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Impact of the Nuclear Equation of State on Models of Rotating Neutron Stars

F. Weber and N.K. Glendenning

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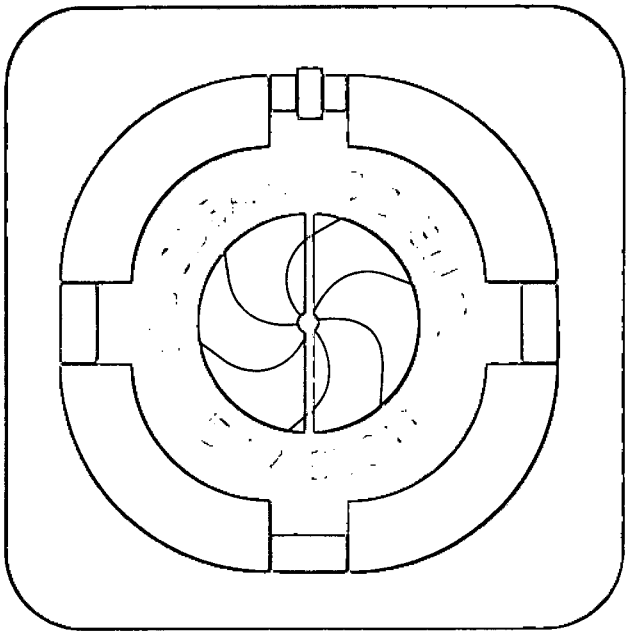

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Abstract

The impact of the nuclear equation of state on the properties of rotating neutron stars from two different sources, (1) stable rotation at the general relativistic Kepler period and (2) rotation at the gravitational radiation-reaction driven instablity mode, is analyzed. For this pupose models of rotating neutron stars are constructed in the framework of Einstein's theory of general relativity by applying a refined version of Hartle's perturbative stellar structure equations. The investigation is based on a representative collection of a total of seventeen nuclear equations of state, covering both non-relativistic as well as relativistic ones.

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IMPACT OF THE NUCLEAR EQUATION OF STATE ON MODELS OF ROTATING NEUTRON STARS

1 Introduction

The number of observed pulsars has increased rapidly since the discovery of the Cambridge Pulsar CP1919 in 1967 [1]. The sources of such objects have been interpreted as rapidly rotating, highly magnetized neutron stars [2]. To date about 500 pulsars are known [3], and the fastest of them have rotational periods in the millisecond range. Table 1 summarizes the observed properties of a selected sample of millisecond pulsars. It is well known that neutron star masses impose a constraint on theories of nuclear matter. Similarly fast rotation can be expected to impose constraints [4], perhaps even more detailed. Against this background we perform a systematic investigation of the influence of rotation on the properties of neutron stars. The application of Einstein's equations to rotating fluid spheres is known to be a complicated and cumbersome task (see, for example, [6, 7]).Simplifications in the treatment however arise if a perturbative solution of Einstein's equations is de-

Table 1: Sample of millisecond pulsars (cf. [5, 3]). The rotation period and its derivative are denoted by P and dP/dt, respectively.

Name	Features†	P [msec]	dP/dt [10^{-19} sec sec^{-1}]	Companion mass [$M_\odot$]	Orbital period [days]
1516+02B	g,b	7.9	–‡		
1953+29	f,b	6.133	0.3	0.2-0.4	117
0021 – 72C	g,nb	5.8	–		
1516+02A	g,nb	5.5	–		
1855+09	f,b	5.436	0.2	0.3-0.4	12.3
0021 – 72E	g,b	4.04	–		≈ 2
1821 – 24	g,nb	3.1	16.2		
1937+21	f,nb	1.6	1.1		
1957+20	f,b	1.6	1.1		0.38

† "g" and "f" refer to respectively globular-cluster pulsars and pulsars in the field (i.e. not in a cluster); b = binary pulsar, nb = non-binary pulsar.

‡ The symbol "–" indicates a measurement that has not yet been made.

veloped that is based on a perturbation of the line element from that of a static star. This has been demonstrated by Hartle [8] (referred to in the following as Hartle's method). It provides an elegant alternative tool to the exact numerical solution of Einstein's equations, which is more transparent and easier to apply. Though being perturbative, Hartle's method has recently been shown [9] to be applicable to the construction of models of rotating neutron stars down to rotational periods $P \approx 0.5$ msec, a value which is by far smaller than the smallest yet observed pulsar period. We apply this method for the construction of general relativistic, rotating neutron star models.

2 Rotating Neutron Stars in General Relativity

Neutron stars are objects of highly compressed matter so that the geometry of space-time is changed considerably from flat space. Thus for the construction of realistic stellar models of neutron stars one has to resort to Einstein's theory of general relativity. Therein the Einstein curvature tensor $\mathcal{G}_{\mu\nu}$ is coupled to the energy-momentum density tensor $\mathcal{T}_{\mu\nu}(\epsilon, P(\epsilon))$ of matter [10]

$$\mathcal{G}_{\mu\nu} \;=\; 8\,\pi\,\mathcal{T}_{\mu\nu}(\epsilon, P(\epsilon))\;. \tag{1}$$

The quantities ϵ and $P(\epsilon)$ denote respectively energy density and pressure (i.e. the equation of state, see Table 2 for the equations of state of this work) of the star's matter, and are derivable from the matter Lagrangian $\mathcal{L}_m$ [4]

$$\frac{\delta \mathcal{L}_m}{\delta \chi} \;-\; \partial_\mu \frac{\delta \mathcal{L}_m}{\delta\,(\partial_\mu \chi)} \;=\; 0\;, \qquad \text{for each matter field} \quad \chi\;. \tag{2}$$

The Lagrangian $\mathcal{L}_m$ is in the case of neutron star matter a complicated function of various baryon ($p, n, \Sigma^{\pm,0}, \Lambda, \Xi^{0,-}, \Delta^{++,+,0,-}$), meson ($\sigma, \omega, \pi, \varrho$), and lepton ($e^-, \mu^-$) fields [11, 12, 13]. Equations of state of neutron matter have been derived by various authors. In this work we study a collection of seventeen representative models which will be introduced in Sect. 4.

In the following Hartle's stellar structure equations are outlined very briefly. Details are given in Ref. [8]. We begin with the perturbed line element,

$$ds^2 \;=\; -\,e^{2\,\nu(r)}dt^2 + e^{2\,\psi(r)}\,(d\phi - \omega\,dt)^2 + e^{2\,\mu(r)}\,d\theta^2 + e^{2\,\lambda(r)}dr^2 + O\left(\Omega^3\right)\;. \tag{3}$$

Here, $\omega(r)$ is the angular velocity of the local inertial frame, which depends on the radial coordinate r. It is proportional to the star's rotational velocity Ω (dragging of the local inertial frame). The frequency Ω is a constant throughout the star's fluid (assumption of uniform rotation). It is convenient to define the angular velocity of the fluid relative to the local inertial frame, $\bar{\omega} \equiv \Omega - \omega$, in terms of which the fluid

inside the star moves. The metric functions in the disturbed line element of Eq. (3) have the form [8, 14, 15]

$$
\begin{aligned}
e^{2\nu(r,\Omega)} &= e^{2\Phi(r)}\left[1+2\left(h_0(r,\Omega)+h_2(r,\Omega)\,P_2(\cos\theta)\right)\right], && (4)\\
e^{2\psi(r,\Omega)} &= r^2\sin^2\theta\left[1+2\left(v_2(r,\Omega)-h_2(r,\Omega)\right)P_2(\cos\theta)\right], && (5)\\
e^{2\mu(r,\Omega)} &= r^2\left[1+2\left(v_2(r,\Omega)-h_2(r,\Omega)\right)P_2(\cos\theta)\right], && (6)\\
e^{2\lambda(r,\Omega)} &= e^{2\Lambda(r)}\left(1+\frac{2}{r}\,\frac{m_0(r,\Omega)+m_2(r,\Omega)\,P_2(\cos\theta)}{1-\Upsilon(r)}\right), && (7)
\end{aligned}
$$

and are independent of time and azimuthal angle ϕ, expressing respectively stationary rotation and axial symmetry about the axis of rotation. The functions h_l, m_l, $(l=0,2)$ and v_2 of Eqs. (4)-(7) stand for the monopole and quadrupole perturbation functions, the quantity P_2 is the second order Legendre polynomial, and $\Upsilon(r)\equiv 2m(r)/r$. In the non-rotating (spherical) limit, the perturbation functions vanish identically and Eq. (3) reduces to the Schwarzschild line element [10, 16].

The (excess) rotational frequency $\bar{\omega}(r)$ is the solution of the differential equation

$$
\frac{d}{dr}\left(r^4\,j(r)\,\frac{d\bar{\omega}(r)}{dr}\right)+4\,r^3\,\frac{dj(r)}{dr}\,\bar{\omega}(r)=0\,, \tag{8}
$$

where

$$
j(r)=e^{-\Phi(r)}\,\sqrt{1-\Upsilon(r)}\,. \tag{9}
$$

The set of coupled monopole ($l=0$) equations can be integrated once $\bar{\omega}(r)$ is known from Eq. (8). The monopole mass and pressure perturbation functions, m_0 and p_0, respectively, are given as the solutions of [8, 14],

$$
\frac{dm_0}{dr}=4\pi\,r^2\frac{d\epsilon}{dP}\,(\epsilon+P)\cdot p_0+\frac{1}{12}\,j^2\,r^4\left(\frac{d\bar{\omega}}{dr}\right)^2+\frac{8\pi}{3}\,r^4\,j^2\,\frac{\epsilon+P}{1-\Upsilon}\,\bar{\omega}^2 \tag{10}
$$

and

$$
\begin{aligned}
\frac{dp_0}{dr} &= -\frac{1+8\pi\,r^2\,P}{r^2\,(1-\Upsilon)^2}\cdot m_0-4\pi\,\frac{(\epsilon+P)\,r}{1-\Upsilon}\cdot p_0\\
&\quad+\frac{1}{12}\,\frac{r^3j^2}{1-\Upsilon}\left(\frac{d\bar{\omega}}{dr}\right)^2+\frac{1}{3}\,\frac{d}{dr}\left(\frac{r^2j^2\bar{\omega}^2}{1-\Upsilon}\right). \qquad (11)
\end{aligned}
$$

The quadruple equations ($l=2$) determine the shape of the rotating star. They are given as the solutions of

$$
\frac{dv_2}{dr}=-\,2\,\frac{d\Phi}{dr}\cdot h_2+\left[\frac{1}{r}+\frac{d\Phi}{dr}\right]\left[-\frac{r^3}{3}\,\frac{dj^2}{dr}\,\bar{\omega}^2+\frac{j^2}{6}\,r^4\left(\frac{d\bar{\omega}}{dr}\right)^2\right], \tag{12}
$$

$$
\begin{aligned}
\frac{dh_2}{dr} &= \left\{-2\frac{d\Phi}{dr} + \frac{2}{1-\Upsilon}\left(\frac{d\Phi}{dr}\right)^{-1}\left[2\pi\,(\epsilon+P) - \frac{m}{r^3}\right]\right\}\cdot h_2 \\
&- \frac{2}{r^2\,(1-\Upsilon)}\left(\frac{d\Phi}{dr}\right)^{-1}\cdot v_2 \\
&+ \frac{1}{6}\left[r\,\frac{d\Phi}{dr} - \frac{1}{2\,r\,(1-\Upsilon)}\left(\frac{d\Phi}{dr}\right)^{-1}\right] r^3\,j^2\left(\frac{d\bar{\omega}}{dr}\right)^2 \\
&- \frac{1}{3}\left[r\,\frac{d\Phi}{dr} + \frac{1}{2\,r\,(1-\Upsilon)}\left(\frac{d\Phi}{dr}\right)^{-1}\right](r\,\bar{\omega})^2\,\frac{dj^2}{dr}\,.
\end{aligned}
\tag{13}
$$

Equations (10) - (13) explicitly show the dependence of the stellar model on the equation of state , $P(\epsilon)$. Next we introduce the redshift of photons emitted at the star's pole. It follows from the expression [17]

$$
z_{\rm p} \;=\; e^{-\nu} \;-\; 1\,, \tag{14}
$$

and can be calculated once the perturbation functions h_0 and h_2 at the star's pole are known (cf. Eq. (4)).

3 Stable Neutron Star Rotation

3.1 Kepler Frequency

No simple stability criteria are known for rotating star configurations in general relativity. An *absolute* upper limit on neutron star rotation is set by the Kepler frequency, $\Omega_{\rm K}$, above which centrifuge overwhelms gravity at the equator of a rotating star. In terms of the metric functions of Eqs. (4) and (5), the Kepler frequency is given as the solution Ω of [17]

$$
\Omega \;=\; \left[\,e^{\nu(\omega)-\psi(\Omega)}\;V(\Omega)\;+\;\omega(\Omega)\,\right]_{\rm eq}\,, \tag{15}
$$

$$
V(\Omega) \;\equiv\; \left[\,\frac{\omega(\Omega)'}{2\,\psi(\Omega)'}\,e^{\psi(\Omega)-\nu(\Omega)} \;+\; \sqrt{\frac{\nu(\Omega)'}{\psi(\Omega)'} + \left(\frac{\omega(\Omega)'}{2\,\psi(\Omega)'}\,e^{\psi(\Omega)-\nu(\Omega)}\right)^2}\,\right]_{\rm eq}\,. \tag{16}
$$

The subscipt "eq" refers to evaluation of Eqs. (15) and (16) at the star's equator. The quantity V denotes the orbital velocity measured by an observer with zero angular momentum in the ϕ-direction. Primes denote derivatives with respect to the radial coordinate.

3.2 Gravitational Radiation Reaction-Driven Instability

Besides the absolute upper limit on rotation set by the Kepler frequency, there is another instability that sets in at a lower rotational frequency, and which therefore sets

a more stringent limit [18]. It originates from counter-rotating surface vibrational modes, which at sufficiently high rotational star frequencies are dragged forward. In this case, gravitational radiation which inevitably accompanies the aspherical transport of matter does not damp the modes, but rather drives them [19, 20]. Viscosity plays the important role of damping such gravitational-wave radiation-reaction instabilities at a sufficiently reduced rotational frequency such that the viscous damping rate and power in gravity waves are comparable [21]. The instability modes are taken to have the dependence $\exp[i\omega_m(\Omega)t + im\phi - t/\tau_m(\Omega)]$, where ω_m is the frequency of the surface mode which depends on the angular velocity Ω of the star, ϕ denotes the azimuthal angle, and τ_m is the time scale for the mode which determines its growth or damping. The rotation frequency Ω at which it changes sign is the critical frequency for the particular mode, m (=2,3,4,...). It is conveniently expressed as the frequency, Ω_m^ν, that solves [18] (ν refers to the viscosity dependence, see below)

$$\Omega_m^\nu = \frac{\omega_m(0)}{m}\left[\tilde{\alpha}_m(\Omega_m^\nu) + \tilde{\gamma}_m(\Omega_m^\nu)\left(\frac{\tau_{g,m}}{\tau_{\nu,m}}\right)^{\frac{1}{2m+1}}\right], \tag{17}$$

where

$$\omega_m(0) \equiv \sqrt{\frac{2\,m\,(m-1)}{2\,m+1}\,\frac{M}{R^3}} \tag{18}$$

is the frequency of the vibrational mode in a non-rotating star. The time scales for gravitational radiation reaction [22], $\tau_{g,m}$, and for viscous damping time [23], $\tau_{\nu,m}$, are given respectively by

$$\tau_{g,m} = \frac{2}{3}\,\frac{(m-1)\,[(2m+1)!!]^2}{(m+1)\,(m+2)}\left(\frac{2m+1}{2m(m-1)}\right)^m\left(\frac{R}{M}\right)^{m+1} R\,, \tag{19}$$

$$\tau_{\nu,m} = \frac{R^2}{(2m+1)\,(m-1)}\,\frac{1}{\nu}\,. \tag{20}$$

The shear viscosity is denoted by ν. It depends on the temperature, T, of the star ($\nu(T) \propto T^{-2}$). It is small in very hot ($T \approx 10^{10}$ K) and therefore young stars and larger in cold ones. A characteristic feature of the above equations (17) - (20) is that Ω_m^ν merely depends on radius and mass (R and M) of the spherical star model.

The functions $\tilde{\alpha}_m$ and $\tilde{\gamma}_m$ contain information about the pulsation of the rotating star models and are difficult to determine [18, 24]. A reasonable first step is to replace them by their corresponding Maclaurin spheroid functions α_m and γ_m [18, 24]. We therefore take $\alpha_m(\Omega_m)$ and $\gamma_m(\Omega_m)$ as calculated in Refs. [25, 26] for the oscillations of rapidly rotating inhomogeneous Newtonian stellar models (polytropic index n=1), and Ref. [18] for uniform-density Maclaurin spheroids (i.e. n=0),

respectively. Managan has shown that Ω_m^ν depends much more strongly on the equation of state and the mass of the neutron star model (through $\omega_m(0)$ and $\tau_{g,m}$, see Eqs. (18) and (19)) than on the polytropic index assumed in calculating α_m [27].

4 Collection of Equations of State

A sample of a total of seventeen nuclear equations of state is used for the construction of models of rotating neutron stars. These are summarized in Table 2. The equations of state are divided into two categories: (1) non-relativisitic potential model equations of state , and (2) relativistic equations of state which are determined in the framwork of relativistic nuclear field theory. An inherent feature of the latter is that they do not violate causality, i.e., $c_s/c = \sqrt{\partial P/\partial \epsilon} < 1$ (c_s denotes the velocity of sound), which is not the case for the potential models. Among the latter only the WFF(UV_{14} + TNI) equation of state does not violate causality. The BJ(I) and Pan(C) equations of state violate causality at ≈ 23 times the energy density of normal nuclear matter, not much above the central density of the limiting star of the seqeuence. The equations of state FP(V_{14} + TNI), WFF(AV_{14} + UVII), and WFF(UV_{14} + UVII) do so at considerably smaller densities of $\approx 6-7$ times normal nuclear matter density, which is less than the central densities encountered in the limiting-mass neutron star models constructed from them. The four equations of state of our collection denoted $\mathrm{G}_{225}^{\mathrm{DCM1}}$, $\mathrm{G}_{265}^{\mathrm{DCM2}}$, $\mathrm{G}_{\mathrm{B180}}^{\mathrm{DCM1}}$, and $\mathrm{G}_{\mathrm{B180}}^{\mathrm{DCM2}}$ have only recently been calculated [28] for electrically charge neutral neutron star matter in generalized β equilibrium from the derivative coupling Lagrangian of Zimanyi and Moszkowski [29]. Those labeled DCM1 correspond to the Lagrangian of Zimanyi and Moszkowski, while those labeled DCM2 correspond to the "hybrid" coupling introduced in [28]. The possibility of a phase transition of the dense core to quark matter is taken into account in equations of state $\mathrm{G}_{\mathrm{B180}}^{\mathrm{DCM1}}$ and $\mathrm{G}_{\mathrm{B180}}^{\mathrm{DCM2}}$. A bag constant of $B^{1/4} = 180$ MeV has been used. It should be noted that not all equations of state of our collection account for neutron matter in generalized β equilibrium (namely entries "13" through "16" in Table 2). These models of the nuclear equation of state treat neutron star matter as being composed of only neutrons ("14"), or neutrons and protons in equilibrium with electrons and muons ("13", "15", "16"), which is however not the true ground-state of neutron star matter predicted by theory [30, 31, 32].

5 Results and Discussion

The general relativistic Kepler periods P_K ($\equiv 2\pi/\Omega_K$), beyond which mass shedding at the star's equator sets in, are graphically depicted in Fig. 1 for a representative sample of equations of state labeled according to Table 2. Pandharipande's equation of state (label 17) leads, for $M \gtrsim 0.7 M_\odot$, to rotating star models which possess the smallest periods by far. The second smallest periods are obtained from equation of state WFF(AV_{14} + UVII). We find that the relativistic equations of state lead *in general to larger rotational periods* than the non-relativistic equations of state [9]. In particular the upper limit on the Kepler period is set by the relativistic HV (label 2) equation of state. The periods obtained from all other equations of state of Table 2 lie between curves "2" and "16". The hatched rectangle in Fig. 1 covers both the approximate range of neutron star masses as determined from observations (cf. [38]), as well as observed rotational periods ($P \geq 1.6$ msec, see Table 1). One clearly sees that all pulsar periods so far observed are larger than the *absolute* limiting Kepler values. On the other hand, equation of state "16" of Fig. 1 demonstrates that periods smaller than $0.60 - 0.75$ msec (depending on the star's mass) are incompatible with those of our representative collection of nuclear equations of state. (Equation of state "17" is not considered for reasons which are given below. Here and in the following it may serve to demonstrate the impact of an extremely soft equation of state on the properties of neutron stars.) If periods below this range were detected it would give a clear hint that the confined hadronic phase of nucleons and nuclei is only metastable [39]. The plausible ground-state in that event is the deconfined phase of (3-flavor) strange-quark-matter.

The rotational masses of sequences of neutron stars rotating at their Kepler periods are shown in Fig. 2. In general, rotation stabilizes the star model against gravitational collapse at masses that are typically $\approx 15 - 20\%$ above the non-rotating mass limit [17, 35]. The central energy density obtained from Pan(C) for the limiting-mass model (the locations are shown by tick marks in Fig. 2) is $\epsilon_c \approx 21\epsilon_0$, a value which lies slightly below the one at which causality is violated for this equation of state (Sect. 4). It is questionable whether the construction of the equation of state to such high densities is valid in the framework of a theory that is based on nucleons interacting through potentials. The star models constructed from the remaining set of equations of state have central energy densities in the range $7 \lesssim \epsilon_c/\epsilon_0 \lesssim 11$. The largest rotational mass is obtained from equation of state $\Lambda^{00}_{\rm HEA}$ + HFV (label 11), which is caused by their rather stiff behavior at larger nuclear densities [35]. The importance of the stiffness of the nuclear equation of state to obtain large enough neutron star masses is a known fact from the literature [4, 35, 40]. This finding is confirmed by the equations of state of Fig. 2 for which

the stiffness increases from bottom to top.

Figure 3 shows the rotational neutron star masses at their Kepler periods of sequences up to their limiting masses (indicated by dots), plotted as a function of polar redshift. Gamma-ray bursters are suspected, but not known, to be neutron stars. If interpreted as such, the measured gamma-ray burst pair annihilation lines give tentative evidence to support a neutron star redshift range of $0.2 - 0.5$, with the highest concentration in the range $0.25 - 0.35$ [41]. The former range, combined with observed neutron star masses, is exhibited by the hatched rectangle in Fig. 3.

Figure 4 shows the limiting rotational neutron star periods P_m^T (i.e. the solution of Eq. (17)) set by the gravitational radiation-reaction instability discussed in Sect. 3.2. Considered are old (and therefore cold) neutron stars of temperature $T = 10^6$ K. For such stars we find that the $m = 2$ mode is largest and thus is exited first. It therefore sets the limit on stable rotation for neutron stars of such temperatures. As in the case of rotation at the Kepler period, the Pan(C) equation of state leads again to the smallest periods obtained from all equations of state of our collection. For the above stated problems inherent in this equation of state, equations of state "16" establishes a more realistic limit on the rotational periods that neutron stars can have before instability against gravitational radiation-reaction sets in: according to this equation of state stable neutron star rotation is restricted to periods $P \geq P_m^T \approx 0.8 - 0.9$ msec (depending on mass). This is shown by means of the hatched area of Fig. 4, which roughly covers the range of stable neutron star periods and observed masses. One sees that the observation of pulsar periods smaller than the above given limit would be in contradiction to this analysis of the gravitational radiation-reaction driven instability (cf. discussion of Fig. 1). We note that the 1.6 msec period of the two fastest yet observed pulsars are compatible with Fig. 4 as long as gravitational masses larger than $\approx 1 M_\odot$ are assumed.

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References

[1] A. Hewish, S. J. Bell, J. D. H. Pilkington, P. F. Scott, and R. A. Collins, Nature **217** (1968) 709.

[2] T. Gold, Nature **218** (1968) 731.

[3] D. C. Backer and S. R. Kulkarni, Physics Today, **43** (1990) 26.

[4] N. K. Glendenning, *Neutron Stars, Fast Pulsars, Supernovae and the Equation of State of Dense Matter*, in: "The Nuclear Equation of State", ed. by W. Greiner and H. Stöcker, NATO ASI Series, Phys. Vol. 216 A, Plenum Press, New York, p. 751.

[5] J. R. Wilson, Astrophys. J. **176** (1972) 195.

[6] E. M. Butterworth and J. R. Ipser, Astrophys. J. **200** (1975) L103.

[7] J. H. Taylor and D. R. Stinebring, Annu. Rev. Astron. Astrophys. **24** (1986) 285.

[8] J. B. Hartle, Astrophys. J. **150** (1967) 1005.

[9] F. Weber and N. K. Glendenning, *Applicability of Hartle's Method for the Construction of General Relativistic Rotating Neutron Star Models*, October, 1990 (LBL-29671).

[10] Ch. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*, W. H. Freeman and Company, San Francisco, 1973.

[11] N. K. Glendenning, Phys. Lett. **114B** (1982) 392;
N. K. Glendenning, Astrophys. J. **293** (1985) 470;
N. K. Glendenning, Z. Phys. A **326** (1987) 57;
N. K. Glendenning, Z. Phys. A **327** (1987) 295.

[12] F. Weber and M. K. Weigel, Nucl. Phys. **A505** (1989) 779, and references contained therein.

[13] B. Friedman and V. R. Pandharipande, Nucl. Phys. **A361** (1981) 502.

[14] J. B. Hartle and K. S. Thorne, Astrophys. J. **153** (1968) 807.

[15] R. C. Kapoor and B. Datta, Mon. Not. R. Astr. Soc. **209** (1984) 895.

[16] F. Weber, N. K. Glendenning, and M. K. Weigel, *Rotating Relativistic Neutron Stars*, May, 1990 (LBL-29106).

[17] J. L. Friedman, J. R. Ipser, and L. Parker, Astrophys. J. **304** (1986) 115.

[18] L. Lindblom, Astrophys. J. **303** (1986) 303.

[19] S. Chandrasekhar, Phys. Rev. Lett. **24** (1970) 611.

[20] J. L. Friedman, Phys. Rev. Lett. **51** (1983) 11.

[21] L. Lindblom and S. L. Detweiler, Astrophys. J. **211** (1977) 565.

[22] S. L. Detweiler, Astrophys. J. **197** (1975) 203.

[23] H. Lamb, Proc. London Math. Soc. **13** (1881) 51.

[24] C. Cutler and L. Lindblom, Astrophys. J. **314** (1987) 234.

[25] J. R. Ipser and L Lindbolm, Phys. Rev. Lett. **62** (1989) 2777.

[26] J. R. Ipser and L. Lindblom, Astrophys. J. **355** (1990) 226.

[27] R. A. Managan, Astrophys. J. **309** (1986) 598.

[28] N. K. Glendenning, F. Weber, and S. A. Moszkowski, *Neutron and Hybrid Stars in the Derivative Coupling Model*, March, 1991 (LBL-30296).

[29] J. Zimanyi and S. A. Moszkowski, Phys. Rev. C **42** (1990) 1416.

[30] V. R. Pandharipande, Nucl. Phys. **A178** (1971) 123.

[31] H. A. Bethe and M. Johnson, Nucl. Phys. **A230** (1974) 1.

[32] N. K. Glendenning, Astrophys. J. **293** (1985) 470.

[33] N. K. Glendenning, Nucl. Phys. **A493** (1989) 521.

[34] N. K. Glendenning, Phys. Rev. Lett. **57** (1986) 1120.

[35] F. Weber, N. K. Glendenning, and M. K. Weigel, Astrophys. J. **373** (1991) 579.

[36] R. B. Wiringa, V. Fiks, and A. Fabrocini, Phys. Rev. C **38** (1988) 1010.

[37] J. L. Friedman, J. R. Ipser, and L. Parker, Phys. Rev. Lett. **62** (1989) 3015.

[38] S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars*, John Wiley & sons, N. Y., 1983.

[39] N. K. Glendenning, Mod. Phys. Lett. **A5** (1990) 2197.

[40] J. M. Lattimer, M. Prakash, D. Masak and A. Yahil, Astrophys. J. **355** (1990) 241.

[41] E. P. Liang, Astrophys. J. **304** (1986) 682.

Table 2: Equations of state (EOS) of this work

Label		EOS	Description†	Reference
		Relativistic field theoretical equations of state		
1		G_{300}	H, K=300	Glendenning 1989 [33]
2		HV	H, K=285	Glendenning 1985 [32, 12]
3		G^{DCM2}_{B180}	Q, K=265, $B^{1/4}$ = 180	Glendenning, Weber, and Moszkowski 1991 [28]
4		G^{DCM2}_{265}	H, K=265	Glendenning,Weber, and Moszkowsi 1991 [28]
5		G^{π}_{300}	H, π, K=300	Glendenning 1989 [33]
6		G^{π}_{200}	H, π, K=200	Glendenning 1986 [34]
7		Λ^{00}_{Bonn} + HV	H, K=186	Weber, Glendenning, and Weigel 1990 [35]
8		G^{DCM1}_{225}	H, K=225	Glendenning, Weber, and Moszkowski 1991 [28]
9		G^{DCM1}_{B180}	Q, K=225, $B^{1/4}$ = 180	Glendenning, Weber, and Moszkowski 1991 [28]
10		HFV	H, Δ, K=376	Weber and Weigel 1989 [12]
11		Λ^{00}_{HEA} + HFV	H, Δ, K=115	Weber, Glendenning, and Weigel 1990 [35]
		Non-relativistic potential model equations of state		
12	C‡	BJ(I)	H, Δ,	Bethe and Johnson 1974 [31]
13		WFF(UV_{14}+TNI)	NP, K=261	Wiringa, Fiks, and Fabrocini 1988 [36]
14	FP‡	FP(V_{14}+TNI)	N, K=240	Friedman and Pandharipande 1981 [13]
15		WFF(UV_{14}+UVII)	NP, K=202	Wiringa, Fiks, and Fabrocini 1988 [36]
16		WFF(AV_{14}+UVII)	NP, K=209	Wiringa, Fiks, and Fabrocini 1988 [36]
17	B‡	Pan(C)	H, Δ, K=60	Pandharipande 1971 [30]

† The following abbreviations are used: N = pure neutron; NP = n, p, leptons; π = pion condensation; H = composed of n, p, hyperons, leptons; Δ = Δ_{1232}-resonance; Q = quark hybrid composition; K = incompressibility in MeV; $B^{1/4}$ = bag constant in MeV.

‡ Notation of Friedman, Ipser, and Parker [17, 37].

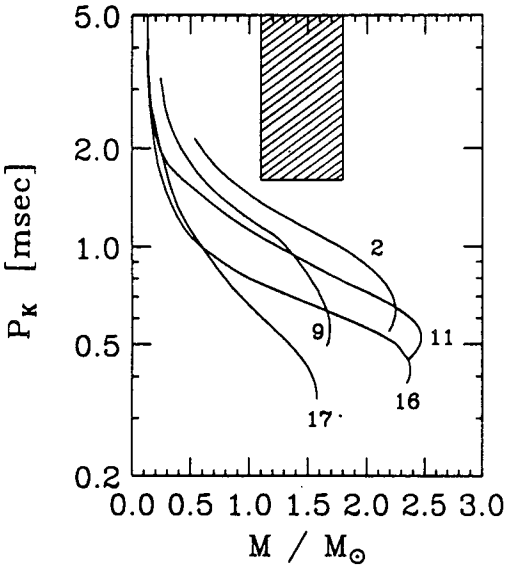

Figure 1: Kepler period as a function of rotational neutron star mass. The labeling of the curves is explained in Table 2.

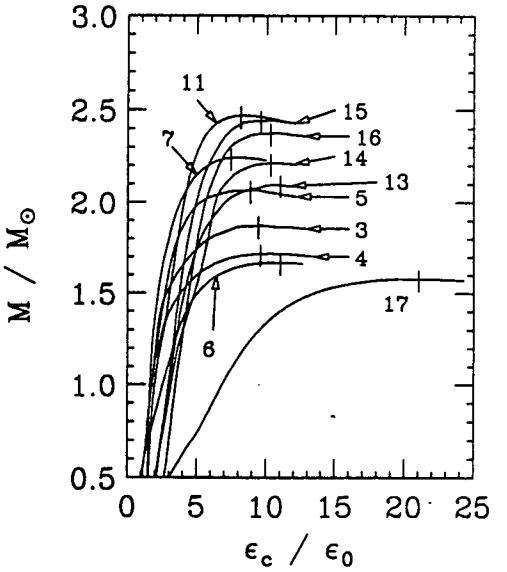

Figure 2: Rotational neutron star mass as a function of central energy density ϵ_c (in units of normal nuclear matter density, ϵ_0).

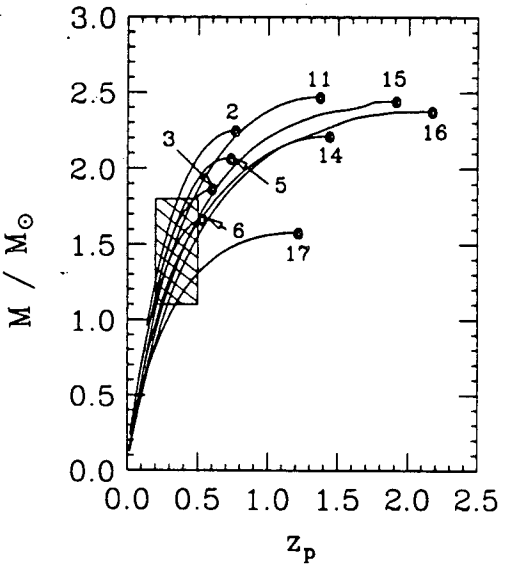

Figure 3: Rotational neutron star mass versus polar redshift. The dots refer to the limiting-mass star models.

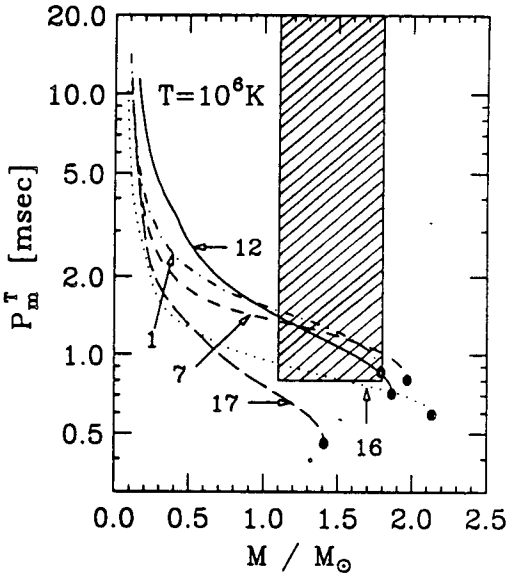

Figure 4: Rotational periods P_m^T at which instability against gravitational radiation-reaction sets in versus neutron star mass. The periods refer to old neutron stars of temperature $T = 10^6$ K.

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