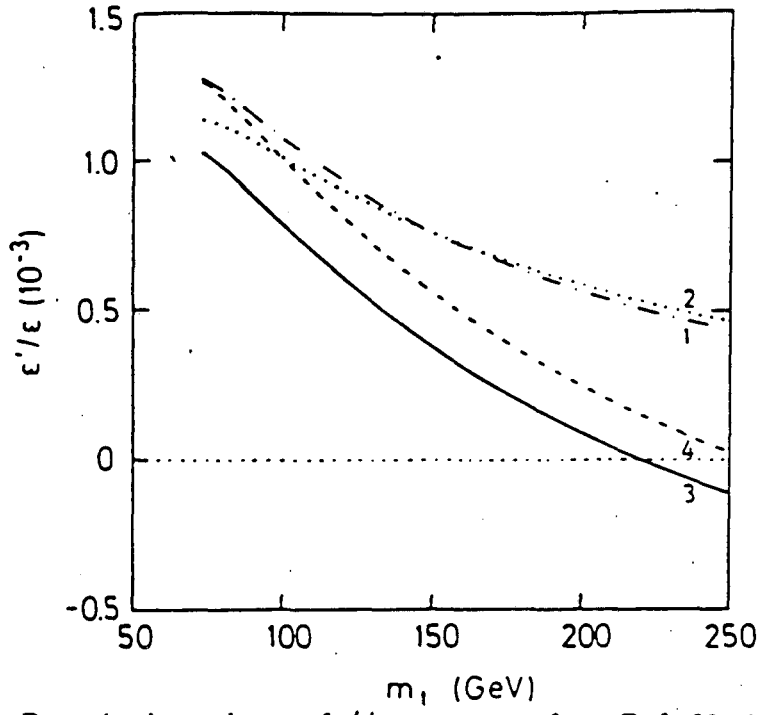

Figure 6. Penguin dependence of ϵ'/ϵ versus m_t, from Ref. 32. 1) corresponds to gluon penguins only; 3) to the full result; see text for details.

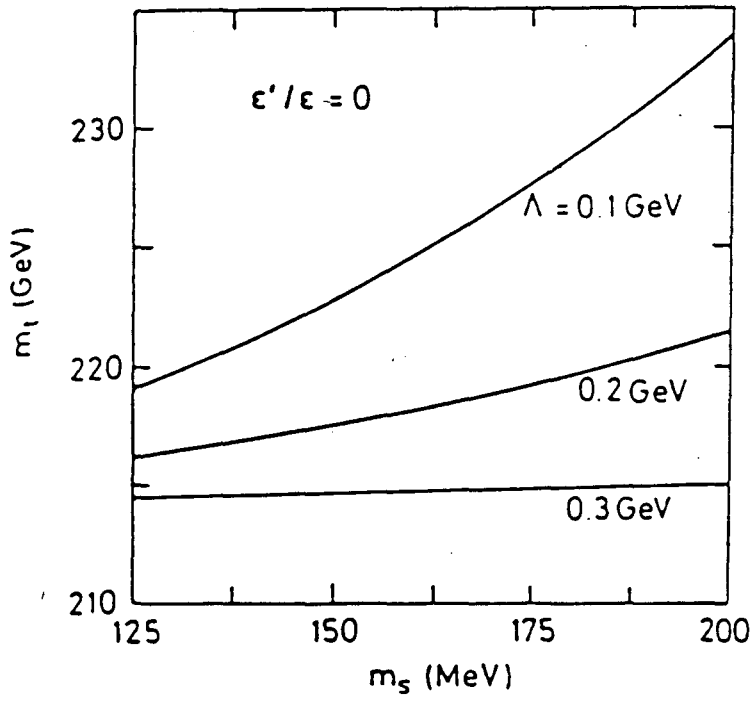

Figure 7. The value of m_t at which ϵ'/ϵ crosses zero for varying m_s(1 GeV) and Λ.

6. State-of-the-Phenomenology

In the rest of this talk we concentrate on showing some of the current phenomenology of ϵ'/ϵ, from Ref. 32.

The inputs are:

1) m_t — enters short distance analysis of ϵ, ϵ'/ϵ, $B\bar{B}$ mixing

2) Λ_{QCD}, $m_s(1\text{ GeV})$ — enters Wilson coefficient functions, and hadronic matrix elements in the pinguini

3) B_K, $\bar{R} = \Gamma(b \to u)/\Gamma(b \to c)$, s_{23} $\left(s_{13} = s_{23}\sqrt{\frac{\bar{R}}{2}}\right)$ — enters into determining $\sin\delta$

4) μ — scale at which Wilson coefficient functions are evaluated.

The constraints are the experimental measurements of ϵ and $B\bar{B}$ mixing, and the expected ranges $0.6 \leq B_K \leq 0.9$, $0.01 \leq \bar{R} \leq 0.03$, and $0.046 \leq s_{23} \leq 0.052$.

Fig. 8 shows the allowed range in the CKM phase δ versus m_t from fitting ϵ_{th} to ϵ_{exp} for $0.046 \leq s_{23} \leq 0.052$ and the narrowed ranges $0.7 \leq B_K \leq 0.8$ and $0.015 \leq \bar{R} \leq 0.025$. Note the two distinct solution sets. The constraint from $B\bar{B}$ mixing will tend to pick one or the other of these two sets.

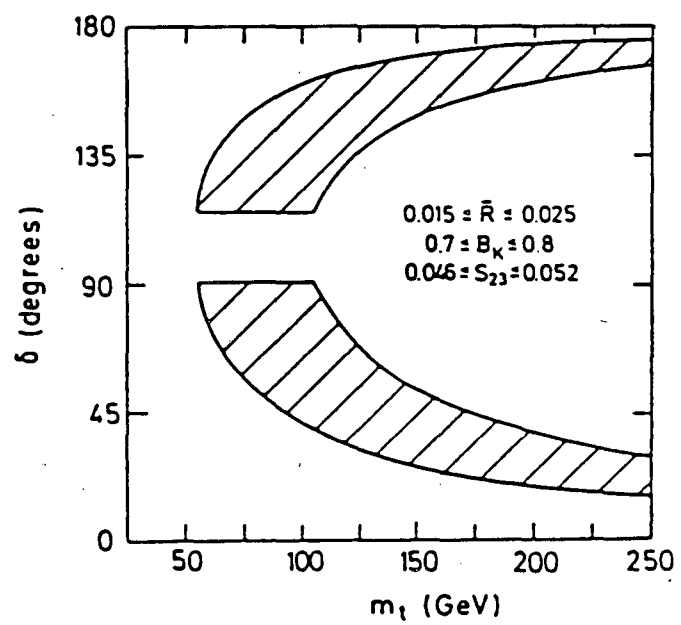

Figure 8. Allowed range in the CKM phase δ versus m_t from fitting ϵ_{th} to ϵ_{exp}.

Fig. 9 shows the lower limit on m_t versus s_{23}, $\bar{R}$ and B_K, from imposing $\epsilon_{th} = \epsilon_{exp}$. Note that the limits have improved due to better determination of $\bar{R}$.

In the next figure, Fig. 10, we show the allowed region in m_t versus f_B from $B\bar{B}$ mixing as we close in on the central values of $\bar{R}$, B_K and s_{23} (B_B is taken to be 1). The number used for $B\bar{B}$ mixing is based on the combined results of the ARGUS measurement[31] and the CLEO measurement,[36] $x_d \equiv (\Delta M/\Gamma)_{B_d} = 0.70 \pm 0.13$. While the region is pretty much unconstrained by current knowledge, we see that improvements in the determination of $\bar{R}$, B_K and s_{23} will impose significant constraints. In particular, if we believe that the central values of $\bar{R}$, B_K and s_{23} are the preferred values (though this is not necessarily true) we see that the allowed region tends to split into a high m_t region (where $\delta < \frac{\pi}{2}$) and

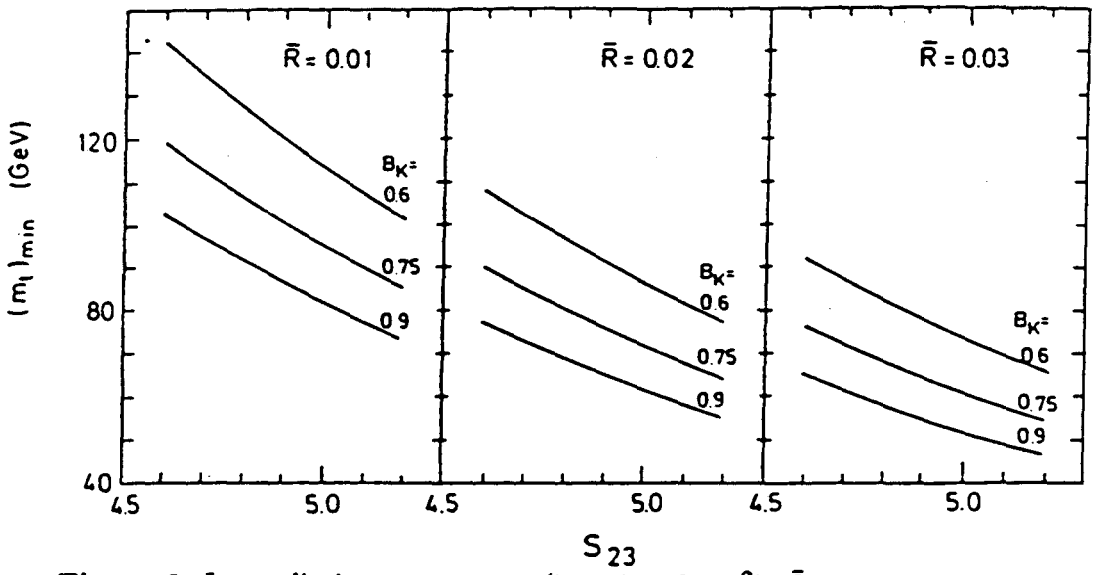

Figure 9. Lower limits on m_t vs s_{23} (in units of 10^{-2}), $\bar{R}$ and B_K from $\epsilon_{th} = \epsilon_{exp}$.

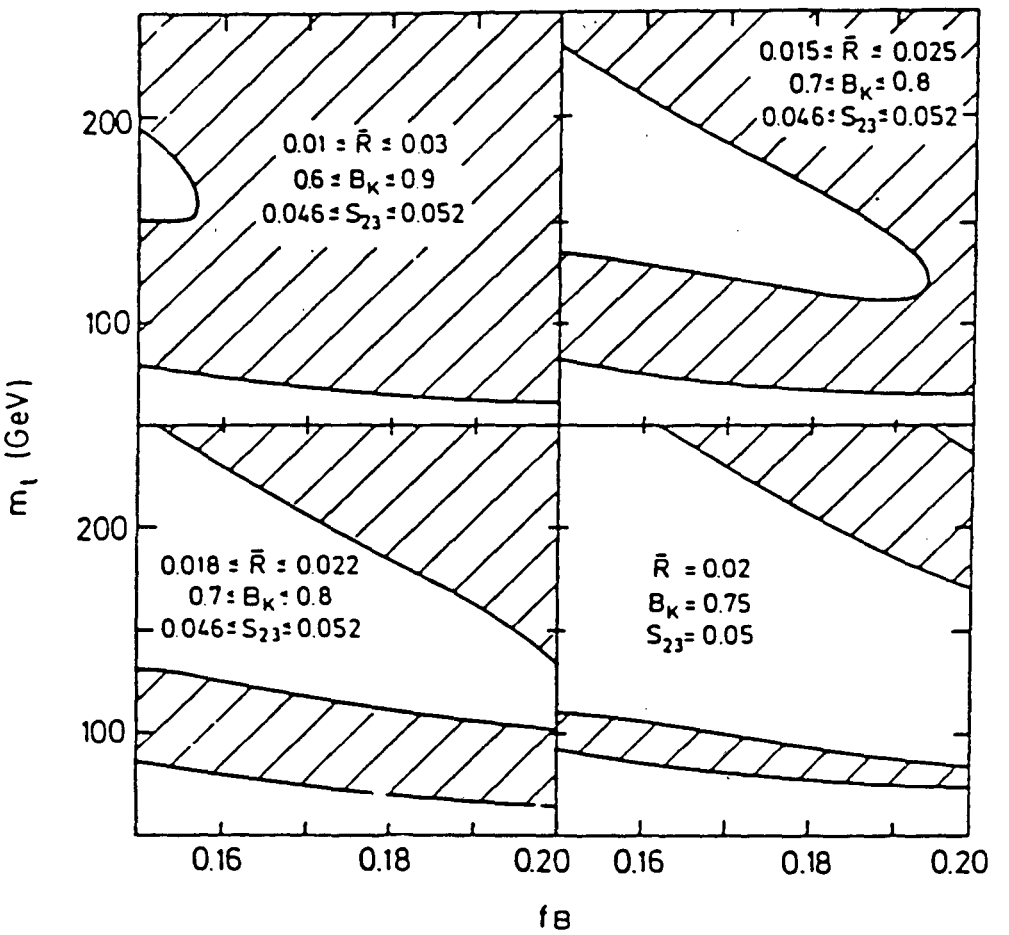

Figure 10. Allowed region in m_t (hatched region) versus f_B from the constraints of the measured values of ϵ and $B\bar{B}$ mixing for increasingly restrictive sets of $\bar{R}$, B_K, and s_{23}.

a low m_t region (where $\delta > \frac{\pi}{2}$). The ϵ'/ϵ graphs shown so far were done with this central

set, hence there was no $\sin\delta$ greater or less than $\pi/2$ uncertainty.

Figure 11 shows the allowed range in ϵ'/ϵ versus $m_s(1\text{ GeV})$ for various values of m_t, and the ranges for B_K, $\bar{R}$ and s_{23} given before, along with $0.1\text{ GeV} \leq \Lambda \leq 0.3\text{ GeV}$.

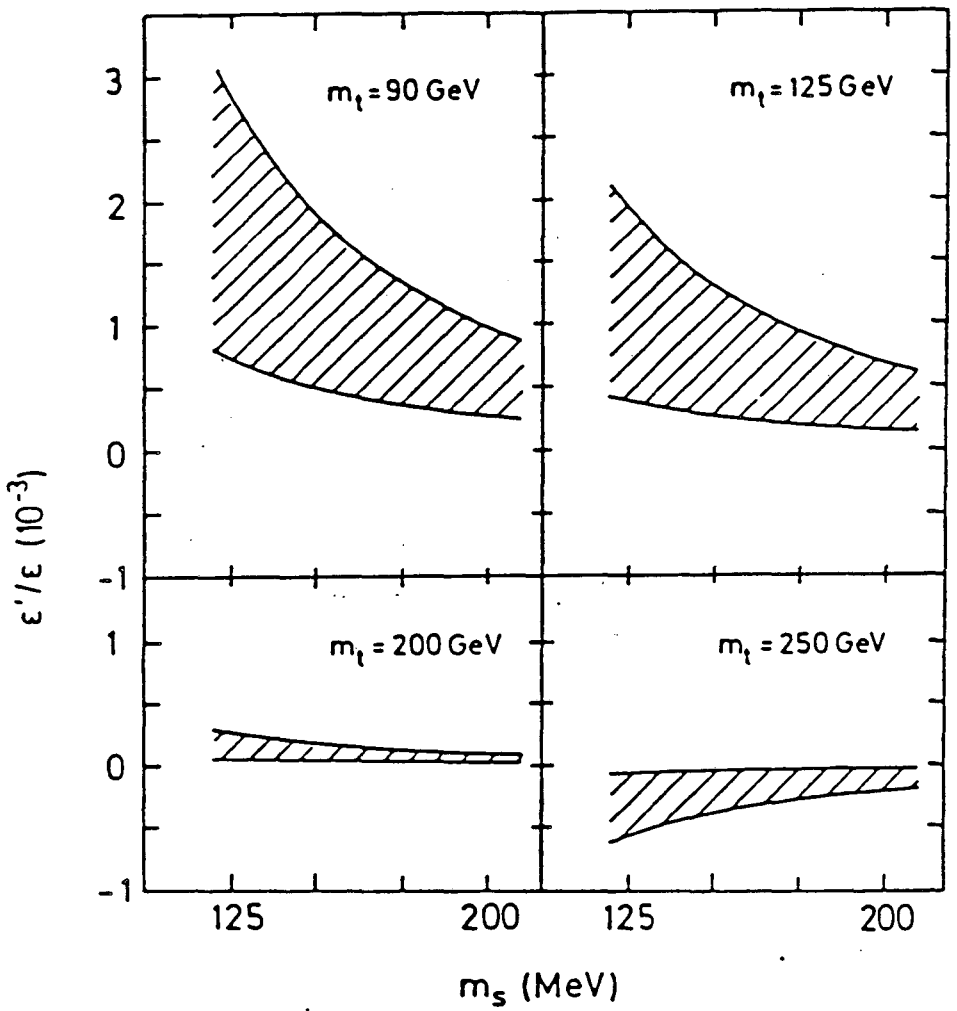

Figure 11. Allowed range (hatched) of ϵ'/ϵ versus $m_s(1\text{ GeV})$ for various m_t, and $0.6 \leq B_K \leq 0.9$, $0.01 \leq \bar{R} \leq 0.03$, $0.046 \leq s_{23} \leq 0.052$, and $0.1\text{ GeV} \leq \Lambda \leq 0.3\text{ GeV}$.

Figure 12 highlights the effect possible due to the first and second quadrant solutions for the phase δ. For various values of m_s the two solutions for ϵ'/ϵ versus m_t are shown. The central values $B_K = 0.75$, $\bar{R} = 0.02$, $s_{23} = 0.05$ and $\Lambda = 0.2$ GeV are used. Here only the measured value of ϵ has been used; the additional constraint of $B\bar{B}$ mixing will tend to favor $\delta > \pi/2$ for m_t small (i.e., the lower curve), and $\delta < \pi/2$ for m_t large. This selection thus further favors small ϵ'/ϵ.

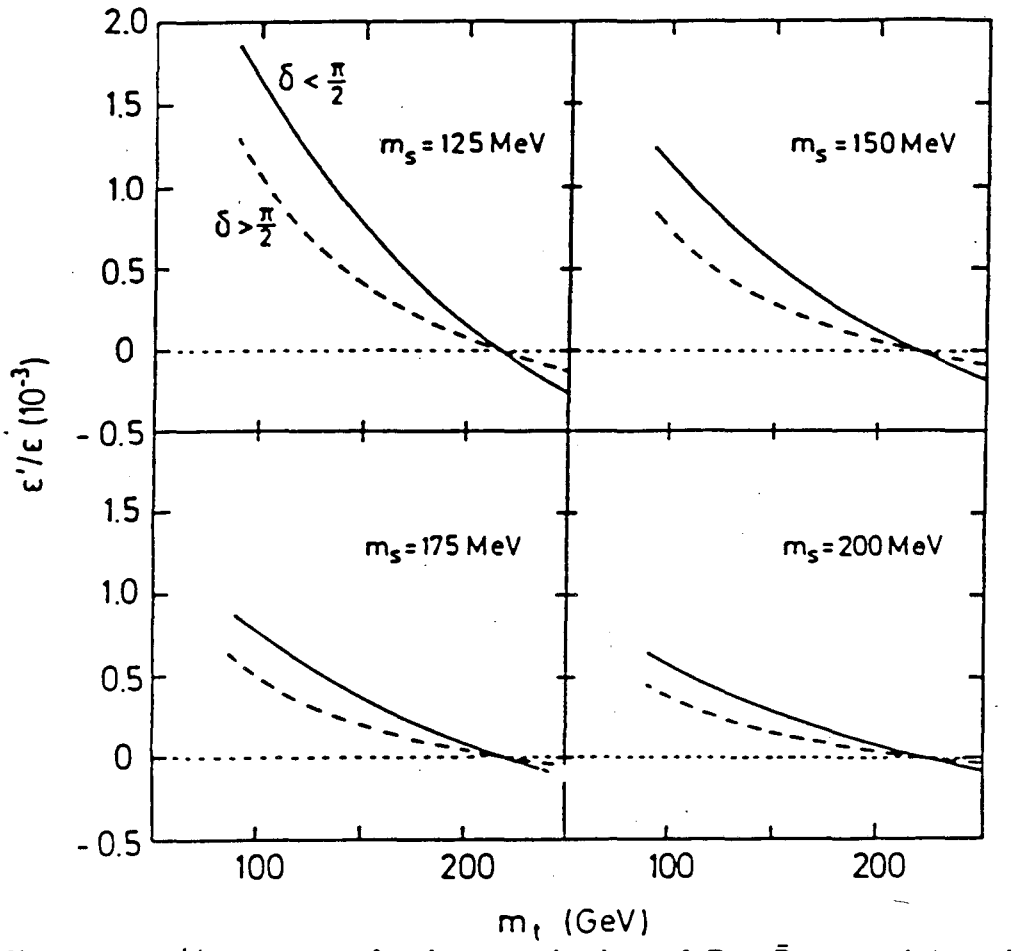

Figure 12. ϵ'/ϵ versus m_t for the central values of B_K, $\bar{R}$, s_{23} and Λ, and the two possible solutions for the CKM phase δ.

7. Exotica

Before concluding a few words about ϵ'/ϵ outside the Standard Model are in order.

- Two-Higgs doublet models — not to be confused with Weinberg's[17] Higgs-CP violating model, which has more than two doublets, and is explicitly designed so that all CP violation comes from the Higgs sector. Instead here we are talking about how the prediction for ϵ'/ϵ changes in a general extension of the standard model with an additional Higgs doublet. In addition to the penguin diagrams discussed previously, we have analogous ones where the W between the quark lines is replaced by a physical charged Higgs. It turns out that ϵ'/ϵ is even more suppressed (see Ref. 37; also Ref. 38) in a two-Higgs doublet extension than in the standard model; it can cross zero for m_t around 100 GeV, for example, if $M_H = 150$ GeV; it crosses zero for lower m_t when M_H is bigger.

- Four generation models — there is of course much greater freedom in CKM type parameters, and a new t' quark. As a result, negative values of ϵ'/ϵ (even for m_t less

than 180 GeV), or moderately larger values, are easier to achieve.[39]

- Superweak scenario — proposed by Wolfenstein in 1964[2] . Since CP violation in this scenario is due to some interaction contributing at lowest order to the $\Delta S = 2$ mass matrix, its contribution to direct CP violation would be essentially zero.

Barr and Freire[40] give a discussion of an explicit diquark model that is superweak, as well as milliweak diquark and leptoquark models.

8. Conclusions

We conclude with Figure 13, the experimental measurements of ϵ'/ϵ that we started with, with the evolution of the theoretical estimates added. We see a trend of convergence, and that ϵ'/ϵ is once more expected to be smaller than previous expectations.

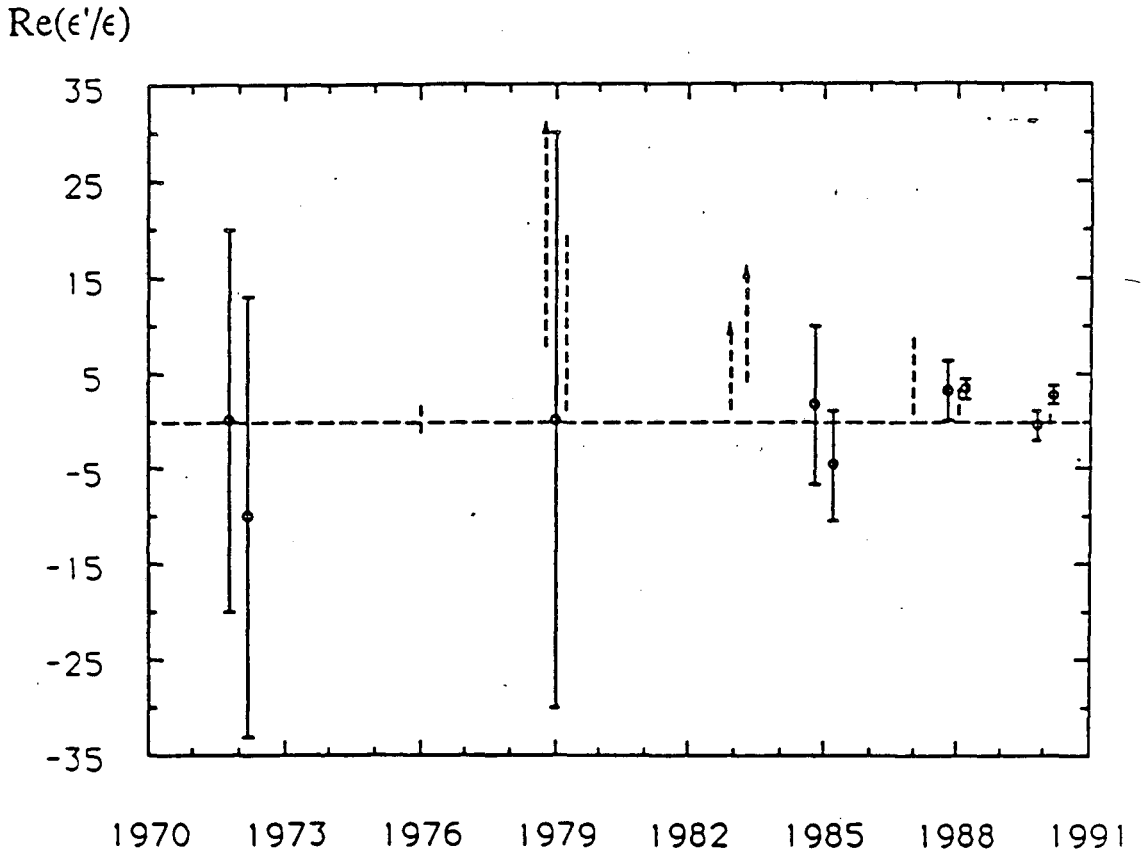

Figure 13. Experimental measurements of $\mathrm{Re}\frac{\epsilon'}{\epsilon}$ (in units of 10^{-3}) from the last two decades (solid), along with evolution of the theoretical expectation for ϵ'/ϵ (dashed).

While the theoretical picture of ϵ'/ϵ has clearly undergone much evolution over the past two decades, there are still many missing pieces. Some of these (e.g., m_t, $\bar{R}$, and $B\bar{B}$

mixing) we will hopefully get further clues on experimentally in the near future, along with the next generation of ϵ'/ϵ measurements. We look toward lattice gauge theory calculations for the other big steps needed, in pinning down parameters like B_K, f_B and in general the uncertainties with which penguin calculations are fraught. In fact, Lusignoli, Maiani, Martinelli and Reina[41] have just re-examined CP-violation in view of the most recent lattice QCD results. They find that, since these results favor larger values of f_B than those considered by Buchalla *et al.*, the $\delta < \pi/2$ solution, which gives larger values of ϵ'/ϵ (see Figure 12), is preferred down to lower values of m_t (around 130 GeV) than predicted by Buchalla *et al.*. This leads to values for ϵ'/ϵ bigger by a factor of 1.5 to 2 for m_t in an intermediate range (roughly 130 to 160 GeV).

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