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Testing the Non-locality of Quantum Theory in Two-Kaon Systems

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Testing the Non-locality of Quantum Theory in Two-Kaon Systems*

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ABSTRACT

An idea for testing the non-local character of quantum theory in systems made of two neutral kaons is suggested. It requires detecting two long-lived or two short-lived neutral kaons in coincidence, when copper slabs are either interposed on or removed from their paths. Tests of this sort may be performed around asymmetric Φ^0-factories. They could answer some questions raised by the EPR paradox and Bell's inequalities. If they are performed and if predictions of quantum mechanics and standard theory of kaon regeneration are verified experimentally, all descriptions of the relevant phenomena in terms of local interactions will be ruled out in principle, with the exception of very peculiar ones, which imply the existence of hidden variables, of different kinds of kaons corresponding to different values of the hidden variables, and, for some of these kaons, of regeneration probabilities enhanced by a factor of the order of 500 or more over the average.

Of course, the experiment may also reveal a break down of quantum theory.

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1 Introduction

1.1 Background

The purpose of this paper is to describe a test of the non-locality of quantum theory with neutral kaons and to show the relevancy of the EPR paradox, [1]. Tests of non-locality in quantum mechanics have been extensively performed in optics and atomic physics, [2]. Their results to date all agree with the predictions of that theory. To be thorough, one should perform tests in kaon physics too. The test suggested here involves systems of two neutral kaons in a C=−1 state, such as those produced in Φ^0-factories. It is shown to probe the same kind of *non-local* effects as those revealed by the EPR paradox and Bell's inequalities, [3, 4]. If such a test is performed and if predictions of the standard theory of kaon regeneration are upheld, the results will provide evidence for non-local phenomena in systems of neutral kaons.

The relation between the EPR paradox and the $K^0\overline{K}^0$ system resulting from Φ^0-decay has been noticed and discussed in the literature, [5]. Recently, it has been claimed, [6], that a fool-proof demonstration of non-locality for systems of two neutral kaons cannot be provided by strangeness measurements at different times using a popular form of Bell inequality, [4], because the quantum mechanical predictions do not violate that inequality. That claim is not contradicted by this paper, because the test proposed here does not involve measurements of strangeness, but of the short- or long-lifetime character of the kaons with different amounts of material interposed on their path. Moreover, the test is not free of all possible loopholes since, if the results confirm quantum theory, hidden-variable theories implying very large enhancement factors in regeneration probabilities will not be ruled out.

Thus, in this test as in tests with supplementary hypotheses, quantum theory is tested only against local theories without hidden variables and local hidden-variable theories with limited enhancement factors.

1.2 The Four Experimental Setups

The test is supposed to be performed at an asymmetric Φ^0-factory, with one electron beam of 2.13 GeV colliding with a positron beam of 122 MeV. Φ^0 states are produced and, for all practical purposes, decay immediately. Some of them decay into two neutral particles, i.e., neutral kaons of mass 0.5 GeV/c^2. These kaons are moving at about the velocity of their CMS, $\beta\gamma \approx 2$, i.e. each one with a momentum of about 1 GeV/c. They separate from each other and take different paths because of their relative motion in the CMS. The kaon along one of the paths will be called κ and the one on the other path will be called κ'.

The experiment consists of measuring the number of events where κ and κ' are identified as both being K_L or both being K_S, in four different setups shown on Fig. 1. K_L is the linear combination of K^0 and $\overline{K}^0$ with a long mean-lifetime and K_S is the combination with a short mean-lifetime. The long-lifetime neutral kaons, K_L, are identified in two detectors, one on the path of the κ particle and the other on the path of κ'. These K_L detectors are detectors of nuclear interactions of neutral particles, located far enough downstream from the setup that the number of short lived K_S reaching them is negligible; because of the K_L long average lifetime, the detection efficiency of K_L's is still approximately 100%. K_S's are identified by their two-π decay mode at short distances. First we neglect ambiguous identifications, but not absorption of kaons in material.

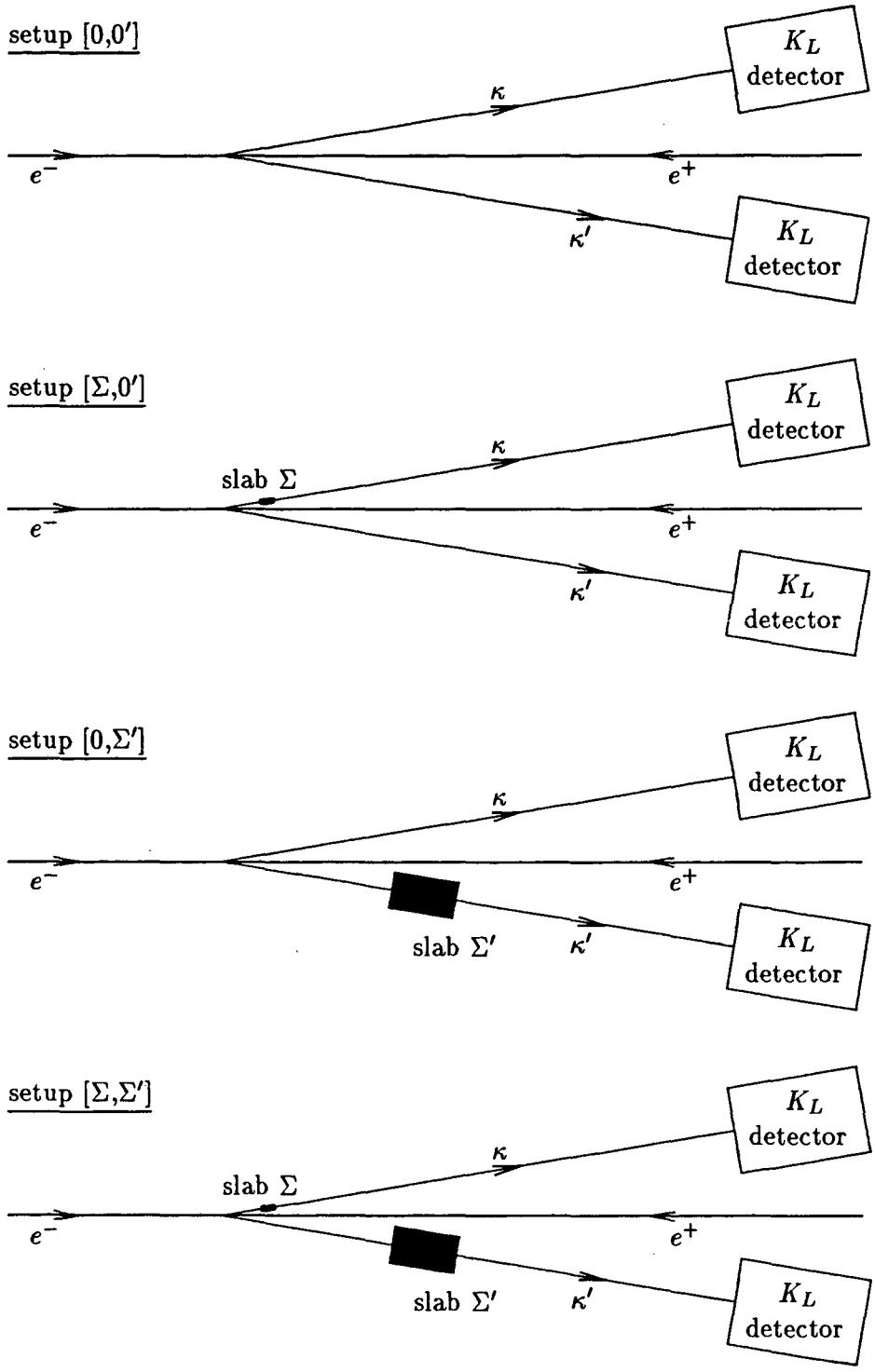

Figure 1: *The four experimental setups.*

As shown on Fig. 1, the four setups are

[0,0′] : no material between the e^+e^--collision point and the neutral particle detectors; theory predicts only events with a kaon being a K_L and the other being a K_S; no two K_L's and no two K_S's;

[Σ,0′] : a 4 mm copper slab, Σ, interposed on the path of particle κ, near the e^+e^--collision point, and no material on κ'; standard theory predicts some two-K_L events because of $K_S \rightarrow K_L$ regeneration of some κ particles in slab Σ;

[0,Σ′] : a 5 cm copper slab, Σ′, on the path of κ', occupying a space between 20 and 25 cm downstream from the collision point, and nothing on κ; theory predicts some two-K_L events because of $K_S \rightarrow K_L$ regeneration of some kaons κ' in slab Σ′;

[Σ,Σ′] : both slabs together, Σ on κ near the collision point and Σ′ on κ', 20 cm downstream from the collision point; theory predicts more two-K_L events than the number of two-K_L events predicted in setup [Σ,0′] plus the number of two-K_L events in setup [0,Σ′], because of a constructive interference effect between the two regeneration processes.

That surplus of two-K_L events is present only if both slabs are interposed, at different locations, on the paths of different particles. This will be shown to be a manifestation of these phenomena called *non-local* in quantum theory and related to the EPR paradox.

2 Regeneration Theory

2.1 Parameters of the Experiment

An asymmetric Φ^0-factory of the type considered here has the advantage over a symmetric one that the average decay range of the K_S's is longer, namely 5.4 cm. This is the same kind of advantage that asymmetric B-factories have over symmetric ones, [7]. For $\Phi^0 \rightarrow$ neutral kaons, copper slabs can be interposed on the kaon paths to regenerate a fraction of the K_S's into K_L's.

In vacuum, K_L and K_S are eigenvectors of the Hamiltonian. If a kaon is a K_L or a K_S, it remains a K_L or a K_S till it decays or interacts in a K_L detector. At this point of the paper, detection inefficiencies and ambiguities are neglected. If a kaon does not interact in a slab, or if there is no slab, the kaon is assumed to end up identified correctly as a K_L or as a K_S. However kaon absorption in the slabs is not neglected.

In each one of the experimental setups, 3×10^9 Φ^0, i.e. about 10^9 events of the type $\Phi^0 \rightarrow$ neutral kaons, are assumed to be produced. Because the Φ^0 is a C=−1 particle, [8], and because of C conservation in Φ^0-decay, the state vector of the two-kaon system at the Φ^0-decay point can be written as

$$\psi = A(K_S K'_L - K_L K'_S) \ , \tag{1}$$

where A is a normalization factor close to $\frac{1}{\sqrt{2}}$; where $K_S K'_L$ refers to a state where particle κ is a short-lived K_S and κ' is a long-lived K_L; and where $K_L K'_S$ refers to the state where κ is a K_L and κ' is a K_S.

In setup [0,0′] of Fig. 1, quantum mechanics predicts that, at all times before either kaon decays, the state vector of the system remains of the form

of Eq. (1). Then 50% of the events end up identified as

$$\kappa\kappa' = K_S K'_L \ , \tag{2}$$

and 50% as

$$\kappa\kappa' = K_L K'_S \ . \tag{3}$$

The number of $K_L K'_L$ events, i.e. with two K_L's, is

$$n_{0,0'} = 0 \ , \tag{4}$$

like the number of $K_S K'_S$ events (with two K_S's),

$$n^*_{0,0'} = 0 \ . \tag{5}$$

2.2 $K_S \rightarrow K_L$ Regeneration

In setup [Σ,0'] as in setup [0,0'], still 50% of the events have particle κ' decaying as a K_S and 50% detected as a K_L, since there is no material interposed on the path of κ'. However, in setup [Σ,0'], among the 50% in which κ' is a K_L, a small fraction has also κ detected as a K_L. That fraction is equal to the probability that κ, assumed to be a K_S initially, is regenerated into a K_L in the copper slab Σ, [9]. For 10^9 events $\Phi^0 \rightarrow K^0\overline{K}^0$, there are

$$n_{\Sigma,0'} = 9200 \tag{6}$$

such events.[1] They are of the $K_L K'_L$ type.

[1] To compute this number, the optical model has been used to get the forward scattering lengths $f(0)$ and $\overline{f}(0)$ of the K^0 and $\overline{K}^0$, respectively, by copper at 1 GeV/c, [9]:
$\frac{|f(0)-\overline{f}(0)|}{1 \text{ GeV/c}} = \frac{42 \text{ mb}}{\hbar}$

The other constants needed are well known, [8].

In setup [0,Σ'] a somewhat symmetrical situation exists. In 50% of the events, the κ particle decays as a K_S and, in the other 50%, it is detected as a K_L. In the latter 50% of the events, particle κ' is assumed to be a K_S initially. In 97.6% of these 50%, it decays as a K_S before it reaches the copper slab Σ'. The remainder may be considered as K_S's reaching the slab. In setups [0,0'] and [Σ,0'], these events are those for which κ' decays as a K_S at a distance of more than 20 cm from the e^+e^--collision point. They amount to

$$n_{0,0'}^{L,S} = 12 \text{ million} \tag{7}$$

events out of the 10^9 $\Phi^0 \rightarrow$ neutral kaons events produced.

In setup [0,Σ'], a fraction of these K_S's are regenerated into K_L's by Σ' on the path of κ', generating events with two K_L. Using the same computation and the same constants as for setup [Σ,0'], one computes their number

$$n_{0,\Sigma'} = 5540 \ . \tag{8}$$

2.3 Two Regenerators

In setup [Σ,Σ'], both regeneration processes, i.e. regeneration of κ in slab Σ and of κ' in Σ', are at work. Since slab Σ is the same in setups [Σ,Σ'] and [Σ,0'], one expects the probability of regeneration of κ from K_S to K_L to be the same in either one of these setups. However, in setup [Σ,Σ'], an event where the particle κ is regenerated from K_S to K_L ends up being a $K_L K_L'$ event only if the associated long-lived κ' remains a K_L after traversing slab Σ'. Let us call η' the probability that a K_L particle impinging on Σ' emerges as a K_L without interacting or being transformed into a K_S;

$\eta' \approx 0.59$. In setup [Σ,Σ'], using Eq. (6), the regeneration process in Σ contributes to the sample of $K_L K'_L$ events by

$$n_{\Sigma,0'} \times \eta' \ = 5430 \text{ events.} \tag{9}$$

Similarly, let us call η the probability that a K_L particle impinging on Σ emerges as a K_L after going through it; $\eta \approx 0.96$. Taking Eq. (8) into account, the regeneration process in Σ' contributes to the sample of $K_L K'_L$ events in setup [Σ,Σ'] by an amount equal to

$$n_{0,\Sigma'} \times \eta \ = 5320 \text{ events.} \tag{10}$$

Whether the regeneration happened to κ in Σ with κ' impinging on and going through Σ' as a K_L, or to κ' in Σ' with κ traversing Σ unmodified, the final state is the same: both κ and κ' are K_L states and there is no recoil particle in any slab. It is impossible to distinguish which regeneration process is responsible for the $K_L K'_L$ event. Therefore the two processes interfere. In setup [Σ,Σ'], slab Σ is close to the e^+e^--collision point, while slab Σ' is located 20 cm downstream. Regeneration of κ in Σ implies that κ' travels the 20 cm to the location of Σ' as a K_L, while, in the process where κ' is regenerated in Σ', it emerges as a K_L only after it has travelled the same distance as a K_S. That distance corresponds to 3.7 K_S mean lifetimes, while the K_L-K_S mass difference is about half the inverse K_S lifetime; it follows that there is a positive interference term, contributing to an additional number of $K_L K'_L$ events, $n_{add} = 8000$. We call these events the additional events. The total number of $K_L K'_L$ events in setup [Σ,Σ'] is

$$n_{\Sigma,\Sigma'} \ = \ n_{\Sigma,0'}\ \eta' + n_{0,\Sigma'}\ \eta + n_{add}\ , \tag{11}$$

$$\text{i.e.} \quad n_{\Sigma,\Sigma'} = 18800 \,, \tag{12}$$

i.e. more than the sum of $n_{\Sigma,0'}$ of Eq. (6) and $n_{0,\Sigma'}$ of Eq. (8). There is an increment of

$$\Delta n = n_{\Sigma,\Sigma'} - n_{\Sigma,0'} - n_{0,\Sigma'} \,, \tag{13}$$

$$\text{i.e.} \quad \Delta n = 4000 \tag{14}$$

$K_L K_L'$ events, present only when both Σ and Σ' are in place.

3 Realistic Pictures of Quantum Systems

3.1 Realism and Local Realism

Along with the computation of Sect. 2, a picture of what is happening was used. At any time, either a kaon <u>is</u> a particle that can be detected in a K_L detector if there is no regenerator further downstream, and then it is called a K_L; or it <u>is</u> a particle that decays at a short distance, and then it is called a K_S. If there is a slab on the path of κ or κ', a regeneration process is invoked, meaning that some kaons born K_S's are transformed into K_L's. This description of the $\kappa\kappa'$ system is foreign to the Copenhagen interpretation of quantum mechanics, [10]. That interpretation gives no physical meaning to descriptions of kaons before these particles actually are seen after they either decay or interact in a detector. In particular, when both κ and κ' are on the stretch between the e^+e^--collision point and the regenerators, the Copenhagen interpretation gives a meaning only to the state vector ψ, which is not an objective, but a mathematical entity depending in principle on both what the system is and what the observer knows about it.

Of course there is no harm in using a picture describing the $\kappa\kappa'$ system as an object independent of any observer, as we did in Sect. 2. It helps visualize what is happening. One can simulate how the system evolves in time from one configuration to another by Monte Carlo and different configurations represent differences in the system itself, not simply in what an alleged observer knows. If a modification in the experimental setup generates a difference in the configuration of a Monte Carlo event, it is an *influence* on the system, not just a change in our information about it.[2]

It is always possible to have an objective picture of any quantum system. Very general models have been constructed that give a picture of what quantum systems may actually be, [11, 12], and these models can reproduce all experimentally verified predictions of quantum mechanics. Physicists believing in models of that type are called *realists.* Their philosophy is called *realism.*

In setup [0,0′] of Fig. 1, there are no $K_L K'_L$ events, i.e. no events with both κ and κ' detected as K_L in the K_L detectors; but there are some in setups [Σ,0′], [0,Σ′], and [Σ,Σ′], when regenerators are interposed on the trajectories of κ and κ'. A realistic description of the phenomenon must involve an influence of these regenerators at least on some of the $\kappa\kappa'$ systems. In setup [Σ,0′], the number $n_{\Sigma,0'}$ of $K_L K'_L$ events can be explained assuming the regeneration of κ particles in slab Σ with a regeneration probability that depends only on the nature, thickness, and location of Σ. The phenomenon can be described using only *local* interactions of κ with its own environment.

[2]The significance of the word *influence* here is the same as in the description of measurements by quantum mechanics, where it is said that measurements not only inform an observer but also *influence* the system itself.

In setup [0,Σ'], similarly, the number $n_{0,\Sigma'}$ of $K_L K'_L$ events can be explained by regeneration processes of κ' involving only local interactions in Σ'. In both setups, there is need only for influences between objects at the same location. Describing phenomena in terms of objects interacting with other objects only in their immediate vicinity is a requirement of a special type of realism called *local realism.* In a Monte Carlo simulation, local realism implies that the random process that decides the fate of κ does not have a mathematical dependence on parameters describing pieces of equipment located on the path of κ'; and vice versa.[3]

In setup [Σ,Σ'], the picture used to compute the interference term n_{add} in Eq. (11) is of a different nature. The term n_{add} is a complicated function of parameters specific of Σ and of others specific of Σ', as if the phenomenon responsible for it involved a *conspiracy* between objects distant from one another. The phenomenon is not described using only local interactions. The question is whether or not there are other pictures than the one of Sect. 2, that could also account for the data but where only local interactions would be involved.

3.2 Comparison with the Two-Slit Experiment

In setup [Σ,Σ'], the computation is similar to the computation of the photon intensity in the two-slit experiment shown on Fig. 2. There, at the center of the screen, the intensity is larger than the sum of the intensity detected if

[3]There are different possible definitions of the words "local" and "locality", [13]. Some do not even mention the concept of "realism", [14]. Others are based on properties that are not Lorentz-invariant, [12]. The one used here, called "local realism", is Lorentz invariant and is most convenient to demonstrate the purpose of the two-kaon test proposed in this paper.

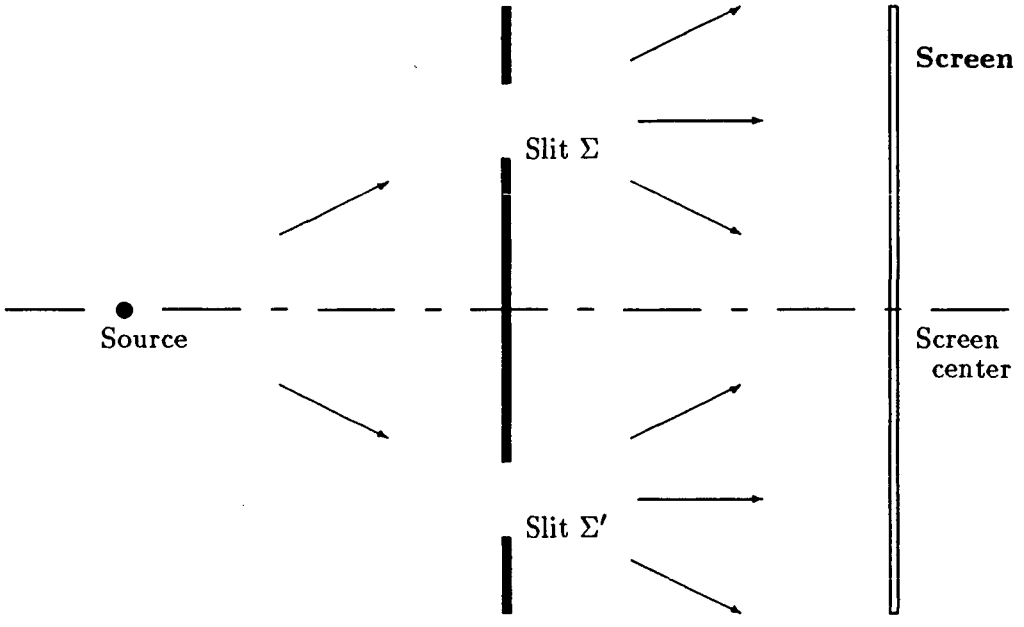

Figure 2: *Sketch of the two-slit experiment.*

only slit Σ is open and of the intensity if only slit Σ' is there. The usual *realistic* description of this effect involves waves propagating through both slits and meeting at the center of the screen where they combine and influence the photon intensity. The waves are shaped by the slits and are the carriers of that influence between the slits and the screen. That interpretation of a wave carrying an influence in the case of the two-slit experiment seems to be corroborated by the experimental fact that the amount of interference changes if a wave shifter is interposed between one of the slits and the center of the screen, suggesting that something is going through that wave shifter.

In the case of the kaons being regenerated in setup $[\Sigma,\Sigma']$, it is different. One could, in principle, place any piece of absorbent material between the two kaons after they left the e^+e^--collision point, even an extended piece of neutron star in a plane containing the e^+e^--beam of Fig. 1, and separate the setup in two parts. Any known wave or particle coming from slab Σ or from any point on the path of κ could not meet another wave or particle emitted by Σ' or any point on the path of κ' without being intercepted and perturbed, but the number of $K_L K'_L$ events including the interference term would be unchanged. If the influence is transmitted by some kind of a wave, that wave has to be far more subtle than in the two-slit experiment.

Furthermore, in a gedanken experiment where both Σ and Σ' would be located far away from the collision point and inserted at the last instant by a free-will experimenter, it is possible to get a similar interference effect but, to carry the influence in this case, one can show that the vehicle would have to travel faster than light if quantum theory is correct. This is why such interference effects are called non-local effects.

3.3 The State Vector as a Description of Reality

To get an objective picture of the system, one could assume, for instance, that a state vector ψ does not depend on the observer's knowledge, but on the system alone, and that no other quantities are needed to describe what the system is. Then ψ is a description of reality as wanted by the realists. However, for a system of two particles, a state vector ψ is an elaborate mathematical entity involving properties of two particles at different points in space. To satisfy not only realism, but also local realism, the description of the system of particles should be made of parts that each can be assigned a location in space, and that do not get modified instantaneously by what happens at a distance. The state vector ψ does not qualify. This was noticed by Heisenberg in 1930 already, [15].

The two-kaon experiment can be used to illustrate Heisenberg's argument. Consider the Monte Carlo simulation of events in setup [0,0′], as they unfold as a function of time. Before the first kaon decays, the state vector is of the form of Eq. (1) for all events. The random process responsible for the decay depends only on ψ, thus it is the same for all events. It gives equal chances to κ and to κ' to decay as a K_S during the next time interval dt. As soon as one particle decays as a K_S, the probability for the other to decay likewise becomes instantaneously zero. The same random process affects both the decay of the first particle and, immediately, the decay probability of the second particle located somewhere else. In effects of this kind, Heisenberg sees an action faster than light.

In setup [0,0′] alone, local realism can be restored easily. One can assume that the description of the two-kaon system requires more physical quantities

than the state vector alone. For instance, in the Monte Carlo simulation, at the time of the Φ^0-decay, one can generate two parameters that later determine the decay modes and the decay times of κ and κ', one as a K_L and one as a K_S. One can consider each of these two parameters as one of the characteristics of the corresponding kaon, to be carried by the particle until it decays or interacts in a detector. Then there is no need for non-local action. However these decay modes and times are not quantities determined by the state vector of the system at the Φ^0-decay time. They are features added for a complete description of the system and they are unknown before decay.[4] They are called *hidden variables.*

3.4 "Local" Monte Carlo Simulations

To describe the impact of *realism* and of *local realism* on a theory, we invoked Monte Carlo simulations of the experiment that the theory describes. Consider the generation of events in setup [0,0′] of Fig. 1, i.e. without copper slab on the path of any particle. The simulation generates a list of events numbered 1 ... j ... 10^9, where the fate of particles κ and κ' is recorded. Each one is either detected in a K_L detector; or it is identified as a K_S because its decay satisfies the experimental criteria set for K_S; or it is ambiguous or not detected at all.[5] Let us call κ_j and κ'_j the κ and the κ'

[4]This is another case like the one described by EPR, [1], where adding quantities to the description of a quantum system can save local realism.

[5]The purpose of taking ambiguous kaon decays into account here is to make the definitions enunciated below appropriate for future analyses in Sect. 5. Except for this reason, ambiguous events are still neglected at this point of the paper.

particles of the jth event, respectively. We write $\kappa_j = K_L$ if κ is identified as a K_L in event j; or $\kappa_j = K_S$ if κ is identified as a K_S; or $\kappa_j = K_{else}$ if κ ends up being an ambiguous or an undetected particle.[6] Similarly, we write $\kappa'_j = K'_L$, or K'_S, or K'_{else}, if κ' is identified as a K_L, or a K_S, or neither.

Our definition of *realism* implies that it is possible to simulate the system itself, not just what we know about it. *Local realism* implies another property in addition; as it is mentioned in Sect. 3.1, *local realism* assumes the existence of a Monte Carlo simulation, where features that can be observed on the path of κ are *independent* of the kind of equipment placed on the path of κ'; and vice versa.[7] After generating events in setup [0,0′], the generation of events in setup [Σ,0′] does not require changing κ'_j since everything on the path of κ' is the same, while, of course, it may require changing κ_j since κ now traverses slab Σ. Therefore the list of κ'_j in setup [Σ,0′] has the same values K'_L, K'_S, or K'_{else}, as in setup [0,0′]. Similarly, in setup [0,Σ′], the list of κ_j is the same as in setup [0,0′], for the same reason. Following the same argument again, once the list of κ_j has been altered to take slab Σ into account in setup [Σ,0′] and once κ'_j has been modified to take Σ′ into account in setup [0,Σ′], the sequence made of these two new lists put together gives a valid simulation of the experiment in setup [Σ,Σ′]. We end up with four sequences of 10^9 Monte Carlo events numbered j where, for each j, the states $\kappa\kappa'$ satisfy the following <u>independence conditions</u>:

Σ′ independence <u>without</u> slab Σ: κ identical in setups [0,0′] and [0,Σ′];

[6]Note that, in Monte Carlo computations, if a particle is not detected, it is still known that it exists.

[7]In the mathematical sense of the word *independent*: the selection of a random number and the algorithm used to generate an observable feature of κ does not depend, *mathematically* on the parameters defining material laying on the path of κ'; and vice versa.

with slab Σ: κ identical in setups [Σ,0′] and [Σ,Σ'];

Σ independence without slab Σ': κ' identical in setups [0,0′] and [Σ,0′];

with slab Σ': κ' identical in setups [0,Σ'] and [Σ,Σ'].

Bell's inequality is a consequence of these independence conditions.

4 Bell's Inequality

4.1 The Additional Events

First let us generalize the concept of additional event of Sect. 2.3. Consider any Monte Carlo simulation of events in the four setups of Fig. 1 and the corresponding four sequences of events $\kappa_j\kappa'_j$. Let us define an *additional* event as one that, for a given event index j, is

a) type $K_L K'_L$ in setup [Σ,Σ'], with both slabs Σ and Σ' in place; but

b) not type $K_L K'_L$ in setup [Σ,0′] with Σ' removed; and

c) not type $K_L K'_L$ in setup [0,Σ'], where Σ is removed instead of Σ'.

In the realistic picture of Sect. 2, such event is an event counted in $n_{\Sigma,\Sigma'}$ of Eq. (11); but neither among the $n_{\Sigma,0'} \times \eta'$ events of Eq. (9) where κ is regenerated into a K_L by Σ and κ' survives as a K_L after traversing Σ'; nor among the $n_{0,\Sigma'} \times \eta$ events of Eq. (10) where κ' is regenerated into a K_L by Σ' and κ survives as a K_L after traversing Σ. In that picture, it is indeed one of those $K_L K'_L$ additional events in setup [$\Sigma\Sigma'$], due to the interference term in Eq. (11). That is the reason why, in any simulation, we give events

with properties a), b), and c) above the same name, *additional events*, as in Sect. 2.3. Their number is still called n_{add}.[8]

The definition given here of *additional* events means that the sample of $n_{\Sigma,\Sigma'}$ events of type $K_L K'_L$ in setup $[\Sigma,\Sigma']$ consists of those n_{add} *additional* events, and of other events, which are among the $K_L K'_L$ events in setups $[\Sigma,0']$ or $[0,\Sigma']$. It follows that

$$n_{\Sigma,\Sigma'} \leq n_{add} + n_{\Sigma,0'} + n_{0,\Sigma'} \ . \tag{15}$$

Therefore, using Eq. (13),

$$n_{add} \geq \Delta n \ . \tag{16}$$

Next let us consider the effect of local realism. If a theory is *local*, there are Monte Carlo simulations producing sequences of events satisfying the independence conditions of Sect. 3.4. Item a) of the definition of *additional* events and the Σ'-independence condition of Sect. 3.4 require that, after removing slab Σ', κ still be a K_L in setup $[\Sigma,0']$ as in setup $[\Sigma,\Sigma']$. Let us again assume 100% efficiency and no ambiguity in the detection of K_L's and K_S's. This assumption and item b) of the definition of *additional* events imply that the kaon κ' of these events is a K_S in setup $[\Sigma,0']$. Then, because of the Σ-independence condition, κ' is a K_S after removing Σ in setup $[0,0']$ as in setup $[\Sigma,0']$. Similarly, using first item a) and the Σ-independence condition, then item c), then the Σ'-independence condition, one sees that κ is a K_S in setup $[0,\Sigma']$ and $[0,0']$. *Additional* events are part of the $n^*_{0,0'}$

[8] Note that n_{add} depends in general on the theory behind the Monte Carlo simulation, even when the different theories predict the same numbers of event types in each setup. See Appendix A.

events of type $K_S K'_S$ in setup [0,0′]. Therefore, whether or not $n^*_{0,0'}$ has the value predicted by Eq. (5), local realism requires

$$n_{add} \leq n^*_{0,0'} \ . \tag{17}$$

From Ineqs. (16) and (17), one can derive

$$\Delta n = n_{\Sigma,\Sigma'} - n_{\Sigma,0'} - n_{0,\Sigma'} \leq n^*_{0,0'} \ . \tag{18}$$

Ineq. (18) is a form of Bell's inequality, [3]. An inequality similar to (18) can be derived with the number of $K_S K'_S$ events substituted to the number of $K_L K'_L$ events, and vice-versa in each setup. Then this new inequality and Ineq. (18) can be added. If the absorption of kaons in the slabs could be neglected, the resulting inequality would be equivalent to the form of Bell's inequality that can be found in Ref. [4].

4.2 Testable Inequality

For a Monte Carlo simulation abiding with the principles of *local* realism, Sect. 3.4 gives a recipe to construct sequences of events satisfying the independence conditions. In such Monte Carlo sequences, *additional* events can be identified and their number n_{add} satisfies Ineqs. (16) and (17). In an experiment, where data is gathered in one setup after the other, the chance to get sequences of events satisfying the independence conditions of Sect. 3.4 is almost zero, even if there is a *local* theory that can explain the data.[9] Furthermore, in general, there are several theoretical pictures that,

[9] However that probability is not zero absolutely, since all outputs of the Monte Carlo are possible outcomes of the experiment. Such four sequences can be considered in a

in the Monte Carlo, give identical numbers of event types in each setup, but different n_{add} when the sequences for the different setups are compared. See Appendix A. Therefore n_{add} is not a quantity provided by experiment. Ineq. (17) which uses n_{add} explicitly, is not adequate for an experimental test.

Ineq. (18), on the other hand, is a constraint imposed on numbers of events $n^*_{0,0'}$, $n_{\Sigma,\Sigma'}$, $n_{\Sigma,0'}$, and $n_{0,\Sigma'}$, which each depends on the Monte Carlo data computed for only one setup. The values obtained by Monte Carlo for these quantities should be equal, within statistical error, to the values obtained in experiments performed with the same setup. In experiments where lack of detection efficiency and ambiguities can be neglected, $n^*_{0,0'}$, $n_{\Sigma,\Sigma'}$, $n_{\Sigma,0'}$, and $n_{0,\Sigma'}$ *can* be measured and introduced in Ineq. (18).[10]

gedanken experiment, as it was considered in Ref. [16]. There, the possibility of such four sequences obtained this way was used in an alternate formulation of the definition of *locality* for a theory. The definition implied the possibility to find an experiment (even gedanken) with such four sequences as an outcome and, at the same time, with averages of number of events in the various setups that do not show excessive deviations from the expectation values of the theory.

This *not quite zero* probability also legitimates a definition of *locality* based on *counterfactual outcomes* of an experiment, [13, 14, 16]. That definition means : suppose an experimenter can set a parameter anyway he chooses to; suppose the experiment is performed with that parameter set at a particular setting and yields a particular value for an observable measured somewhere else; if the parameter has *no influence* on the observable, the same value would have been taken by the observable.

This interpretation of the word *influence* implies what is called *operational determinism* by some philosophers, [17], though this philosophy is often adopted when no deterministic theory is even assumed to exist, e.g. in cases of human beings taking decisions and assumed not to be *influenced* by facts that they do not know about.

[10]Note that absorption of kaons in the slabs does not have to be neglected though.

Ineq. (18) is an inequality that the data must satisfy if the physical processes involved can be simulated by Monte Carlo with the independence conditions of Sect. 3.4, i.e. with κ simulated identically whether slab Σ' is in place or not, and κ' independently of Σ.

If the data agrees with the predictions of Eqs. (5) and (14), Ineq. (18) is violated. The predictions of standard regeneration theory and quantum mechanics are not compatible with those of any local theory. In principle the test suggested here should should reveal a violation either of regeneration theory and quantum mechanics, or of local realism. However there are uncertainties associated with the experimental determinations of $n^*_{0,0'}$, $n_{\Sigma,\Sigma'}$, $n_{\Sigma,0'}$, and $n_{0,\Sigma'}$ because of ambiguous decays, which we have ignored so far. These ambiguities may spoil the violation of Bell's inequality, (18), because, among the 10^9 $\Phi^0 \rightarrow K^0\overline{K}^0$ events involved in the experiment, the number of ambiguous decays is of the order of or greater than Δn, predicted by Eq. (14). The effect of ambiguous decays has to be taken into account in the experimental determination of Δn.

4.3 Direct Observations

From now on we stop neglecting ambiguous kaon decays. Contrarily to what we have assumed so far, in an experiment, one cannot always tell for sure if a kaon is a K_L or a K_S.[11] However, assuming that K_L's and K_S's behave as in standard theory, the number of K_L's and the number of K_S's can be determined statistically by analyzing time decay-distributions and decay modes. Let us distinguish three possible direct observations on kaons :

[11] In quantum theory, K_L's and K_S's states are not even completely orthogonal in Hilbert space when CP is not conserved.

"K_L" : kaon surviving without decay for more than 80 cm, or via the semi-leptonic mode at a distance of more then 50 cm from the e^+e^--collision point, or decaying via the three-π mode;

"K_S" : kaon decaying via the two-π mode at a distance inferior to 80 cm from the collision point;

"K_{else}" : anything else.

From the known decay properties of kaons, [8], one can estimate that 99% of the states said to be K_L contribute to the "K_L" sample and 8×10^{-5} of them to "K_S"; 99.9% of the states said to be K_S contribute to the "K_S" sample and 3×10^{-7} of them to "K_L". From the numbers of direct observations of "K_L" and "K_S", making adequate corrections, one can infer the number of K_L's and of K_S's in each conditions. This can be done for κ and for κ', therefore for each event type, to determine the numbers $n^*_{0,0'}$, $n_{\Sigma,\Sigma'}$, $n_{\Sigma,0'}$, and $n_{0,\Sigma'}$ of $K_S K'_S$ or $K_L K'_L$ events in the four setups.

4.4 Supplementary Assumption

Ineq. (18) for local theories was arrived at by analyzing *additional* events, which are defined by properties a) to c) of Sect. 4.1, and that satisfy the independence conditions of Sect. 3.4. Such *additional* events can always be identified in a *local* Monte Carlo simulation where κ and κ' can be recognized as either a K_L or a K_S in each event. For local theories, let us make the following supplementary assumption :

> κ and κ' particles can be given a K_L or a K_S label in the record of each Monte Carlo event, with probabilities of K_L and K_S to end

up as "K_L" or "K_S" equal to the ones assumed when analyzing the experimental data; or

the simulation deviates from this but by so little that the difference is within the statistical error of the experiment.

Then *additional* events can be identified in the Monte Carlo and the demonstration of Sect. 4.1 is relevant. If the theory is *local*, the numbers $n^*_{0,0'}$, $n_{\Sigma,\Sigma'}$, $n_{\Sigma,0'}$, and $n_{0,\Sigma'}$ determined from the Monte Carlo should satisfy Ineq. (18), and so should the experimental determinations of $n^*_{0,0'}$, $n_{\Sigma,\Sigma'}$, $n_{\Sigma,0'}$, and $n_{0,\Sigma'}$ by the statistical analysis described above of the data of the experiment. If the experimentally determined values of $n^*_{0,0'}$, $n_{\Sigma,\Sigma'}$, $n_{\Sigma,0'}$, and $n_{0,\Sigma'}$ turn out to agree with the standard predictions of Eqs. (5), (6), (8), and (12), they violate Ineq. (18). Thus *local* theories that satisfy the supplementary assumption are ruled out.

One can imagine theories and simulations where some of the kaons end up as "K_L", "K_S", or "K_{else}" without necessarily being K_L or K_S in an intermediate state. In particular, leptonic decays at short distances are ambiguous and do not have be assigned to a K_S or a K_L particle. Such theories may not abide with the supplementary assumption and, even if they are *local*, they are not necessarily ruled out if the experimental data violates Ineq. (18).

The class of local theories satisfying our supplementary assumption is limited but not empty. If it were not for that 1% of ambiguous decays where K_L does not end up as "K_L", and that 0.1% where K_S does not end up as "K_S", each K_L and K_S states could be identified. Then $n^*_{0,0'}$, $n_{\Sigma,\Sigma'}$, $n_{\Sigma,0'}$, and $n_{0,\Sigma'}$ could be measured directly. The predictions of quan-

tum theory would have to be wrong, if any *local* theory were correct. In the presence of ambiguous decays, if nature designed kaon physics in conformity with the principles of *local realism*, it would have to manipulate these "ambiguous" modes very carefully not to let this show by a violation of quantum mechanics in a test like this one. Nature may just not be so "tricky". Therefore a test by Ineq. (18) is a *bona fide* test of quantum theory in kaon physics, in circumstances where the non-local effects of quantum mechanics are involved.

In the same spirit, one could make the test less difficult experimentally, by adding another supplementary assumption. If quantum theory is right and there is no violation of charge conjugation in Φ^0-decay, the number of $K_S K'_S$ events is the same as the number of $K_L K'_L$ events in setup [0,0']. Then $n^*_{0,0'}$ is the same as $n_{0,0'}$. This is true even if the Φ^0 particle has a large branching ratio into $K^0\overline{K}^0\ \gamma$. Nature may abide by the principles of *local realism* in kaon physics and not hide the violation of quantum mechanics that this entails behind another violation, i.e. behind a violation of C. Therefore one can test local realism using $n_{0,0'}$, which is probably easier to measure than $n^*_{0,0'}$, and introduce it instead of $n^*_{0,0'}$ in Ineq. (18).

5 Tests Using Direct Observations

The issues are clearer if the test is expressed by inequalities imposed on quantities that are directly observable and not, like $n^*_{0,0'}$, $n_{\Sigma,\Sigma'}$, $n_{\Sigma,0'}$, and $n_{0,\Sigma'}$, inferred from the data using a supplementary assumption. In the record of the Monte Carlo event, there must be parameters that specify what is finally observed. Then, using such parameters, entities like *additional* events

can be defined in every simulation and analyzed as in Sect. 4.1. Unfortunately predictions concerning these direct observations do not have such clear cut properties as for K_L and K_S. Because of ambiguous decays, some complications arise.

5.1 Modified Bell's Inequality

First let us adjust Ineq. (18) so it can be used with directly measurable quantities.

5.1.1 "Additional" Events

Consider the possible direct observations of kaons, "K_L", "K_S", and "K_{else}", defined in Sect. 4.3. Let us define two-kaon states "$K_L K'_L$", "$K_L K'_S$", "$K_L K'_{else}$", "$K_S K'_L$", and so on, as states where κ is observed like the unprimed K-symbol and κ' like the primed one.

In a Monte Carlo simulation of experimental data in the four setups of Fig. 1, "additional" events are defined as those that are "$K_L K'_L$" in setup [Σ,Σ']; but not in setup [Σ,0$'$] for the same event number j; and not in setup [0,Σ'] either. There are "n_{add}" such events. Let us also define "$n_{0,0'}$", "$n_{\Sigma,0'}$", "$n_{0,\Sigma'}$", and "$n_{\Sigma,\Sigma'}$" as the numbers of "$K_L K'_L$" events in setups [0,0$'$], [Σ,0$'$], [0,Σ'], and [Σ,Σ'], respectively,

$$\text{“}\Delta n\text{”} \;=\; \text{“}n_{\Sigma,\Sigma'}\text{”} \;-\; \text{“}n_{\Sigma,0'}\text{”} - \;\text{“}n_{0,\Sigma'}\text{”}\;. \tag{19}$$

Using a similar argument as in Sect. 4.1, one gets relations analogous to Ineq. (15), thus to Ineq. (16),

$$\text{“}n_{add}\text{”} \;\geq\; \text{“}\Delta n\text{”}\;. \tag{20}$$

Category	Setup [0,0′]	Setup [Σ,0′]	Setup [0,Σ′]	Setup [Σ,Σ′]
I_{add}	"$K_S K'_S$"	"$K_L K'_S$"	"$K_S K'_L$"	"$K_L K'_L$"
II_{add}	"$K_{else} K'_S$"	"$K_L K'_S$"	"$K_{else} K'_L$"	"$K_L K'_L$"
III_{add}	"$K_S K'_{else}$"	"$K_L K'_{else}$"	"$K_S K'_L$"	"$K_L K'_L$"
IV_{add}	"$K_{else} K'_{else}$"	"$K_L K'_{else}$"	"$K_{else} K'_L$"	"$K_L K'_L$"

Table 1: *Categories of "additional" events and their types in the four different setups.*

Now, let us assume that the Monte Carlo simulation satisfies the independence conditions of Sect. 3.4, and let us follow the same reasoning as in Sect. 4.1. Table 1 shows all the possible types that an "additional" event can have in each setup. It is clear that all "additional" events are included in the sample formed by the union of three sets: the "$n^{S,S}_{0,0'}$" events of type "$K_S K'_S$" in setup [0,0′], which include category I_{add}; the "$n^{else,S}_{0,0'}$" events of type "$K_{else} K'_S$" in setup [0,0′], which include category II_{add}; and the "$n^{L,else}_{\Sigma,0'}$" events of type "$K_L K'_{else}$" in setup [Σ,0′], which include categories III_{add} and IV_{add}. One can write

$$\text{“}n_{add}\text{”} \quad \leq \quad \text{“}n^{*}_{0,0'}\text{”} \; + \; \text{“}n^{L,else}_{\Sigma,0'}\text{”} \; , \tag{21}$$

$$\text{where} \quad \text{“}n^{*}_{0,0'}\text{”} \quad = \quad \text{“}n^{S,S}_{0,0'}\text{”} \; + \; \text{“}n^{else,S}_{0,0'}\text{”} \; . \tag{22}$$

Note that, to include all categories, one may omit events in which the particle κ' decays at a distance smaller than 20 cm from the e^+e^--collision point. The reason is that, in a local theory, the kaon κ' is assumed not to be affected by the presence or absence of copper slab Σ' before it reaches the place where that slab is or would be. If κ' decays at a distance inferior to

20 cm in setups [0,0′] or [Σ,0′], it would do so in setup [Σ,Σ′] too, thus cannot belong to an "additional" event according to the definition of this section. From now on we will discard κ' decays at less than 20 cm altogether.

From Ineqs. (20) and (21), another form of Bell inequality is obtained,

$$\text{“}\Delta n\text{”} \leq \text{“}n^{*}_{0,0'}\text{”} \quad + \quad \text{“}n^{L,else}_{\Sigma,0'}\text{”} \ . \tag{23}$$

where "$n^{L,else}_{\Sigma,0'}$" is equal within statistical error to a quantity measurable in an experiment with only one setup, and where "Δn" and "$n^{*}_{0,0'}$" can be determined from quantities shown in Eqs. (19) and (22), each of which is measurable because it depends on events counted in only one setup. Ineq. (23) is an experimentally verifiable inequality, involving only direct observations and requiring no supplementary assumption.

5.1.2 Predictions of Regeneration Theory

A likely outcome of the experiment is one that upholds the predictions of quantum mechanics and standard regeneration theory. To have an impact on the largest possible range of local theories, the best experiments are those where quantum mechanical predictions would violate Ineq. (23). Then all *local* theories would be ruled out. The interpretation of such experiments is *loophole-free.*

Taking ambiguous decay modes into account but still assuming the detector to be perfect in its identification of the decay times and the decay modes, standard theory of Sects. 2.2 predicts :

$$\text{“}n_{0,0'}\text{”} \quad = \quad 440 \ ; \tag{24}$$

$$\text{“}n_{\Sigma,0'}\text{”} \quad = \quad 9360 \ ; \tag{25}$$

$$\text{“}n_{0,\Sigma'}\text{”} = 5680 \; ; \tag{26}$$

$$\text{“}n_{\Sigma,\Sigma'}\text{”} = 18570 \; ; \tag{27}$$

$$\text{thus} \quad \text{“}\Delta n\text{”} = 3530 \; ; \tag{28}$$

$$\text{and} \quad \text{“}n_{0,0'}^{S,S}\text{”} = 28800 \; ; \tag{29}$$

$$\text{“}n_{0,0'}^{else,S}\text{”} = 126000 \; ; \tag{30}$$

$$\text{“}n_{\Sigma,0'}^{L,else}\text{”} = 13600 \; ; \tag{31}$$

$$\text{thus} \quad \text{“}n_{0,0'}^{*}\text{”} = 155000 \; . \tag{32}$$

Equations (28), (31), and (32) show that, for the experiment and the experimental parameters described in this paper, predictions based on regeneration theory do not violate Bell's inequality (23). This experiment is not *loophole-free*. However Ineq. (23) would be violated if one could find conditions in which there were more $K_S \rightarrow K_L$ regeneration before the K_S's decay. Since all parameters have not been optimized, it may turn out that a loophole-free experiment of this kind, but with different parameters, is possible. Further study is necessary before one can know whether or not this is true.

However, if the predictions of standard regeneration are upheld, the local theories satisfying the supplementary assumption of Sect. 4.4 will be ruled out by the test described in Sect. 4.4. In addition, other local theories will also be ruled out if the tests described in the next sections are performed.

5.2 Theories without Hidden Variables

5.2.1 Samples of Single Kaons

Consider now *local* theories that describe kaons without hidden variables, i.e. as linear superpositions of two basic states, as in one-particle quantum mechanics applied to single kaons. With such theories, four sequences of Monte Carlo events, one for each setup, can be generated satisfying the independence conditions of Sect. 3.4 and, for each configuration of particle κ, the state of κ' is described by a density matrix in a two-dimensional Hilbert space. By selecting Monte Carlo events on criteria based on observations on κ, one defines samples of κ' particles with decay properties determined by a density matrix. Then any two observations made on any two samples of particles κ' satisfy an inequality, (59), shown in Appendix B. That inequality and *local* realism impose constraints on measurable quantities, which can be tested in our experiment.

For Ineq. (59) of Appendix B, let us consider two simulated "measurements" made on particles κ' generated by Monte Carlo :

$M_{\Sigma'}$: slab Σ' *in place*; outcome considered: "K'_L"; because of local realism, the outcome is the same in setups [0,Σ'] and [Σ,Σ'];

M_0: slab Σ' *removed*; outcome considered: "K'_L"; because of local realism, the outcome is the same in setups [0,0$'$] and [Σ,0$'$].

For Ineq. (59), let us also consider two samples, (A) and (B), of κ' particles that belong to events abiding with the independence conditions of Sect. 3.4 and that reach the location of slab Σ' before they decay. That decay criterion should not invalidate the description of the samples of surviving

kaons κ' by a density matrix. Aside of that decay requirement, the selection of the samples of κ' is based only on observations made on the other particle, κ. The selection criteria are :

(A) : κ' particles associated with κ's seen as "K_L" when slab Σ is not there, i.e. in setups [0,0′] and [0,Σ′];

(B) : κ' particles belonging to events where, in the four sequences of Monte Carlo events, κ is a "K_S" or a "K_{else}" without slab Σ, i.e. in setups [0,0′] and [0,Σ′], but where κ is a "K_L" with slab Σ in place, i.e. in setups [Σ,0′] and [Σ,Σ′].

5.2.2 Probabilities and Limits on Probabilities

The probabilities $p^{(A)}_{\Sigma'}$ and $p^{(B)}_{\Sigma'}$ we want to introduce in Ineq. (59) can be determined from selecting the events belonging to sample (A) or (B), then dividing the number of events where κ' is a "K'_L" in setups with slab Σ' by the total number of events. Similarly for $p^{(A)}_0$ and $p^{(B)}_0$ without slab Σ. For the numbers of events selected with criteria defined in more than one setup, no determination is possible from experiment. However, for them, in the Monte Carlo, one can derive limits that are functions of quantities depending on criteria in only one setup thus equal to the same quantities measured in the experiment.

The limits are justified from properties of *local* Monte Carlo simulations according to Sect. 3.4. Their numerical value depends on the experimental data and can be obtained using expressions given hereafter. The value expected for them from standard regeneration theory is also given, after a $\stackrel{?}{=}$ sign.

The number of events in sample (A), $n^{(A)}$, is equal to the sum of the numbers "$n_{0,0'}$" of "$K_L K'_L$", "$n^{L,S}_{0,0'}$" of "$K_L K'_S$", and "$n^{L,else}_{0,0'}$" of "$K_L K'_{else}$" events, in setup [0,0'] or [0,Σ']. The predicted value is given in Eq. (7).

$$n^{(A)} = \text{“}n_{0,0'}\text{”} + \text{“}n^{L,S}_{0,0'}\text{”} + \text{“}n^{L,else}_{0,0'}\text{”} \overset{?}{\approx} n^{L,S}_{0,0'} \overset{?}{=} 12 \text{ million} . \tag{33}$$

The probability that is associated with $M_{\Sigma'}$, i.e. of κ' being observed as a "K_L" with slab Σ' *in place*, is given by the ratio of the number of "$K_L K'_L$" events in setup [0,Σ'] to $n^{(A)}$:

$$p^{(A)}_{\Sigma'} = \frac{\text{“}n_{0,\Sigma'}\text{”}}{n^{(A)}} \overset{?}{=} \frac{5680}{12 \text{ million}} = 5\ 10^{-4} . \tag{34}$$

The probability corresponding to M_0, i.e. of κ' being observed as a "K_L" when slab Σ' is *removed*, is the probability of having a "$K_L K'_L$" event in setup [0,0'] :

$$p^{(A)}_{0} = \frac{\text{“}n_{0,0'}\text{”}}{n^{(A)}} \overset{?}{=} \frac{440}{12 \text{ million}} = 4\ 10^{-5} . \tag{35}$$

As to sample (B), Table 2 shows all the categories of events belonging to that sample and their event types in the various setups, taking the independence conditions of Sect. 3.4 into account. The number $n^{(B)}$ of events in sample (B) is less than the sum of all events that are "$K_S K'_S$" or "$K_{else} K'_S$" in setup [0,0'], which include categories $\mathrm{II}_{(B)}$ and $\mathrm{V}_{(B)}$ of Table 2, and of all events that are "$K_L K'_L$" or "$K_L K'_{else}$" in setup [Σ,0'], which include categories $\mathrm{I}_{(B)}$ and $\mathrm{IV}_{(B)}$, as well as $\mathrm{III}_{(B)}$ and $\mathrm{VI}_{(B)}$. Eqs. (25), (29), (30) and (31) give the predicted value,

$$n^{(B)} \leq \text{“}n^{S,S}_{0,0'}\text{”} + \text{“}n^{else,S}_{0,0'}\text{”} + \text{“}n_{0,\Sigma'}\text{”} + \text{“}n^{L,else}_{\Sigma,0'}\text{”} \overset{?}{=} 178000 . \tag{36}$$

For sample (B) and $M_{\Sigma'}$, the probability $p^{(B)}_{\Sigma'}$ that κ' ends up as a "K_L" with slab Σ' *in place* is the sum of the contributions of categories $\mathrm{I}_{(B)}$, $\mathrm{II}_{(B)}$, and

Category	Setup [0,0′]	Setup [Σ,0′]	Setup [0,Σ′]	Setup [Σ,Σ′]
$\mathrm{I}_{(B)}$	"$K_{S/else}K'_L$"	"$K_L K'_L$"	"$K_{S/else}K'_L$"	"$K_L K'_L$"
$\mathrm{II}_{(B)}$	"$K_{S/else}K'_S$"	"$K_L K'_S$"	"$K_{S/else}K'_L$"	"$K_L K'_L$"
$\mathrm{III}_{(B)}$	"$K_{S/else}K'_{else}$"	"$K_L K'_{else}$"	"$K_{S/else}K'_L$"	"$K_L K'_L$"
$\mathrm{IV}_{(B)}$	"$K_{S/else}K'_L$"	"$K_L K'_L$"	"$K_{S/else}K'_{S/else}$"	"$K_L K'_{S/else}$"
$\mathrm{V}_{(B)}$	"$K_{S/else}K'_S$"	"$K_L K'_S$"	"$K_{S/else}K'_{S/else}$"	"$K_L K'_{S/else}$"
$\mathrm{VI}_{(B)}$	"$K_{S/else}K'_{else}$"	"$K_L K'_{else}$"	"$K_{S/else}K'_{S/else}$"	"$K_L K'_{S/else}$"

Table 2: *Categories of events belonging to sample (B) and their types in the four different setups;* "$K_{S/else}$" *means* "K_S" *or* "K_{else}".

$\mathrm{III}_{(B)}$ of Table 2. Events of type "$K_L K'_L$" in setup [Σ,Σ′] are either events of these categories, or events that do not belong to sample (B) because they have κ as a "K_L" without slab Σ, i.e. events of type "$K_L K'_L$" in setup [0,Σ′] too. Predicted values are given by Eqs. (26) and (27).

$$p^{(B)}_{\Sigma'} \geq \frac{\text{“}n_{\Sigma,\Sigma'}\text{”} - \text{“}n_{0,\Sigma'}\text{”}}{n^{(B)}} \stackrel{?}{=} \frac{12900}{178000} = 0.072 \ . \tag{37}$$

Finally, for M_0 on sample (B), the probability of κ' being a "K_L" when slab Σ′ is *removed*, is given by events of categories $\mathrm{I}_{(B)}$ and $\mathrm{IV}_{(B)}$ of Table 2, which are fewer than all "$K_L K'_L$" events in setup [Σ,0′]. Eqs. (25) and Ineq. (37) give a prediction.

$$\frac{p^{(B)}_0}{p^{(B)}_{\Sigma'}} \leq \frac{\text{“}n_{\Sigma,0'}\text{”}}{n^{(B)}} \frac{n^{(B)}}{\text{“}n_{\Sigma,\Sigma'}\text{”} - \text{“}n_{0,\Sigma'}\text{”}} = \frac{\text{“}n_{\Sigma,0'}\text{”}}{\text{“}n_{\Sigma,\Sigma'}\text{”} - \text{“}n_{0,\Sigma'}\text{”}} \stackrel{?}{=} \frac{9360}{12900} = 0.73 \ . \tag{38}$$

5.2.3 Experimental Test

If the local theory describes kaons without hidden variables, it has to predict probabilities $p_{\Sigma'}^{(B)}, p_0^{(B)}, p_0^{(A)}$, and $p_{\Sigma'}^{(A)}$ that would satisfy Ineq. (59), i.e. an inequality that can be rewritten as

$$\left(1 - \sqrt{\frac{p_0^{(B)}}{p_{\Sigma'}^{(B)}}}\right) \sqrt{p_{\Sigma'}^{(B)}} \leq \sqrt{p_0^{(A)}} + \sqrt{p_{\Sigma'}^{(A)}} \ . \tag{39}$$

If the data agrees with the predictions of standard regeneration theory, a local theory cannot fit the data because, if Eqs. (34) and (35), as well as Ineqs. (37) and (38) are verified by the data, the left-hand side of Ineq. (39) is larger than 0.039 and the right-hand side is 0.030. Then no Monte Carlo abiding with the independence conditions of Sect. 3.4 and describing kaons by a two-component wave function can simulate the data. All local theories without hidden variables in their description of kaons will be ruled out, regardless of the description of Φ^0-decay or kaon evolution, decay and regeneration.

5.3 Theories with Enhanced Regeneration for Some Kaons

In local hidden-variable theories, it is possible to define different kinds of kaons, as if there were another quantum number than strangeness to describe the kaon basic states. Ineq. (59) is not a constraint anymore. However, if the result of the experiment proposed here are those predicted by standard regeneration theory, very different properties will be required from those different kinds of kaons. The larger the difference in properties required to fit the data, the larger the class of *local* theories eliminated.

The property of relevance here is the "K_S"$\rightarrow$"K_L"-regeneration probability. For Monte Carlo kaons observed as "K_S" or "K_{else}"when no slab is

on their path, it is defined as the probability to be observed as a "K_L" when, on its path, a slab is added to the setup. It is possible to determine such probability in Monte Carlo simulations, but not in experiments. However, it is possible, for those regeneration probabilities defined for samples of simulated events, to find limits computable from quantities that each depends on a sequence of events in only one setup. These quantities should be equal to values obtained by experiment. Thus, from the experimental data, one can compute limits which then become constraints on *local* theories. From the high and the low limits established for different samples of events, the need for large variations in regeneration probabilities can be demonstrated. Hereafter, expressions are given as how to compute those limits from data and, along with them, the predictions of standard regeneration theory after a $\stackrel{?}{=}$ sign.

5.3.1 Samples of Two-Kaon Events

In the Monte Carlo sequences, this time, five samples of events are considered. They are

(1) : in setup [0,0′], type "$K_S K'_L$" or "$K_{else} K'_L$"; the number $n^{(1)}$ of events in this sample depends on events defined in only one setup; it can be measured, $n^{(1)} \stackrel{?}{=} 500$ million;

(2) : in setup [0,0′], type "$K_L K'_S$"; the number $n^{(2)}$ of such events can also be measured; it is $n^{(2)} \approx n^{L,S}_{0,0'} \stackrel{?}{=} 12$ million according to Eq. (7);

(3) : in setup [0,0′], type "$K_S K'_S$" or "$K_{else} K'_S$"; the number of events here is measurable and equal to "$n^{*}_{0,0'}$" $\stackrel{?}{=}$ 155000; sample (3) contains all

so-called "additional" events of categories I_{add} and II_{add} defined in Table 1;

(4) : same criteria as for sample (3) but, in addition, κ is a "K_L" in setups [Σ,0'] and [Σ,Σ']; thus the number of events in sample (4) is not measurable in an experiment; sample (4) contains fewer events than sample (3), but it still contains all "additional" events of categories I_{add} and II_{add};

(5) : in setup [Σ,0'], type "$K_L K'_{else}$"; the number of events in this sample is measurable; it is equal to "$n^{L,else}_{\Sigma,0'}$" $\stackrel{?}{=}$ 13600; sample (5) contains all "additional" events of categories III_{add} and IV_{add}.

From these samples again, events where κ' decays upstream from the location of slab Σ were eliminated.

5.3.2 Limits on Enhancement Factors

In the samples of Monte Carlo events (1) and (3), where κ is a "K_S" or a "K_{else}" in setups without slab Σ, a $K_S \rightarrow K_L$-regeneration probability for κ can be defined as the probability of κ being a "K_L" when Σ is in place. The averages of these regeneration probabilities over samples (1) and (3) are calling p_{regen} and $f \times p_{regen}$, respectively. Similarly, regeneration probabilities for κ' when slab Σ', p'_{regen}, $f' \times p'_{regen}$, and $f'_{(5)} \times p'_{regen}$ can be defined for sample (2), (4), or (5) respectively. The probabilities p_{regen} and p'_{regen} are essentially the <u>average</u> regeneration probabilities. The factors f are enhancement factors. They are ratios between the regeneration probabilities of at least some kaons in samples (3) or (5) and the average regeneration probabilities. By showing that these enhancement factors have to be high,

one shows how different the regeneration probabilities have to be for a *local* hidden-variables theory not to be ruled by the data.

Regenerated κ's in events of sample (1) are of the type "$K_L K'_L$" in setup [Σ,0'], and regenerated κ''s of sample (2) are "$K_L K'_L$" in setup [0,Σ']. Therefore, using Eqs. (25) and (26),

$$p_{regen} \leq \frac{\text{“}n_{\Sigma,0'}\text{”}}{n^{(1)}} \stackrel{?}{=} \frac{9360}{500 \text{ million}} = 1.9\ 10^{-5} , \tag{40}$$

$$p'_{regen} \leq \frac{\text{“}n_{0,\Sigma'}\text{”}}{n^{(2)}} \stackrel{?}{=} \frac{5680}{12 \text{ million}} = 4.7\ 10^{-4} . \tag{41}$$

By definition of sample (4) and of $f \times p_{regen}$, the number of events in sample (4) is $f \times p_{regen} \times$ "$n^*_{0,0'}$". "Additional" events are those events from sample (4) and (5) where κ' has become a "K_L" when slab Σ' has been added to setup [Σ,0']. Therefore,

$$\text{“}n_{add}\text{”} = f f' p_{regen} p'_{regen} \text{“}n^*_{0,0'}\text{”} + f'_{(5)} p'_{regen} \text{“}n^{L,else}_{\Sigma,0'}\text{”} \tag{42}$$

and, using Ineqs. (20), (40) and (41),

$$\text{“}\Delta n\text{”} \leq f f' \frac{\text{“}n_{\Sigma,0'}\text{”}}{n^{(1)}} \frac{\text{“}n_{0,\Sigma'}\text{”}}{n^{(2)}} \text{“}n^*_{0,0'}\text{”} + f'_{(5)} \frac{\text{“}n_{0,\Sigma'}\text{”}}{n^{(2)}} \text{“}n^{L,else}_{\Sigma,0'}\text{”} , \tag{43}$$

where all the terms "Δn", "$n_{\Sigma,0'}$", "$n_{0,\Sigma'}$", $n^{(1)}$, $n^{(2)}$, "$n^*_{0,0'}$", "$n_{0,\Sigma'}$", and "$n^{L,else}_{\Sigma,0'}$" are measurable.

The result of the test consists of showing how large at least one of these enhancement factors f, f' and $f'_{(5)}$ has to be. The minimum value compatible with Ineq. (43) will be given by making the values of f, f' and $f'_{(5)}$ all equal. That value is the minimum value for the ratio between regeneration probabilities that a *local* Monte Carlo simulation of the experiment has to assign to the different kinds of kaons. And the kaons with the large probability have also to be made responsible for the "additional" events where both κ and κ' get regenerated in the same event.

If the standard predictions are correct, Eqs. (28), (31), and (32) together with Ineq. (40) and (41) show that Ineq. (43) reduces to

$$3530 \overset{?}{\leq} 1.4\ 10^{-3} f f' + 6.8\ f_{(5)}\ . \tag{44}$$

If the factors f are all equal, they must be larger than 470. If they are not equal, one of them at least must be more than 470.

5.4 Apparatus Ambiguities

All the ambiguities that have been considered so far are due to ambiguous kaon decays, i.e. decay modes and decay times for which both K_L and K_S can decay with a non-negligible probability. There are other ambiguities due to detection inefficiencies, particle mis-identification, and position measurement errors of kaon decay-products in the detectors. A complete analysis of these apparatus ambiguities is beyond the scope of this paper. Here, we will just point out that detector ambiguities do not depend on kaon physics but, instead, on the behavior of pions and leptons in matter. One can assume that the behavior of such particles is well understood and can be simulated accurately in the detectors. This assumption further restricts the local theories that are tested by the experiment; it is called a "fair sampling" assumption; it means that, if hidden variables are used for the description of kaons, they are not correlated with apparatus ambiguities due to physical processes outside of kaon physics. Then apparatus ambiguities can be corrected for in order to get the numbers of events "$n_{\Sigma,\Sigma'}$", "$n_{\Sigma,0'}$", "$n_{0,\Sigma'}$", "$n^{S,S}_{0,0'}$", "$n^{else,S}_{0,0'}$", "$n^{L,else}_{\Sigma,0'}$", and "$n_{0,0'}$" in Eqs. (19), (22), (34), and (35), as well as in Ineqs. (23), (36), (37), (38), and (43).

Even with a good detector simulation, some apparatus ambiguities may

still impact on the results and increase the number of event types with κ or κ' ending up as "K_{else}" in setup [0,0′]. Then, if the predictions of standard regeneration theory are upheld, the enhancement factor required for hidden-variable theories that survive the test may be expected to be reduced below the value of about 500 deduced from Ineq. (44).

6 Summary and Conclusions

6.1 Local Realism

Realism is defined as implying the existence of a Monte Carlo simulation of what the system is, i.e. where changes in the description of the system correspond to changes of the system, not only of what a alledged observer knows about it. *Local realism* implies that, in that simulation, one can generate different sequences of events corresponding to changes in the experimental setup and only what is observed on waves and particles traversing the part of the setup that has been modified is changed. Here it has been shown that a *local* theory, i.e. a theory abiding with the principles of local realism, makes predictions that obey inequalities, some of them requiring a supplementary assumption. The test suggested in this paper investigates a case where some of these inequalities are violated by the predictions of quantum mechanics and standard regeneration theory.

6.2 Significance of the Experiment

If the predictions of the standard theory of kaon regeneration are upheld, the experiment described in this paper will give results that will rule out any local theory describing the kaon by a quantum-theoretical superposition of

two basic states without hidden variables, regardless of how Φ^0-decay or kaon time-evolution and decay are accounted for. The experiment will also rule out a class of local hidden-variable theories, which assume the existence of different kinds of kaons for different values of the hidden variables. Defining a regenerated kaon in the Monte Carlo simulation as one that satisfies the "K_L" criteria of Sect. 4.3 when a copper slab is placed on its path and does not satisfy them when the slab is not there, the only local theories that survive are those

> where most kaons are described as being regenerated with a probability smaller than or equal to what is predicted by regeneration theory; and
>
> where a less frequent kind of kaons is regenerated with a probability enhanced by a factor of the order of 500 or more over the standard prediction. The latter kind of kaons would be the ones involved in the "additional events" defined in Sect. 5.1.1, where the two kaons get regenerated in the same event.

Unless a Monte Carlo simulation is based on a theory implying hidden variables with such enhancement factors, that simulation would have to be *non-local*, i.e. what is observed in one part of the apparatus has to be changed when parts of the apparatus are changed elsewhere.

With the parameters described in this paper, the standard predictions for this experiment do not violate Bell's inequality (23). Therefore, the experiment would not rule out all hidden variable theories; it is not *loophole-free*. To know if a different choice of parameters could make the experiment *loophole-free*, further study is necessary.

It has been assumed that hidden variables, if there are any, are not correlated with the efficiencies of the apparatus to detect and measure kaon decay products. A good detector simulation is needed to correct for detector inefficiencies and ambiguities. Even with a good detector simulation, the enhancement factor for the hidden-variables theories that are not ruled out by the test may be reduced below the value of about 500 mentioned above and in Sect. 5.3.2. The experiment has to be designed so that the conclusion concerning enhancement factors still be significant. Also, the experiment should be able to rule out local theories without hidden variables.

6.3 Possible Violations of Quantum Theory

This entire paper was written to show the usefulness of the experiment if it gives results in agreement with the standard predictions. It should not be forgotten that kaon physics has been the source of surprises in the past, e.g. CP-violations. It may happen that the predictions of the standard theory of regeneration are not verified in this experiment. This may be evidenced already by the test of Ineq. (18), analyzing the data as recommended in Sect. 4.3. When everything is cleared, some failure of orthodox quantum mechanics may be discovered. Such an occurrence may be improbable but it would be so significant that, regardless of its small likelihood, it may still be a good justification of this test .

For the same reason, interesting tests of quantum theory in kaon physics can be performed around Φ^0-factories operating in the symmetric mode.

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Appendices

A Dependence of n_{add} on the Theoretical Picture

Suppose a Monte Carlo simulation has been made and, in each category shown in Table 3, suppose there are, let us say, more than 200 events with the indicated event types corresponding to the different setups. One can imagine another Monte Carlo simulation based on another theoretical picture, where the categories marked with a sign + in the last column have a larger population by 200 events and where the categories marked with a − sign have a smaller population by 200 events. The new Monte Carlo simulation gives the same number of event types in each setup, thus gives the same predictions for the measurable quantities. However, the number of "additional" events, Category IV, is increased by 200. Thus n_{add} does not depend only on the measurable numbers of event types in each setup. It also depends on the theory behind the Monte Carlo generation.

Note that all four categories of Table 3 satisfy the independence conditions of Sect. 3.4. The change in n_{add} without changing the numbers of

Category	Setup [0,0′]	Setup [Σ,0′]	Setup [0,Σ′]	Setup [Σ,Σ′]	+/−
I	$K_L K'_S$	$K_L K'_S$	$K_L K'_L$	$K_L K'_L$	−
II	$K_L K'_S$	$K_S K'_S$	$K_L K'_L$	$K_S K'_L$	+
III	$K_S K'_S$	$K_S K'_S$	$K_S K'_L$	$K_S K'_L$	−
IV	$K_S K'_S$	$K_L K'_S$	$K_S K'_L$	$K_L K'_L$	+

Table 3: *Example of categories of events that can be altered without changing the number of event types in any setup, while n_{add} is changed.*

event types can be done and the theory be kept *local*, if it were *local* initially.

B Inequality for Two-Components Systems

Suppose a quantum system can be described by a state vector with only two components, i.e. belonging to a two-dimensional Hilbert space. Operators can be expressed as linear combinations of the identity operator I and the three Pauli matrices σ_x, σ_y, and σ_z. Consider a measurement M and one of the possible outcomes. Let ρ be the system density matrix. The probability for the outcome is related to the elements of ρ by a linear expression of the form of a trace, $Tr(\rho \cdot M)$, where M is a positive-definite hermitian operator.

Let us now consider two measurements. One will be called M_0 and the other $\mathrm{M}_{\Sigma'}$.[12] There is one outcome singled-out for each of them. Let M_0 and $M_{\Sigma'}$ be the operators associated with the probability computation of the singled-out outcomes of the two measurements. Also consider two samples (A) and (B) of such quantum systems and their associated density matrices $\rho^{(A)}$ and $\rho^{(B)}$. The probabilities of the singled-out outcome when the measurement M_0 is performed on samples (A) and (B) are called $p_0^{(A)}$ and $p_0^{(B)}$; they are called $p_{\Sigma'}^{(A)}$ and $p_{\Sigma'}^{(B)}$ when $\mathrm{M}_{\Sigma'}$ is performed instead. There are real three-dimensional vectors $\vec{m}_0$, $\vec{m}_{\Sigma'}$, $\vec{r}^{(A)}$, and $\vec{r}^{(B)}$ such that

$$M_0 = \frac{1}{2}(I - \vec{m}_0 \cdot \vec{\sigma}) \; ; \tag{45}$$

$$M_{\Sigma'} = \frac{1}{2}(I - \vec{m}_{\Sigma'} \cdot \vec{\sigma}) \; ; \tag{46}$$

$$\rho^{(A)} = \frac{1}{2}(I + \vec{r}^{(A)} \cdot \vec{\sigma}) \; ; \tag{47}$$

$$\rho^{(B)} = \frac{1}{2}(I + \vec{r}^{(B)} \cdot \vec{\sigma}) \; . \tag{48}$$

[12]These names are convenient when used in the core of the paper.

The coefficient of I for the operators M_0 and $M_{\Sigma'}$ is $\frac{1}{2}$ as for the density operators and the norms m_0, $m_{\Sigma'}$, $r^{(A)}$, and $r^{(B)}$ of the vectors $\vec{m}_0$, $\vec{m}_{\Sigma'}$, $\vec{r}^{(A)}$, and $\vec{r}^{(B)}$ are all inferior to 1. Then,

$$p_0^{(A)} = \frac{1}{2}(1 - \vec{r}^{(A)} \cdot \vec{m}_0) \; ; \tag{49}$$

$$p_{\Sigma'}^{(A)} = \frac{1}{2}(1 - \vec{r}^{(A)} \cdot \vec{m}_{\Sigma'}) \; ; \tag{50}$$

$$p_0^{(B)} = \frac{1}{2}(1 - \vec{r}^{(B)} \cdot \vec{m}_0) \; ; \tag{51}$$

$$p_{\Sigma'}^{(B)} = \frac{1}{2}(1 - \vec{r}^{(B)} \cdot \vec{m}_{\Sigma'}) \; . \tag{52}$$

Let us also define

$$q = \frac{1}{2}(1 - \vec{m}_0 \cdot \vec{m}_{\Sigma'}) \; . \tag{53}$$

An inequality can be written for the three sides of the triangle formed by the vectors $\vec{m}_0$, $\vec{m}_{\Sigma'}$, and $\vec{r}^{(A)}$.

$$|\vec{m}_{\Sigma'} - \vec{m}_0|^2 \leq \left(|\vec{r}^{(A)} - \vec{m}_0| + |\vec{r}^{(A)} - \vec{m}_{\Sigma'}|\right)^2 \; . \tag{54}$$

Developing both sides of Ineq. (54), using Eqs. (49), (51), and (53), as well as the normalization conditions for the vectors :

$$\begin{aligned} & m_0^2 + m_{\Sigma'}^2 - 2 + 4q \leq \\ & \quad (r^{(A)})^2 + m_0^2 - 2 + 4p_0^{(A)} + (r^{(A)})^2 + m_{\Sigma'}^2 - 2 + 4p_{\Sigma'}^{(A)} \\ & \quad +2\sqrt{\left((r^{(A)})^2 + m_0^2 - 2 + 4p_0^{(A)}\right)\left((r^{(A)})^2 + m_{\Sigma'}^2 - 2 + 4p_{\Sigma'}^{(A)}\right)} \; ; \end{aligned} \tag{55}$$

$$\begin{aligned} \text{i.e. } q \leq\; & p_0^{(A)} + p_{\Sigma'}^{(A)} - \frac{1}{2}\left(1 - (r^{(A)})^2\right) \\ & +2\sqrt{\left(p_0^{(A)} - \frac{2 - (r^{(A)})^2 - m_0^2}{4}\right)\left(p_{\Sigma'}^{(A)} - \frac{2 - (r^{(A)})^2 - m_{\Sigma'}^2}{4}\right)} \end{aligned}$$

$$\leq p_0^{(A)} + p_{\Sigma'}^{(A)} + 2\sqrt{p_0^{(A)} p_{\Sigma'}^{(A)}} \; ; \tag{56}$$

$$\sqrt{q} \leq \sqrt{p_0^{(A)}} + \sqrt{p_{\Sigma'}^{(A)}} \; . \tag{57}$$

An inequality similar to Ineq. (57) can be derived replacing $\vec{m}_0$ by $\vec{r}^{(B)}$ and $\vec{r}^{(A)}$ by $\vec{m}_0$, and also using Eqs. (50), (52), and (53),

$$\sqrt{p_{\Sigma'}^{(B)}} \leq \sqrt{p_0^{(B)}} + \sqrt{q} \; . \tag{58}$$

Taking Ineq. (57) into account,

$$\sqrt{p_{\Sigma'}^{(B)}} \leq \sqrt{p_0^{(B)}} + \sqrt{p_0^{(A)}} + \sqrt{p_{\Sigma'}^{(A)}} \; . \tag{59}$$

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