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APPLICATION OF CLASSICAL THEORY OF ELECTRONS IN GASES TO DRIFT PROPORTIONAL CHAMBERS*

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ABSTRACT

We show that it is possible to understand both qualitatively and quantitatively, in the framework of the classical theory of electrons in gases, the behavior of drift velocities in gases commonly used in proportional drift chambers. We study in detail argon-isobutane mixtures and comment especially on the contribution of diffusion to the spatial resolution and the modification of drift velocities in magnetic field.

*This work was performed in part under the auspices of the U.S. Atomic Energy Commission.

†Part of this work was done at CERN, Geneva, Switzerland.

1. Introduction

After multiwire proportional chambers (MWPC)[1]), which have been intensively developed in the last years, multiwire drift chambers (MWDC)[2-7]) have roused remarkable interest, more recently especially as very accurate particle position detectors. Many groups have developed working drift chamber systems and these appear today to be very promising. As for the older MWPC, however, the MWDC development has not so far been backed by a satisfactory understanding of the properties of the commonly used gas mixtures. Although the phenomena involved are intrinsically complicated, a better knowledge of the basic mechanisms may be useful in understanding their properties. The present paper is a summary of a more extensive work, reviewing the classical theory of electrons in gases and its usefulness in the design of MWPC and MWDC[8]). Such a theory does exist and reached a fairly satisfactory status in about the 1950's.[9-12]) Here we will omit the review aspect and most of the lengthy calculations of our previous work and will stress mainly those points which appear to be of practical interest to those working on drift chambers.

In the first section, we briefly review the theory of electrons in gases. After introducing the classical expressions for transport coefficients, we present a simple minded method of calculation based on energy conservation. We then summarize the rigorous theory based on the Boltzmann transport equation. The capability of this theory to compute the energy distribution of the free electrons and to reproduce drift velocities and diffusion data in noble gases had already been stated by Phelps[6]) et al. We have written a FORTRAN program† to solve the relevant differential equations, which we used first to obtain the Phelps

†This program can be forwarded to any interested physicist.

results again. We then applied it to commonly used gas mixtures, namely isobutane-argon and methane-argon mixtures. We compared the results of these calculations to experimental data and give a qualitative discussion of the mechanisms controlling the behavior of drift in particular the very important saturation effects. The second section will deal more closely with the application of these results to drift chambers. We comment on the importance of diffusion, the absolute value and the stability of the drift velocities and the behavior of electrons in a magnetic field.

2. Classical theory of electrons in gases[8,13-15])

2.1 TRANSPORT COEFFICIENTS

The ionization of gas by a ionizing radiation produces free electrons (and ions). If the ionized gas is not subject to any electric field, the free electrons will move randomly with an average agitation energy given by the Maxwell formula, 3/2 kT (about 3.7×10^{-2} eV at 18°C). If an electric field is applied, the free electron will continue to have a nondirectional velocity v (and energy ϵ) but they will exhibit a small drift along the field direction, with a much lower mean drift velocity. This velocity of electrons in gases, under the influence of an external electric field is entirely determined by the cross sections and the electric field. It can be shown rigorously[8]) to be

$$\vec{w}_{\parallel} = \frac{2}{3}\frac{e\vec{E}}{m}\left\langle \frac{\ell_e(v)}{v}\right\rangle + \frac{1}{3}\frac{e\vec{E}}{m}\left\langle \frac{d\ell_e(v)}{dv}\right\rangle \qquad (1)$$

where m and e are the electron mass and charge, $\ell_e(v)$ is the mean free path[‡] for collision of the electrons with the gas molecules as a

[‡]Strictly speaking ℓ_e is the mean free path corresponding to the "momentum transfer cross-section"

$$\sigma_e = \int_{-1}^{+1} (1 - \cos\theta)\frac{d\sigma}{d\cos\theta}\, d\theta .$$

function of the electron random velocity (which is different and higher than the drift velocity w!). The averages are to be done on the distribution f(v) of the electron random velocities. When $\frac{d\ell_e}{d\epsilon} = \frac{\ell_e}{v}$ (constant collision time) one obtains the naive result

$$w_{\parallel} = \frac{eE}{m} \left\langle \frac{\ell_e}{v} \right\rangle ,$$ where one recognizes the product of acceleration by time between collisions.

If a constant magnetic field B (perpendicular to E, for simplicity) is present too, the electrons will exhibit a nonzero component of their drift velocity along the direction $\vec{E} \times \vec{B}$. This is given by (again averages are to be carried out)

$$w_{\perp} = \frac{1}{3} \frac{eE}{m} \frac{eB}{m} \left\langle \frac{\ell_e^2(v)}{v^2} \right\rangle + \frac{2}{3} \frac{eE}{m} \frac{eB}{m} \left\langle \frac{\ell_e(v) d\ell_e}{v\, dv} \right\rangle . \tag{2}$$

To first approximation, $w_{\parallel}$ is unchanged. It is to be noticed, however, that if the average time between collisions $t = \langle \ell_e / v \rangle$ is not small, the particles can turn over an appreciable angle and the above expression have to be divided by the following term

$$1 + \frac{e^2 B^2 \ell_e^2(v)}{m^2 v^2} \approx 1 + \frac{3}{2} \frac{B^2}{E^2} w^2 \tag{3}$$

where the last approximate expression gives the order of magnitude of the correction and has been obtained from eq. (1) neglecting the $d\ell/dv$ term and the dispersion in v. We will treat this question later quantitatively.

It is customary to define a third drift velocity, the so called magnetic drift velocity:

$$w_M = \frac{w_{\perp}}{w_{\parallel}} \frac{E}{B} \tag{4}$$

When $\frac{d\ell_e}{dv} = \frac{\ell_e}{v}$, note that

$$w_M = \frac{eE}{m} \frac{\ell_e}{v} = w .$$

The angle α at which the electrons drift with respect to the electric field is obviously

$$\alpha = \arctan\left(\frac{w_\perp}{w_\parallel}\right) = \arctan\left(B \frac{w_M}{E}\right) .$$

The diffusion coefficient D, very important because the rms σ of the dispersion along each of the three axes of the electrons after the drift length x is given by $\sigma = \sqrt{\frac{2Dx}{w}}$ is

$$D = \frac{1}{3} \langle \ell_e(v)/v \rangle . \tag{5}$$

Of course, for the velocity distribution $f(\vec{v})$ one can define a mean energy $\langle \epsilon \rangle = \frac{1}{2} mv^2 f(\vec{v})\, d\vec{v}$. More commonly used is a similar parameter, called characteristic energy ϵ_k directly measurable, defined as

$$\epsilon_k = \frac{eDE}{w_\parallel} . \tag{6}$$

It is a function of the field E, through the field dependence of w and D. Notice that this gives for σ

$$\sigma = \sqrt{\frac{2\epsilon_k}{eE} x}$$

whose behavior we will investigate.

2.2 ENERGY CONSERVATION: A SIMPLIFIED THEORY

As we pointed out, in order to compute the various transport coefficients, one has to know the energy distribution of the electrons. We will show in the next two sections how it can be obtained. We first present a very simple argument based on the energy conservation that allows a rough determination of the velocity. By neglecting the dispersion in

velocity one may then obtain an order of magnitude of the various transport coefficients.

An electron, continuously accelerated by a constant field E very quickly reaches (in 10^{-11} sec) a stable drift velocity, exactly as does any body accelerated in a viscous medium. If we call $\Lambda(\epsilon)$ the mean fractional energy loss in a collision as a function of the energy, the equilibrium between the energy gained from the field acceleration and the energy lost in atomic collisions can be written per unit time as

$$eEw = \left\langle \frac{\Lambda \epsilon v}{\ell_e} \right\rangle . \tag{8}$$

In order to make a rough estimation of transport coefficients let us assume that the momentum transfer mean free path is independent of v and that the distribution in energy is narrow (this is not too true, in general). Then

$$eEw \approx \frac{1}{2} \Lambda m \frac{v^3}{\ell_e}$$

$$w \approx \sqrt{\frac{2}{3} \sqrt{\frac{\Lambda}{3}} \frac{eE\ell_e}{m}}$$

$$\langle \epsilon \rangle \approx \sqrt{\frac{1}{3\Lambda}} \, eE\ell_e$$

$$D = \sqrt{\frac{2}{9} \sqrt{\frac{1}{3\Lambda}} \frac{eE\ell_e^3}{m}}$$

$$\epsilon_k = \langle \epsilon \rangle$$

$$w_\perp = \frac{1}{2\sqrt{3}} \frac{eB}{m} \sqrt{\Lambda} \, \ell_e$$

$$w_M = \frac{3}{4} w. \tag{9}$$

We see that the shorter the mean free path is, the lower the drift velocity, the mean thermal velocity, and the diffusion coefficient are.

On the other hand, we need to have a great mean fractional loss per collision to get a great drift velocity, and a small thermal velocity and diffusion. In spite of the crude approximation, those results are essentially true. We shall refer to them extensively in the following.

2.3 THE BOLTZMANN TRANSPORT EQUATION

The classical (and successful) theory of electrons in gases is based on the Boltzmann transport equation, which expresses the conservation of the number of electrons (in case of absence of a significant contribution of ionization). If one introduces the distribution function $f(\vec{v},\vec{r},\vec{t})$ of electrons at the point $\vec{r},\vec{v}$ of phase space at time t, we have

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \vec{r}}\frac{d\vec{r}}{dt} + \frac{\partial f}{\partial \vec{v}}\frac{d\vec{v}}{dt} - \frac{\partial f}{\partial t}\Bigg|_{\text{via collisions}} = 0 .$$

Then we go through the following steps of derivation[9,8].

a) We express the various terms of this equation in terms of the applied electric field E, the electron energy $\epsilon = \frac{1}{2}mv^2$, the momentum transfer mean free path $\ell_e(\epsilon)$, the mean free path $\ell_h(\epsilon)$ for excitation of the h-th level of energy ϵ_h.

b) In the stationary case, there is no x and t dependence, and if $\vec{E}$ is along the x axis, symmetry arguments restrict the dependence only to ϵ and $\cos\theta$. We may then work only with the distribution function $F(\epsilon, \cos\theta)$ normalized such that

$$\int_0^{\epsilon_{max}} F(\epsilon, \cos\theta)\, d\epsilon \frac{d\cos\theta}{2} = 1 .$$

c) In order to solve the resulting equation, we expand $F(\epsilon, \cos\theta)$ in Legendre polynomials. Limiting ourselves to the first two terms

$$F(\epsilon, \cos\theta) = F_0(\epsilon) + F_1(\epsilon)\cos\theta + \cdots$$

we get two coupled differential equations for F_0 and F_1

$$\frac{\partial F_0/v}{\partial \epsilon} = - \frac{F_1}{eE\ell_e v}$$

$$\frac{\partial F_1(\epsilon)}{\partial \epsilon} = \frac{3\Lambda}{eE} \left[\epsilon \frac{\partial}{\partial \epsilon}\left(\frac{1}{\ell_e}\right) + \frac{2}{\ell_e}\right] F_0(\epsilon)$$

$$+ \sum_h \frac{3}{eE} \left[\frac{\sqrt{\epsilon + \epsilon_h}}{\sqrt{\epsilon}} \frac{F_0(\epsilon + \epsilon_h)}{\ell_h(\epsilon + \epsilon_h)} - \frac{F_0(\epsilon)}{\ell_h(\epsilon)}\right]$$

$$- \left[\frac{1}{2\epsilon} + \frac{3\Lambda\epsilon}{(eE\ell_e)^2}\right] F_1(\epsilon) , \qquad (10)$$

where m and M are the masses of electron and gas molecules respectively,

$$v = \sqrt{2\epsilon/m} \quad \text{and} \quad \Lambda = 2m/M .$$

It is possible to solve such an equation by numerical methods[12]). The present authors have developed a FORTRAN program to perform this job[8]) enabling calculation of the various transport coefficients. In presence of magnetic field eqs. (10) have to be modified[8]).

3. Results of Numerical Calculations

3.1 RESULTS FOR NOBLE GASES

With cross sections very similar to those directly measured, Phelps et al. were able to get the transport coefficients in various gases. Fig. 1 (solid curve) give the cross section they used for argon, with which they were able to reproduce the complex behavior of the data on a range of electric field of 5 orders of magnitude. In fig. 2 we give the results of our own program which are in complete agreement with theirs, not unexpectedly since we are using essentially the same

cross sections (fig. 1 dotted line and Appendix).

3.2 FIT TO REALISTIC GAS MIXTURES

Unfortunately the gas used in multiwire proportional counters or drift chambers are more complex than pure argon: argon-CO_2, argon-isobutane, argon-methane or pure organic vapor such as methane or ethylene.

The fact that some of these gases are mixtures can be dealt with very easily: the various cross sections and energy losses, are just the appropriate weighted average of the cross sections for each component. On the other hand, the fact that we are using complex molecules introduces important modifications: in addition to electronic excitation, we now have excitation of the rotational and vibrational modes of the molecules. Fig. 3 gives the excitation spectrum of CO_2 as deduced by Hake and Phelps[16]) and shows two general effects which are very important for understanding the behavior of electrons in complex molecular gases:

a) The vibrational excitation cross sections Q_v are very important compared to the momentum transfer elastic cross section Q_m in the energy range 0.1 to 1 eV. In other words, the mean fractional energy loss per collision Λ is large, and from eq. (9) the drift velocities will be large and the mean energies low. So low in fact that they are of the order of the thermal energy kT.

b) The important vibrational energies are limited to a few tenths of an electron volt. This leads for higher fields to an effective $\Lambda = \epsilon_{max}/\epsilon$ where ϵ_{max} is the highest important vibrational energy. This will generate as we will see, constant or even decreasing drift velocities.

Of course when speaking of realistic mixtures we encounter the usual problem that we have not enough data on cross sections to predict through the use of the above theory the various transport coefficients. We have therefore attempted to deduce these cross sections from a rough fit to the drift velocities in various mixtures of argon and the gas considered. We have done this for isobutane using data kindly given to us before publication by Charpak and Sauli[2]) and for methane where drift velocity measurements[17]) exist for the pure gas and one mixture with argon (10% CH_4- 90% Ar). In both cases, we can compare the predicted ϵ_k with either the ϵ_k which can be estimated from the measurement by Charpak, Sauli and Duinker[2]) of the position accuracy in a drift chamber filled with 25% isobutane and 75% argon or the ϵ_k measurements[18]) for pure methane.

a) Argon - Isobutane Mixtures

Figs. 4 and 5 show our results for w, w_M and ϵ_k (see Appendix). The agreement for drift velocities is fair, taking into account the crudeness of our model and the experimental difficulties of absolute normalization of gas concentration. As will be shown later, we predict rather well the position accuracy obtained by Charpak et al. in their drift chamber[2]).

From our fit we can predict the drift velocity in pure isobutane. On the small range of E/p where it is measured[19]), we agree quite well with experiment (see fig. 10).

b) Methane

In order to have more confidence in the mechanisms proposed to explain the data, we have played the same game for methane. With the same adjustment by hand of the total and excitation cross sections (see Appendix) we obtained the drift velocities shown in fig. 6. The

data are from Bortner et al.[17]) Cottrel and Walker[18]), and Wagner et al.[17]). The experimental points of Hurst et al.[17]) for pure CH_4 seem too high and are usually not quoted in the literature.

The agreement is worse than in isobutane. In particular our assumption of flat cross section below 1 eV does not allow us to describe very well the sharp increase of drift velocity for pure methane below 900 V/cm, and we predict too sharp a peak for the mixture 10% CH_4-90% argon. We may note that in this region of concentration the drift velocity predicted by our program depends very much on the exact percentage of methane as shown in fig. 6 (dashed-dotted curve). There are also problems in our analysis due to the fact that F_1 is of the same order of magnitude as F_0 and that therefore the first two terms of the Legendre expansion of the distribution function $F(\epsilon, \cos\theta)$ are not sufficient.

We may however, compared our ϵ_k with the measured values of Cochran and Forester[18]) (fig. 7) which agree for pure CH_4 with Cottrel and Walker. The agreement is rather good.

We think therefore that in spite of its problems, our model is able to describe reasonably well the qualitative features of the data both for isobutane and methane mixtures with argon: the saturation of the drift velocity, the dramatic decrease of drift velocity at low concentration, and the small values of ϵ_k.

4. Intuitive Interpretation

The strange behavior of the drift velocities encountered above are somewhat difficult to understand intuitively. We therefore would like to conclude this section by the discussion of their origin. Let us assume that the dependence of Λ and ℓ_e on ϵ is given by a power law (which will always at least locally be true).

Namely

$$\Lambda(\epsilon) = \Lambda_0 \epsilon^m$$

$$\ell_e(\epsilon) = \ell_0 \epsilon^{-n}$$ (The cross section σ behaves as ϵ^n).

In this case using eqs. (1) and (8) (the conservation of energy) to eliminate ϵ one gets

$$w \approx E^{m + 1/(m + 2n + 2)} \quad \text{and} \quad \epsilon_K \approx E^{2/(m+2n+2)}$$

These results can also be deduced by the more general approach à la Boltzmann[4]). This shows that ϵ_k is always a rising function of E, but may rise very slowly, in agreement with intuition if σ or Λ are fast rising functions of ϵ (m and n large). On the other hand, w will rise slowly if σ rises rapidly or if Λ decreases with ϵ. Eventually for $m \leq -1$, w will decrease. We will now study how in practice we have a rising cross section or a decreasing Λ.

4.1 PURE GASES

In order to exemplify these effects let us first look at argon. The cross section has been given in fig. 1. The cross section below 0.3 eV is decreasing. Therefore, from eq. (11) for $\epsilon_k \leq 0.3$ eV we expect the strong rise of ϵ_k and w that is seen experimentally. Then the cross section rises sharply and we expect w and ϵ_k to be nearly constant with ϵ_k rising twice as fast as w. This will continue until an appreciable proportion of the tail of the energy distribution is beyond the excitation potential of 11.5 eV. This happens around $E/N = 3\times10^{-17}$ V cm^2. Λ then is effectively increasing very much, producing a sharp rise in w and a leveling off of ϵ_k.

This qualitative discussion may also be applied to the other heavy noble gases where the Ramsauer dip in the cross section at low ϵ occurs, producing basically the same structure or to CO_2 where the cross section is falling off rapidly at small ϵ, giving at relatively high fields

($E/N > 10^{-16}$ V/cm^2) a very fast rising w and ϵ_k. In the last case for small fields, ϵ_k is limited by ambient temperature and, as in the case of thermal massive ions, w is then proportional to the field (constant mobility).

However, the rising cross sections cannot alone explain the saturation of the drift velocity in organic vapors. The upper limit to the important vibrational energies is the responsible mechanism: it leads for high ϵ to an effective fractional energy loss

$$\Lambda = \epsilon_{max}/\epsilon$$

where ϵ_{max} is the highest vibrational energy. Therefore $m \simeq -1$ and w is approximately constant as soon as $\epsilon_k > \epsilon_{max}$.

4.2 MIXTURES

Another mechanism may occur in mixtures especially with the heavy noble gases (argon, krypton, xenon) where there is a Ramsauer minimum in the cross section. Let us add to argon a hypothetical gas of constant cross section and constant Λ in such a proportion that the Ramsauer dip is filled partially. For small enough ϵ, the added gas will dominate, and ℓ_e and Λ are constant. ϵ_k and w rise with the field until the cross section of argon becomes comparable to the cross section of the added gas. From that point on ℓ_e and Λ begin to decrease. This results in the leveling off of the drift velocity, which then becomes constant or even decreases. If the proportion of the additional gas is increased, the Ramsauer dip is filled completely and such a behavior no longer occurs.

This is the mechanism responsible, when combined with the maximum vibrational energy effect, for the behavior of mixtures of argon-isobutane (fig. 4) and argon-methane (fig. 6).

5. Application to drift chambers

We will attempt in this section to apply the theoretical results reviewed in the preceding sections to practical problems encountered in designing and operating drift chambers.

At least four properties are desirable for a gas to be used in a drift chamber:

- Minimum diffusion
- Low drift velocity
- Good behavior in magnetic field and especially low drift magnetic velocity.
- Constant drift velocities and stability

5.1 MINIMUM DIFFUSION

Apart from the "trival" problems of imperfection of electronics and mechanics, the accuracy of a drift chamber is limited by three different processes:

a) The dispersion of primary electrons. It is well known (see for instance ref. 8) that about only 30 primary ion-electron pairs per cm are produced by a minimum ionizing particle (for argon). They are distributed in a random way along the track, and clearly this effect will contribute to the spatial resolution: Let us consider a particle going through the sense wire. If the density of primary electrons per unit length is ρ, the distribution of the distance δ to the wire of the closest electron is

$$2\rho\, e^{-2\rho\delta}\, d\delta$$

since electrons may be on both sides and the rms of δ is

$$\sqrt{\int \left(\delta - \frac{1}{2\rho}\right)^2 2\rho\, e^{-2\rho\delta}\, ds} = \frac{1}{2\rho} \approx 150\ \mu m\ .$$

This effect prevails (see refs. 2b, 7) close to the wire and will rapidly decrease when the particle is passing farther from the wire.

b) δ rays. The high energy particle has a certain chance to extract a relatively high energy δ ray which will travel an appreciable distance in the chamber. The theory of this effect is difficult since the knowledge of the stopping distance of the electrons taking into account their random walk (and not their total range usually found in tables) would be needed in order to compute its contribution to the final accuracy. This may, in fact, be the ultimate limit on drift chamber accuracy. However, present results (refs. 2 and 7) seem to indicate that this effect is not important at low pressure in argon-isobutane mixtures.

c) Diffusion in the gas. The accuracy of a drift chamber is also limited by the diffusion in the gas. Let us consider the simplified case of a constant electric field E along $O\vec{x}$. After a drift length x the r.m.s. of the dispersion of electrons in x is [eq. (7)]

$$\sigma_x = \sqrt{\frac{2Dx}{w}} = \sqrt{\frac{2\epsilon_k x}{eE}} \tag{12}$$

In order to decrease σ_x one should decrease ϵ_k and increase E in the drift region. However they are not independent as shown in fig. 8, where we give experimental results for pure argon[16]), CO_2 [22]), CH_4, C_2H_4, and C_3H_6 (ref. 18).[§] We have also drawn (dotted lines) our theoretical estimate for pure isobutane, a 25% isobutane-75% argon mixture. We have chosen the field region of 1 kV/cm since the fields in the drift space of drift chambers are usually of this order of magnitude.

[§]There are some questions about the measurement by Cochran and Forester. See ref. 8.

This figure shows how easy it is to decrease the effective temperature of the electrons by addition to argon of multiatomic molecules or using pure organic vapor. Because of the excitation of a high number of rotational and vibrational modes, the effective fractional energy loss Λ is increased and from eq. (8) ϵ_k is decreased. However many limits are encountered in that direction: ϵ_k is bounded anyway from below by $kT \approx 0.025$ eV at ambient temperature: If the electrons are thermal then the drift velocity is proportional to the field as in CO_2 for $E < 1$ kV/cm, and finally, the cooling should not be so efficient as to prevent any amplification before breakdown because of too high an electric field at the cathode.

It should be noted that the σ_x given in eq. (12) will not be the contribution of diffusion to the spatial accuracy of a drift chamber.

The electronics will detect the time of arrival of the mth fastest electron. Depending on the conditions, m may vary between 1 (sensitivity to a single electron pair, e.g., in magic gas[23]) and 10 (~1/10 of the total number of electrons if the threshold is set at 1/10 of the mean pulse height and there is no saturation effect).

In order to estimate this effect let us consider the group of n electrons created at a distance x from the sense wire in a volume small compared to σ_x. The dispersion on the first electron[24]), for large n, is given by

$$\sigma_f \simeq \frac{1.28\,\sigma_x}{\sqrt{2\,(\log n)}} .$$

Let us consider a particle going through the chamber at a right angle at 2 cm from the sense wire (Fig. 9). The n electrons produced along 4 mm of the track will be at the same distance from the wire within 100 μm which is small compared to the usual σ_x and may be considered to originate for the same point. In argon-isobutane mixtures or similar

mixtures

$$n \simeq 0.4 \times 100 \simeq 40$$

and

$$\sigma_f \simeq 0.5\, \sigma_x . \tag{13}$$

For a mixture of 25% isobutane and 75% argon at E = 500 V/cm, our theoretical estimate was $\epsilon_k = 1.1 \times 10^{-1}$ eV which leads for a drift of 2 cm to

$$\sigma_x \simeq \sqrt{\frac{2 \times .11 \times 2}{500}} \simeq 0.03 \text{ cm}$$

and

$$\sigma_f \simeq 150\, \mu$$

to be compared with the result of Charpak, Sauli and Duinker[2]) of 175 ± 15 μ.[||]

Eq. (13) represents an upper limit for σ_f, since taking the mth fastest electron, the dispersion will be decreased and taking the center of gravity

$$\sigma_f \approx \frac{\sigma_x}{\sqrt{n}} \approx 0.16\, \sigma_x .$$

However these more elaborate methods based on constant fraction discriminators, may be limited in practice by the fact that the rise time of the pulses are not constant.

5.2 LOW DRIFT VELOCITIES

One potential advantage of drift chambers is the relatively low cost of electronics for a great accuracy. This advantage may be cancelled by too high a value of drift velocity. Unfortunately the cooling of electrons by complex molecules dramatically increases the drift velocity. The value obtained with argon-isobutane mixtures ($w \approx 4 \times 10^6$ cm/s^2) for instance, leads to 500 MHz electronics if one wants to exploit the

[||]More recent results by the same authors (ref. 2) give for E~ 1.4 kV/cm 30% of isobutane, x = 2 cm, $\sigma_f \simeq 135 \pm 10\, \mu$m. We predict 150 μ also. Note that our predictions are for gas mixtures without methylal. The presence of methylal may cool down the electrons.

intrinsic accuracy of the chamber.

In the case where the drift velocity is not imposed by other considerations (e.g., dead time problems), it is interesting to cool the electrons down, not through an increase of Λ, but through an increase of the cross section at small ϵ. From eq. (8) this will, for a given ϵ_k, decrease w. Experimentally higher cross sections and smaller Λ are achieved through the use of big molecules such as isobutane. It may be profitable, although to our knowledge no one has tried it yet, to use pure isobutane.¶ Fig. 10 gives our theoretical estimate for w and fig. 5 gives our estimated ϵ_k; although these predictions are presumably not very accurate, they show that low values of drift velocities and low diffusion might be achieved at the same time.

5.3 BEHAVIOR IN MAGNETIC FIELD

Another disadvantage of large drift velocities is seen in the operation in magnetic fields. In a constant electric field E and transverse displacement of the electrons after a drift x is

$$\Delta y = \frac{w_{\parallel}}{w_{\perp}} x = w_M \frac{B}{E} x \tag{14}$$

and w_M for low field is usually close to w (see figs. 2 and 4.). For instance with $E = 5\times 10^4$ V/m, $w_M \approx w \approx 4\times 10^4$ m/sec

$$\begin{aligned} \Delta y &\approx 0.8\ Bx; && \text{(B in tesla)} \\ &\approx 0.3\ x && \text{for } B = 4\ \text{kG} \\ &\approx 1.2\ x && \text{for } B = 15\ \text{kG} \quad . \end{aligned}$$

This dramatically distorts the trajectory of electrons as shown in fig. 11 for 15 kG in a drift chamber of Walenta's type. The dimensions are given in the figure, the cathode plane is at -2150 V with respect to

¶The drift velocity may not be saturated however. See Section 5.4.

the field wire and the field wire at -2710 V. The gas used in this simulation is 38% isobutane—52% argon and the w_M have been taken from the theoretical computation described in Section 3.2(a). If the drift length is too large, such an effect will lead to the loss of electrons, thus requiring special methods for overcoming this problem (e.g., large gap or tilting of electric field[2]).

In addition to the distortion of the mean electron trajectory, high magnetic fields may

$$\frac{1}{1 + \frac{(eB\ell)^2}{2m\epsilon}}$$

encountered in Section 2. This factor from the simplified theory is of the order of

$$\frac{1}{1 + \left(\frac{3}{2}\frac{wB}{E}\right)^2}$$

and for $w \simeq 5 \times 10^4$ m/s, B = 1 tesla, and $E = 10^5$ V/m is of the order of 2/3. The effect is therefore not negligible, and it would be interesting to investigate it more thoroughly. In figs. 12(a) and (b) we give the values of $w_{\parallel}$ and w_M for pure isobutane and two mixtures of isobutane with argon for B = 0, 10, and 20 kG as deduced from the solution of the relevant form of Boltzmann equation[15]). As could be guessed beforehand, $w_{\parallel}$ is reduced while w_M is increased and the effect is more pronounced for large concentrations of argon where the Ramsauer minimum plays a large role. Figure 12(c) gives the absolute value of the drift velocity:

$$|w| = \sqrt{w_{\parallel}^2 + \frac{w_{\parallel}^2 w_M^2 B^2}{E_2}} \; .$$

For magnetic fields up to 20 kG and electric fields higher than 1000 V/cm the variation of $|w|$ is small.

5.4 CONSTANCY AND STABILITY OF THE DRIFT VELOCITY

Another property which may be useful in practice is a small dependence of drift velocity with respect to the electric field. This allows an easier correlation between time and position. We have seen in Section 2 that this happens for some organic vapor such as methane, ethylene (fig. 10) etc., and for argon-isobutane mixtures[27]) (fig. 4).

However, one should not exaggerate the importance of the linearity of the relation between position and time. There are many other factors than drift velocity that will distort this curve, for example:

a) Geometrical effects: In particular, even if the dependence of time on position is linear for particles normal to the drift chamber, it is no more linear for particles at an angle[2,7]). Thus software corrections have to be made anyway.

b) Variation of the shape of the signal: Depending on the position of the primary particle, the number of electrons arriving in the avalanche region and their time intervals vary. This leads to a variation in the shape of the signal. If the threshold is not low enough there will be time delays dependent on the position of the primary particle and these will induce threshold and chamber-dependent nonlinear effects in the discriminator.

We think that a better way of looking at this question is to speak of stability. A smooth dependence of w on the field decreases the dependence of the position accuracy on the mechanical inaccuracies of the chamber construction where imperfections may slightly change the field in the drift region. It will also decrease the dependence on the temperature: fig. 13 gives the estimated variation of drift velocity for isobutane-argon mixtures between 0°C and 18°C. In first approximation the shape is unchanged and there is only a slight displacement to higher

field at lower temperature since the transport coefficient is a function of the ratio of field to density E/ρ and ρ has increased slightly. Clearly it is then desirable to have dependence of w as small as possible with respect to E/ρ.

A method to achieve this result is to optimize the mixture composition:[6] however, this method for achieving stability with respect to electric field and temperature, may introduce more serious instabilities due to too large a dependence of the drift velocity on the exact composition of the mixture.

Note that if the variation of the drift velocity due to various factors is not too important, it can be calibrated away by the use of two chambers displaced by half a wire spacing. In addition to the solution of the left-right ambiguity, this method gives the sum of the two drift times which is, for a properly designed system, a smooth function of wire position; this enables computation of the scale factor necessary for the correction of possible small variations of w.

Acknowledgments

The authors wish to thank G. Charpak for many discussions and suggestions about this work. We are also greatly indebted to F. Bourgeois, S. Derenzo, D. Nygren, S. Parker, and F. Sauli for fruitful discussions.

Appendix -- Cross Sections

1. Argon

On fig. 1 we have plotted (dotted line) the momentum transfer cross section section, we have used:

$$\epsilon > 11.5 \text{ eV} \quad \sigma = 1.52\ 10^{-15}/\sqrt{(\epsilon/11.5)} \quad \text{cm}^2$$

$$1.15 < \epsilon < 11.5 \quad \sigma = 1.52\ 10^{-15}\ \epsilon/11.5 \quad \text{cm}^2$$

$$.3 < \epsilon < 1.15 \quad \sigma = 1.46\ 10^{-17} + 1.9\ 10^{-16}(\epsilon - .3)^2 \text{cm}^2$$

$$\epsilon < .3 \quad \sigma = 1.46\ 10^{-17} + 1.24\ 10^{-12}(.3-\epsilon)^{6.5} \text{cm}^2$$

The excitation cross section has been taken as

$$\sigma = 9\ 10^{-17}\sqrt{\epsilon/11.5}\ (\epsilon/11.5-1)\ \text{cm}^2$$

$$\epsilon_h = 11.5 \text{ eV}$$

2. Isobutane

The total cross section for isobutane is only known above 1 eV and not very accurately[20]). The data on drift velocities seems to be incompatible with the simple extrapolation of the measured cross section to zero (over estimation of w at low field). As a first guess we took, therefore, a flat total cross section below 1 eV. Above, we assume a straight line joining the point at 1 eV with the known maximum at 8 eV. We also took a flat excitation cross section and from infrared spectra[21]) we fixed the upper limit to vibrational energies at 0.36 eV.

We then fitted by hand the data at the lower and the upper concentrations (7% and 38% isobutane) varying only the value of the total cross section below 1 eV and the excitation cross section. Reasonable values seem to be

$$\sigma_{tot} = \begin{cases} 1.1 \;\; 10^{-15} \text{ cm}^2 & \text{for } \epsilon \leq 1 \text{ eV} \\ [1.1 \;\; 10^{-15} (8.-\epsilon) + 4.8 \;\; 10^{-15}(\epsilon-1)] / 7 \text{ cm}^2 & \\ & \text{for } 1 \leq \epsilon \leq 8 \text{ eV} \\ (4.8 \;\; 10^{-15}/\sqrt{\epsilon/8.})\text{cm}^2 & \text{for } \epsilon > 8 \text{ eV} \end{cases}$$

$$\sigma_{exc} = 8 \;\; 10^{-17} \text{ cm}^2 \qquad \text{for } \epsilon_h \leq 0.36 \text{ eV}$$

3. Methane

For methane cross sections we use the following[20, 21]):

$$\sigma_{tot} = \begin{cases} 1.4 \; 10^{-16} \text{ cm}^2 & \text{for } \epsilon \leq 1 \\ [1.4 \; 10^{-16} (8.-\epsilon) + 2.8 \;\; 10^{-15} (\epsilon-1)] / 7 \text{ cm}^2 & \\ & \text{for } 1 \leq \epsilon \leq 8 \text{ eV} \\ (2.8 \; 10^{-15}/\sqrt{\epsilon/8.}) & \text{for } \epsilon > 8 \text{ eV} \end{cases}$$

$$\sigma_{exc} = 5. \;\; 10^{-17} \text{ cm}^2 \qquad \text{for } \epsilon_h \leq 0.36 \text{ eV}.$$

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Fig. 1. Momentum transfer cross section in argon. Notice the Ramsauer dip at 0.3 eV electron energy. The dotted line shows the fit we have made for practical computer purposes.

Fig. 2. Results of the numerical calculation of the argon transport coefficients compared to experimental data.

Fig. 3. Excitation spectrum for CO_2 (Q_m is momentum transfer cross section, Q_v vibrational cross section, Q_x the electronic excitation cross section, Q_i the ionization cross section). From Ref. 16.

Fig. 4. Results for computed $w_{\parallel}$, w_M for different concentrations in argon-isobutane mixtures. Comparison is done with Charpak et al. measurements.

Fig. 5. Results for computed ϵ_k for different concentrations in argon-isobutane mixtures.

Fig. 6. Results for computed $w_{\parallel}$ for pure methane and 10% methane in argon. Comparison is done with data by Bortner et al.

Fig. 7. Results for computed ϵ_k for CH_4. Experimental data for CH_4, CO_2, and pure argon are shown as well.

Fig. 8. σ_x (diffusion after 1-cm path of the electron swarm) from ϵ_k data for Ar, CH_4, CO_2, C_2H_4, C_3H_6 and theoretical estimate for Ar-isobutane mixture (25%), pure isobutane, and argon-methane 10% mixture.

Fig. 9. Determination of σ_f from σ_x.

Fig. 10. Experimental drift velocities for CH_4, C_2H_3 and C_2H_6. Theoretical drift velocity for C_4H_{10} and CH_4 are sketched as well (dotted lines).

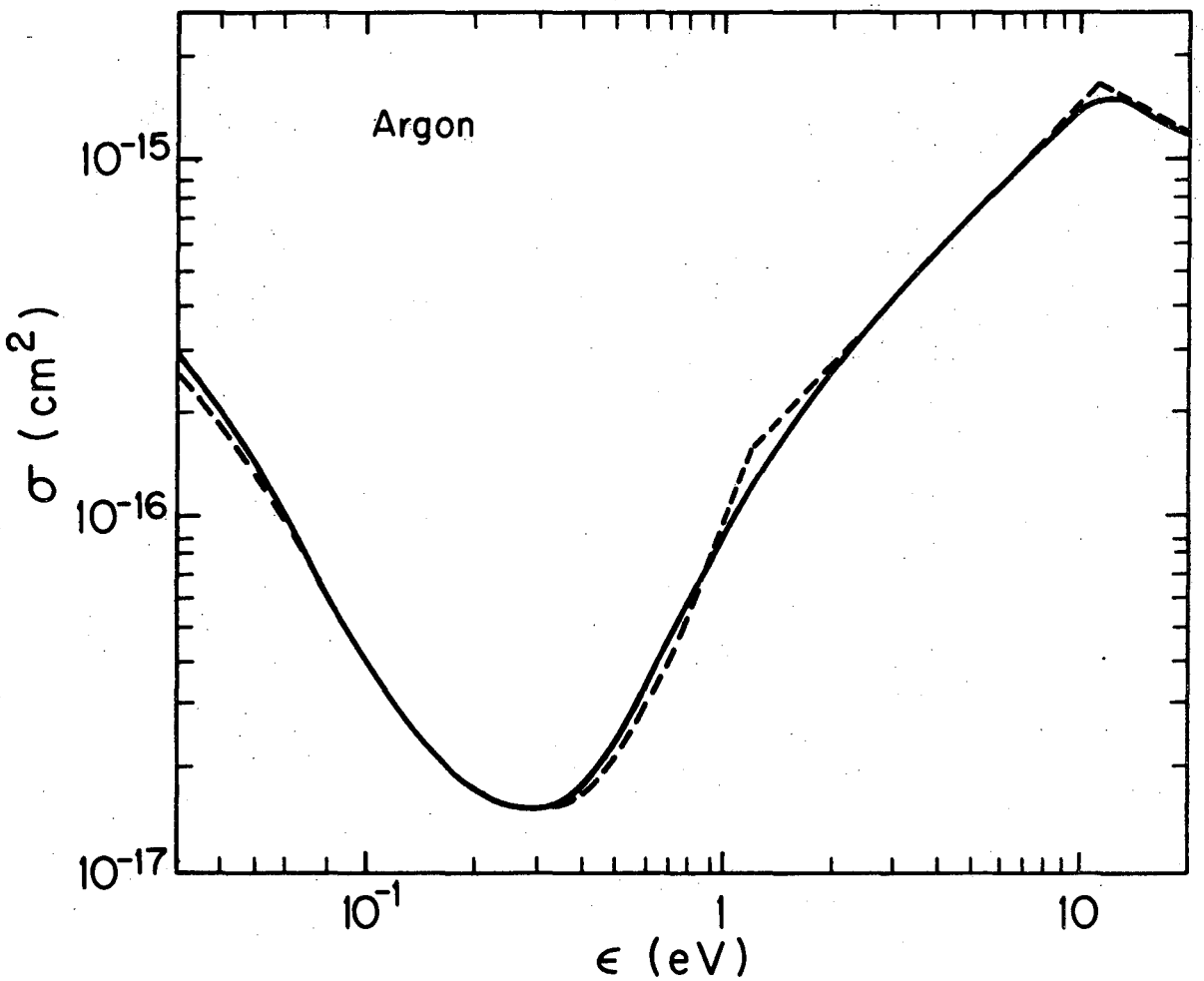
Fig. 11. Trajectories of electrons (dotted lines) in a MWDC operating in a 1.5 tesla magnetic field. Full lines are lines of equal drift time to the sense wire.

Fig. 12. (a) Computed behavior of $w_{\parallel}$ in isobutane-argon mixtures in presence of different magnetic fields.

(b) Same for w_M.

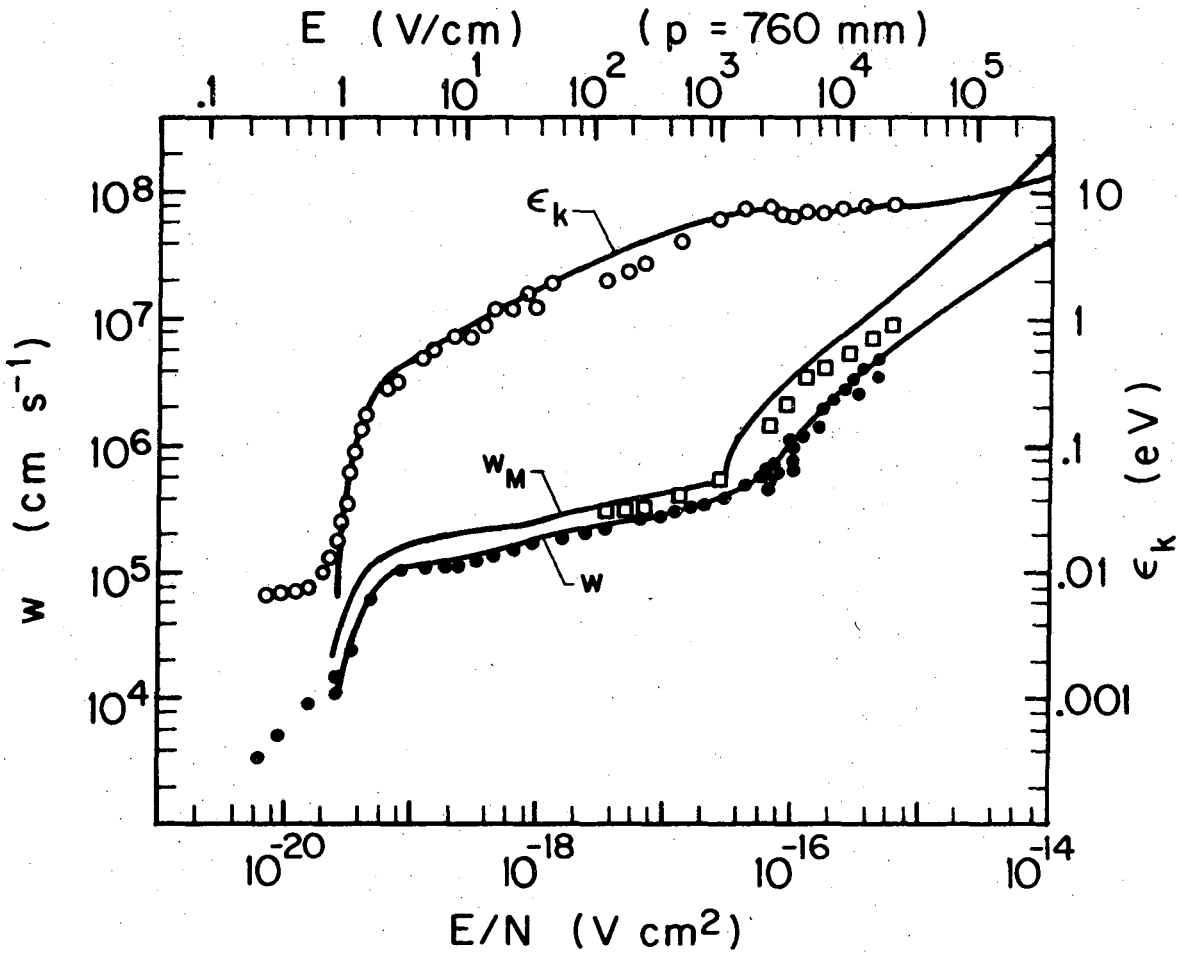
(c) Same for absolute value of the drift velocity $|w|$.

Fig. 13. Simulated temperature dependence (0° and 18°C) of the drift velocity in three argon-isobutane mixtures.



XBL 745-3166

Fig. 1.



XBL 745-3165

Fig. 2.

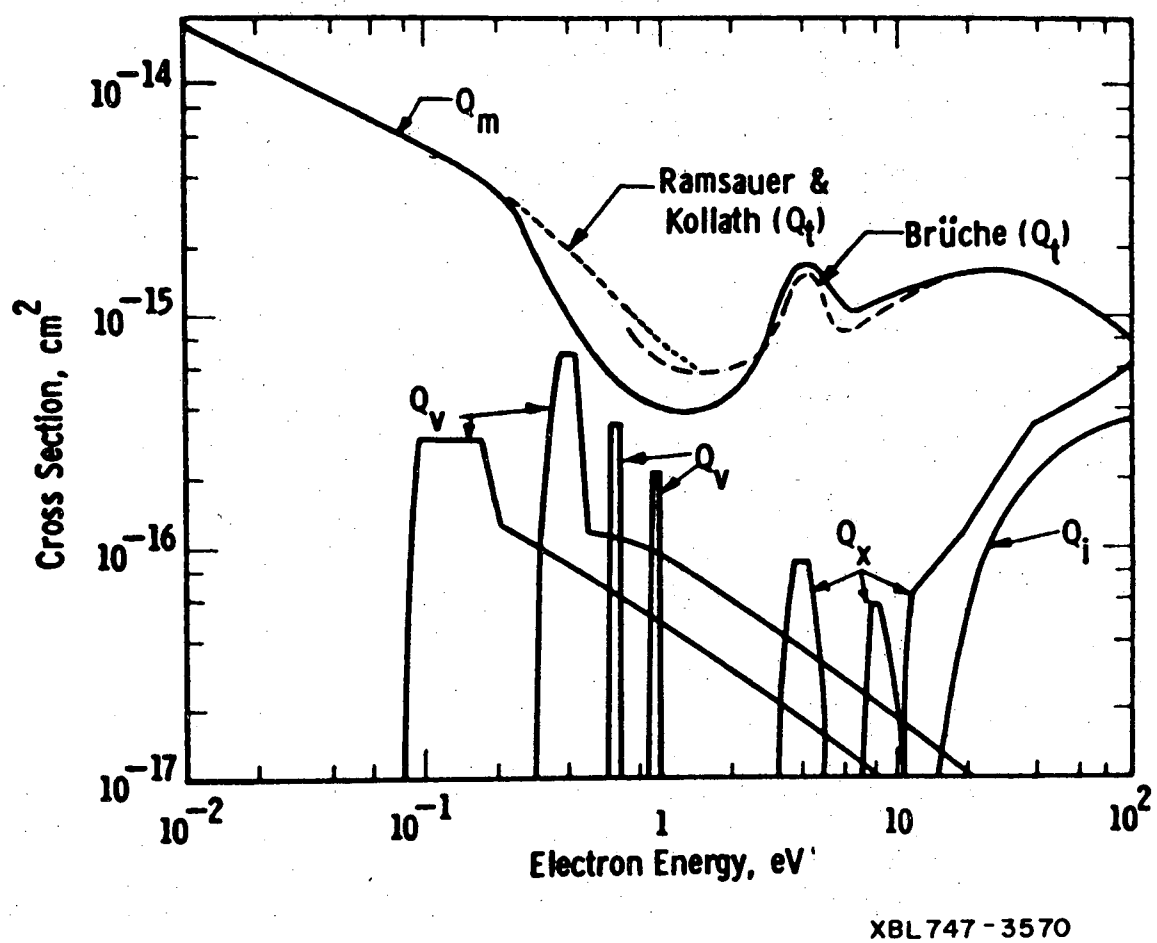
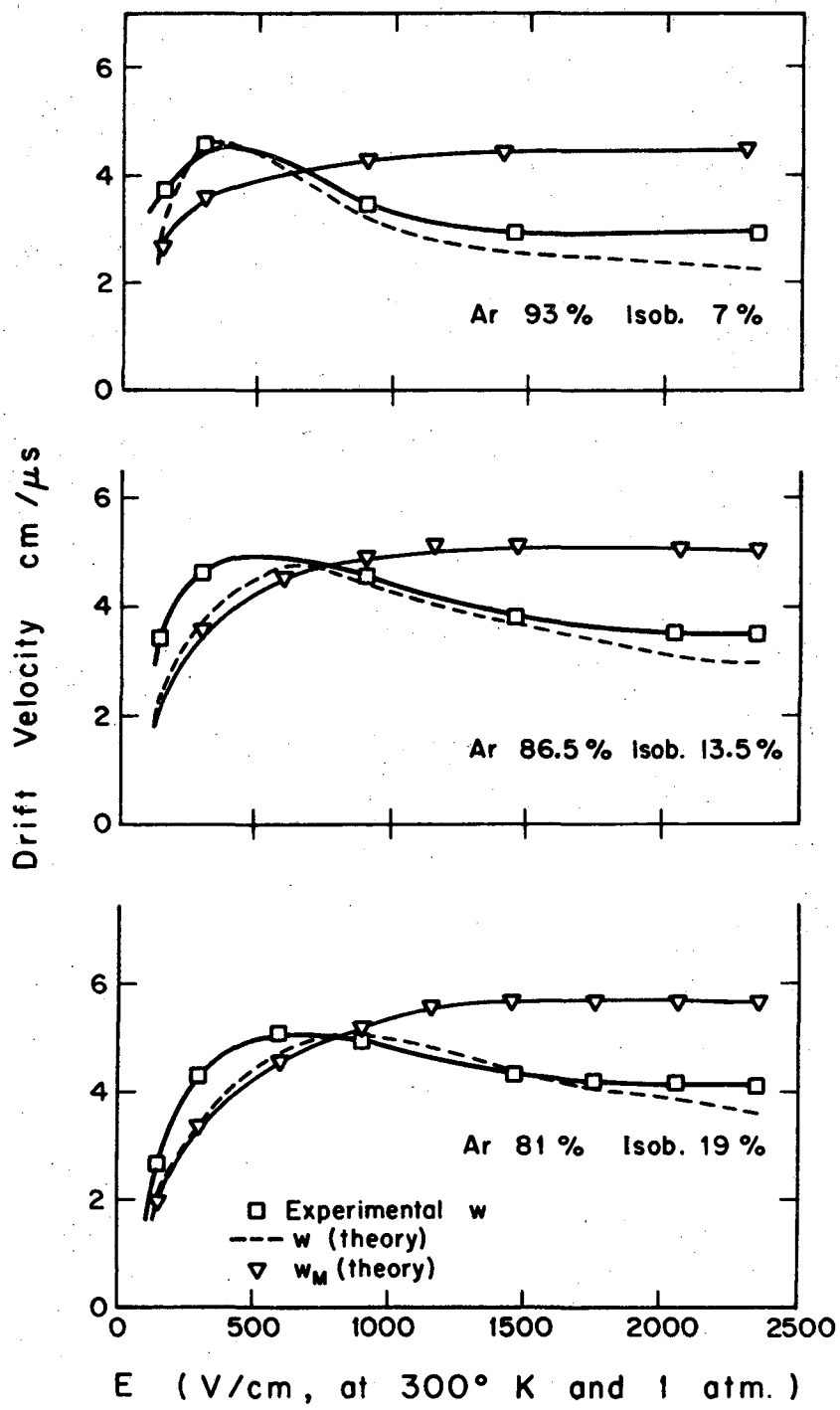

10^{-14}
10^{-15}
10^{-16}
10^{-17}
Cross Section, cm^2
Q_m
Ramsauer &
Kollath (Q_t)
Brüche (Q_t)
Q_v
Q_v
Q_x
Q_i
10^{-2}
10^{-1}
1
10
10^2
Electron Energy, eV
XBL747-3570

Fig. 3.

0 0 0 0 4 1 0 7 8 0 9



XBL 745-3161

Fig. 4a

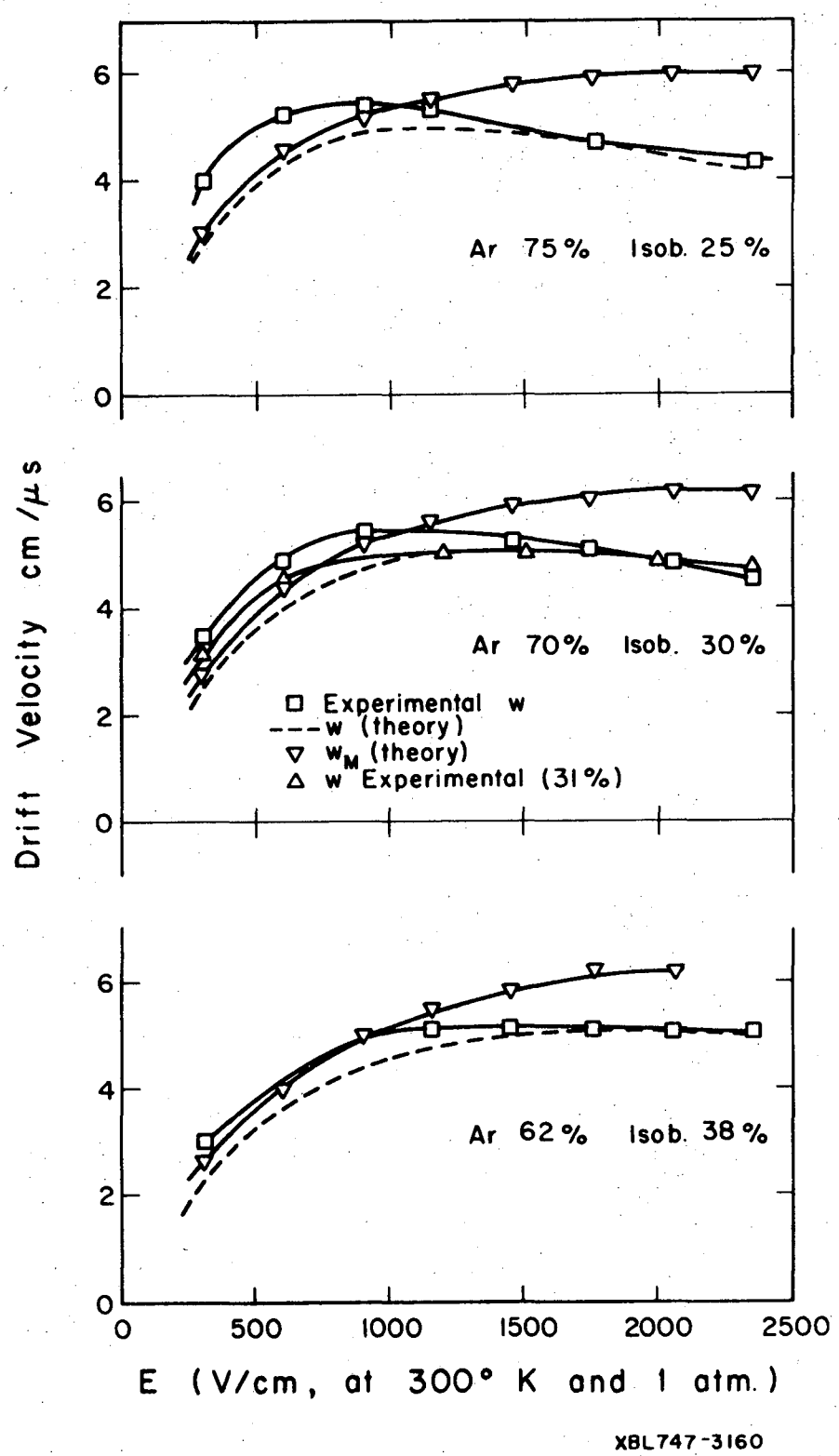

Drift Velocity cm/μs
Ar 75% Isob. 25%
Ar 70% Isob. 30%
Ar 62% Isob. 38%
Experimental w
w (theory)
w_M (theory)
w Experimental (31%)
0
2
4
6
0
500
1000
1500
2000
2500
E (V/cm, at 300° K and 1 atm.)
XBL747-3160

Fig. 4b.

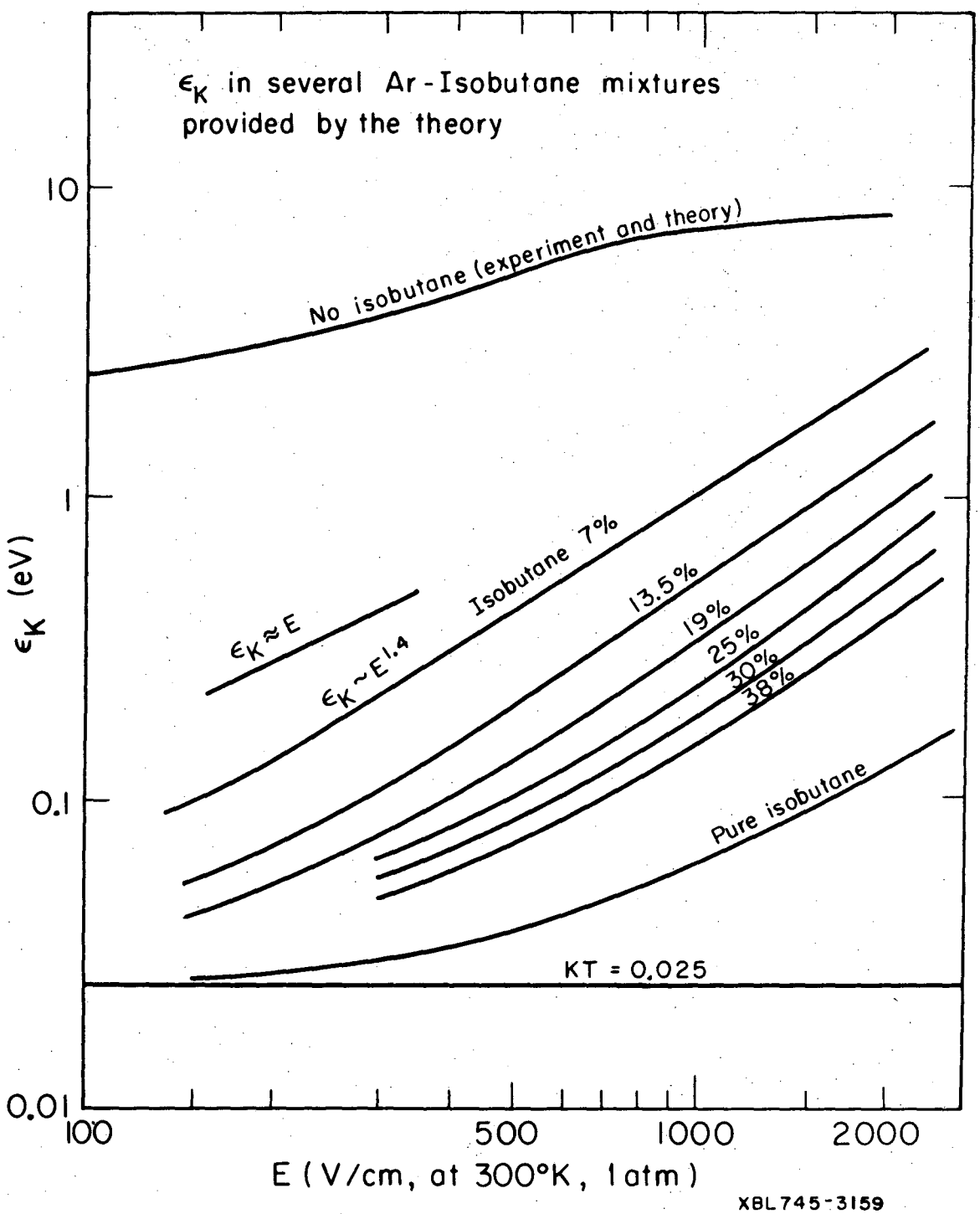

ϵ_K in several Ar-Isobutane mixtures provided by the theory
No isobutane (experiment and theory)
Isobutane 7%
13.5%
19%
25%
30%
38%
Pure isobutane
$\epsilon_K \approx E$
$\epsilon_K \sim E^{1.4}$
KT = 0.025
ϵ_K (eV)
10
1
0.1
0.01
100
500
1000
2000
E (V/cm, at 300°K, 1atm)
XBL745-3159

Fig. 5.

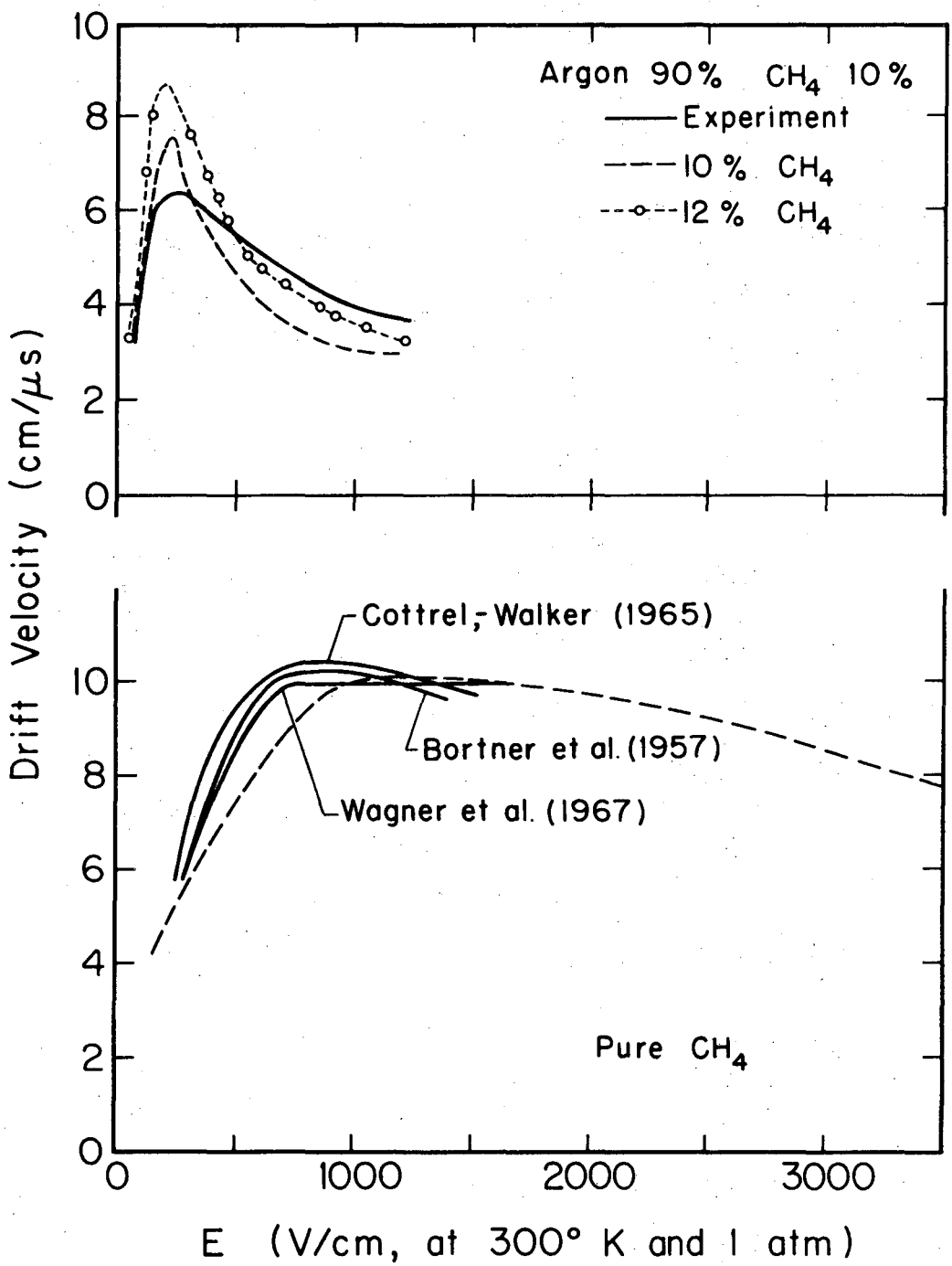


Fig. 6.

XBL 745-3158

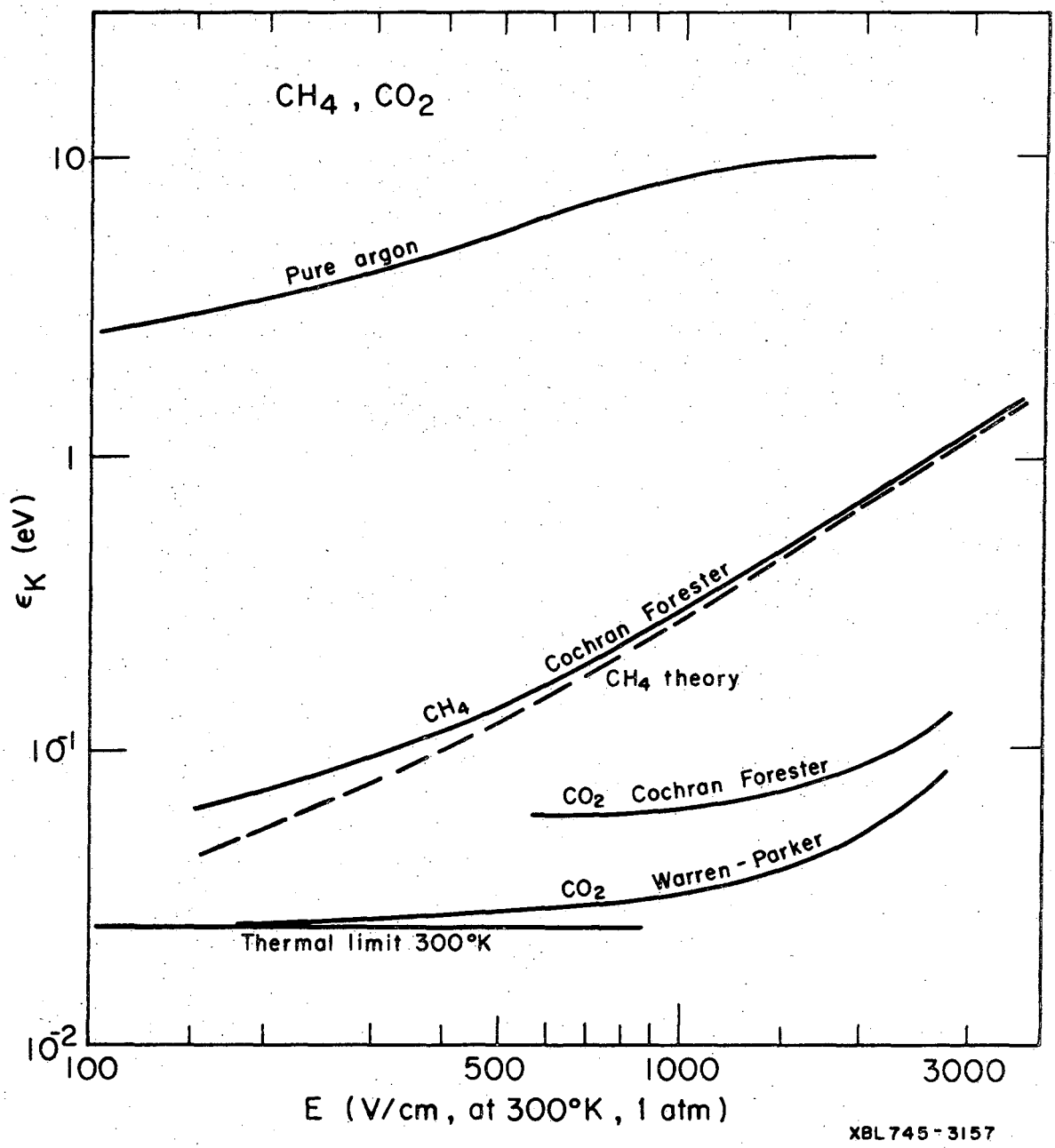

CH4 , CO2
10
1
10-1
10-2
εK (eV)
Pure argon
Cochran Forester
CH4 theory
CH4
CO2 Cochran Forester
CO2 Warren - Parker
Thermal limit 300°K
100
500
1000
3000
E (V/cm, at 300°K, 1 atm)
XBL745-3157

Fig. 7.

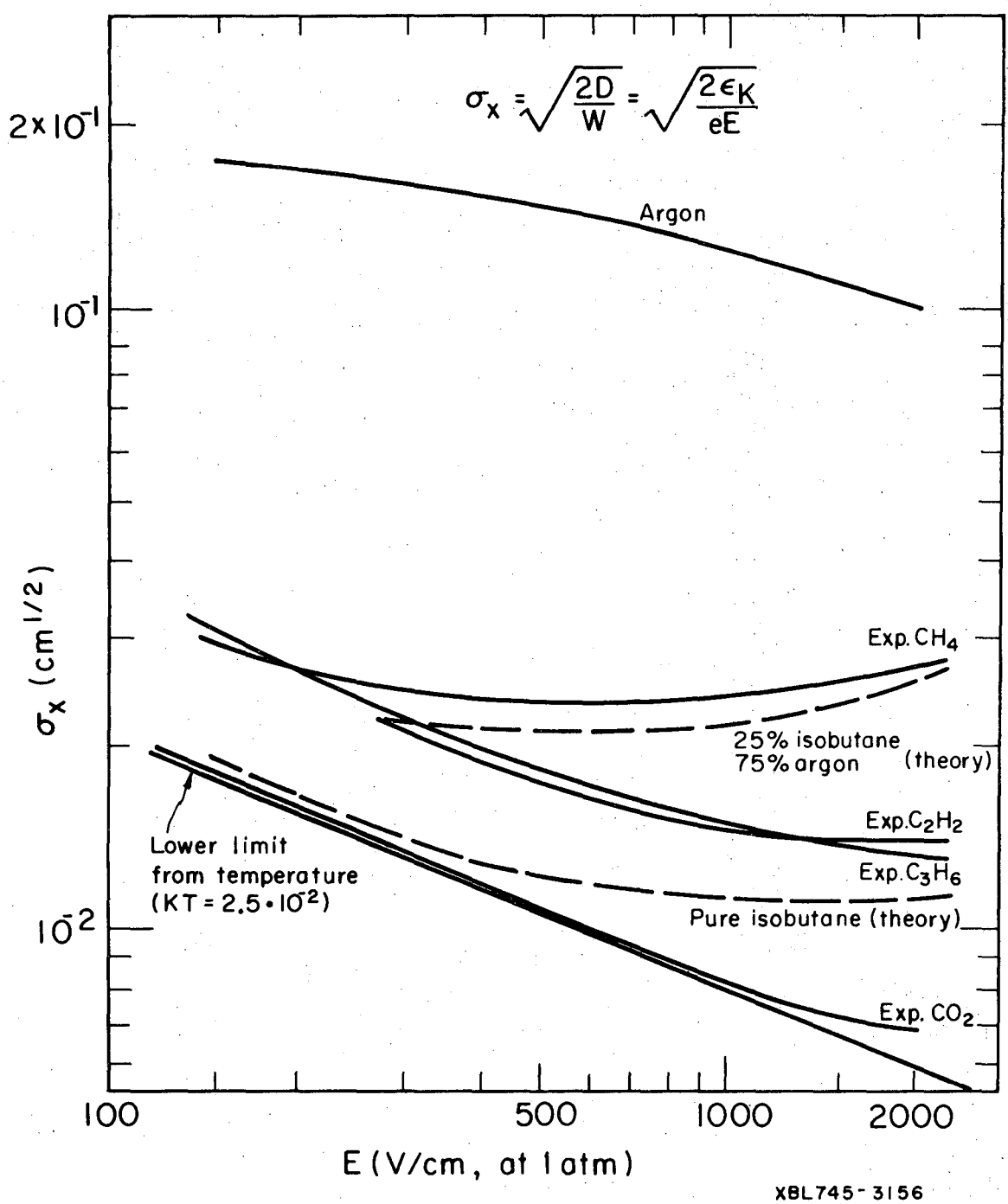
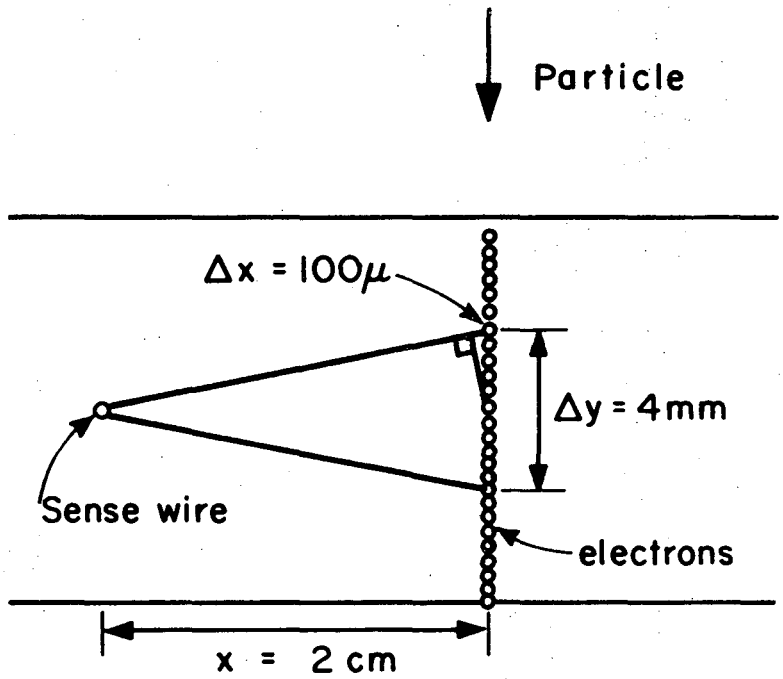

$\sigma_x = \sqrt{\frac{2D}{W}} = \sqrt{\frac{2\epsilon_K}{eE}}$
2×10^{-1}
10^{-1}
10^{-2}
σ_x (cm$^{1/2}$)
Argon
Exp. CH_4
25% isobutane
75% argon
(theory)
Exp. C_2H_2
Exp. C_3H_6
Pure isobutane (theory)
Exp. CO_2
Lower limit
from temperature
(KT = 2.5 • 10^{-2})
100
500
1000
2000
E (V/cm, at 1 atm)
XBL745-3156

Fig. 8.



XBL 745 - 3155

Fig. 9.

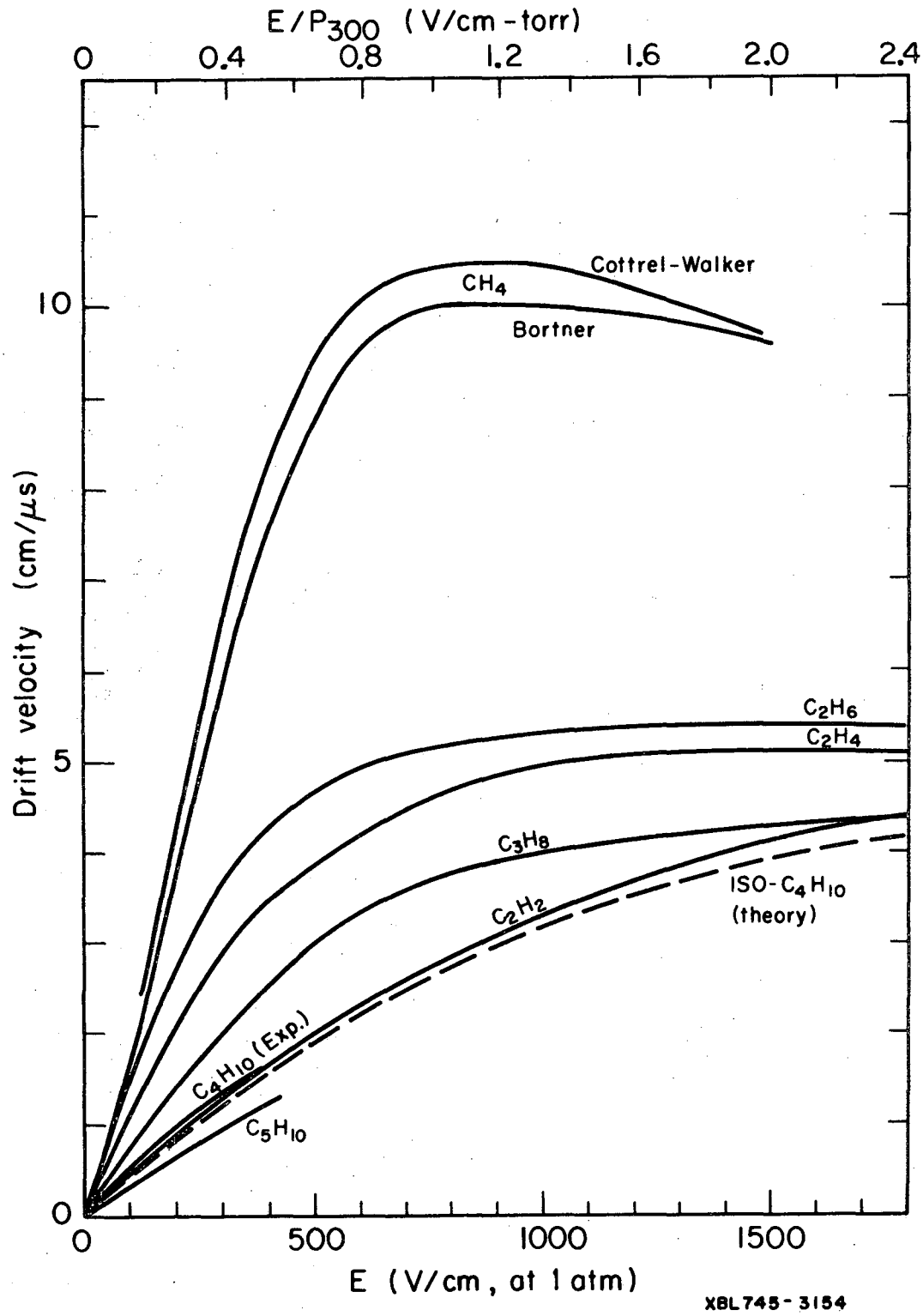
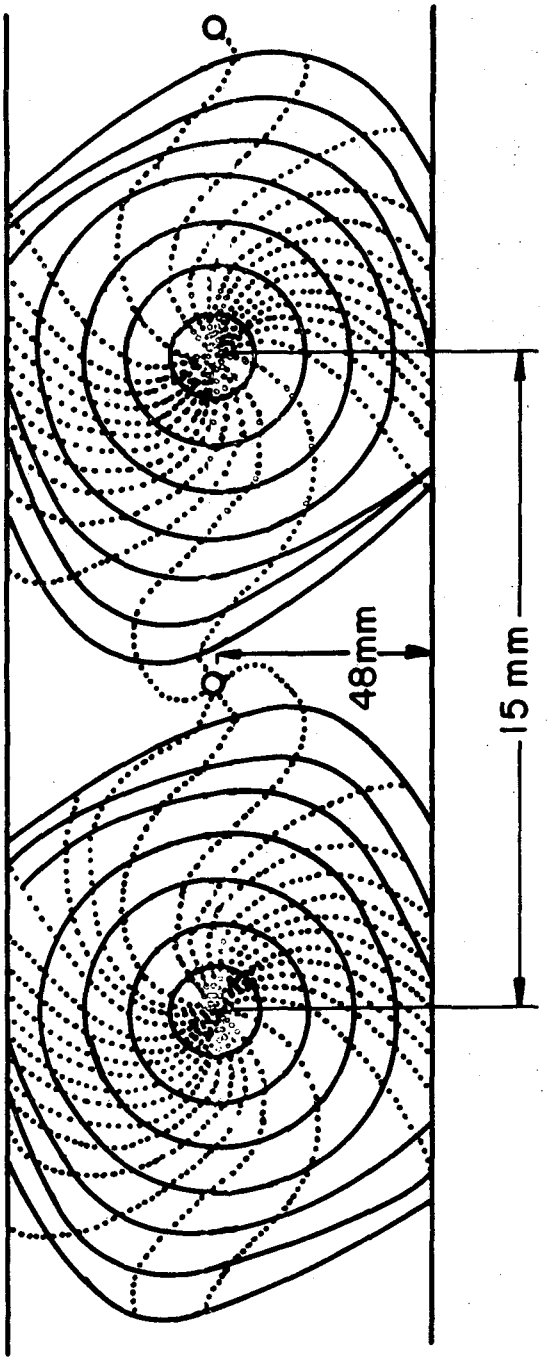
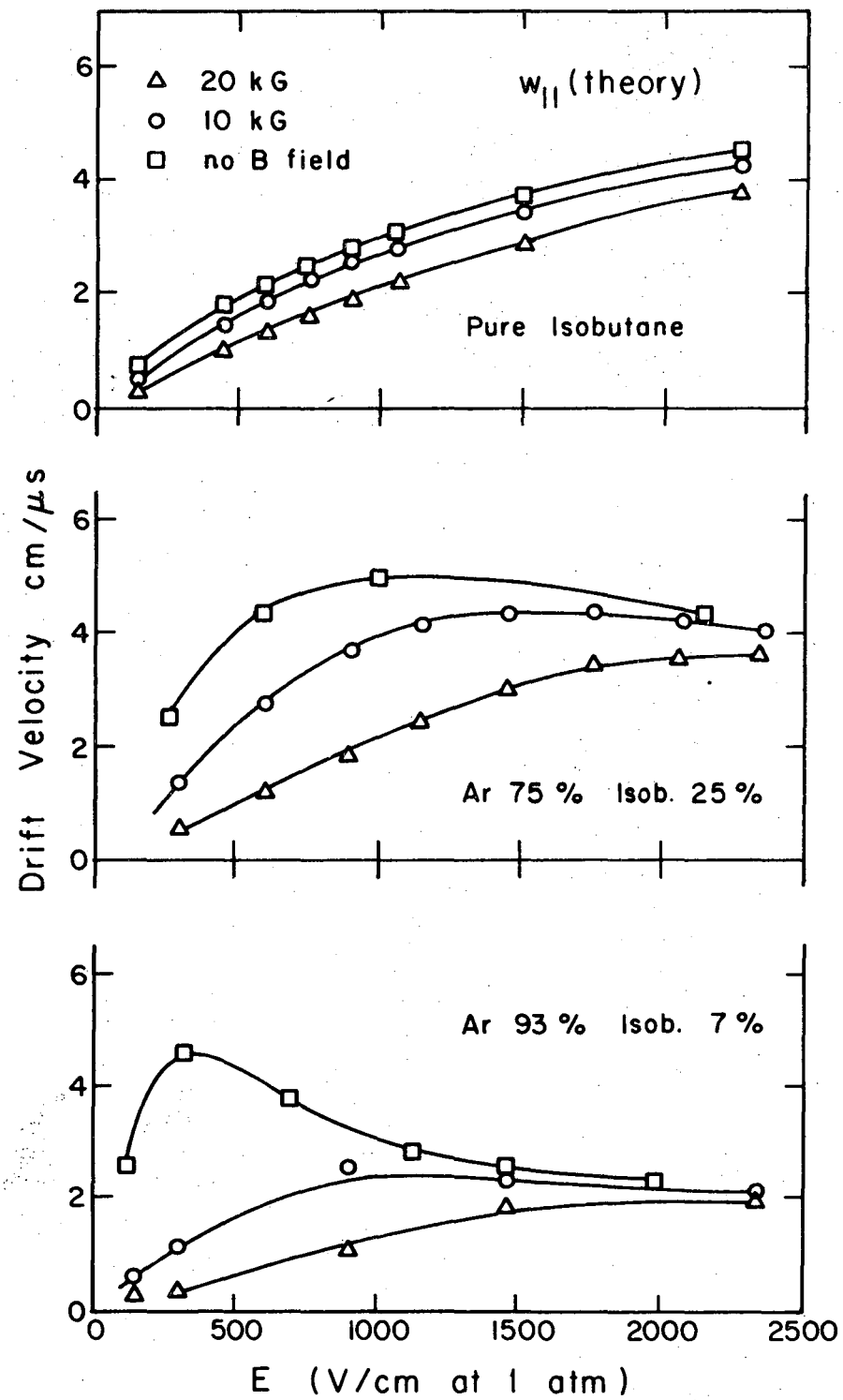

E/P300 (V/cm-torr)
0
0.4
0.8
1.2
1.6
2.0
2.4
Cottrel-Walker
CH4
Bortner
10
5
0
Drift velocity (cm/μs)
C2H6
C2H4
C3H8
C2H2
ISO-C4H10
(theory)
C4H10 (Exp.)
C5H10
0
500
1000
1500
E (V/cm, at 1 atm)
XBL745-3154

Fig. 10



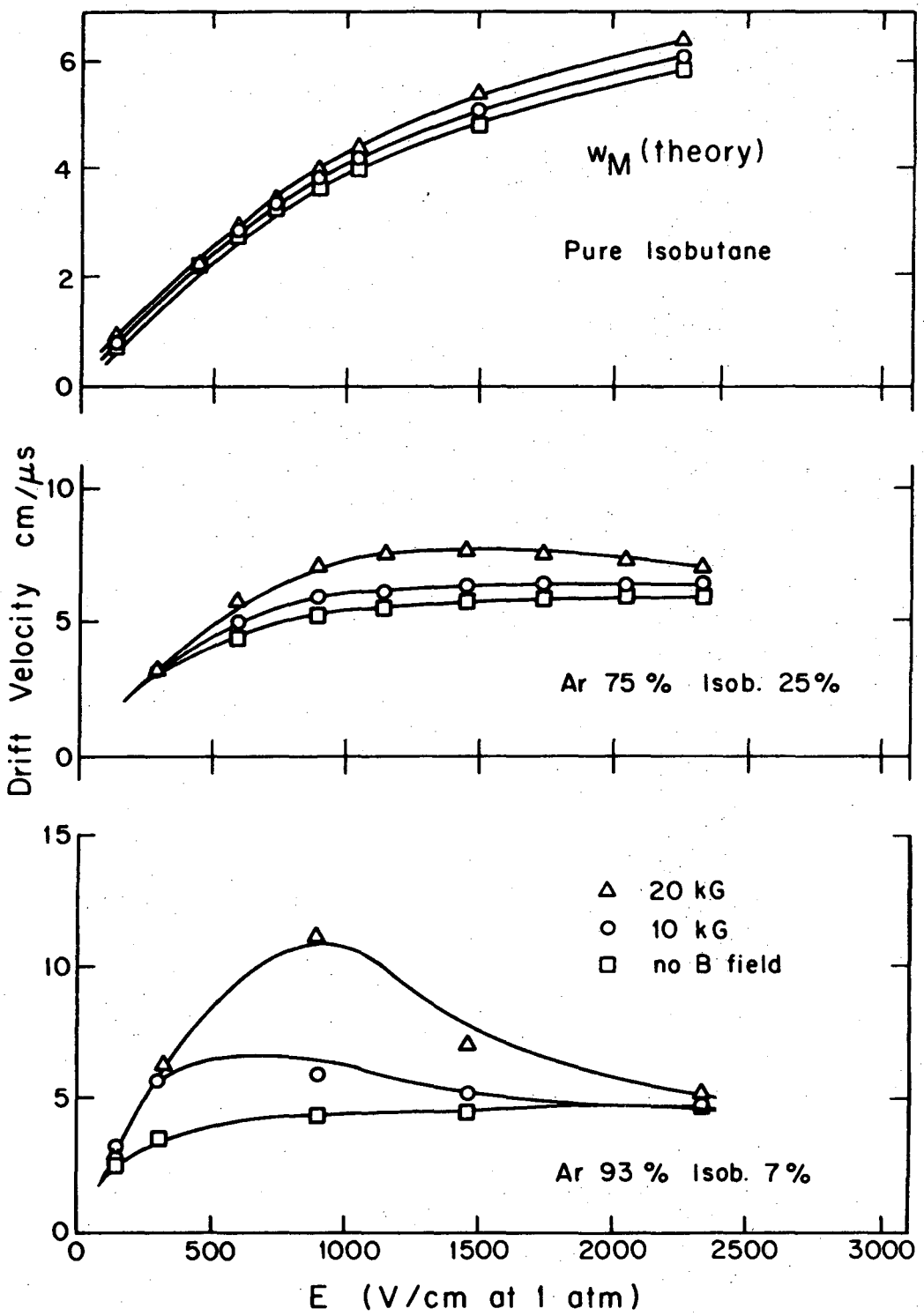
XBL745-3153

Fig. 11.



XBL 745-3152

Fig. 12a.



XBL 745-3151

Fig. 12b.

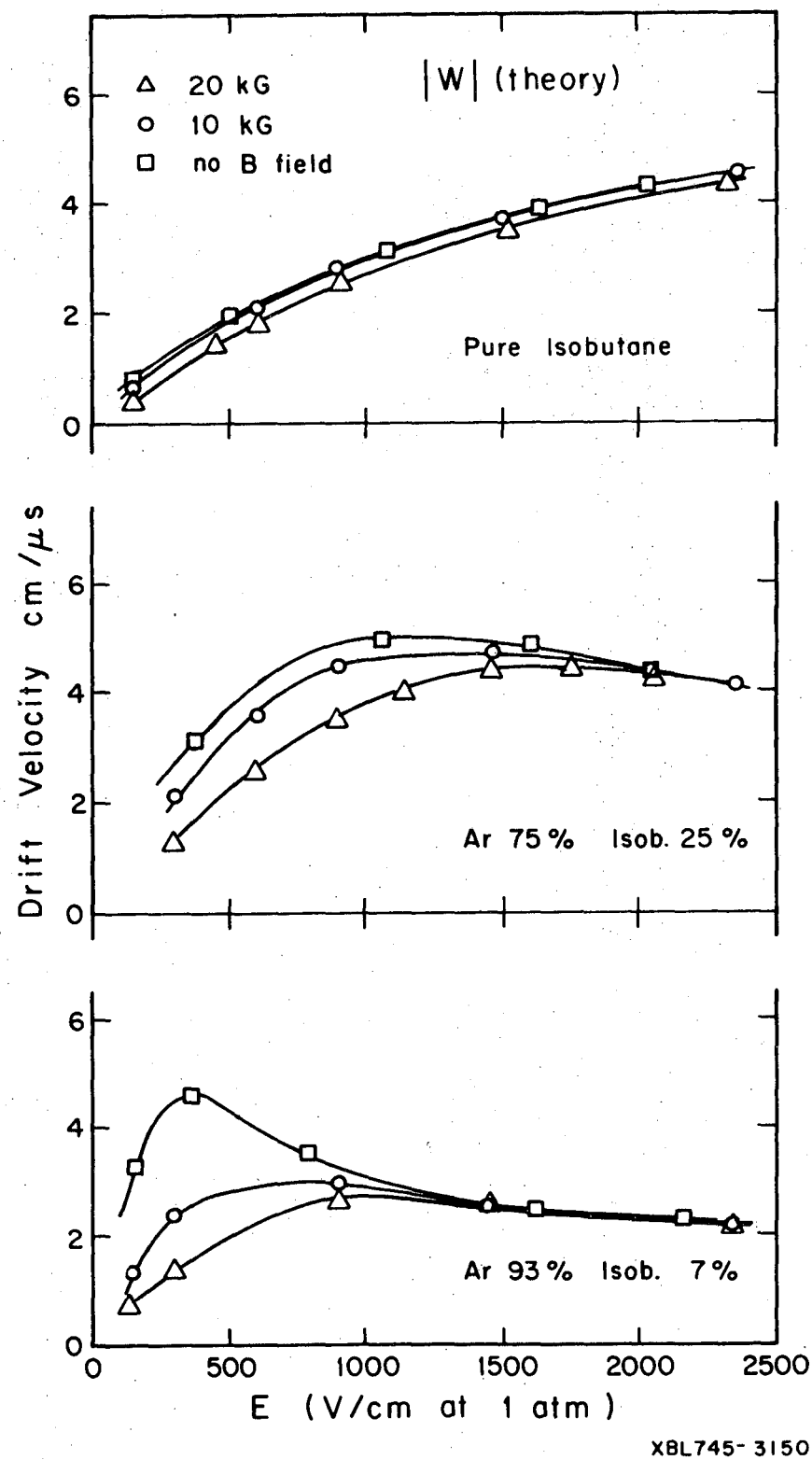
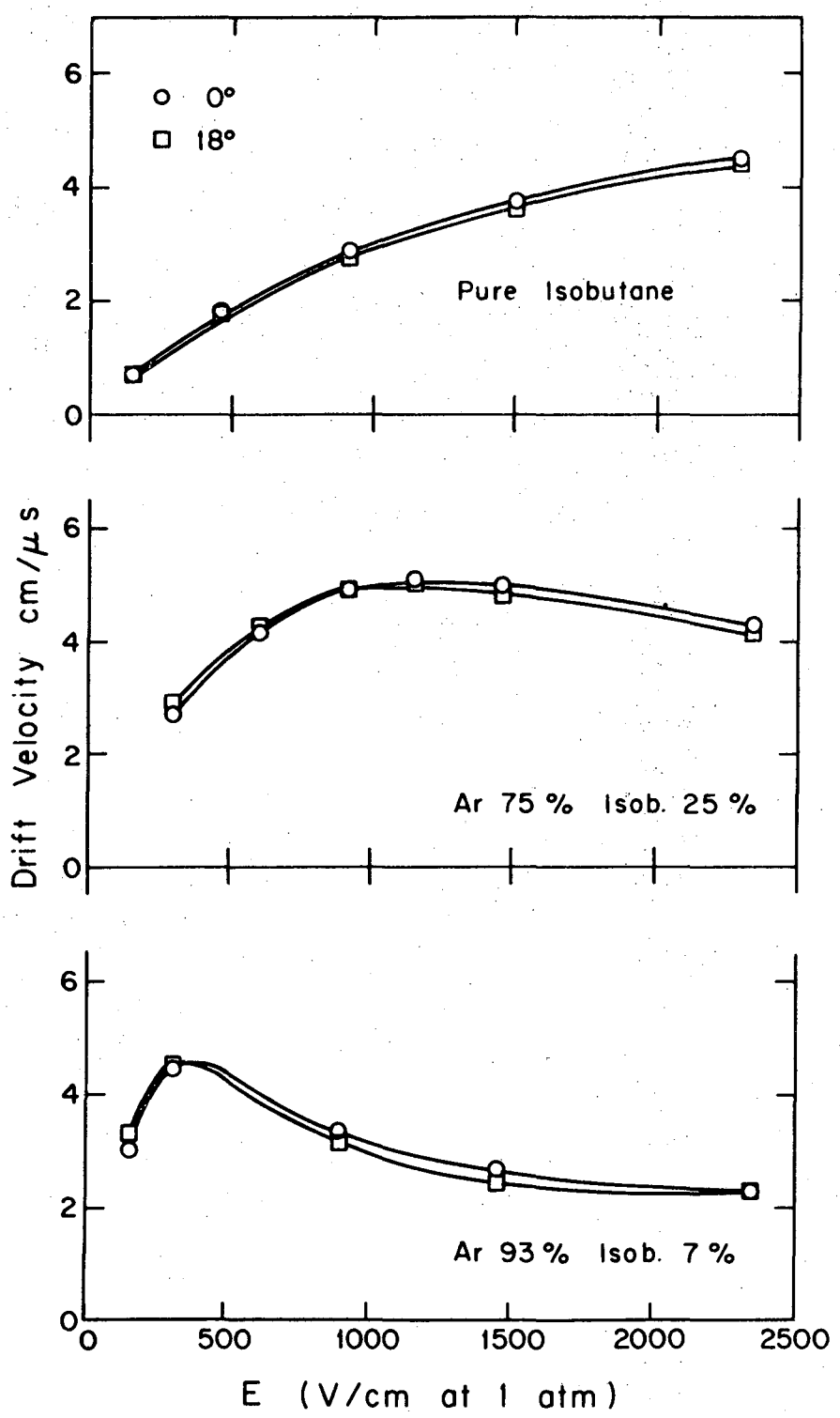

|W| (theory)
20 kG
10 kG
no B field
Pure Isobutane
Ar 75 % Isob. 25 %
Ar 93 % Isob. 7 %
Drift Velocity cm/μs
E (V/cm at 1 atm)
0
500
1000
1500
2000
2500
2
4
6
XBL745-3150

Fig. 12c.



XBL 745-3148

Fig. 13.

0 0 0 0 4 1 0 7 8 1 5

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