LBL-32507
UC-122
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

EARTH SCIENCES DIVISION

Submitted to Water Resources Research

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June 1992

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LBL-32507

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This work was supported by the Assistant Secretary for Fossil Energy, Office of Technical Coordination, Advanced Extraction and Process Technology, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

CORRELATION MODEL OF TRANSPORT IN RANDOM FIELDS

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ABSTRACT

This paper is an investigation of the correlation model for transport of non-reactive solutes in media with random porosity and permeability. The method of perturbation is used to obtain a second order approximate, non-local (integro-differential) equation for mean concentration. An approximate method of localization and regularization is used to convert to with the same order of approximation differential equations of transport. Exact averaged equations for one-dimensional transport are examined, and the question of the consistency and asymptotic behavior for approximate averaged equations is discussed. A detailed investigation of transport in a stratified system has been carried out. The second moment of concentration is examined, the variance of the concentration is computed, and cross-correlation moments are obtained for random fields of porosity and velocity including solute concentration.

INTRODUCTION

The principal source of flow dispersion is the heterogeneity in liquid velocities, which is caused by the irregular nature of the real geometry in porous and fractured media. Theoretical models of dispersion have a distinct dependence on the scale of the heterogeneities in the flow field. On the pore scale, dispersion has been represented by phenomenological models. The first investigation in this area was by Scheidegger (1957) and further developments came from Nikolaevskiy (1959) and Saffman (1959). They used a random walk model for a particle and the mathematical methods of random process theory. However, the statistical theory of dispersion on the pore-scale is incomplete because the parameter functions for dispersion are difficult to determine from media properties.

By contrast, the process of dispersion on the macroscale can be described using the theory of flow in random porous or fractured media. For this case, the theoretical equations of flow and transport in porous and fractured media can be used (Darcy's law, conservation of mass, conservation of energy, etc...). If the parameters of the heterogeneous media are assumed to be random fields, then flow and transport can be described statistically. This can be done in terms of the mean fields, correlations, and effective characteristics of the process on meso- and macro-scales.

Different variations of this approach have been widely developed during the last several years, (for example Shvidler, 1963, 1964, 1985b; Matheron, 1967; Gelhar, 1987; Dagan, 1989; Sudicky and Huyakorn, 1991). Problems described by averaging lead to stochastic non-linear equations, that are

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independent of whether an Eulerian or Lagrangian method is used to analyze the random fields. Except for the case of an exact solution, a descriptive realization requires a linearization of the problem, and linearization is effective only if the problem has appropriately small parameters. Usually this involves such things as the coefficient of variation for the case of weak heterogeneity, the dimensionless scale of correlation for quickly oscillating fields, etc... In any case, independent of the method of analysis, an approximate description of the process can be obtained.

The purpose of this paper is to present a correlation model of transport. This will be done by considering the first and second moments of the concentration field, as well as the cross-correlations between the porosity and velocity fields over the concentration field. In addition to finding the mean concentration, the correlation theory of transport provides a method of finding the variance of concentration and the cross correlations.

AVERAGING THE EQUATION OF TRANSPORT

Basic Equations

For local concentrations of a nonreactive solute, the macroscopic concentration, c(x,t), satisfies the advection-dispersion equation

$$m(x)\frac{\partial c(x,t)}{\partial t} + \nabla [c(x,t)v(x,t) - d(x,t)\nabla c(x,t)] = \varphi(x, t) \tag{1}$$

where m(x) is the porosity, v(x,t) is the flow velocity vector, and d(x,t) is the tensor of microdispersion. If m(x), v(x,t), and d(x,t) are considered preassigned random fields, then it is possible to search for a solution to (1) in infinite space assuming that a non-random initial concentration is known

$$c(x,0) = f(x). \tag{2}$$

The method of operators (Shvidler, 1985b) can be used for transport of a nonreactive solute, and (1) can be written as

$$Lc = \varphi \qquad L = m\frac{\partial}{\partial t} + v\nabla + \nabla(v - d\nabla) \tag{3}$$

Suppose the inverse operator L^{-1} exists, such that

$$L^{-1}L = I \tag{4}$$

where I is the identity operator. If (3) is multiplied by L^{-1}, then

$$c = L^{-1}\varphi \tag{5}$$

and after averaging (5), we have

$$\langle c\rangle = \langle L^{-1}\rangle\, \varphi \tag{6}$$

Multiplying (6) by an operator $L^* = \langle L^{-1}\rangle^{-1}$, results in the equation for mean concentration

$$L^*\langle c\rangle = \varphi. \tag{7}$$

Operator L^* is usually called the effective operator. Substitution of (7) into (5) and (6) determines the fluctuations within the concentration field over the field of mean concentration

$$\tilde{c} = c - \langle c\rangle = L_1\langle c\rangle \tag{8}$$

and

$$L_1 = L^{-1}\, L^* - I \tag{9}$$

Thus, to describe the effective operator L^*, it is necessary to find the inverse operator L^{-1}, the mean inverse operator $\langle L^{-1}\rangle$, as well as the inverse operator $\langle L^{-1}\rangle^{-1}$. If the mean field can be determined, then the field of fluctuations and its moments can be calculated. Of course, in the general case it is unlikely that this be possible. Concrete results are ordinarily obtained using modifications to perturbation methods. Let us suppose that the operator L is in some sense near to another non-random operator L_O, for which the operations of inversion, self multiplication, and other operations can be easily performed. Then the fields m, v, d, and c can be written in the form

$$\begin{aligned} m &= m_o + \varepsilon m', & m_o &= \langle m\rangle \\ v &= W + \varepsilon v', & W &= \langle v\rangle \\ d &= d_o + \varepsilon d', & d_o &= \langle d\rangle \\ c &= u + \varepsilon c', & u &= \langle c\rangle\ , \end{aligned} \tag{10}$$

where ε is a small positive parameter, and the operator can be written in the form

$$L = L_o + \varepsilon L'\ , \tag{11}$$

where $L_o = m_o\dfrac{\partial}{\partial t} + W\nabla + \nabla(W - d_o\nabla)$ and $L' = m'\dfrac{\partial}{\partial t} + v'\nabla + \nabla(v' - d'\nabla)$.

If the non-random operator L_O has an inverse operator, then

$$L = L_o\,(I + \varepsilon L_o^{-1} L')\tag{12}$$

and

$$L^{-1} = (I + \varepsilon L_o^{-1} L')^{-1} L_o^{-1}.\tag{13}$$

If the first factor in (13) is written in the form of a geometric series, then after averaging

$$\langle L^{-1}\rangle = [I + \sum_{k=1}^{\infty} (-1)^k \langle (\varepsilon L_o^{-1} L')^k \rangle]\, L_o^{-1}.\tag{14}$$

Inversion of (14) and conversion to a geometric series results in

$$L^* = \langle L^{-1}\rangle^{-1} = L_o\,\{I + \sum_{n=1}^{\infty} [\sum_{k=1}^{\infty} (-1)^{k+1} \langle (\varepsilon L_o^{-1} L')^k \rangle]^n\}.\tag{15}$$

This results in a binary correlation approximation given by

$$L^* = L_o - \varepsilon^2 \langle L' L_o^{-1} L' \rangle.\tag{16}$$

For fluctuations of concentration, the equation is

$$L_o \tilde{c} = -\varepsilon L' \langle c \rangle\tag{17}$$

hence,

$$L_1 = -\varepsilon L_o^{-1} L'.\tag{18}$$

This method can be simplified, to a marked degree, if (1) is modified to account for the various transport mechanisms, such as microdispersion. From dimensional analysis and comparison of the micro and macro scales, the contribution of microdispersion to the process of macrodispersion is usually essentially negligible. Also, the effects of physical diffusion can usually be neglected when dealing with microdispersion.

This formally determines that $d \to 0$ in (1). If the fluid is assumed incompressible and the flow is steady with W = constant and m_o = constant, then

$$L_o = m_o \frac{\partial}{\partial t} + W\nabla \quad \text{and} \quad L' = m' \frac{\partial}{\partial t} + v' \nabla\tag{19}$$

$$L_o^{-1} = m_o^{-1} \int \int_0^t (\ )\, \delta[x - x_1 - W m_o^{-1}(t-\tau)]\, dx_1\, d\tau \tag{20}$$

where δ [ ] is the dirac delta function. Substituting (20) into (18) and assuming that the mean concentration is differentiable, we solve for $c'(x, t)$ to obtain

$$c'(x, t) = - m_o^{-1} \int_0^t [m'(z) \frac{\partial u\,(z,\tau)}{\partial \tau} + v'\,(z)\, \nabla u\,(z,\tau)] d\tau + O(\varepsilon) \tag{21}$$

where $z = x - W m_o^{-1}(t - \tau)$.

The bracketed term in (21) can be rewritten

$$[\ \ ] = [m'(y) \frac{\partial u\,(y,\tau)}{\partial \tau} + v'\,(y)\, \nabla u\,(y,\tau)]_{y = z} \tag{22}$$

To compute the value of the integrand, consider τ fixed and y to be independent variable. Then make the displacement transformation of $y \rightarrow z$ and calculate the integral in (21). Equation (21) physically represents fluctuations in the field concentration at a time t and a point x. This fluctuation is a result of integrating fluctuations in the field porosity and field velocity with weighting functions that depend on the trajectory at a time t and a point x of a fictitious particle X, whose velocity is W / m.

In obtaining the mean concentration, u (x, t), in the correlation approximation, (16) can be used to determine the effective operator L^* and (20) can be used to determine L_o^{-1}. Then, from Shvidler (1976) we have

$$\begin{aligned} m_o \frac{\partial u}{\partial t} + W\, \nabla u = \varphi + m_o^{-1} [\frac{\partial}{\partial t} \int_0^t M\,(x,z) \frac{\partial u\,(z,\tau)}{\partial \tau}\, d\tau \\ + \frac{\partial}{\partial t} \int_0^t N\,(x,z) \nabla_z\, u(z, \tau)\, d\tau + \int_0^t \mathrm{div}_x\, N\,(z,x) \frac{\partial u\,(z,\tau)}{\partial \tau}\, d\tau \\ + \int_0^t N\,(x,z) \frac{\partial}{\partial \tau} \nabla_x u\,(z,\tau) d\tau + \int_0^t B^{ij}\,(x,z) \frac{\partial^2 u\,(z,\tau)}{\partial x_i \partial z_j} d\tau] + O(\varepsilon^3) \end{aligned} \quad , \tag{23}$$

where the correlation function M, correlation vector N, and correlation tenzor B are

$$M(x,z) = \varepsilon^2 \langle m'\,(x)\, m'\,(z) \rangle$$

$$N(x,z) = \varepsilon^2\langle m'(x)\, v'(z)\rangle \qquad (24)$$
$$B^{ij}(x,z) = \varepsilon^2\langle v'_i(x)\, v'_j(z)\rangle.$$

If the fields $m'(x)$ and $v'(x)$ are statistically homogeneous and homogeneous-connected, then these correlations depend on the argument $x - z = W m_o^{-1}(t - \tau)$.

Equation (23) for the mean concentration $u(x, t)$ is non-local. The right-hand side of (23) summarizes the derivatives of $u(x, t)$ with the weighting functions M, N, and B for the fictitious particle X that migrates with velocity W/m_o and arrives at point x at some time t. Thus, (23) connects the derivatives of the average concentration $u(x, t)$ that is defined along the particle path starting at point $x - W m_o^{-1} t$ and moving to point x at time t. It is obvious that the non-local measure of the functional (23) is the "memory" of the correlation M, N, B.

It should be noted that the method of analysis that reduces to (23) is discussed by Dagan and Neuman (1991). They have drawn their conclusions from the inconsistency and non-asymotic behavior of a common Eulerian approximation. In my opinion, the examples from their paper and their interpretation do not provide a sufficient basis for their general conclusion. I will consider this problem in detail and provide some comments below. Here, it should be noted that the Eulerian approximation of Dagan and Neuman (1991) as written in their final integro-differential equation (21) is not correct. The correct equation is equation (23) of this paper.

Localization of Functional Equations of Transport

Equation (23) is an approximate integro-differential equation and it is non-local. We can derive another form that is local and has the same order of approximation.

If the first and second derivatives of mean concentration $u(x, t)$ are integrable, we can represent (23) by

$$m_o \frac{\partial u}{\partial t} + W\nabla u = \varphi + O(\varepsilon^2) \qquad (25)$$

Along characteristics of equation (25) when $\varphi=0$ we can write *

$$u(z,\tau) = u(x,t) + 0(\varepsilon^2) \qquad (26)$$

Therefore if third derivatives of $u(x, t)$ are integrable, we have from (26)

$$\nabla_z u(z,\tau) = \nabla_x u(z,\tau) = \nabla_x u(x,t) + 0(\varepsilon^2) \qquad (27)$$

$$\frac{\partial u(z,\tau)}{\partial \tau} = \frac{du(z,\tau)}{d\tau} - \frac{W}{m_o}\nabla_z u(z,\tau) = -\frac{W}{m_o}\nabla_x u(x,t) + 0(\varepsilon^2) \tag{28}$$

$$\frac{\partial u(z,\tau)}{\partial \tau} = \frac{\partial u(x,t)}{\partial t} + 0(\varepsilon^2) \tag{29}$$

$$\nabla_z \frac{\partial u(z,\tau)}{\partial \tau} = \nabla_x \frac{\partial u(x,t)}{\partial t} + 0(\varepsilon^2) = -\nabla_x \frac{W}{m_o}\nabla_x u(x,t) + 0(\varepsilon^2) \tag{30}$$

*For the general case $\varphi \neq 0$, a localized form can also be developed and will be presented in a separate paper.

Thus, we obtain from (23)

$$\begin{aligned} m_o \frac{\partial u}{\partial t} + W\nabla u = m_o^{-1} \{ & \frac{\partial}{\partial t}[\frac{\partial u(x,t)}{\partial t}\int_0^t M(x,z)d\tau] \\ & + \frac{\partial}{\partial t}[\nabla u(x,t)\int_0^t N(x,z)d\tau] + \frac{\partial u(x,t)}{\partial t}\int_0^t \mathrm{div}_x N(x,z)d\tau \\ & + \frac{\partial \nabla u(x,t)}{\partial t}\int_0^t N(x,z)d\tau + \frac{\partial^2 u(x,t)}{\partial x_i \partial x_j}\int_0^t B^{ij}(x,z)d\tau\} + O(\varepsilon^3) \end{aligned} \tag{31}$$

and further,

$$\int_0^t \mathrm{div}_x N(z,x)d\tau = -\int_0^t \mathrm{div}_z N(z,x)d\tau = 0\ . \tag{32}$$

Let us introduce the following variables for the time scale of M, N, and B

$$\begin{aligned} \theta_M(t) &= M_o^{-1}\int_0^t M(x,z)d\tau, & M_o &= M(x,x) \\ \theta_N^i(t) &= N_{io}^{-1}\int_0^t N_i(x,z)d\tau, & N_{io} &= N_i(x,x) \\ \theta_{ij}(t) &= (B_o^{ij})^{-1}\int_0^t B^{ij}(x,z)d\tau, & B_o^{ij} &= B^{ij}(x,x). \end{aligned} \tag{33}$$

We can then introduce(32), and (33) into (31) to obtain

$$m^* \frac{\partial u}{\partial t} + W^* \nabla u = m_o^{-1} [M_o \theta_M(t) \frac{\partial^2 u}{\partial t^2} + 2N_o \theta_N(t) \frac{\partial \nabla u}{\partial t} + B_o^{ij} \theta_{ij}(t) \frac{\partial^2 u}{\partial x_i \partial x_j}] + O(\varepsilon^3) \tag{34}$$

$$m^*(t) = m_o - \frac{M(t)}{m_o} \tag{35}$$

$$W^*(t) = W - m_o^{-1} N(t)] \tag{36}$$

Thus, by a second order approximation, the non-local equation (23) is transformed into the local equation (34). All the coefficients of this equation are functions of time. Equation (34) is a second order derivative, but note that the stochastic equation (1) is a first order derivative. It is obvious, that at large times the coefficients of (34) in the limit are constant, but the equation is non-invariant to displacements on the time. The cause of this effect is that at (a) t=0, the concentration field is non-random, and at (b) t >0, the field is random.

If a non-random porosity is assumed by using $M_o = 0$ and $N_o = 0$ in (34), the result is

$$m_o \frac{\partial u}{\partial t} + W\nabla u = B_o^{ij} \theta_{ij}(t) \frac{\partial^2 u}{\partial x_i \partial x_j} + O(\varepsilon^3) . \tag{37}$$

This equation is equivalent to the linearized dispersion equation obtained by Dagan (1982) in a Lagrangian framework, with the assumption that the velocity v(x) is Gaussian. A similar equation of dispersion was obtained by Mendelson and Shvidler (1967), Shvidler (1985b), in a Lagrangian framework with the assumption that the random walk of the solute particle is some Markovian process. It is important to note that the hypothesis of a Gausgian velocity was not used in deriving (23) and the localized variants (34) and (37). It should also be noted that the assumption of a Gaussian velocity field gives rise to very long tail in the dispersion both in the forward and backward directions of the mean flow. This is not consistent with real transport in porous media. The same effects are seen in (37), although it does not assume the field is Gaussian. This contradiction can be explained by the fact that (23) and (37) are, in general, approximations.

Equation (37) was also obtained by Sposito and Barry (1987). In their approach they used an Eulerian framework, linearization, and the method of cumulant expansion. They did not assume a Gaussian flow velocity v(x).

If t is small, the function $\theta_{ij} \sim t$. If t is large and the scale of the correlation velocity approaches the limit, then (37) is approximately a parabolic equation with constant coefficients (Mendelson and Shvidler 1967, Shvidler 1976, Dagan 1982, Gelhar and Axness 1983).

The discriminant of the partial differential equation (34) with respect to the variables x_i and t is

$$\tilde{d}_i = M_o B_o^{ii} \theta_M(t) \theta_{ii}(t) [k_i^2(t) \tilde{\gamma}_i^2 - 1] \tag{38}$$

where $\tilde{\gamma}_i$ is the correlation coefficient of fields m and v_i', and the parameter $k_i^2(t)$ is determined from

$$k_i^2(t) = \frac{\theta_{Ni}^2(t)}{\theta_M(t)\, \theta_{ii}(t)}$$

One would expect $\theta_{Ni}(t) \le \min\{\theta_M(t), \theta_{ii}(t)\}$, and hence $k_i \le 1$. Thus, the discriminant, d_i, is less than zero since $|\tilde{\gamma}_i| \le 1$. Equation (34) is elliptic in space (x, t) when M_o, $B_o^{ii} \ne 0$. Cauchy's problem for (1) transforms to a Dirichlet problem for (34) with the condition on the hyperplane of t = 0. This is a correct problem. If $M_o = 0$, (34) is parabolic, and a correct Cauchy problem is obtained. If $B_o^{ii} = 0$, but $M_o \ne 0$, the problem of Cauchy is incorrect, since the hyperplane t = 0 is a characteristic of this equation. In this case the regularization procedure, discussed in detail below, may be used.

Localization of Equation of Transport for Exponential Correlation Function

Suppose that correlations M, N, and B are dependent on (x - z), and they can be approximated by

$$M = M_o E,\ N = N_o E,\ B^{ij} = B_o^{ij} E,\ E = \exp\left(\frac{\tau - t}{\theta_o}\right),\ \theta_o = \text{constant}. \tag{39}$$

In this case (23) may be localized for any value of θ_o (Shvidler, 1985a; 1985b). Differentiating of the integrals in (23) with respect to t, and applying the conditions

$$\frac{\partial M}{\partial t} = -\frac{M}{\theta_o}, \quad \frac{\partial N}{\partial t} = -\frac{N}{\theta_o} \tag{40}$$

eliminates all of the integrals in (23). This leads to the following second order differential equation

$$\begin{aligned} m^* \frac{\partial u}{\partial t} + W \frac{\partial u}{\partial x_1} = \theta_o \Big[& m_o \left(\frac{M_o}{m_o^2} - 1\right) \frac{\partial^2 u}{\partial t^2} + 2 \left(\frac{N_o}{m_o} - W\right) \frac{\partial^2 u}{\partial x_1 \partial t} \\ & + (B_o^{11} - W^2)\, m_o^{-1} \frac{\partial^2 u}{\partial x_1^2} + \frac{B_o^{22}}{m_o} \frac{\partial^2 u}{\partial x_2^2} + \frac{B_o^{33}}{m_o} \frac{\partial^2 u}{\partial x_3^2} \Big] \end{aligned} \tag{41}$$

where $m^* = m_o + \frac{N_o}{W}$, and the mean vector velocity W is parallel to the x_1 axis. The discriminant of (41) for variables x_1 and t is

$$\tilde{d} = \theta_o^2 W^2 \left[\zeta_v^2 \zeta_m^2 (\tilde{\gamma}_1^2 - 1) + \zeta_m^2 + \zeta_v^2 - 2\zeta_m^2 \zeta_v^2 \tilde{\gamma}_1 \right]$$

$$\zeta_m = \frac{M_o^{1/2}}{m_o}, \qquad \zeta_v = \frac{(B_o^{11})^{1/2}}{W}. \tag{42}$$

If $\tilde{\gamma}_1 < 0$, then $\tilde{d} > 0$ and hence (41) is hyperbolic. For $\tilde{\gamma}_1 \geq 0$, either of the conditions $\zeta_m < 1$ or $\zeta_v < 1$ is sufficient for (41) to be hyperbolic.

The correct representation of the Cauchy problem for the hyperbolic equation occurs when the initial concentration and the derivative over time are set on the hyperplane $t = 0$. The initial value of the derivative can be determined from (23), and the resulting initial conditions are

$$u(x,0) = f(x), \quad \frac{\partial u(x,0)}{\partial t} = -\frac{W}{m_o}\left(1 - \frac{N_o}{m_o W}\right)\left(1 - \frac{M_o}{m_o^2}\right)^{-1} f'(x). \tag{43}$$

When the conditions of (43) are applied, (41) can be evaluated for any value of θ_o.

Consider two variations of a one-dimensional problem (Shvidler, 1985a; 1985b). In the first problem, the porosity is a non-random function and the velocity is a random function of time. In this case, the exponential form of the correlation B, in the moving coordinate system $\eta = x - W m_o^{-1} t$, results in the equation for $u(\eta, t)$

$$\frac{m}{\chi^2}\frac{\partial u}{\partial t} + \frac{1}{b^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial \eta^2}, \quad \chi^2 = B\theta_o m_o^{-1}, \quad b^2 = B m_o^{-2} \tag{44}$$

This so called "telegraph" equation is hyperbolic. The parameter b is the mean square deviation of the velocity of a particle in the liquid. The finite velocity of propagation and the perturbation relative to a moving coordinate system are both determined by b. The parameter χ^2 is a measure of dispersion. Thus (44) describes both the wave and dispersion mechanisms.

Analysis of the solution to (44) with initial concentration $f(x) = \delta(x)$ shows that the plume remains with the spatial interval $\left[(\frac{W}{m_o} - b)t, \ (\frac{W}{m_o} + b)t \right]$. At the front and rear boundaries of the moving plume, there are spikes of very high concentration. The amount of solute in these spikes, however, is $(1/2)\exp(-t/2\theta_o)$, so that when $t >> \theta_o$, these portions of the total plume are very small. Between the fronts, the distribution of concentration is essentially Gaussian for large values of t/θ_o. On the other hand, for small values of t/θ_o, the portion of the plume in the spikes is dominant and the movement is approximately that of a wave mechanism.

In the second version of the one-dimensional problem, the velocity is non-random and porosity is a random function of space (Shvidler, 1985a; 1985b). In this case the equation is also hyperbolic, but the behavior of the solution is different from the first problem. One difference is that the concentrations on the fronts are asymmetric. The leading front has a large decrease, with a smaller decrease in the other portion. The mean velocity for the velocity fronts is larger than the W/m_o-flow velocity.

The same problem was studied by Indel'man (1986) using the Lagrangian approach. He obtained an exact solution for the case where m^{-1} has a
γ-distribution. At large times, the mean square of the shift of the particle grows linearly with time, but the third and higher odd moments are positive and increase with time. For a "Gaussian" particle, the third and higher odd moments are equal to zero. So that the hyperbolic local (41) describes the asymmetry of the distribution. The approximate localization describes only the Gaussian symmetric distribution.

Regularization of Local Equations

We have derived the non-local equation (23) and the local equations (39) and (41), all of which are second order approximations from the method of perturbation. Now we shall derive yet another form to the same order. An equivalent representation of these equations is (25), when $\varphi=0$. If we differentiate the equation (25) with respect to x_i, the result is

$$m_o \frac{\partial^2 u}{\partial t \partial x_i} = -\sum_j W_j \frac{\partial^2 u}{\partial x_i \partial x_j} + O(\varepsilon^2). \tag{45}$$

Alternatively, if (25) is differentiated over t, we obtain

$$\frac{\partial^2 u}{\partial t^2} = \frac{1}{m_o^2} \sum_{i,j} W_i W_j \frac{\partial^2 u}{\partial x_i \, \partial x_j} + O(\varepsilon^2) \tag{46}$$

Computation of the right hand side of (35), using (45) and (46) to eliminate the time derivatives, results in

$$M_o \theta_M(t) \frac{\partial^2 u}{\partial t^2} + 2N_o \theta_N(t) \nabla \frac{\partial u}{\partial t} + \mathrm{div}\,(B_o \theta_B(t) \nabla u) = \mathrm{div}\,[D(t) \nabla u] \tag{47}$$

where $$D^{ij}(t) = \frac{M_o W_i W_j}{m_o^2} \theta_M(t) - 2\frac{N_{oi} W_j}{m_o} \theta_{Ni}(t) + B_o^{ij} \theta_{ij}(t) .$$

Since the vector N is a cross-correlation of a vector and a scalar, the component N_i is proportional to W_i. The tensor D^{ij} is symmetric, because the tensor $B\theta_B$ is symmetric by definition. For example, if the first coordinate axis is parallel to vector W, then the system is defined for $B^{ij} = 0$ when $i \neq j$. Thus,

$$D^{11} = \frac{M_o W^2}{m_o^2}\theta_M - 2\frac{N_{o1} W}{m_o}\theta_{n1} + B_o^{11}\theta_{11}$$
$$D^{22} = B_o^{22}\theta_{22}, \quad D^{33} = B_o^{33}\theta_{33} . \tag{48}$$

If $\theta_{n1} \leq \min(\theta_M, \theta_{11})$, then $D^{11} > 0$, since $|N_{01}| \leq |M_0||B_{01}|$

Substituting (47) into (40) results in the parabolic equation of transport

$$m^* \frac{\partial u}{\partial t} + W^* \nabla u = m_o^{-1} \operatorname{div} D \nabla u$$
$$m^*(t) = m_o - M(t)\, m_o^{-1}, \qquad W^*(t) = W - m_o^{-1} N(t) \tag{49}$$
$$D^{ij}(t) = \frac{M_o W_i W_j}{m_o^2}\theta_M(t) - 2\frac{N_{oi} W_j}{m_o}\theta_{Ni}(t) + B_o^{ij}\theta_{ij}(t) .$$

For (49), the Cauchy problem with condition u (x, 0) = f (x) is correct. For small values of θ, the hyperbolic equation (41) can be regularized to the parabolic equation (49), where

$$D^{ii}(t) = \theta_o \left(\frac{M_o W_i^2}{m_o^2} - 2\frac{N_{oi} W_i}{m_o} + B_o^{ii} \right) . \tag{50}$$

DISCUSSION

As can be seen from the second order approximation in this method of perturbation, the equation for mean concentration is non-local. The right hand side of (23) depends on the correlation functions of the porosity field, the correlation tensor of the velocity field, and the cross correlation for these two fields. The method that is proposed here of developing an approximate localization of the integro-differential equation (23) reduces, in the general case, to a differential equation of the elliptic type.

When the correlation parameters can be represented using a special form of the exponential functions, the proposed method of exact localization reduces (23) to the averaged equation (41) hyperbolic type. A method of regularizing this problem can produce a second approximation in the form of a parabolic equation for the mean concentration. Thus, the approximations for the transport equation can be reduced to equivalent asymptotic equations of either a differential or an integro-differential type. These are typical results from the theory of asymptotic averaging and require

careful analysis. For this, one needs exact results from the theory of averages and a precise numerical analysis.

It should be noted that before the averaging process takes place, we have a hyperbolic equation of first order with variables and random coefficients. However, the effect of the averaging process for different combinations of the random porosity and velocity fields is to produce an equation that has: (a) a quasi wave mechanism, and (b) a quasi diffusive mechanism. The forms of these two mechanisms may be non-classical. Therefore, it is hardly possible to expect any random field to generate a unique averaged equation that is not a differential equation of large order.

It should be obvious that since the exact equation in the general case must contain complete information on the random fields, an infinite number of parameters is required to satisfy all the moments of these fields. In the case of Gaussian fields, there is another problem. In describing this case, the first two moments are sufficient, but the negative values of flow velocity and porosity in a Gaussian model are physically incorrect. Thus, the existence of different approximations for these different correlation models is quite understandable. We shall examine some exact results of averaging the one-dimensional equation of transport (Indelman and Shvidler, 1985). Consider the averaged one-dimensional equation

$$\frac{\partial c(x,t)}{\partial t} + v\frac{\partial c(x,t)}{\partial x} = 0 \tag{51}$$

with the condition $c(x, 0) = c_o(x)$.

In the general case for the power moments functions, $u_k(x, t) = \langle c^k(x, t)\rangle$,we have by V=const an equation

$$\frac{\partial u_k}{\partial t} + \sum_{n=0}^{\infty} (-t)^n \frac{\overline{K}_{n+1}}{n!}\frac{\partial^{n+1} u_k}{\partial x^{n+1}} = 0 \tag{52}$$

under the initial condition $u_K(x, 0) = c_o^K(x)$, where $\overline{K}_n$, are the cumulant values of the random velocity v. Thus, the averaged equation in the general case is first order in time and infinite order in x, which means (52) is non-local.

There are several cases where (52) has a simplified form as shown by the following examples:

(1) Let velocity $v = nv_o$, where v_o is a non-random constant and n has a Poisson distribution with parameter λ_o. In this case, the series (52) has an exact sum, and we can write a different non-local equation

$$\frac{\partial u_k(x,t)}{\partial t} + \lambda_o v_o \frac{\partial u_k(x-v_o t, t)}{\partial x} = 0 \tag{53}$$

(2) Let v be Gaussian with mean velocity V and variance B. In this case, we have the parabolic equation

$$\frac{\partial u_k(x,t)}{\partial t} + V\frac{\partial u_k(x,t)}{\partial x} = Bt\frac{\partial^2 u_k(x,t)}{\partial x^2} \tag{54}$$

(3) If velocity $v > 0$ and has a γ distribution given by

$$\psi(v) = \frac{\lambda^\gamma}{\Gamma(\gamma)} v^{\gamma-1} \exp(-\lambda v) \tag{55}$$

where $\frac{\gamma}{\lambda} = V = \langle v \rangle$ and $\gamma^{-1} = \zeta^2 = \frac{\sigma_v^2}{v^2}$, we obtain from (55) the hyperbolic equation

$$\frac{\partial u_k(x,t)}{\partial t} + V\frac{\partial u_k(x,t)}{\partial x} + \zeta^2 Vt\frac{\partial^2 u_k(x,t)}{\partial t\,\partial x} = 0 \tag{56}$$

The solution of (56) for mean concentration $u = u_1 = \langle c \rangle$ with the initial condition $c_o = \delta(x)$ is $u(x,t) = t^{-1}\psi(x/t)$. The solid lines in Figure 1 show that dependence of the function $u(x,t)Vt$ on the parameter x/Vt for four different values of γ. It can be seen that there is considerable asymmetry in the distribution of mean concentration.

If the algorithm to regularize (56) is used, one can obtain (54), which for a Gaussian distribution of velocity v, is exact. The dashed lines on Figure 1 show the dependence of the function $\tilde{u}(x,t)Vt$, which is the solution to (54). It can be seen that when $\gamma \leq 10$, the functions $u(x,t)$ and $\tilde{u}(x,t)$ are essentially different. As the parameter γ becomes large, both solutions are similar to the Dirac δ function, but since the function $u(x,t)$ is asymmetrical, the maximum value of $u(x,t)$ for any γ is larger than the maximum value of $\tilde{u}(x,t)$. As $\gamma \to \infty$, the ratio of $u_{max}/\tilde{u}_{max}$ tends to $\sqrt{2}$.

(4) Let the velocity v have a Cauchy distribution given by

$$\psi(v) = \pi^{-1}\lambda_*[(v-V)^2 + \lambda_*^2]^{-1} \tag{57}$$

In this case, all the moments and summations are infinite, and (52) can not be used. However, we can write the elliptic equation

$$\frac{\partial^2 u_k(x,t)}{\partial t^2} + 2V\frac{\partial^2 u_k(x,t)}{\partial x\,\partial t} + (V^2 + \lambda_*^2)\frac{\partial^2 u_k(x,t)}{\partial x^2} = 0 \tag{58}$$

and investigate the Dirichlet problem on the half plane $t \geq 0$.

If we now let v = v(t), we can write the exact non-local equation for the probability density function for concentration $P^c(x, t) = \langle\delta[c(x, t) - c]\rangle$

$$\frac{\partial P^c(x,t)}{\partial t} + \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^t \cdots \int_0^t \overline{K}_{n+1}(t_1, \tau_1, \ldots, \tau_n) d\tau_1, \ldots, d\tau_n \frac{\partial^{n+1} P^c(x,t)}{\partial x^{n+1}} = 0 \tag{59}$$

Here, $\overline{K}_{n+1}(t_1, \tau_1, \ldots, \tau_n)$ are cumulants of the random function v(t) subject to the following:
(1) If the velocity v(t) is Gaussian, we have $\overline{K}_1 = \langle v(t)\rangle = V(t)$ and $\overline{K}_2(t, \tau) = B(t, \tau)$, $\overline{K}_3 = \ldots = 0$. In this case, we obtain the parabolic equation with variable coefficients.

$$\frac{\partial P^c(x, t)}{\partial t} + V(t)\frac{\partial P^c(x, t)}{\partial x} = \int_0^t B(t, \tau)\, d\tau \frac{\partial^2 P^c(x, t)}{\partial x^2} \tag{60}$$

(2) If the random velocity v(t) is a delta-correlated process, $\overline{K}_{n+1}(t_1, \tau_1, \ldots, \tau_n) = \overline{K}^*_{n+1}(t)\delta(t - \tau_1)\delta(\tau_1 - \tau_2)\ldots\delta(\tau_{n-1} - \tau_n)$, then, in the general case, we have the non-local equation

$$\frac{\partial P^c(x,t)}{\partial t} + \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!} \overline{K}^*_{n+1}(t) \frac{\partial^{n+1} P^c(x,t)}{\partial x^{n+1}} = 0 \tag{61}$$

(3) Now let us consider the Poisson process where $v(t) = \sum_{i=1}^{n} \xi_i g_i(t - t_i)$. Here ξ_i are the stochastic independent random values with a probability density function $\phi(\xi)$. The points t_i are equally distributed over the interval (0, T), and the number n has a Poisson distribution with $n^* = \upsilon T$. The step function $g(\tau) = 1$ if $\tau > 0$ and 0, if $\tau < 0$. If the process v(t) is delta-correlated, and $\phi(\xi)$ is an exponential distribution with parameter λ, we have the hyperbolic equation for $P^c(x, t)$

$$\frac{\partial P^c(x, t)}{\partial t} + \frac{v}{\lambda}\frac{\partial P^c(x, t)}{\partial x} + \frac{1}{\lambda}\frac{\partial^2 P^c(x, t)}{\partial t\, \partial x} = 0 \tag{62}$$

And finally, consider the averaged stochastic equation with a dispersive term

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial x} = \kappa\frac{\partial^2 c}{\partial x^2} \tag{63}$$

where κ is non-random. In this case, the equation for $u = \langle c \rangle$ is

$$\left[\frac{\partial}{\partial t} - \frac{\partial}{\partial t}\ln\left\langle \exp(-tv\frac{\partial}{\partial x})\right\rangle - \kappa\frac{\partial^2}{\partial x^2}\right]u(x,t) = 0 \tag{64}$$

If v is Gaussian, we have

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x} = (\kappa + Bt)\frac{\partial^2 u}{\partial x^2}. \tag{65}$$

When the velocity in the averaged equation has a γ distribution, one has a derivative of third order

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x} = \kappa\frac{\partial^2 u}{\partial x^2} - \zeta^2 Vt\frac{\partial^2 u}{\partial x\,\partial t} + \zeta^2 V\kappa t\frac{\partial^3 u}{\partial x^3} \tag{66}$$

If κ is random and proportional to v, i.e. $\kappa = \beta v$, we have a non-local equation

$$\frac{\partial u}{\partial t} + \sum_{n=0}^{\infty}\frac{\overline{K}_{n+1}}{n!}(-tA)^n Au = 0 \tag{67}$$

where $A = (1-\beta\frac{\partial}{\partial x})\frac{\partial}{\partial x}$. For a γ-distributed velocity, one has the averaged equation

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x} = \beta V\frac{\partial^2 u}{\partial x^2} - V\zeta^2 t\left(\frac{\partial^2 u}{\partial t\,\partial x} - \beta\frac{\partial^3 u}{\partial t\,\partial x^2}\right) \tag{68}$$

The examples presented here characterize the complications of the averaging problem and reveal the variety of equations involved. In those cases where the random fields depend on the space variables, the general situation is not simple because the problem has the additional complication of depending on the correlation scales for the random fields. However, it should be noted that, for large values of time, there is a weak dependence of the averaged equations on the detailed behavior of the random field distributions. In specific situations, the principal effects are those of the first moments of the random fields.

Based on this analysis of the different aspects of the problem of averaging, it should be noted that the procedure used by Dagan and Neuman (1991) in comparing approximate solutions is not fully convincing. They used an approach involving Gaussian velocities, where the Lagrangian approach and the parabolic equation are exact for all times. In this case, the three first central spatial moments are identical using either approach, and differences only appear in the fourth moment. However, it is obvious that computation of the fourth moment (which is proportional to ε^4) with

the help of an approximate equation of second order (where the error is proportional to ε^3) is not correct.

FLOW AND TRANSPORT IN STRATIFIED FORMATIONS

As an example of the process under consideration, we shall examine the relatively simple case of flow and transport in a stratified formation. Let the random fields for porosity, m, and permeability, k, depend on one variable x_2

$$m = m(x_2), \quad k = k(x_2) \tag{69}$$

In the stratified formation, we have

$$\begin{aligned} &v = kh, \quad h = -\nabla P \\ &W = \langle v \rangle = k_\perp h_\perp + k_\parallel h_\parallel \\ &h_\parallel = \langle h_1 \rangle, \ h_\perp = \langle h_2 \rangle, \ k_\parallel = k_0 = \langle k \rangle, \ k_\perp = \langle k^{-1} \rangle^{-1} \\ &h_1 = h_\parallel, \ h_2 = k_\perp h_\perp / k \end{aligned} \tag{70}$$

The values of v_2 and h_1 are non-random, while v_1 and h_2 are random. Therefore, the fluctuations in velocity are

$$\begin{aligned} &\tilde{v}_1 = kh_\parallel - k_0 h_\parallel = \tilde{k} h_\parallel, \tilde{k} = k - k_0 \\ &\tilde{v}_2 = kh_2 - k_\perp h_\perp = 0 \end{aligned} \tag{71}$$

and since $z = x - Wm_0^{-1}(t - \tau)$, we have the correlation function of the velocity

$$\begin{aligned} &B^{11}(x,z) = \left\langle \tilde{k}(x) h_\parallel \tilde{k}(z) h_\parallel \right\rangle = K(x,z) h_\parallel^2 \\ &K(x,z) = \left\langle \tilde{k}(x) \tilde{k}(z) \right\rangle \\ &B^{22}(x,z) = 0, \quad B^{12}(x,z) = 0 \end{aligned} \tag{72}$$

For the cross-correlation functions of porosity and velocity, we can write

$$\begin{aligned} &N_1(x,z) = \left\langle \tilde{m}(x) \tilde{k}(z) \right\rangle h_\parallel = K_1(x,z) h_\parallel \\ &N_2(x,z) = 0 \\ &K_1(x,z) = \left\langle \tilde{m}(x) \tilde{v}_1(z) \right\rangle \end{aligned} \tag{73}$$

If both porosity and permeability are statistical homogeneous functions of x_2, then we have:

$$K(x, z) = K_o f_k\left(\frac{W_2(t - \tau)}{m_o}\right), \quad M(x, z) = M_o f_m\left(\frac{W_2(t - \tau)}{m_o}\right)$$
$$N(x, z) = N_o f_N\left(\frac{W_2(t - \tau)}{m_o}\right) \tag{74}$$

from which

$$\theta_{11}(t) = \frac{m_o}{W_2}\int_0^{W_2 t/m_o} f_K(y)\,dy, \quad \theta_m(t) = \frac{m_o}{W_2}\int_0^{W_2 t/m_o} f_m(y)\,dy$$
$$\theta_{N1}(t) = \frac{m_o}{W_2}\int_0^{W_2 t/m_o} f_{N1}(y)\,dy \tag{75}$$

Thus the time θ is a function of the transverse velocity W_2. Since $f(0) = 1$, by $W_2 = 0$ for all $\theta(t) = t$.

Let us consider this stratified system as a binary system with layers whose parameters are k_1, m_1 and k_2, m_2. The randomly distributed layers occupy portions Ω_1 and Ω_2 in the space Ω. We shall define the random indicator function

$$z_i(x) = \left\{ \begin{array}{l} 1, x \in \Omega_i \\ 0, x \in \Omega/\Omega_i \end{array} \right\} \quad i = 1, 2 \tag{76}$$

Then, $\langle z_i(x)\rangle = P_i$ is the probability that point x falls within the subspace Ω_i and $P_1 + P_2 = 1$. For the random function k(x), we have

$$k(x) = k_1 z_1 + k_2 z_2$$
$$\tilde{k}(x) = k_1(z_1 - P_1) + k_2(z_2 - P_2) \tag{77}$$

$$K(x', x'') = K_o \overline{\psi}(x', x'') = K_o \overline{\psi}(|x' - x''|)$$
$$K_o = \sigma_K^2 = P_1 P_2 (k_1 - k_2)^2, \quad \overline{\psi}(x', x'') = P_1^{-1} P_2^{-1}\langle z_1(x') z_1(x'') - P_1^2\rangle \tag{78}$$

Similarly,

$$M(x', x'') = M_o \overline{\psi}(|x' - x''|), \quad M_o = \sigma_m^2 = P_1 P_2 (m_1 - m_2)^2 \tag{79}$$

$$N_1(x', x'') = N_{01}\,\overline{\psi}(|x' - x''|), \quad N_{01} = \sigma_k \sigma_m \tilde{\gamma} h_{\|} = P_1 P_2 (k_1 - k_2)(m_1 - m_2) h_{\|} \tag{80}$$

$$\tilde{\gamma} = \mathrm{sign}[(k_1 - k_2)(m_1 - m_2)]$$

Hence, $\theta_{11}(t) = \theta_M(t) = \theta_N(t) = \theta(t) =$

$$\theta(t) = \frac{m_o}{W_2} \int_0^{W_2 t/m_o} \overline{\psi}(x)dx \tag{81}$$

and from (34), we can write the equation for mean concentration as

$$m^* \frac{\partial u}{\partial t} + W^* \nabla u = m_o^{-1} \theta(t) \left(M_o \frac{\partial^2 u}{\partial t^2} + 2N_{01} \frac{\partial^2 u}{\partial x_1 \, \partial t} + B_o^{11} \frac{\partial^2 u}{\partial x_1^2} \right) \tag{82}$$

If the porosity is non-random, $M_o = 0$, $N_o = 0$, $m^* = m_o$, $W^* = W$, equation (82) results in

$$m_o \frac{\partial u}{\partial t} + W \nabla u = m_o^{-1} \theta(t) B_o^{11} \frac{\partial^2 u}{\partial x_1^2} \tag{83}$$

Since $B_o^{11} = \sigma_K^2 h_{\|}^2$ and $W_1 = k_o h_{\|}$, then $B_o^{11} = \sigma_K^2 k_o^{-2} W_1^2$ and we can write

$$m_o \frac{\partial u}{\partial t} + W \nabla u = \frac{\theta(t) \sigma_k^2}{m_o k_o^2} W_1^2 \frac{\partial^2 u}{\partial x_1^2} \tag{84}$$

The expression for dispersion from (84) is equivalent to equation (7) in the article by Salandin et. al. (1991), and if $\theta(t)$ has a limit as $t \to \infty$, one has the coefficient of dispersion as obtained by Matheron and de Marsily (1980). If $W_2 \to 0$, we have $\theta(t) \to t$, and the equation for dispersion is

$$m_o \frac{\partial u}{\partial t} + W_1 \frac{\partial u}{\partial x_1} = \frac{t\, \sigma_k^2}{m_o k_o^2} W_1^2 \frac{\partial^2 u}{\partial x_1^2} \tag{85}$$

In regularizing (82), we need to eliminate the derivative with respect to time from the right hand side. Substituting (45) and (46) into (82) and neglecting the small values of fourth order, we obtain

$$m^*\frac{\partial u}{\partial t} + W^*\nabla u = m_o^{-1}\theta(t)\left(\frac{M_o W_1^2}{m_o^2} - \frac{2N_{01}W_1}{m_o} + B_o^{11}\right)\frac{\partial^2 u}{\partial x_1^2}$$
$$+ 2W_2\left(\frac{M_o W_1}{m_o^2} - \frac{N_{01}}{m_o}\right)\frac{\partial^2 u}{\partial x_1 \partial x_2} + \frac{M_o W_2^2}{m_o^2}\frac{\partial^2 u}{\partial x_2^2} \tag{86}$$

which can be written in the following form

$$m^*\frac{\partial u}{\partial t} + W^*\nabla u = m_o^{-1}\theta(t)$$
$$\left[W_1^2(\xi_m - \tilde{\gamma}\xi_k)^2\frac{\partial^2 u}{\partial x_1^2} + 2W_1W_2\,\xi_m(\xi_m - \tilde{\gamma}\xi_k)\frac{\partial^2 u}{\partial x_1 \partial x_2} + W_2^2\xi_m^2\frac{\partial^2 u}{\partial x_2^2}\right] \tag{87}$$

Here,

$$\xi_m^2 = \frac{\sigma_m^2}{m_o^2} = \frac{(m_1 - m_2)^2 P_1P_2}{(m_1P_1 + m_2P_2)^2}, \quad \xi_k^2 = \frac{\sigma_k^2}{k_o^2} = \frac{(k_1 - k_2)^2 P_1P_2}{(k_1P_1 + k_2P_2)^2} \tag{88}$$

The discriminant on the right hand side of (87) is zero, and it is convenient to transform the coordinate system using

$$y_1 = x_2 - \frac{W_1}{W_2}\frac{(\xi_m - \tilde{\gamma}\xi_k)}{\xi_m}x_1$$
$$y_2 = x_2 + \frac{W_2}{W_1}\frac{\xi_m}{(\xi_m - \tilde{\gamma}\xi_u)}x_2 \tag{89}$$

The equation of transport then has the form

$$m^*\frac{\partial u\,(y,t)}{\partial t} + W^*\nabla u\,(y,t) = m_o^{-1}\theta(t)\mu\frac{\partial^2 u\,(y,t)}{\partial y_1^2}$$
$$\mu = W_2^{-2}\left[W_1^2(\zeta_m - \tilde{\gamma}\zeta_k)^2 + W_2^2\zeta_m^2\right]^2 \tag{90}$$

Thus by the interaction of two random fields (porosity and permeability), the dispersion is one-dimensional along the axis y_1 while advection is along vector W^*. In the general case, these two directions are not the same.

For large times, $W^* \rightarrow W$, and we can determine the angle φ between vector W and y_1, the axis of dispersion from

$$\varphi = \arctan\left[\frac{\tilde{\gamma}\tilde{\lambda}\tan\varphi_o}{\tan^2\varphi_o + \tilde{\gamma}\tilde{\lambda} - 1}\right], \quad \tilde{\lambda} = \frac{\xi_k}{\xi_m} \tag{91}$$

Here, φ_o is the angle between vector W and the x_1 axis which is the principal direction along the layers. The variation of φ with φ_o for different values of parameter $\tilde{\lambda}$ is shown for two different cases on Figure 2. The first case is shown in Figure 2A and 2B where the coefficient of correlation between porosity and permeability is positive, i.e., $\tilde{\gamma} = 1.0$. The second case is shown in Figure 2C where the correlation is negative and $\tilde{\gamma} = -1.0$.

It can be seen on Figure 2 that when W is parallel ($\varphi_o = 0°$) or perpendicular ($\varphi_o = 90°$) to the layers, the direction of dispersion and advection are identical ($\varphi = 0°$) independent of the values for λ and γ. If $\xi_m \to 0$, then $\lambda \to \infty$ and $\varphi = \varphi_o$ i.e. the axis of dispersion is the same as x_1, the longitudinal axis of the layered system. This case was studied by Matheron and de Marsily (1980) and Salandin et. al. (1991).

For the particular case where $\tilde{\varphi}_o \leq 45°$, Figure 2A shows that the axis of dispersion and vector of advection are orthogonal ($\varphi = 90°$) for any $\tilde{\lambda} < 1$ and $\tilde{\gamma} = 1$. On Figure 2B where $\tilde{\gamma} = -1$, the same orthogonal condition holds, but now at $45° \leq \tilde{\varphi} \leq 90°$. The function $\tilde{\varphi}_o = \tilde{\varphi}_o(\tilde{\lambda})$ is plotted in Figure 3.

CORRELATION FUNCTIONS FOR RANDOM FIELDS

To obtain a complete description within the framework of correlation theory for the equations of mean concentration, it is necessary to develop expressions for the cross-correlation random concentration field c(x, t) with random fields for porosity m(x) and velocity v(x). It is also necessary to find the auto-correlation function of the concentration field, in particular the variance of the concentration. (This problem has been discussed by Shvidler, 1990.) So, we must compute the functions

$$K_1(x, t, y) = \varepsilon^2\langle m'(y)\, c'(x, t)\rangle \tag{92}$$

$$K_2(x, t, y) = \varepsilon^2\langle v'(y)\, c'(x, t)\rangle \tag{93}$$

$$K_3(x, t_1, y, t_2) = \varepsilon^2\langle c'(x, t_1)\, c'(y, t_2)\rangle \tag{94}$$

To compute the K_i in the correlation approximation, we use the explicit expression (21) for $c'(x, t)$. After multiplying (92), (93), (94) by this expression and averaging, we have

$$K_1(x, t, y) = -m_o^{-1}\int_0^t [M(z, y)\frac{\partial u(z, \tau)}{\partial \tau} + N(z, y)\nabla u(z, \tau)]d\tau + O(\varepsilon^3) \tag{95}$$

$$K_2(x, t, y) = -m_o^{-1} \int_0^t [N(z, y)\frac{\partial u(z, \tau)}{\partial \tau} + B(z, y)\nabla u(z, \tau)]d\tau + O(\varepsilon^3) \tag{96}$$

$$K_1(x, t_1, y, t_2) = -m_o^{-2} \int_0^{t_1}\int_0^{t_2} M(z_1, z_2)\frac{\partial u(z_1, \tau_1)}{\partial \tau_1}\frac{\partial u(z_2, \tau_2)}{\partial \tau_2}$$

$$+N(z_1, z_2)\nabla u(z_2, \tau_2)\frac{\partial u(z_1, \tau_1)}{\partial \tau_1} + N(z_1, z_2)\nabla u(z_1, \tau_1)\frac{\partial u(z_2, \tau_2)}{\partial \tau_2} \tag{97}$$

$$+B^{ij}(z_1, z_2)\frac{\partial u(z_1, \tau_1)}{\partial z_{1i}}\frac{\partial u(z_2, \tau_2)}{\partial z_{2j}}d\tau_1 d\tau_2 + O(\varepsilon^3)$$

$$z_1 = x - W m_o^{-1}(t_1 - \tau_1),\ \ z_2 = y - W m_o^{-1}(t_2 - \tau_2) \tag{98}$$

It must be understood that (95), (96), and (97) are, like (21), expressions for $c'(x, t)$.

Using equations (17) or (19) we can find equations for the moments K_i. After multiplying and averaging, the expression for the functions corresponding to (92), (93), and (94) are

$$m_o\frac{\partial K_1(x, t, y)}{\partial t} + W\nabla_x K_1(x, t, y) = -M(x, y)\frac{\partial u(x, t)}{\partial t} - N(x, y)\nabla u(x, t) + O(\varepsilon^3)$$

$$x \neq y \tag{99}$$

$$m_o\frac{\partial K_2(x, t, y)}{\partial t} + W\nabla_x K_2(x, t, y) = -N(x, y)\frac{\partial u(x, t)}{\partial t} - B(x, y)\nabla u(x, t) + O(\varepsilon^3)$$

$$x \neq y \tag{100}$$

$$m_o\frac{\partial K_3(x, t_1, y, t_2)}{\partial t_1} + W\nabla_x K_3(x, t_1, y, t_2)$$

$$= -K_1(x, t_1, y)\frac{\partial u(x, t_1)}{\partial t} - K_2(x, t_1, y)\nabla u(x, t_1) + O(\varepsilon^3) \tag{101}$$

$$x \neq y \quad t_1 \neq t_2$$

The equations for moments K_i are similar to those for the transport equation. The right hand side of (99) and (100) are dependent on the mean concentration $u(x, t)$ and the correlation fields $m(x)$ and $v(x)$. The right hand side of (101) is depends on mean concentration and the functions K_1 and K_2.

The initial conditions for K_i are a consequence of the condition $c'(x, 0) = 0$, and

$$K_1(x, 0, y) = 0, \qquad K_2(x, 0, y) = 0$$
$$K_3(x, 0, y, t_2) = 0, \qquad K_3(x, t_1, y, 0) = 0 \tag{102}$$

Equations (99) and (100) can be used only when $x \neq y$, and (101) can be used only when $x \neq y$ and $t_1 \neq t_2$. When $x = y$, we have other equations

$$m_o \frac{\partial K_1(x, t, x)}{\partial t} + W\nabla K_1(x, t, x) = -M(x, x)\frac{\partial u(x, t)}{\partial t} - N(x, x)\nabla u(x, t)$$
$$-m_o^{-1}W\int_0^t \nabla_x M(x,z)\frac{\partial u(z, \tau)}{\partial \tau}\partial\tau - m_o^{-1}W_i\int_0^t \frac{\partial N_j(x,z)}{\partial x_i}\frac{\partial u(z,\tau)}{\partial z_j}\partial\tau + O(\varepsilon^3) \tag{103}$$

For the i th component vector $K_2(x, t, x)$, we have

$$m_o \frac{\partial K_{2i}(x, t, x)}{\partial t} + W\nabla K_{2i}(x, t, x) = -N_i(x, x)\frac{\partial u(x, t)}{\partial t} - B^{ij}(x, x)\frac{\partial u(z,t)}{\partial x_j}$$
$$-m_o^{-1}W_j\int_0^t \left[\frac{\partial N_i(x,z)}{\partial x_j}\frac{\partial u(x,\tau)}{\partial \tau} + \frac{\partial B^{il}(z,x)}{\partial x_j}\frac{\partial u(z,\tau)}{\partial x_l}\right]\partial\tau + O(\varepsilon^3) \tag{104}$$

The equation for $K_3(x, t)$ differs from equation (101) in that there is a factor of 2 on the right hand side

$$m_o \frac{\partial K_3(x, t)}{\partial t} + W\nabla K_3(x, t) = -2\left[K_1(x, t)\frac{\partial u(x, t)}{\partial t} + K_2(x, t)\nabla u(x, t)\right] + O(\varepsilon^3) \tag{105}$$

Thus, in addition to the explicit expressions in (95), (96), and (97), we can compute the moments K_i from (99), (100), and (101) or from (103), (104), and (105) with initial conditions given in equation (102).

In order to compute the mean concentration, $u(x, t)$, one can use finite differences and a step by step procedure to compute K_i , from which the mean concentration and mean square deviation of concentration can be obtained.

Cross-Correlation Function $K_1(x, t, y)$

Let us consider in more detail the moment $K_1(x, t, y)$

If the expressions in (30) are used, we have from (95)

$$K_1(x, t, y) = -m_o^{-1}\left[M_o\tilde{\theta}_M(x-y, t)\frac{\partial u(x, t)}{\partial x} + N_o\tilde{\theta}_N(x-y, t)\nabla u(x, t)\right] + O(\varepsilon^3), \tag{106}$$

$$\text{where} \quad \tilde{\theta}_M(x-y, t) = -M_o^{-1}\int_0^t M(z, y)d\tau, \;\; \tilde{\theta}_N(x-y, t) = N_o^{-1}\int_0^t N_i(z, y)d\tau \tag{107}$$

When $x = y$, $\tilde{\theta} = \theta$ and

$$K_1(x, t) = -m_o^{-1}\left[M_o\theta_M(t)\frac{\partial u(x, t)}{\partial t} + N_o\theta_N(t)\nabla u(x, t)\right] + O(\varepsilon^3) \tag{108}$$

If the derivative with respect to time is eliminated in (108), we have another approximation

$$K_1(x, t) = -m_o^{-1}\left[N_o\theta_N(t) - M_o W m_o^{-1}\theta_M(t)\right]\nabla u(x, t) + O(\varepsilon^3) \tag{109}$$

Cross-Correlation Function $K_2(x, t, y)$

Using (30) and following the same procedure for K_2 from (96), we have

$$K_2(x, t, y) = -m_o^{-1}\left[N_o\,\tilde{\theta}_M(x-y, t)\frac{\partial u(x, t)}{\partial t} + B_o\,\tilde{\theta}_B(x-y, t)\,\nabla u(x, t)\right] + O(\varepsilon^3) \tag{110}$$

$$\begin{aligned} \tilde{\theta}_{ij}(x-y, t) &= \left(B_o^{ij}\right)^{-1}\int_0^t B^{ij}(z, y)\,d\tau \\ K_{2i}(x, t, x) &= -m_o^{-1}\left[N_o\,\theta_N(t)\frac{\partial u}{\partial t} + B_o\,\theta_B(t)\,\nabla u\right] + O(\varepsilon^3) \end{aligned} \tag{111}$$

If the time derivative in (110) is eliminated, when $x = y$ we have

$$K_{2i}(x, t) = -m_o^{-1}\left[-\frac{N_{oi}\,\theta_N(t)}{m_o}W\nabla u + B_o^{ij}\,\theta_{ij}(t)\frac{\partial u(x, t)}{\partial x_j}\right] + O(\varepsilon^3) \tag{112}$$

Auto-Correlation Function $K_3(x, t_1, y, t_2)$

If we introduce a new variable scale

$$\tilde{\tilde{\theta}}_M(x - y, t_1, t_2) = M_o^{-1} \int_0^{t_1} \int_0^{t_2} M(z_1, z_2)\, d\tau_1 d\tau_2$$

$$\tilde{\tilde{\theta}}_{Ni}(x - y, t_1, t_2) = N_{oi}^{-1} \int_0^{t_1} \int_0^{t_2} N_i(z_1, z_2)\, d\tau_1 d\tau_2 \tag{113}$$

$$\tilde{\tilde{\theta}}_{ij}(x - y, t_1, t_2) = \left(B_o^{ij}\right)^{-1} \int_0^{t_1} \int_0^{t_2} B^{ij}(z_1, z_2)\, d\tau_1 d\tau_2 \quad ,$$

where

$$z_1 = x - Wm_o^{-1}(t_1 - \tau_1), \;\; z_2 = y - Wm_o^{-1}(t_2 - \tau_2)$$

we can write the moment K_3 from (97) as

$$\begin{aligned} K_3(x, t_1, y, t_2) = m_o^2 \, [\, & M_o \tilde{\tilde{\theta}}_M \frac{\partial u(x, t_1)}{\partial t_1} \frac{\partial u(y, t_2)}{\partial t_2} \\ & + N_o \tilde{\tilde{\theta}}_N \, \nabla u(y, t_2) \frac{\partial u(x, t_1)}{\partial t_1} + N_o \tilde{\tilde{\theta}}_N \, \nabla u(x, t_1) \frac{\partial u(y, t_2)}{\partial t_2} \\ & + B_o^{ij} \tilde{\tilde{\theta}}_{ij} \frac{\partial u(x, t_1)}{\partial x_i} \frac{\partial u(y, t_2)}{\partial y_j} \,] + O(\varepsilon^3) \end{aligned} \tag{114}$$

If $x = y = x$ and $t_1 = t_2 = t$, the variance of concentration at point x at time t is obtained from (114) as

$$K_3(x, t) = m_o^2 \, [\, M_o \tilde{\tilde{\theta}}_M \left(\frac{\partial u}{\partial t}\right)^2 + 2N_o \tilde{\tilde{\theta}}_N \frac{\partial u}{\partial t} \nabla u + B_o^{ij} \tilde{\tilde{\theta}}_{ij} \frac{\partial u}{\partial x_i} \frac{\partial u}{\partial x_j} \,] + O(\varepsilon^3) \tag{115}$$

$$K_3(x, t) = m_o^2 \, \nabla u \, \tilde{\tilde{D}} \, \nabla u + O(\varepsilon^3) \tag{116}$$

$$\tilde{\tilde{D}}^{ij}(t) = \frac{M_o \tilde{\tilde{\theta}}_M(t) W_i W_j}{m_o^2} - \frac{2N_{oi}\tilde{\tilde{\theta}}_{Ni}(t)}{m_o} + B_o^{ij}\tilde{\tilde{\theta}}_{ij}(t)$$

$$\tilde{\tilde{\theta}}_M(t) = 2\int_0^t \theta(\tau)d\tau, \quad \tilde{\tilde{\theta}}_{Ni}(t) = 2\int_0^t \theta_{Ni}(\tau)d\tau, \quad \tilde{\tilde{\theta}}_{ij}(t) = 2\int_0^t \theta_{ij}(\tau)d\tau \tag{117}$$

In an analysis of the behavior of the functions $\tilde{\tilde{\theta}}_{ij}$ for a series of correlation functions of permeability, Dagan (1989) showed that:(a) the longitudinal component of tensor $\tilde{\tilde{\theta}}_{ij}$ at large time is proportional to time t, and (b) the transverse components have a finite limit in 3D space and are proportional to ln t in 2D space. The unlimited growth of longitudinal and transverse (D = 2) components of tensor $\tilde{\tilde{\theta}}_{ij}$ does not contradict the concept of a limited (finite) variation for concentration because the decrease in $\nabla u(x, t)$ at large time is more rapid.

Let us return to (103) and (104) and write them in local form. After introducing (105) for K_3 and (49) for u(x, t), we have a full correlation description for the process of dispersion given by

$$m^*\frac{\partial u}{\partial t} + W^*\nabla u = m_o^{-1}\,\mathrm{div}\, D\nabla u$$

$$m^* = m_o - M(t)m_o^{-1}, \quad W^* = W - m_o^{-1}N(t)$$

$$D^{ij} = \frac{M_o W_i W_j}{m_o^2}\theta_M(t) - 2\frac{N_{oi}W_j}{m_o}\theta_{Ni}(t) + B_o^{ij}\theta_{ij}(t) \tag{118}$$

$$u(x, 0) = f(x)$$

$$m_o\frac{\partial K_1(x, t)}{\partial t} + W\nabla K_1(x, t) = -\left[M(t)\frac{\partial u(x, t)}{\partial t} + N(t)\nabla u(x, t)\right] \tag{119}$$

$$K_1(x, 0) = 0$$

$$m_o\frac{\partial K_{2j}(x, t)}{\partial t} + W\nabla K_{2j}(x, t) = -\left[N_j(t)\frac{\partial u(x, t)}{\partial t} + B^{nj}(t)\frac{\partial u(x, t)}{\partial x_n}\right] \tag{120}$$

$$K_{2j}(x, 0) = 0$$

$$m_o\frac{\partial K_3(x, t)}{\partial t} + W\nabla K_3(x, t) = -2\left[K_1(x, t)\frac{\partial u(x, t)}{\partial t} + K_2(x, t)\,\nabla u(x, t)\right] \tag{121}$$

$$K_3(x, 0) = 0$$

If we use K_1 and K_2 from (109) and (110), respectively, then (121) becomes

$$m_o\frac{\partial K_3(x, t)}{\partial t} + W\nabla K_3(x, t) = 2\nabla u(x, t)D(t)\nabla u(x, t) \tag{122}$$

For example, we shall consider the problem of transport in a 1D field with random porosity. If B = 0 and N = 0 are inserted in (49), we have

$$m^* \frac{\partial u}{\partial t} + W \frac{\partial u}{\partial x} = m_o^{-1} D(t) \frac{\partial^2 u}{\partial x^2} \tag{123}$$

If we assume that

$$M(x, z) = M_o \exp\left(\frac{\tau - t}{\theta_o}\right) \tag{124}$$

then

$$m^* = m_o - \frac{M_o}{m_o} e^{-(t/\theta_o)} \tag{125}$$

$$D(t) = \frac{M_o W^2}{m_o^2} \theta_o \left(1 - e^{-(t/\theta_o)}\right) \tag{126}$$

$$\tilde{\tilde{D}}(t) = \frac{2M_o W^2 \theta_o}{m_o^2} \left[t + \theta_o \left(e^{-(t/\theta_o)} - 1\right)\right] \tag{127}$$

If $t >> \theta_o$, then from (125) and (126), we have

$$m_o \frac{\partial u}{\partial t} + W \frac{\partial u}{\partial x} = \frac{M_o W^2 \theta_o}{m_o^3} \frac{\partial^2 u}{\partial x^2}$$
$$\tilde{\tilde{D}}(t) = \frac{2M_o W^2 \theta_o t}{m_o^2} \tag{128}$$

For example, if we assume an initial concentration given by

$$u(x, 0) = f(x) = \begin{matrix} 1, & x \le 0 \\ 0, & x > 0 \end{matrix} \tag{129}$$

Then the solution to (128) and (129) can be rewritten as

$$u(x, t) = \frac{1}{2}(1 - \operatorname{erf} z), \quad \operatorname{erf} z = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt, \quad z = \frac{x - Vt}{2a\sqrt{t}}$$
$$V = \frac{W}{m_o}, \quad a^2 = \frac{M_o V^2 \theta_o}{m_o^2} \tag{130}$$

It must be kept in mind that the solution given in (130) is valid when $t >> \theta_o$. Using (130) we can write the variance of concentration as

$$K_3(x, t) = \frac{2M_o V^2 \theta_o t}{m_o^2} \left(\frac{\partial u}{\partial x}\right)^2 = \frac{1}{2\pi} e^{-z^2} \tag{131}$$

Since the initial concentration has two values as shown in (129) and the mean concntration u(x, t) is the probability that local concentration is 1, we can write an exact expression for $\overline{K}_3 = u(1 - u)$.

Figure 5 shows the solution for u(x, t) given by (130). The solid line for $\overline{\sigma}_c$ is the mean square deviation from the exact solution and the dashed line shows the approximate solution for σ_c from (130). We see that the latter solution is a reasonable approximation.

SUMMARY

A full description of flow dispersion in a correlation approximation of second order perturbation theory is given. In the field with time and space depended source and random porosity and flow velocity the functional equations for average concentration are suggested. The approximate non-local integral-differential equation for mean concentration is asymptotical equivalent another equations the same order approximation, but differential and local. Correct problems for hyperbolic, elliptic and parabolic transport differential equations are set.

Autocorrelation and cross-correlation moments of concentration, in particular the variency of concentration, porosity and flow velocity fields are presented.

ACKNOWLEDGMENTS

The research reported in this paper was primarily carried out at the All-Union Scientific-Research Oil and Gas Institute (Moscow, Russia) prior to the author coming to the United States. The continuation of this research and the translation into English at the Lawrence Berkeley Laboratory was supported by the Assistant Secretary for Fossil Energy, Office of Technical Coordination, Advanced Extraction and Process Technology, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098. I am particularly indebted to Drs. G. Moridis, K. Pruess and Y.W. Tsang for critically reviewing this paper and providing suggestions for improvements. The kind assistance of Ms. Maureen McGraw and Dr. P. A. Witherspoon and their participation in the preparation of this paper in final form is gratefully acknowledged.

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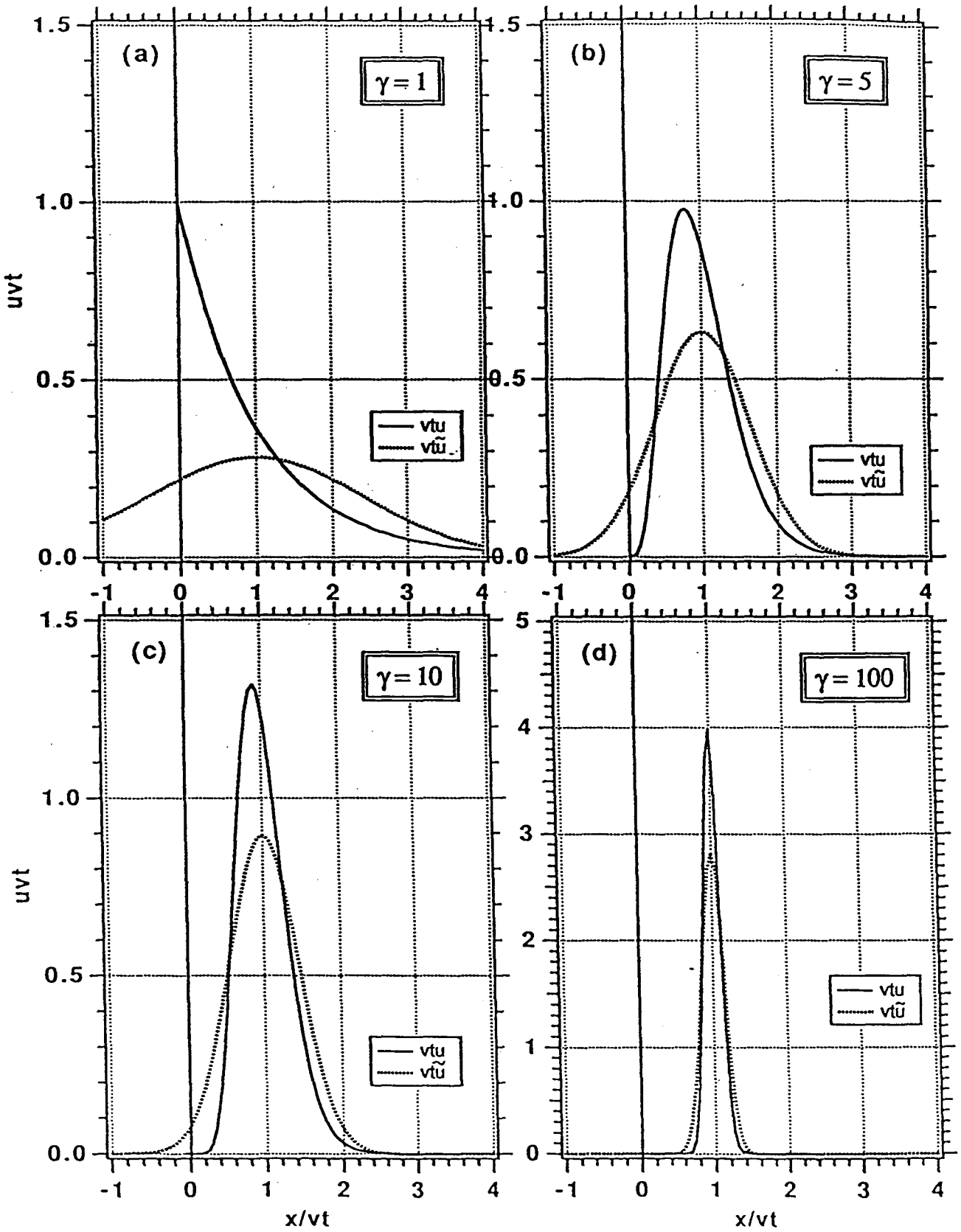

Fig. 1. Comparison of mean concentration distribution obtained from the hyperbolic equation 56 (solid line) with that from the parabolic equation 54 (dashed line) for $\gamma = 1, 5, 10, 100$.

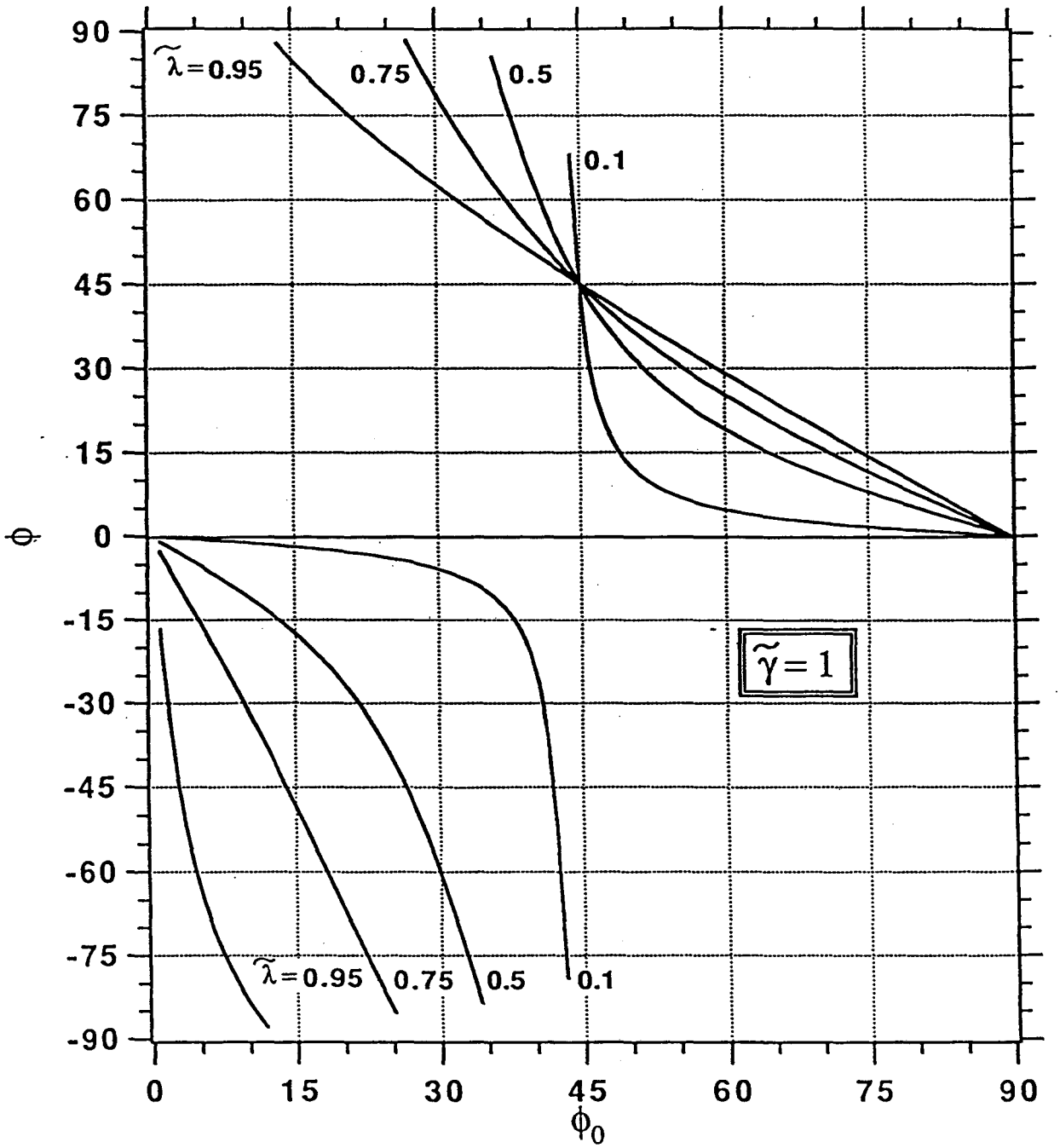

Fig. 2A. Angle ϕ between direction of advection vector W and the y_1-axis of dispersion where the coefficient of correlation between porosity and permeability is positive, $\tilde{\gamma} = 1$, for $\tilde{\lambda} < 1$.

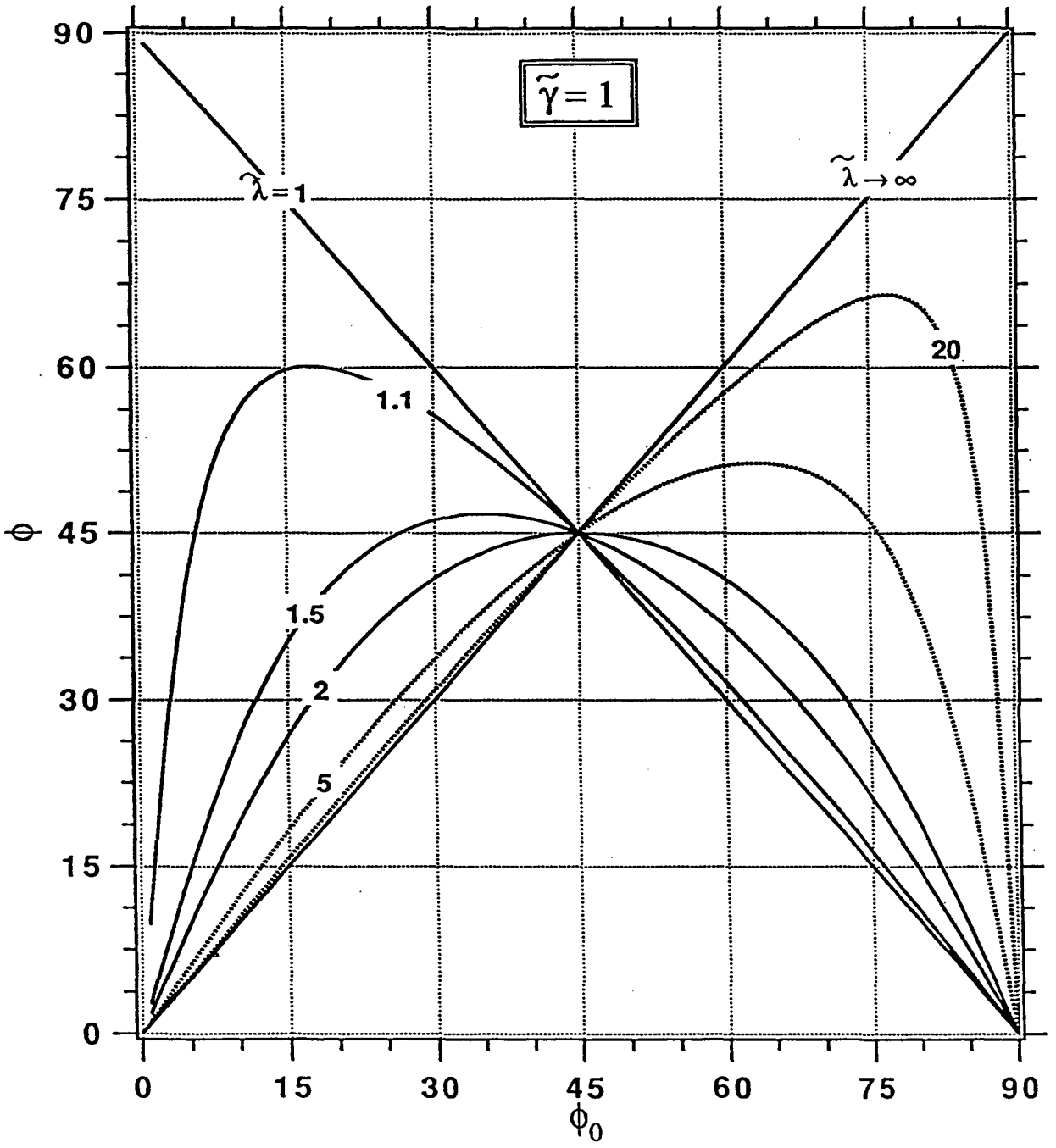

Fig. 2B. Angle ϕ between direction of advection vector W and the y_1-axis of dispersion where the coefficient of correlation between porosity and permeability is positive, $\tilde{\gamma} = 1$, for $\tilde{\lambda} \geq 1$.

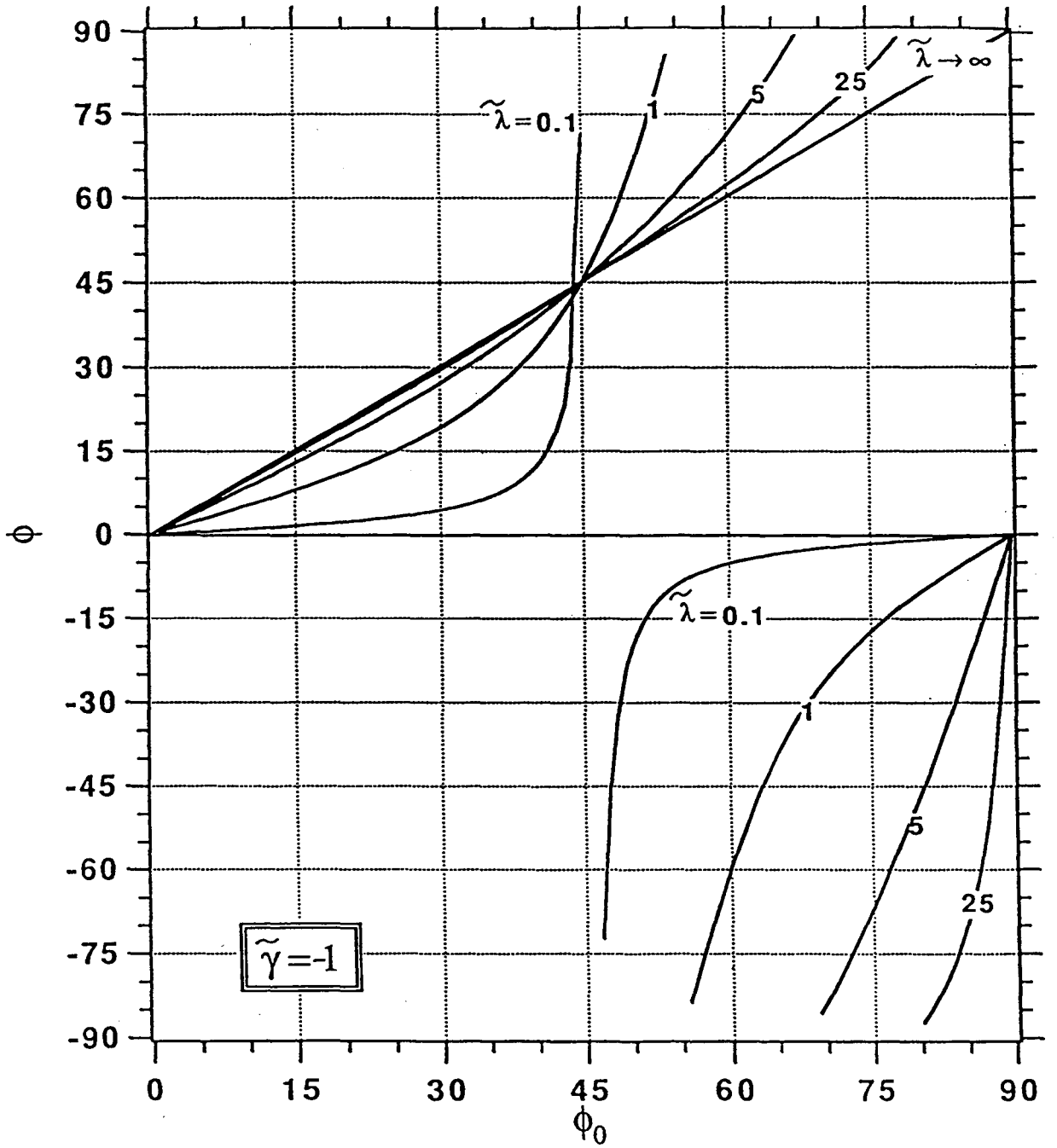

Fig. 2C. Angle ϕ between direction of advection vector W and the y_1-axis of dispersion where the coefficient of correlation between porosity and permeability is negative, $\tilde{\gamma} = -1$.

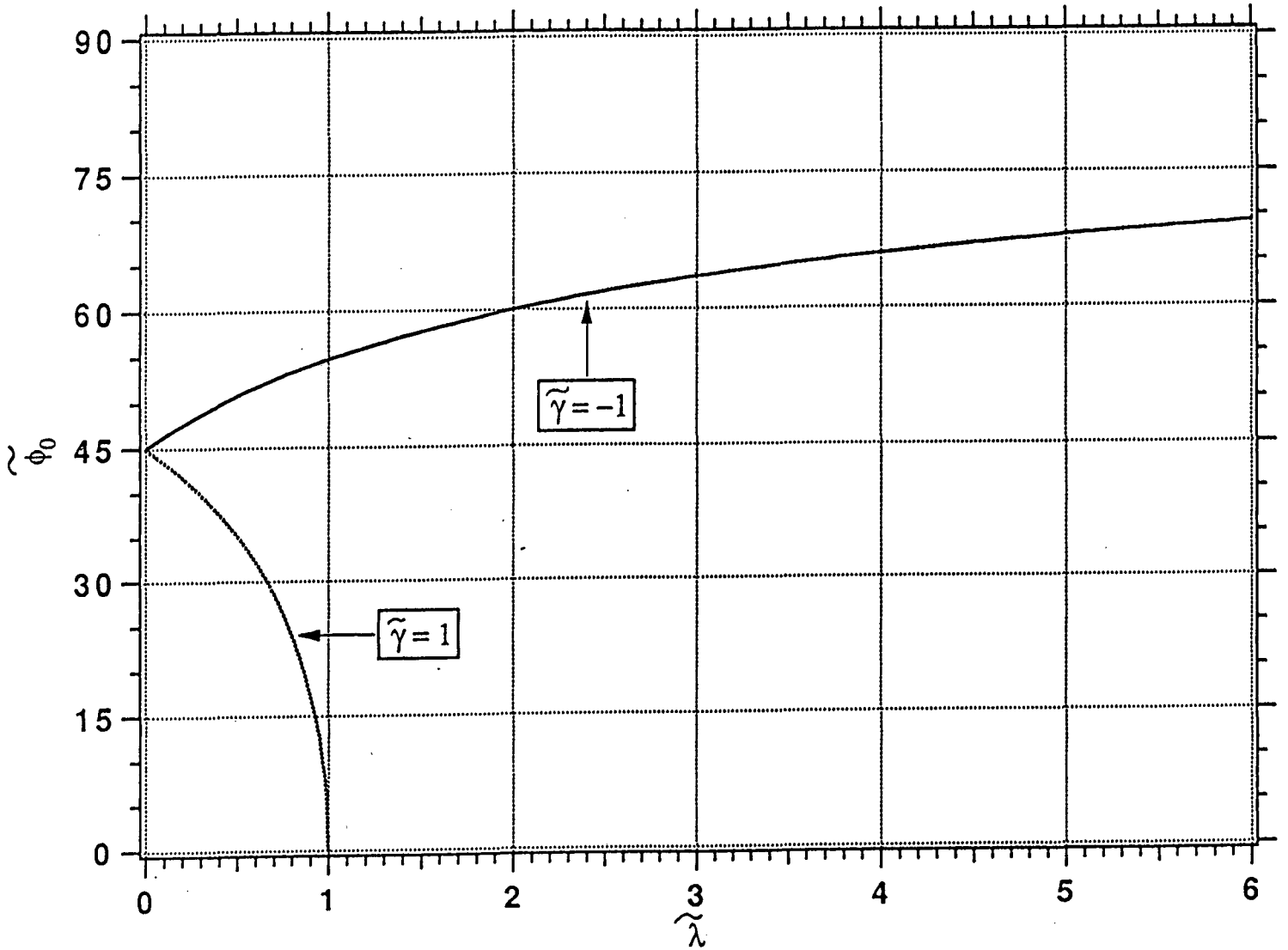

Fig. 3. Dependence of $\tilde{\phi}_0 = \tilde{\phi}_0(\tilde{\lambda})$ for the special condition where the direction of advection vector W is orthogonal to the y_1-axis of dispersion.

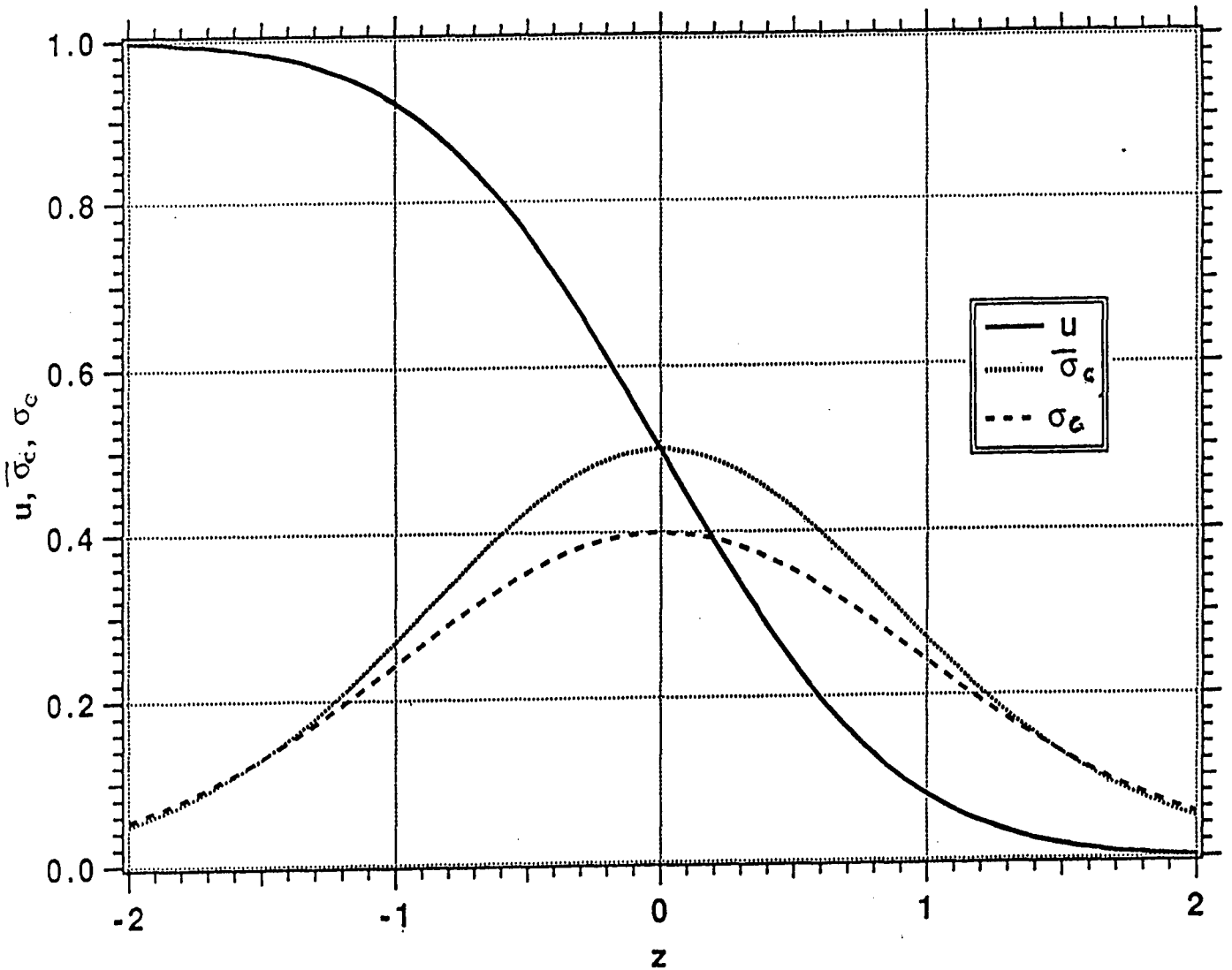

Fig. 4. Distribution of mean concentration u(z) (solid line). The dotted line shows $\bar{\sigma}_c$, the mean square deviation of concentration as computed from u(z), and the dashed line σ_c, is an approximation computed from (131).

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