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Abstract

Precession can modulate pulsar frequencies in much the same way as the doppler shift caused by orbital motion with a planetary companion. We derive a relation among observables for situations where a companion is suspected, which if satisfied, would make it difficult to distinguish planetary modulation of pulsar frequency from that caused by free precession of an isolated pulsar. Conversely, if the relation cannot be satisfied, precession is ruled out, and a planetary hypothesis thereby strenthened.

The first planets outside the solar system were discovered recently in orbit around a pulsar [1]. It seems most unlikely that this is a singular system; – surely other pulsars with one or more planetary companions exist [2, 3]. Of course planets cannot be directly seen in such distant systems. What is observed is a modulation of the pulsar period, which is interpreted to be a doppler shift due to orbital motion of the pulsar. No doubt timing residuals will now be closely scrutinized for evidence of planets as companions to pulsars. That is the main motivation for this note. Because there is another possible cause of period modulation, – the free precession of an isolated pulsar. It is possible to confuse this with modulation by a single companion if the modulation is sinusoidal. We have discussed free precession in a different context altogether, in great detail, showing how pulse drifting, nulling and mode switching might be manifestations of precession [4, 5].

Our purpose here is different and quite simple; it is (1) to show that there are circumstances in which free precession of a solitary pulsar can mimic a single planetary companion, (2) to establish a relationship involving three observable quantities, the pulsar period, the modulation period, and its amplitude, that has to hold if free precession is occuring. If the relation is satisfied, then no definite conclusion concerning a planetary companion could be readily made. If the relation cannot be satisfied, then precession is ruled out, and one has clearer evidence for a planetary hypothesis.

In cases where the timing residuals are not sinusoidal, they might still be caused by precession because it can cause a multiple infinity of *quasi-periodic* modulations. However we have not been able to devise a simple test in this case. The origin of modulation, – several planets *or* precession in the general case, – have such different

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mechanics that it seems plausible that if the orbital elements of several putative planets continue to describe the timing residuals over an extended era, then the case for planets has been made.

The most familiar example of precession is that of forced precession in an external field, such as a top, rotating and precesing on a plane surface in the earth's field. *Free* precession of an aspherical body, which a rotating pulsar must be, is less familiar and we refer to our papers, [4, 5], which deal with the case of modulation of pulses emitted around the direction of the magnetic axis of a pulsar whose symetry axis is missaligned with the angular velocity vector and hence with the angular momentum vector. Such a missalignment could be caused by accident of birth or by the action of the radiation torque on the magnetic field of the rotating pulsar. We idealize the precesion to that of a rigid body [6, 7], which is simpler to treat than the less familiar mechanics of a fluid body possibly with a crust [8, 9]. Moreover, the mixed phase of hadronic and quark matter in neutron stars is now believed to span a wide density range beginning around twice nuclear density, and it is a *Coulomb solid* [10, 11, 12, 13]. So in fact much of the interior of a neutron star can be solid. Indeed for plausible equations of state it may extend from the center to about 8 km [11].

Regular precession will occur in a rigid symmetric star (moments of inertia $I_1 = I_2 \neq I_3$) if the symmetry axis is misaligned with the angular momentum axis, say by angle β, the constant value of the Euler angle θ [6, 7]. Modulation of the pulsar signal will occur whenever the magnetic axis is not aligned with the symmetry axis. Such an alignment, for which there is no apparent cause, would be highly unlikely. Let the angle between symmetry and magnetic axis be γ. As the symmetry axis of the pulsar precesses about the fixed angular momentum axis, say with angular velocity Ω which is the constant time derivative of the Euler angle ϕ, the pulsar itself spins about its symmetry axis with the very slow angular velocity

$$n = \Omega \frac{e}{1+e} \cos\beta, \tag{1}$$

compared to the precession or pulsar angular velocity, Ω, because of the factor e, the eccentricity of the momental ellipsoid. (n is referred to, appropriately, as the "spin" in ref. [7], and is the constant time derivative of the Euler angle ψ.) This motion carries the magnetic axis, fixed in the pulsar, about the precessing symmetry axis [4]. Recall that it is generally assumed that the magnetic axis is fixed in the pulsar and that the radiation is beamed in its direction. Although the *symmetry* axis reaches the observer's azimuth at the angular frequency of the precession, because of the star's slow "spin" about its symmetry axis, the azimuth of the *magnetic* axis precedes for a few cycles, then follows, the passage of the symmetry axis across the observer's azimuthal direction as measured from the pulsar. The pulse emission times, being the times of coincidence of the azimuthal directions of observer and magnetic axis, are therefore sometimes advanced, sometimes retarded with respect to the actual period of the pulsar [4, 5, 14]. Because the spin frequency is slow compared to the precession (eq. (1)) there will be a sequence of many advanced pulses followed by a sequence of retarded ones. This introduces a modulation of the observed (not emitted) signal of a complicated form which sometimes resembles pulse drifting, nulling and mode

switching [4, 5], all of which have been observed in some pulsars [15, 16]. This is not a proof that these observed phenomena are due to precession. Perhaps there are other explanations.

The pulse frequency of a freely precessing pulsar is the time derivative of the azimuthal angle of the magnetic axis, Φ, given by eq. (6) of Ref. [4] whose constant part is the precession frequency, $\Omega/2\pi$. The modulation is given by the time dependent part of the derivative which in general is a complicated function of nt [4, 5]. If precession occurs in some pulsars, the parameters which describe it, including the above two angles, determine the form of the modulation and they are consequences of the circumstances of the pulsar's birth and later evolution. In general therefore there is a multiple infinity of modulations, and as discussed in Ref. [5], the precise intensity, timing and structure of pulses is never exactly repeated over any sequence (see eq. (2) below). There is however a quasi-periodicity, and under particular conditions it is sinusoidal [4]. Very importantly in this case, there is a relationship between the pulsar period, the modulation period and its amplitude. All three are measureable.

The general expression for the time dependent frequency of the azimuthal angle of the magnetic axis, assuming only that the star precesses with angular velocity Ω and as a consequence spins about its symmetry (or reference) axis with angular velocity n is,

$$\Omega(t) \equiv \frac{d\,\Phi(t)}{dt} = \Omega + \left\{1 + \left(\frac{\tan\gamma\sec\beta\sin(nt)}{\tan\beta + \tan\gamma\cos(nt)}\right)^2\right\}^{-1} \times \frac{n\tan\gamma\sec\beta}{\tan\beta + \tan\gamma\cos(nt)}\left\{\cos(nt) + \frac{\tan\gamma\sec\beta\sin^2(nt)}{\tan\beta + \tan\gamma\sec\beta\cos(nt)}\right\} \tag{2}$$

The modulation reduces to sinusoidal form if $\tan\gamma \ll \tan\beta$. Then

$$\Omega(t) \approx \Omega + \frac{n\tan\gamma}{\sin\beta}\cos(nt) \tag{3}$$

From this time-dependent frequency, we can read, what, for a freely precessing pulsar, would be interpreted as its period, $P = 2\pi/\Omega$, and its modulation period, $P_m = 2\pi/n$. According to (1), $P_m << P$. The amplitude of the frequency modulation is

$$\Delta\Omega \approx \frac{n\tan\gamma}{\sin\beta}, \tag{4}$$

giving the amplitude of period modulation,

$$|\Delta P| = P\frac{\Delta\Omega}{\Omega} = 2\pi\frac{n}{\Omega^2}\frac{\tan\gamma}{\sin\beta} = \frac{P^2}{P_m}\frac{\tan\gamma}{\sin\beta}. \tag{5}$$

There are altogether five observations that are relevant to the question of precession vs. planetary companion as a cause of pulsar period modulation. Three are quantitative observations; (1) pulsar period, (2) modulation period, (3) the amplitude of the modulation, ΔP. In addition there are two qualitative observations (4) the

modulation of the pulsar period is approximately sinusoidal which as noted above requires

$$\tan\gamma \ll \tan\beta \tag{6}$$

and (5) the pulsar does not undergo quasi-periodic disappearances. This restricts the angle γ to lie sufficiently close to the symmetry axis, with the observer's assumed polar angle to be also near β, so that the modulation of the polar angle of the magnetic axis (given by eq. (7) of Ref. [5]) does not carry the direction of the beam (magnetic axis) further from the observer's polar angle by more than the half-width of the emitted beam. If it did, since $n/\Omega \ll 1$ pulsar periods being much greater than orbital ones, the pulsar would be out of sight for a part of each of the long (modulation) periods on a quasi-periodic basis [4]. Widths of pulsar beams are typically 20 degrees or less [15]. So for typical pulsar half-widths we can write $\gamma \sim \pi/18$. The relation, eq. 5, which holds for sinusoidal modulation and follows from the general results if eq. (6) holds, can be rewritten

$$|\Delta P| = \frac{P^2}{P_m}\frac{\tan\gamma}{\sin\beta} = \frac{P^2}{P_m}\frac{\tan\gamma}{\tan\beta}\sqrt{1+\tan^2\beta}\,, \tag{7}$$

where we use a trigonometric identity. There are two cases here:

Case (a) $\tan^2\beta \gg 1$.
Then

$$|\Delta P| \approx \frac{P^2}{P_m}\tan\gamma < \frac{P^2}{P_m}\,. \tag{8}$$

since γ must be small, as discussed above. This is one relation that must be satisfied by observables under the case stated above.

Case (b) $\tan^2\beta \leq 1$.
In this case the trigonometric factors are small by reason of the condition of sinusoidal modulation, eq. (6) so again,

$$|\Delta P| < \frac{P^2}{P_m}\,. \tag{9}$$

Neither can any intermediate value of $\tan\beta$ subject to the sinusoidal condition provide a trigonometric factor that is anything but small. This is the test of which we spoke, where P is the pulsar period and P_m the moduation period.

The relation, (9), specifies the condition that three observables must satisfy so that precession can mimic the slow sinusoidal pulsar period modulation that might otherwise be attributed to the Doppler shift due to orbital motion. If the relation is not satisfied, precession can be ruled out and a planetary hypothesis is therefore strengthened.

We note that (1) imposes the condition on the eccentricity, $e > P/P_m \sim 10^{-9}$, for a 10 ms pulsar and a hundred day modulation period if precession were occuring.

This is a satisfactorily small lower bound. Under the same circumstances, precession could be ruled out immediately if the modultion were sinusoidal and its amplitude more than 10^{-11} sec. So the criterion is most interesting in cases where low-mass companions with tight orbits are suspected.

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