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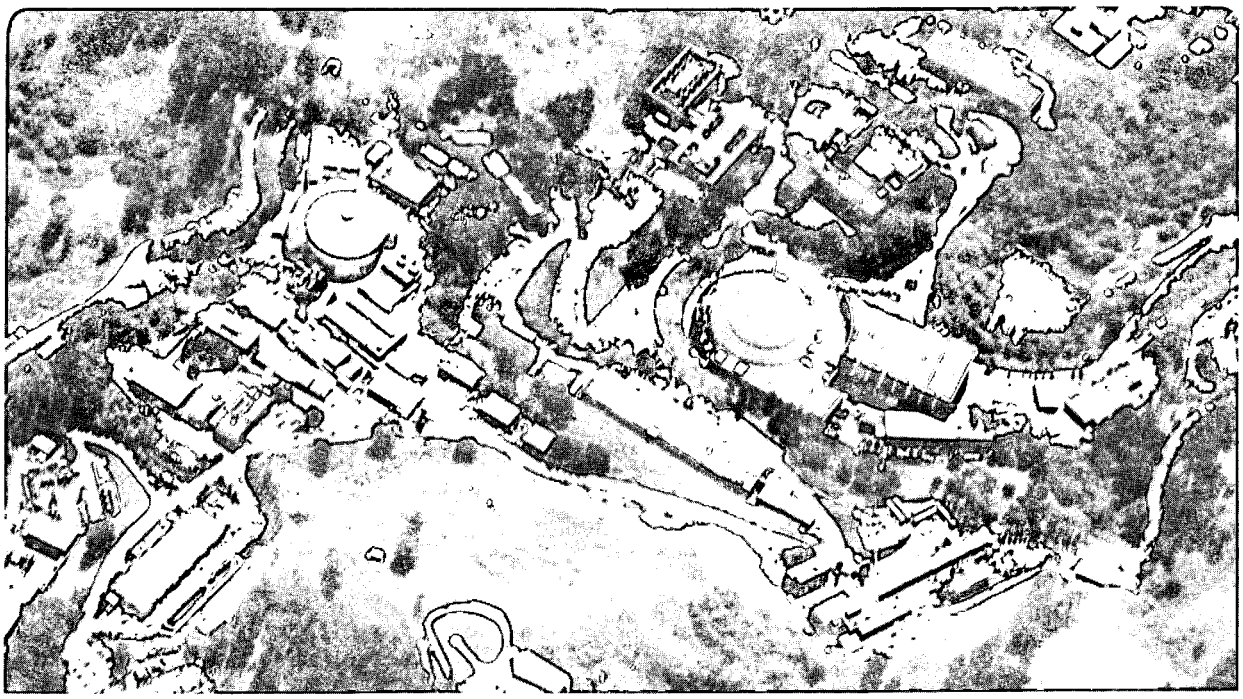

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Minimizing Systematic Errors on the Measurement of $\sin^2\theta_W$ *

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ABSTRACT

A method of analyzing information from the reaction $e^+e^- \to \tau^+\tau^-$ is described. It consists of determining a combination of parameters specific of this reaction, such that systematic errors on $\sin^2\theta_W$ be minimized. The impact on statistiscal errors is discussed.

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Minimizing Systematic Errors on the Measurement of $\sin^2\theta_W$

Introduction.

There are many ways to determine the electroweak mixing parameter $\sin^2\theta_W$. From the data of the reaction $e^+e^- \to \tau^+\tau^-$ alone, at a center-of-mass energy equal to the Z^0 mass and with unpolarized beams, three observables yielding a value of $\sin^2\theta_W$ can be measured: the forward-backward asymmetry of the τ production cross section A_{FB}, the mean τ polarization P, and the forward-backward asymmetry of the τ polarization A^P_{FB}. This article describes a method of determining $\sin^2\theta_W$ by combining various informations obtained from the production of τ's near the Z^0 mass and their subsequent decay, in a way that minimizes systematic errors[1].

A major source of systematic errors is generated by the variations of detection efficiencies in the detector, which bias angular and energy distributions. The present method relies on the measurement of only the fraction of positive and negative particles produced by τ decay in each direction at each energy, assuming only that, in each direction at each energy, the detectors have the same detection efficiency for positive and negative particles. The detection efficiency thus falls off in the expression of the likelihood function and the determination of $\sin^2\theta_W$ does not depend on that efficiency.

Notation and definitions

According to Ref. [1], the production cross section $\frac{d\sigma_\pm}{d\Omega}(\theta)$ and the average helicity $P_\pm(\theta)$ of $\tau^\pm$'s produced at an angle θ with respect to the incident e^- direction are :

$$\frac{d\sigma_\pm}{d\Omega}(\theta) = F_0(1+\cos^2\theta) \mp 2F_1\cos\theta \tag{1}$$

[1]In the same spirit, the determination of $\sin^2\theta_W$ by the measurement of the left-right asymmetry parameter of any e^-e^+ cross section, in an experiment using polarized e^- or e^+ beams, has been advocated because of its small systematic error.

and :

$$\frac{d\sigma_\pm}{d\Omega}(\theta)P_\pm(\theta) = \pm F_2(1+\cos^2\theta) - 2F_3\cos\theta \tag{2}$$

where :

$$F_0(s) = \frac{\pi\alpha^2}{2s}[1 + 2g_V^2 Re\chi(s) + (g_V^2 + g_A^2)^2|\chi(s)|^2] \tag{3}$$

$$F_1(s) = \frac{\pi\alpha^2}{2s}[2g_A^2 Re\chi(s) + 4g_V^2 g_A^2|\chi(s)|^2] \tag{4}$$

$$F_2(s) = \frac{\pi\alpha^2}{2s}[2g_V g_A Re\chi(s) + 2g_V g_A(g_V^2 + g_A^2)|\chi(s)|^2] \tag{5}$$

$$F_3(s) = \frac{\pi\alpha^2}{2s}[2g_V g_A Re\chi(s) + 2g_V g_A(g_V^2 + g_A^2)|\chi(s)|^2] \tag{6}$$

and :

$$\chi(s) = \frac{s}{s - M_Z^2 + is\Gamma_Z/M_Z} \tag{7}$$

$$s = 4\,E_{\text{beam}}^2 \tag{8}$$

where E_{beam} is the beam energy.

We assume the standard model, i.e. lepton universality and expressions of coupling constants between Z^0 and leptons satisfying :

$$g_A = g_A^e = g_A^\tau = -\frac{1}{2} \tag{9}$$

$$g_V = g_V^e = g_V^\tau = -\frac{1}{2} + 2\sin^2\theta_W\ . \tag{10}$$

In Born approximation, the production of the τ lepton is characterized entirely by its charge and its production angle θ. The charge of the τ is that of its decay particle (we consider only decays into one single charged particle) and we assume that the angle θ_d of the decay particle in the laboratory is a good enough approximation for θ.

The decay of the τ is described by a number of kinematical quantities which include angles in the τ center-of-mass system, effective masses of combinations of

particles (in the case of a multi-particle decay), etc... In the case of τ decays into leptons ($\tau \to e\nu\nu$ or $\tau \to \mu\nu\nu$), or into $\pi\nu$, the subset of quantities that are related to the τ helicity reduces to the ratio x of the decay particle energy E to the beam energy E_{beam} [2],[3],[4]:

$$x = E/E_{\text{beam}} \ . \tag{11}$$

Decays into leptons and $\pi\nu$ are the only ones analyzed in this article, though, in principle, the method described can be applied to more complicated decay modes such as $\tau \to \rho\nu$ [5].

Let $u(x)$ be the x distribution expected from unpolarized τ's decaying into neutrino(s) and the charged decay particle under consideration (e, μ, or π). Let $v(x)$ be the difference between the x distribution expected for τ^-'s with helicity $+1$ and for unpolarized τ's, and let :

$$\omega(x) = \frac{v(x)}{u(x)} \ . \tag{12}$$

The decay functions $u(x)$ and $v(x)$, and their ratio $\omega(x)$, are given in Table 1 for the three decay modes considered, in Born approximation. For other decay modes, what follows is true if one uses the proper expression of ω, which depends on the kinematical quantities characteristic of the decay [6],[5].

Decay functions	$\tau \to \pi$ decays	τ leptonic decays
$u(x)$	1	$\frac{1}{3}(5 - 9x^2 + 4x^3)$
$v(x)$	$2x - 1$	$\frac{1}{3}(1 - 9x^2 + 8x^3)$
$\omega(x) = v(x)/u(x)$	$2x - 1$	$(1 + x - 8x^2)/(5 + 5x - 4x^2)$

Table 1: Decay functions $u(x)$ and $v(x)$.

Fractions of positively and negatively charged particles

From Eqs. (1) and (2), the cross section for a τ emitted at angle θ and decaying into a charged particle of energy E can be derived :

$$\frac{d^2\sigma}{d\Omega dx} = \frac{d\sigma}{d\Omega}\,[u(x) \mp P(\theta)v(x)] \tag{13}$$

Thus the charge asymmetry of decay particles at θ and E is :

$$Q(\theta, x) = \frac{\frac{d^2\sigma_-}{d\Omega dx} - \frac{d^2\sigma_+}{d\Omega dx}}{\frac{d^2\sigma_-}{d\Omega dx} + \frac{d^2\sigma_+}{d\Omega dx}} = G(x)\ H(\theta) \tag{14}$$

where :

$$G(x) = \frac{F_1 - F_3\,\omega(x)}{F_0 - F_2\,\omega(x)} \tag{15}$$

and :

$$H(\theta) = \frac{2\cos\theta}{1 + \cos^2\theta} \tag{16}$$

The fractions of negatively and of positively charged particles, $f_-(\theta, x)$ and $f_+(\theta, x)$, at θ and x, are given by :

$$f_q(\theta, x) = \frac{1}{2}[1 - q\ Q(\theta, x)] \tag{17}$$

where the electric charge of the particle is $q = \pm 1$.

$G(x)$, thus $Q(\theta, x)$ and $f_q(\theta, x)$, are functions of $\sin^2\theta_W$ which can be used in a fit to determine $\sin^2\theta_W$. The functions f_+ and f_- are conditional probabilities, for an event with a τ produced at angle θ and decaying with an energy ratio x, that the decay particle be positive and negative, respectively. Let us assume that the detection efficiency η at θ and x is independent of the electric charge. Then f_+ and f_- are independent of that efficiency entirely. Advantage can be taken of this independence with respect to detection efficiency in a fit maximizing the log-likelihood function $\mathcal{L}$ based on these conditional probabilities. Assuming that the difference between the angle θ of the τ produced and the angle of its charged decay product θ_d can be neglected, one can compute the likelihood function

$$\mathcal{L} = \sum_{\tau's} \ln[\infty - \Pi\ \mathcal{Q}(\theta_\Gamma, \S)] \tag{18}$$

(the sum being on all τ's of a given decay mode) and maximize it as a function of $\sin^2\theta_W$.

Correction for charge ambiguity

To justify the method described above, ideal conditions have been assumed : no electric charge or decay channel ambiguity, no radiative correction, no difference between θ and θ_d, no error in the measurement of θ or E, and no background. With these ideal conditions, the determination of $\sin^2\theta_W$ is insensitive to the variations of detection efficiency η throughout the detector, as long as η is the same for positively and negatively charged particles.

One advantage of this method is that it stays valid regardless of any cut introduced in the data, as long as the cut is made symmetrically on positive and negative particles. Any cut is equivalent to having the efficiency η equal to zero in a given domain of the space of event configuration. Therefore, one may cut out any domain where corrections (therefore uncertainties on those corrections) are substantial. However, even after restricting the domain of the data, it is likely that a last small correction still has to be introduced.

The effect of electric charge ambiguity can be expressed analytically. In general, the electric charge of the decay particle is determined experimentally from its curvature c in a magnetic field. Then, instead of charge asymmetry, one should consider curvature asymmetry. There is a probability $p_+(c)$ to observe the curvature c when the particle is positive, and another, $p_-(c)$, to observe the same curvature when the particle is negative. The probabilities $p_+(c)$ and $p_-(c)$ are functions of the quantities ξ characterizing the event. Because of measurement errors, they are both non zero in general.

The cross section for producing an event with curvature c is :

$$\frac{d\sigma}{d\xi} = [\frac{d^2\sigma_+}{d\Omega dx}\, p_+(c) + \frac{d^2\sigma_-}{d\Omega dx}\, p_-(c)]\, \eta(\xi) \tag{19}$$

where $\eta(\xi)$ is the detection efficiency. The probabilities for a positively charged particle to produce an identical event but with opposite curvature $-c$, $p_+(-c)$, is assumed to be the same as $p_-(c)$ for a negatively charged particle to produce the event with curvature c ; and vice versa: $p_-(-c) = p_+(c)$. Therefore the cross section for producing the event with opposite curvature is :

$$\frac{d\overline{\sigma}}{d\xi} = [\frac{d^2\sigma_+}{d\Omega dx}\, p_-(c) + \frac{d^2\sigma_-}{d\Omega dx}\, p_+(c)]\, \eta(\xi) \tag{20}$$

with the same efficiency $\eta(\xi)$ according to our basic assumption. Therefore the observed curvature asymmetry now is

$$\frac{\frac{d\overline{\sigma}}{d\xi} - \frac{d\sigma}{d\xi}}{\frac{d\overline{\sigma}}{d\xi} + \frac{d\sigma}{d\xi}} = \frac{\frac{d^2\sigma_-}{d\Omega dx} - \frac{d^2\sigma_+}{d\Omega dx}}{\frac{d^2\sigma_-}{d\Omega dx} + \frac{d^2\sigma_+}{d\Omega dx}} \frac{p_+ - p_-}{p_+ + p_-} = q\ Q(\theta, x) \tag{21}$$

where :

$$q = \frac{p_+ - p_-}{p_+ + p_-} \tag{22}$$

and Q is given by Eq. (14).

The fraction of events with curvature $\pm c$ that have actual curvature c is :

$$f(c) = \frac{\frac{d\sigma}{d\xi}}{\frac{d\overline{\sigma}}{d\xi} + \frac{d\sigma}{d\xi}} = \frac{1}{2}\ [1 - q\ Q(\theta, x)] \tag{23}$$

The fraction $f(c)$ is a conditional probability that the particle has curvature c under the condition of an absolute value $|c|$ of c. That conditional probability can be used to construct a likelihood function $\mathcal{L}$. That $\mathcal{L}$ is formely equivalent to Eq. (18), but with a fractional value for q given by Eq. (22).

It should be pointed out that the values of the probabilities p_+ and p_- can be best obtained from the data of the whole event, i.e. the data about both the τ lepton under study and the other τ in the pair produced, which we know has the opposite sign. Let us define c and c' the curvatures of the τ under study and of the other τ, respectively ; δc and $\delta c'$ the errors on c and c' ; C and C' the absolute values of the curvatures obtained from the measurement of θ_d and of the energies E and E' in any part of the detector [2]. We use the convention that the curvature is positive for a positive particle and negative for a negative one.

The probability to have both measured curvatures c and c' is, in the Gaussian approximation :

$$p_+(c) = \frac{1}{2\pi\ \delta c\ \delta c'}\ \exp[-\frac{(c - C)^2}{2\delta c^2}]\ \exp[-\frac{(c' + C')^2}{2\delta c'^2}] \tag{24}$$

if the τ under study is a τ^+, and :

[2]For L3, for instance, it would be in the electromagnetic or hadronic calorimeters, or in the muon chambers.

$$p_-(c) = \frac{1}{2\pi\ \delta c\ \delta c'}\ \exp[-\frac{(c+C)^2}{2\delta c^2}]\ \exp[-\frac{(c'-C')^2}{2\delta c'^2}] \tag{25}$$

if the τ under study is a τ^-.

It follows that :

$$q = \frac{p_+ - p_-}{p_+ + p_-} = \tanh[\frac{cC}{\delta c^2} - \frac{c'C'}{\delta c'^2}] \tag{26}$$

Other corrections

Elaborate analytic procedures can be introduced to take care of decay channel ambiguities, radiative corrections, and measurement errors. A more pedestrian approach will be described here.

Monte Carlo programs have been written to simulate :

a) τ production with and without radiative corrections for any value of $\sin^2\theta_W$.

b) the propagation, reconstruction and measurement of trajectories in all four detectors installed at LEP.

With those programs, it is possible to generate events with radiative corrections and to simulate ambiguities and measurement errors for several values of $\sin^2\theta_W$. To these simulated events, the analysis suggested in this paper, with all its approximations, can be applied to determine a new value of $\sin^2\theta_W$. The difference between the original value of $\sin^2\theta_W$ used in the Monte Carlo generation and the one deduced from the analysis of the Monte Carlo sample is a correction to be applied to the value obtained from the analysis of the real data.

The correction for background can also be obtained by generating background events by Monte Carlo and adding these events to the Monte Carlo sample of τ-leptons events. Indeed, the background events are due to e^+e^- interactions mistaken for τ pairs, and there are programs to generate these events too. The difference between the values of $\sin^2\theta_W$ obtained by applying this method to the sample without and to the sample with background is the correction due to the background to be applied to the value of $\sin^2\theta_W$ obtained from the data.

This technique to compute corrections is essentially the one applied to any analysis of the data. The advantage of the method advocated in this paper is that the sample can be weeded out of events belonging to domains where the corrections are large. Therefore the uncertainty on these corrections can be made negligible. This is how systematic errors can be reduced.

Comments about statistical errors

This method deliberately ignores information that requires knowledge of the detector efficiency distribution. Therefore it is expected to lead to a larger statistical error than one that uses all information.

In general, the statistical error ε on $\sin^2\theta_W$ can be estimated from the second derivative of the logarithm of the likelihood function at its maximum.

$$\frac{1}{\varepsilon^2} = \frac{\partial^2}{(\partial \sin^2\theta_W)^2} \sum_{\text{events}} \ln[1 - q\ Q(\theta, x)] \approx \frac{N_\tau}{\sigma_{\text{eff}}} \int \frac{(q\frac{\partial Q}{\partial \sin^2\theta_W})^2}{(1 - qQ(\theta, x))^2} \frac{d^2\sigma}{d\Omega dx} \eta d\Omega dx \tag{27}$$

where N_τ is the number of τ's, and :

$$\sigma_{\text{eff}} = \int \frac{d^2\sigma}{d\Omega dx} \eta d\Omega dx \tag{28}$$

To give an idea of the loss of statistical information with the method described here, we compute Eq. (27) above setting $\eta = 1$ over the entire domain of angles θ and energy ratios x, for $\sin^2\theta_W \approx \frac{1}{4}$. Then :

$$\frac{1}{\varepsilon^2} = 24\ N_\tau \int \frac{4\cos^2\theta}{1 + \cos^2\theta} d\cos\theta \int \frac{v^2(x)}{u(x)} dx = 41\ N_\tau \int \frac{v^2(x)}{u(x)} dx \tag{29}$$

For $\tau \longrightarrow \pi\nu$ decays, this gives :

$$\varepsilon = \frac{0.28}{\sqrt{N_\tau}} \tag{30}$$

and for $\tau \longrightarrow$ leptons :

$$\varepsilon = \frac{0.70}{\sqrt{N_\tau}} \tag{31}$$

From a study of τ production and decay [1],[7], one can derive the error $\varepsilon_{\min}$ on the value of $\sin^2\theta_W$ determined from all the available information about τ production and decay. In the same conditions, i.e. at the Z^0 peak and for $\sin^2\theta_W \approx \frac{1}{4}$, that error is, for $\tau \longrightarrow \pi\nu$ decays :

$$\varepsilon_{\min} = \frac{0.17}{\sqrt{N_\tau}} \tag{32}$$

and for $\tau \rightarrow$ leptons :

$$\varepsilon_{\min} = \frac{0.44}{\sqrt{N_\tau}} \tag{33}$$

i.e. a factor 1.6 times smaller than the statistical error of the method of this paper. Therefore the method of this paper does not improve the error on $\sin^2\theta_W$ unless the combined statistical and systematic error is 1.6 times the statistical error, i.e. unless the systematic errors is already 1.25 times the statistical one.

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