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# EQUILIBRIUM THEORY OF TOROIDAL MAGNETIC CUSP CONFINEMENT (TORMAC) WITH INTERNAL CIRCULATION

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# EQUILIBRIUM THEORY OF TOROIDAL MAGNETIC CUSP CONFINEMENT (TORMAC) WITH INTERNAL CIRCULATION*

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June 1975

## ABSTRACT

Plasma confined in the Tormac configuration can be made to rotate about the minor axis so that guiding-center drifts do not carry the particles to the boundary layer even in a purely toroidal field B(R). In this way the anisotropy in velocity space caused by losses at the cusps will not be rapidly communicated to the interior. Rotation can be enhanced and controlled, in principle, by neutral injection with angular momentum about the minor axis. The properties and stability characteristics of such a state are theoretically investigated in the present paper. The conditions for a stationary state are shown to require that the number of particles on a flux tube remains constant, yielding the relation $n(R)R/B(R) \approx$ constant. The equilibrium properties of the plasma are observed with the use of two different models: (1) The plasma is treated as consisting of two fluids with negligible centrifugal effects and interspecies interactions, where electrons are considered as isotropic with the equation of state $d/dt(P_e n^{-\gamma}) = 0$, while the ions are anisotropic and obey the CGL equations. (2) The plasma is treated as a single fluid with negligible centrifugal effects and scalar pressure, i.e., obeys $d/dt(Pn^{-\gamma}) = 0$. In each case the equilibrium solutions n(R) are found to be marginally interchange-stable.

## I. INTRODUCTION

Levine and coworkers have described the confinement of high-beta plasma in the Tormac device.[1-3] In this configuration the bulk of the plasma is contained within a toroidal volume of closed magnetic flux tubes (see Fig. 1). A thin boundary layer separates this space from an outer region, the so-called outer "sheath," which is characterized by strong poloidal field components produced by external coils in an annular cusp arrangement. Particles with guiding centers in the outer sheath are only mirror confined on the open field lines. The boundary layer can not measure more than a mean ion-cyclotron-orbit in thickness, because it is the region in which many ions with guiding centers on both open and closed flux surfaces coexist. While the structure and detailed properties of such a boundary layer and outer sheath have not yet been fully analyzed, it is certain that the major pressure gradients have to be concentrated there. In the following we shall assume that such thin layers exist and are stable.

## II. GUIDING-CENTER CIRCULATION

In the basic Tormac concept, the internal field lines are concentric circles, i.e., no toroidal current is present so that there is no magnetic rotational transform. It follows that, unless internal electric fields modify the motion, guiding-center drifts parallel to the torus major axis carry all particles to the plasma edge where they have to be deflected and recirculated promptly along the boundary if toroidal equilibrium is to be maintained. Obviously, this requirement must be incorporated in the prescription for the cusp confinement

of plasma in Tormac. Unfortunately, the losses through the cusps produce an anisotropy in the velocity distribution of the ions in a part of the boundary layer which then is propagated throughout the interior by the particle circulation mentioned above. This leads to the unavoidable conclusion that either a magnetic rotational transform or a net mass rotation is required in Tormac if the confinement is to be better than that expected in a minimum-B stabilized mirror configuration. The function of the rotation here is simply to force most particle guiding centers to circulate well inside the boundary and thereby to reduce drastically the communication as regards the velocity distribution between the interior and the sheath region. Assuming such rotation is maintained, interior particles with velocity vectors in the cusp loss-cones can reach the open field lines only by cross-field diffusion, so that the expected confinement time is much improved over that of ordinary open-ended magnetic configurations.[2] Although the sheath may remove angular momentum from the internal region, it should be possible, as described below, to control and maintain the internal rotation.

Some internal rotation around the minor torus axis by E x B drifts may be expected to arise spontaneously because of initial preferential loss of particles of one sign or the other. Ordinary open systems acquire a positive potential because of the shorter electron collision time. In fact, most electrons are confined mainly by the electrostatic potential created by the magnetically trapped ions in such devices. In the Tormac arrangement, on the other hand, excess positive charge in the interior would give rise to electric fields along the minor

radius, which here, instead of forcing ions out along field lines, would enhance confinement by causing internal circulation of guiding centers. If such spontaneous motion is insufficient for our purpose, rotation about the minor axis can also be driven by application of a torque from the outside, such as for instance by neutral-beam injection with net angular momentum about the minor axis. In fact, if neutral-beam injection is used to feed or heat a Tormac-confined plasma, it may be difficult to avoid adding any angular momentum. In short, we conclude that the plasma in Tormac is likely to be in a state of rotation, and the description of the equilibrium must take this mass motion into account, i.e., we are dealing with a dynamic rather than a static equilibrium.

## III. CONDITION FOR THE STATIONARY STATE

In the basic Tormac the internal plasma is assumed to be uniform in temperature and density so that the internal magnetic field would have to be force-free. The simplest case is the one with $\nabla \times \underline{B} = 0$ so that the field lines are circles, as mentioned above, and $BR = \text{const}$, if R denotes the distance from the major axis. This would be the state of magnetostatic equilibrium with minimal free energy. However, such an ideal uniform-pressure condition cannot be maintained in the presence of even the slightest guiding-center rotation around the minor axis because this would lead to a finite divergence in the mass flow. The condition that must be satisfied in the presence of any rotation is readily derived using a purely macroscopic two-fluid description.

In the stationary state we must have $\nabla \cdot n\underline{u} = 0$ for both ions and

electrons, as well as $\nabla \times \underline{E} = 0$. In this paper we restrict ourselves to the simple case without toroidal current, i.e., with purely circular field lines:

$$\underline{B} = B(R,z)\hat{\theta}, \tag{3.1}$$

so that the parallel flow, $nu_{\parallel}$, does not enter the problem. The mean perpendicular flow velocities, $\underline{u}_{\perp}$, in the steady state follow from the equation of motion for each species (particle mass m, charge Ze, and stress tensor $\underline{\underline{P}}$) and can be written in the form[4]

$$\underline{u}_{\perp} = \underline{u}_E + \underline{u}_c + \underline{u}_{In} + \underline{u}_{\nabla}, \tag{3.2}$$

where

$$\underline{u}_E \equiv c\,\frac{\underline{E} \times \underline{B}}{B^2},$$

$$\underline{u}_c \equiv -\frac{c}{Zen}\,\frac{(\nabla\cdot\underline{\underline{P}}) \times \underline{B}}{B^2},$$

$$\underline{u}_{In} \equiv -\frac{mc}{Ze}\,\frac{(\underline{u}\cdot\nabla)\underline{u} \times \underline{B}}{B^2},$$

$$\underline{u}_{\nabla} \equiv \frac{c}{Ze}\,\frac{\underline{F} \times \underline{B}}{B^2}.$$

The last term, $\underline{u}_{\nabla}$, represents cross-field diffusion, which is caused either by the pressure and temperature gradients in the presence of electron-ion friction $\underline{F}$, or by anomalous dissipation. The term $\underline{u}_{\nabla}$ is assumed negligible here as compared to the first two, $\underline{u}_E$ and $\underline{u}_c$, because we are mostly concerned with the conditions in the collisionless limit, and gradients in the interior are too small to drive turbulent

transport. The inertial drift, $\underline{u}_{In}$, is primarily caused by centrifugal effects and is also negligible as long as the bulk rotation frequency is small compared to the ion gyrofrequency. The diamagnetic flow (or "net gyration"), $n\underline{u}_c$, may of course be large, but by its nature it is always free of divergence under conditions of hydromagnetic equilibrium. We, therefore, are only concerned with the E x B guiding-center drift $\underline{u}_E$. The requirement $\nabla \cdot n\underline{u}_E = 0$ and $\nabla \times \underline{E} = 0$ leads to

$$\frac{\underline{E} \times \underline{B}}{B^2} \cdot \nabla \ln n = -\nabla \cdot \frac{\underline{E} \times \underline{B}}{B^2} = \underline{E} \cdot \nabla \times \frac{\underline{B}}{B^2}, \tag{3.3}$$

which is satisfied for all $\underline{E}$ if

$$\nabla \times \underline{B} = \underline{B} \times \nabla \ln (n/B^2). \tag{3.4}$$

If the field $\underline{B}$ is purely toroidal, i.e., if (3.1) holds, Eq. (3.4) can be solved immediately: The R component of Eq. (3.4) yields

$$n(R,z)/B(R,z) = f(R), \tag{3.5}$$

while the z component of (3.4) has the solution

$$\frac{n(R,z)R}{B(R,z)} = g(z),$$

so that we must have $g(z) = \text{const}$, and $f(R) = \text{const}/R$. The condition for stationary flow can thus be expressed simply as

$$\frac{nR}{n_0 R_0} = \frac{B}{B_0}, \tag{3.6}$$

where $n_0 = n(R_0,z_0)$ and $B_0 = B(R_0,z_0)$ at an arbitrary point $(R_0,z_0)$ inside the plasma. Relation (3.6) merely states formally the obvious fact that the number of particles in a flux tube must remain constant during the motion under our conditions when cross-field diffusion is negligible. This means, of course, that $n(R,z) = \text{const}$ is not an acceptable solution if $\underline{E} \times \underline{B} \neq 0$ anywhere in this torus. To find the specific required dependence of n (or B) on R and z we must use the equations for the dynamic equilibrium to obtain a second and independent relation between these variables.

## IV. DYNAMIC EQUILIBRIUM

Further progress towards determining the equilibrium requires adoption of a model. In any physically reasonable model, which must be valid for times $\lesssim \tau_{ii}$ (the ion-ion collision time), the electrons are isotropic due to their fast collision rate and the collisionless whistler instability.[5] The latter takes place on a very fast time scale, i.e., $\tau_e \approx (\beta_{e\perp}/2)^{-1/2}|\omega_{ce}|^{-1}$. The ions are subject to a similar instability, the electromagnetic ion-cyclotron instability,[6] which is capable of rendering the ions isotropic within a few time constants, $\tau_i$. For this instability the time constant $\tau_i$ may or may not be small compared to $\tau_{ii}$. Two limiting cases are treated here:

(1) A two-fluid plasma with isotropic electrons obeying $d/dt\ (P_e n^{-\gamma}) = 0$ and anisotropic ions obeying the CGL equations;[7]

(2) A single-fluid plasma with a scalar pressure obeying $d/dt\ (Pn^{-\gamma}) = 0$, where $P = P_e + P_i$.

We shall first treat the two fluids since the one-fluid result will then follow as a special case. The equilibrium equations for the

rotating plasma are (neglecting interspecies collisions):

$$(nm_i + nm_e)\underline{u}\cdot\nabla\underline{u} + \nabla\cdot\underline{\underline{P}}_i + \nabla P_e = (\nabla \times \underline{B}) \times \underline{B}/4\pi,$$

$$\underline{u}\cdot\nabla(P_e n^{-\gamma}) = 0,$$

$$\underline{u}\cdot\nabla\left(\frac{P_{i\perp}^2 P_{i\parallel}}{n^5}\right) = 0,$$

$$\underline{u}\cdot\nabla(P_{i\perp}/nB) = 0,$$

$$\underline{\underline{P}}_i = \begin{bmatrix} P_{i\perp} & & \\ & P_{i\perp} & \\ & & P_i \end{bmatrix}. \tag{4.1}$$

Quasi-neutrality has also been assumed ($n_i = n_e = n$). Under conditions of slow (very subsonic) rotation, $nm_i|\underline{u}\cdot\nabla\underline{u}| \ll |\nabla P|$, the inertial term may be dropped. The dynamic character of the equilibrium is, of course, still preserved by the additional constraints $\underline{u}\cdot\nabla(P_e n^{-\gamma}) = 0$, etc. Using the conditions $\underline{B} = B(R,z)\hat{\theta}$ and $nR/B = \text{const}$, the above set reduces to all quantities being functions of R only (z-independent), and:

$$\frac{\partial}{\partial R}\left(\frac{B^2}{8\pi} + P_{i\perp} + P_e\right) = -\frac{B^2}{4\pi R}\left(\frac{P_{i\perp} - P_{i\parallel}}{B^2/4\pi} + 1\right), \tag{4.2}$$

$$\frac{P_{i\perp}R}{B^2} = \frac{P_{i0\perp}R_0}{B_0^{\,2}}, \qquad \frac{P_{i\parallel}R^3}{B} = \frac{P_{i0\parallel}R_0^{\,3}}{B_0},$$

$$P_e\left(\frac{B}{R}\right)^{-\gamma} = P_{e0}\left(\frac{B_0}{R_0}\right)^{-\gamma}.$$

Defining $\beta_{i\perp} = 8\pi P_{i\perp}/B^2$, etc., we find

$$(1 + \beta_{i\perp} + \beta_e) \frac{\partial B^2}{\partial R} + B^2 \frac{\partial}{\partial R} (\beta_{i\perp} + \beta_e) = \frac{B^2}{R} (\beta_{i\parallel} - \beta_{i\perp} - 2), \qquad (4.3)$$

and hence

$$(1 + \beta_{i\perp} + \beta_e) 2B \frac{\partial B}{\partial R} + B^2 \left[ - \frac{\gamma}{R} \beta_e + (\gamma - 2) \frac{\beta_e}{B} \frac{\partial B}{\partial R} \right] = \frac{B^2}{R} (\beta_{i\parallel} - 2). \qquad (4.4)$$

Hence, in equilibrium, B satisfies

$$\frac{\partial \ln B}{\partial \ln R} = \frac{\beta_{i\parallel}/2 - 1 + \beta_e \gamma/2}{\beta_{i\perp} + 1 + \beta_e \gamma/2} . \qquad (4.5)$$

The equilibrium relation between density and radius may be obtained explicitly. Letting $b = B/B_0$, $\eta = n/n_0$, and $r = R/R_0$, Eq. (4.3) becomes

$$(1 + \beta_{i\perp} + \frac{\gamma}{2} \beta_e) \frac{\partial b}{\partial r} = \frac{b}{r} \left( \frac{\beta_{i\parallel}}{2} + 1 + \frac{\gamma}{2} \beta_e \right) . \qquad (4.6)$$

From Eq. (4.2),

$$\beta_{i\perp} = \frac{\beta_{i0\perp}}{r} ,$$

$$\beta_{i\parallel} = \frac{\beta_{i0\parallel}}{\eta r^4} ,$$

$$\beta_e = \frac{\beta_{e0}}{\eta r^2}^{\gamma-2}$$

$$b = \eta r, \qquad (4.7)$$

and hence (4.6) becomes

$$\frac{\partial\eta}{\partial r}\left(1 + \frac{\beta_{i0\perp}}{r} + \frac{\gamma}{2}\beta_{e0}\frac{\eta^{\gamma-2}}{r^2}\right) = \frac{\eta}{r}\left(\frac{\beta_{i0\parallel}}{2\eta r^4} - \frac{\beta_{i0\perp}}{r} - 2\right). \tag{4.8}$$

Multiplying through by $r^2$ yields a perfect differential, i.e., Eq. (4.8) is equivalent to

$$\frac{\partial}{\partial r}\left[\frac{\gamma}{2(\gamma - 1)}\beta_{e0}\eta^{\gamma-1} + \eta r^2 + \beta_{i0\perp} r\eta + \frac{\beta_{i0\parallel}}{4r^2}\right] = 0, \tag{4.9}$$

and hence we obtain:

$$\frac{\gamma}{2(\gamma - 1)}\beta_{e0}\left(\frac{n}{n_0}\right)^{\gamma-1} + \frac{n}{n_0}\left(\frac{R}{R_0}\right)^2 + \beta_{i0\perp}\frac{n}{n_0}\left(\frac{R}{R_0}\right) + \frac{\beta_{i0\parallel}}{4}\left(\frac{R_0}{R}\right)^2$$
$$= \frac{\gamma}{2(\gamma - 1)}\beta_{e0} + 1 + \beta_{i0\perp} + \frac{\beta_{i0\parallel}}{4}. \tag{4.10}$$

The B field corresponding to this n is plotted for several cases, including high $\beta$, in Fig. 2. The fact that, for high $\beta$, B is monotonically increasing should make Tormac confinement easier in that the curvature and gradient-B drifts of the particle guiding centers tend to cancel rather than add (as is the case in current-free fields). In the opposite limit $\beta_{e0}$, $\beta_{i0\perp}$, $\beta_{i0\parallel} \ll 1$, Eq. (4.10) gives $n \propto 1/R^2$, or $B \propto 1/R$ as expected for the current-free case.

Proceeding to the one-fluid MHD approach, we see that the equations follow from the two-fluid equations if we let $\underline{\underline{P}}_i \to 0$, and $P_e \to P_e + P_i = P$; i.e.,

$$(nm_i + nm_e)\underline{u}\cdot\nabla\underline{u} + \nabla P = (\nabla \times \underline{B}) \times \underline{B}/4\pi,$$

$$\underline{u}\cdot\underline{\nabla}(Pn^{-\gamma}) = 0. \tag{4.11}$$

Making the same approximations of slow rotation and quasi-neutrality, but dropping the assumption of negligible interspecies interactions, we arrive at Eq. (4.10) with $\beta_{i0\perp} = \beta_{i0\parallel} = 0$, $\beta_{e0} \to \beta_0$:

$$\frac{\gamma}{2(\gamma - 1)}\beta_0\left(\frac{n}{n_0}\right)^{\gamma-1} + \frac{n}{n_0}\left(\frac{R}{R_0}\right)^2 = \frac{\gamma}{2(\gamma - 1)}\beta_0 + 1. \tag{4.12}$$

Of special interest is the high-$\beta$ situation, when $\beta_0 \gg 2(\gamma - 1)R^2/\gamma R_0^{\ 2}$, so that the term in $R^2$ in Eq. (4.12) can also be neglected as a first approximation. In that case we see that $n \approx n_0$ everywhere, which means that, because of (3.6),

$$\underline{B}(R) \approx (B_0R/R_0)\hat{\theta}. \tag{4.13}$$

In other words, in the high-$\beta$ limit, the field is modified so that the volume of a toroidal flux tube remains constant as it changes its major radius R. It should be noted, also, that $n \approx$ const does not mean that the plasma is current-free. On the contrary, the equilibrium here requires a uniform internal current density large enough to balance the finite residual pressure gradient that is consistent with the continuity requirement, Eq. (3.6). Again we see that the curvature and grad-B drifts are in opposition, due to the linearly increasing nature of the field. Also, as $\beta \to 0$, we again find that $B \propto 1/R$.

The applicability of each model depends on the degree of isotropization of the ion pressure over the time-scales of interest (i.e., for $\Delta t \approx |R - R_0|/u_\perp$). On time scales shorter than the ion-ion collision time, the primary agent for achieving isotropy is the previously mentioned ion-cyclotron instability.

The growth rate for the instability increases rapidly with magnitude of $\beta_{i\perp}$ and $\beta_{i\perp}/\beta_{i\parallel}$.[6] Typical values are given in Table I. For judicious choice of $\beta_{i0\perp}$, $\beta_{i0\parallel}$, $\beta_{e0}$, and $R/R_0$ such that $\left[(\beta_{i\perp}/\beta_{i\parallel}) - 1\right]$ is fairly small throughout the inner Tormac region, it is possible that the time constant for the instability, $\tau_i$, is of the same order as $\tau_{ii}$ and much longer than $|R - R_0|/u_\perp$ so that the two-fluid theory is valid. For large initial perpendicular and parallel pressure differences $\beta_{i\perp}/\beta_{i\parallel} \gg 1$, the instability will, in linear approximation, start to isotropize the ions on a fast time scale $\gamma_{inst} \gg |\omega_{ci}|$. Presumably once this process is initiated it will continue to completion and the MHD theory will be appropriate. Of course, there is an intermediate region where neither theory is valid.

## V. HYDROMAGNETIC STABILITY

To investigate the hydromagnetic stability of the system, we use a method developed by Newcomb.[8] We consider a single flux tube displaced radially by an amount $\xi$; the change in perpendicular pressure in the tube, $\Delta(P_{i\perp} + P_e + B^2/8\pi)$, must balance the change in the ambient value of the perpendicular pressure, $\xi d/dR(P_{i\perp} + P_e + B^2/8\pi)$. The adiabatic and double adiabatic equations of state, as well as $n(R)R/B(R) = \text{const}$, link $\Delta P_e$, $\Delta P_{i\perp}$, $\Delta P_{i\parallel}$ to $\xi$ and $\Delta B$. The equilibrium equation (4.2) yields $d/dR(P_{i\perp} + P_e + B^2/8\pi)$. Equating the change in perpendicular pressure in the tube to the change in the ambient per-

pendicular pressure we find, eliminating $\Delta P_{i\perp}$, $\Delta P_{i\parallel}$, $\Delta P_e$:

$$\frac{\Delta B}{B} = \frac{\xi}{R} \frac{(P_{i\parallel} + \gamma P_e - B^2/4\pi)}{(2P_{i\perp} + \gamma P_e + B^2/4\pi)} . \tag{5.1}$$

Next we consider the effective tension along the field lines, which is given by $(-P_{i\parallel} - P_e + B^2/8\pi)$. If the change in tension of a flux tube exceeds the change in tension of the ambient medium, the tube will experience a net restoring force, i.e., the plasma will be stable. If the change in tension of the tube is less than that of the surrounding medium, the tube will continue to expand or contract radially resulting in instability. To obtain the marginally stable case we equate $\Delta(-P_{i\parallel} - P_e + B^2/8\pi)$ to $\xi d/dR(-P_{i\parallel} - P + B^2/8\pi)$. This, in combination with the equations of state and Eq. (5.1), yields

$$\frac{d \ln B}{d \ln R} = \frac{\beta_{i\parallel}/2 - 1 + \beta_e\gamma/2}{\beta_{i\perp} + 1 + \beta_e\gamma/2} \tag{5.2}$$

for marginal stability. Note that the marginal stability condition (5.2) is just the expression earlier obtained as the equilibrium condition for $(d \ln B)/(d \ln R)$. The result is not surprising in view of the fact that the slow rotation about the minor axis, which we have proscribed for our hydromagnetic equilibrium state, is simply a pure interchange process.

The centrifugal effects, which we neglected in going from Eq. (4.1) to Eq. (4.2) must then be expected to render the system unstable. However, any dissipative process, e.g., as caused by heat flow, viscosity, or electrical resistivity, will undoubtedly have a stabilizing

effect. We conclude that slow mass rotation about the minor axis is probably permissible in Tormac without causing difficulties.

Fortunately, not much rotation is needed to dominate the guiding-center drifts when the particles' mean gyroradius $r_L$ is small compared to R. The conditions that the mass velocity u must satisfy for this purpose, and at the same time must introduce only negligible centrifugal effects, are

$$\frac{r_L}{R} \ll \frac{u}{v_{th}} \ll \left(\frac{r_c}{R}\right)^{1/2} . \tag{5.3}$$

Here $v_{th}$ is the mean thermal speed, and $r_c$ denotes the radius of curvature of the rotating flow, i.e., $r_c$ is of the order the plasma minor radius, so that the centrifugal acceleration is $|(\underline{u}\cdot\nabla)\underline{u}| \approx u^2/r_c$. For conditions appropriate for a thermonuclear reactor this requirement should be easily accommodated. If we wish to restrict the displacement of a drift orbit to the order of a few gyroradii, the left inequality in (5.3) must be more specific, i.e., it must be replaced by

$$r_c/R \leq u/v_{th} . \tag{5.4}$$

In other words, as is easily understood, the problems associated with the toroidal geometry are more readily dealt with when the aspect ratio $R/r_c$ is large.

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## FOOTNOTES AND REFERENCES

*Work performed under the joint auspices of the Electric Power Research Institute and the U. S. Energy Research and Development Administration.

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7. N. A. Krall and A. W. Trivelpiece, Principles of Plasma Physics (McGraw-Hill Book Co., New York, 1973), p. 118 ff. Note that on p. 121 in the last bracket of the first equation (3.10.11) the term $B^2/8\pi$ should be $B^2/4\pi$. Also, in the first equation of Problem 3.10.1, on p. 120, there should be a plus sign between the two major terms.

8. W. A. Newcomb, Lawrence Livermore Laboratory, private communication.

## FIGURE CAPTIONS

Fig. 1. The Tormac configuration, showing a typical ion-guiding center drift in the interior.

Fig. 2. Normalized B $\left[B_0 = B(R_0)\right]$ for two-fluid and one-fluid equilibria and various $\beta$'s. In both cases $\gamma = 5/3$.

Fig. 3. Normalized pressures, $P_{i\perp}/P_{i0\perp}$, $P_{i\parallel}/P_{i0\parallel}$, $P_e/P_{e0}$ for the case $\gamma = 5/3$ and $\beta_{i0\perp} = \beta_{i0\parallel} = \beta_{e0} = 1/2$.

Table I. Growth rates for the electromagnetic ion-cyclotron instability.

| $\beta_{i\perp}$ | $\beta_{i\perp}/\beta_{i\parallel}$ | $\gamma_{inst}$ |
|---|---|---|
| $\ll 1$ | $\sim 1$ | 0 |
| 10 | 1 | $\ll 1\lvert\omega_{ci}\rvert$ |
| 10 | 2 | $\approx 0.1\lvert\omega_{ci}\rvert$ |
| $\gg 1$ | $\gg 1$ | $\left(\frac{\beta_{i\perp}}{2}\right)^{1/2}\lvert\omega_{ci}\rvert$ |

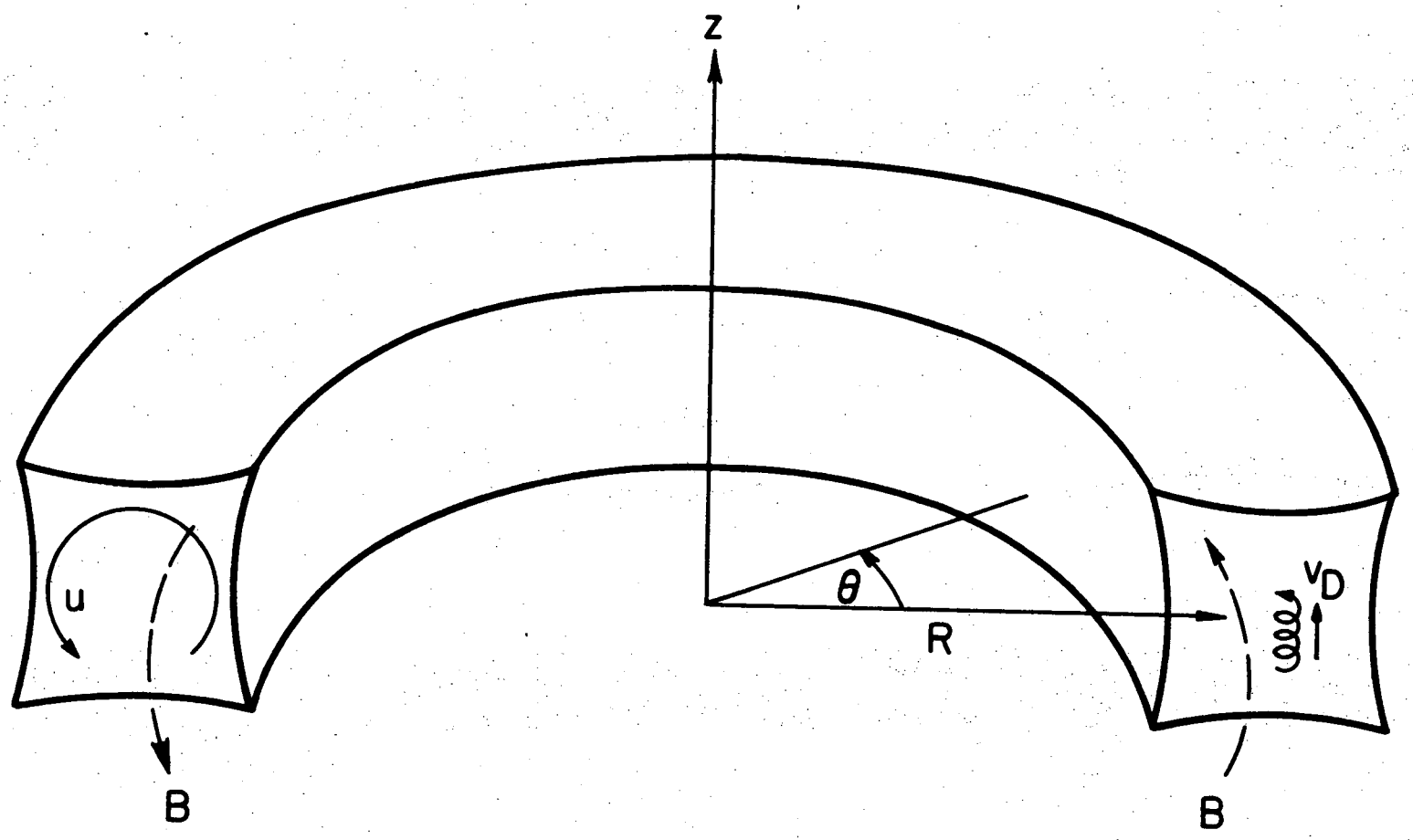


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Fig. 1

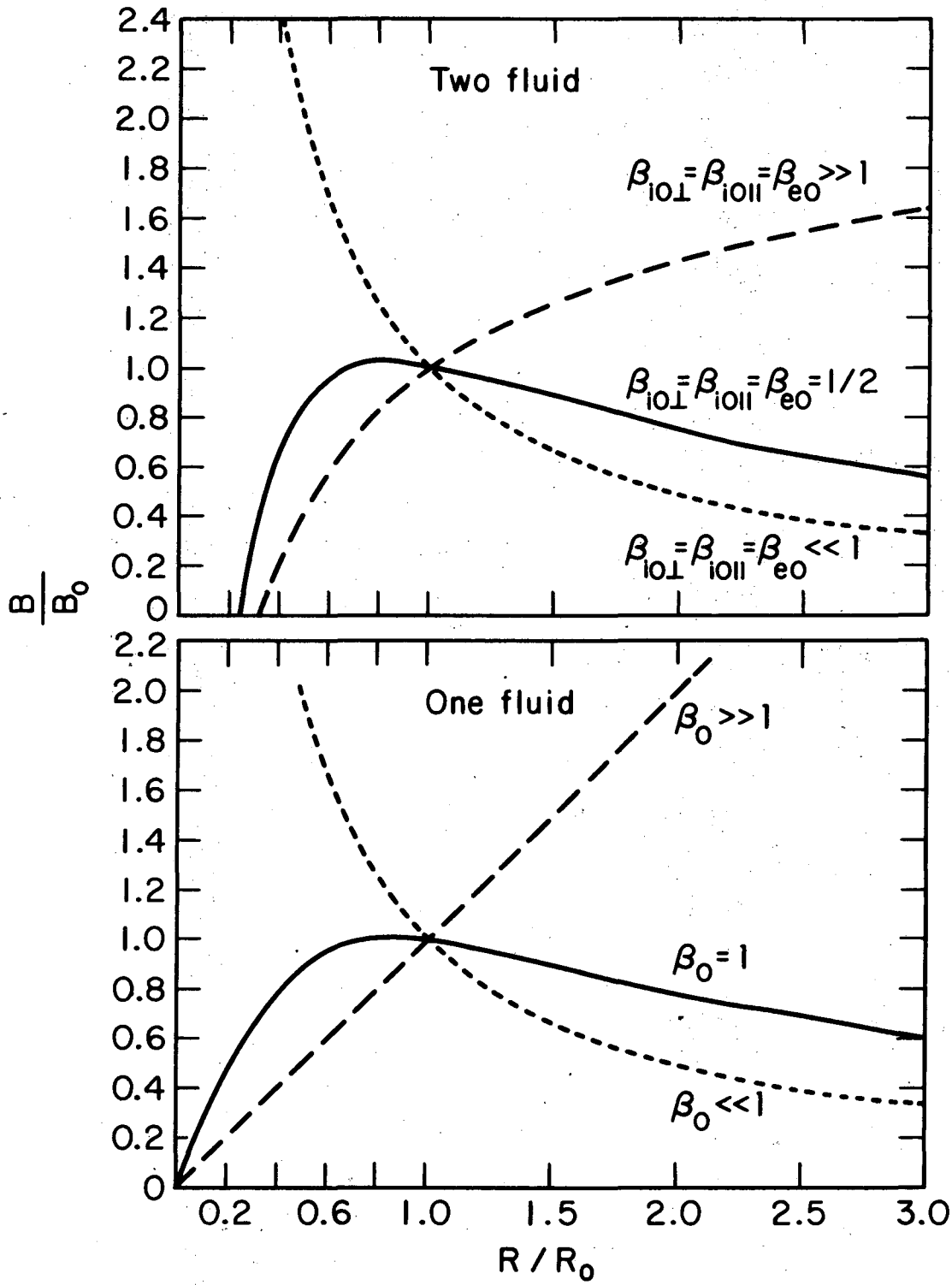


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Fig. 2

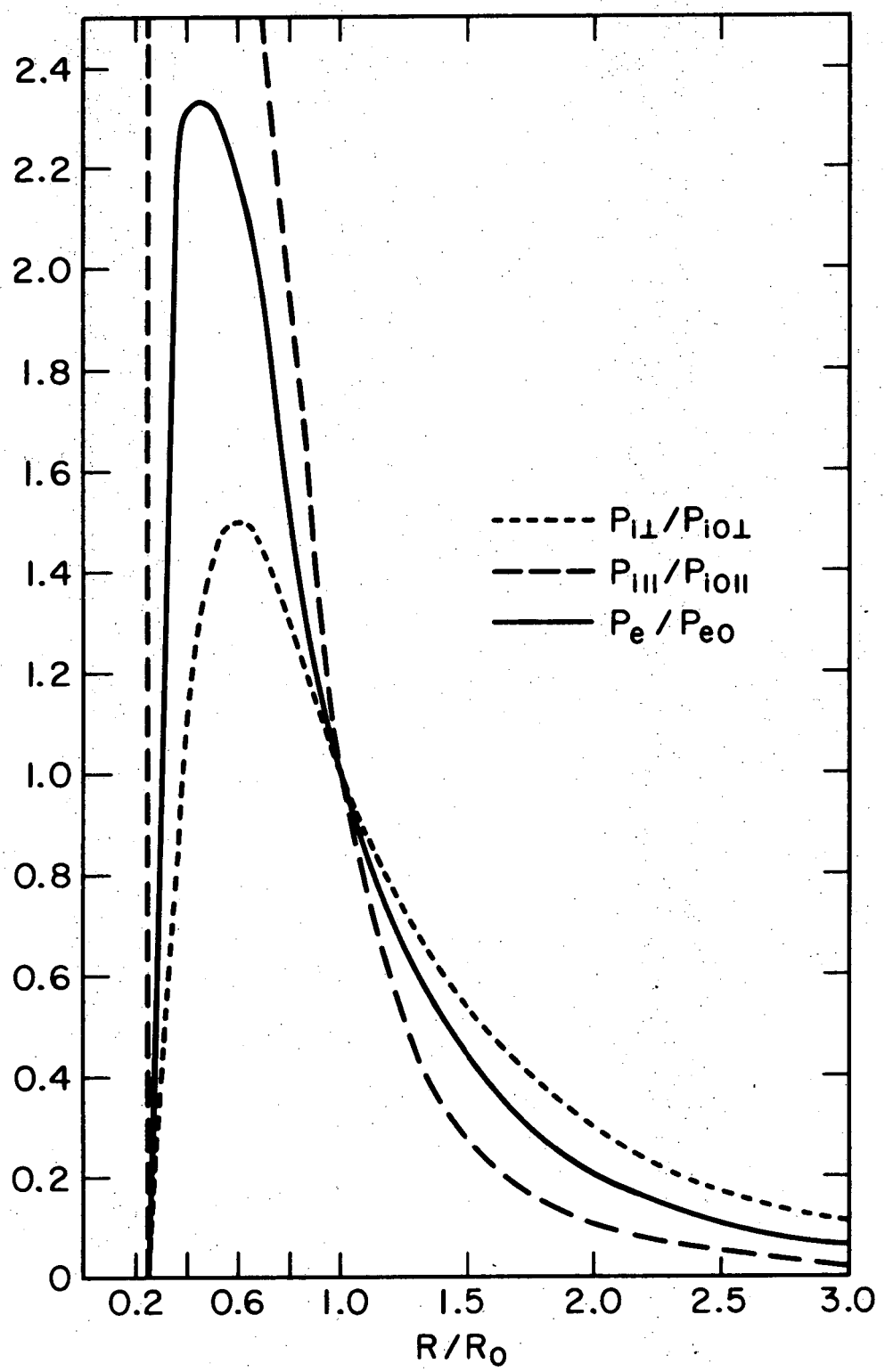


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Fig. 3

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