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Spin Polarization in Nuclear and Particle Physics

H.E. Conzett

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H. E. Conzett

Nuclear Science Division, Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720, USA

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SPIN POLARIZATION IN NUCLEAR AND PARTICLE PHYSICS

H. E. Conzett
Nuclear Science Division, Lawrence Berkeley Laboratory
University of California, Berkeley, CA 94720

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Abstract

This review has two aims: i), to present the formalism, which describes the spin-polarization observables in terms of spin-state transition amplitudes, in a manner that reveals the required correspondence between the theoretical and experimental definitions of the observables; then ii), to emphasize that spin physics, the experimental and theoretical investigations of spin-polarization effects in scattering and reactions, has become a clear unifying element among the otherwise seemingly disparate fields of nuclear, particle, and electron-scattering physics. Illustrative examples of research results in these fields are used to demonstrate this commonality. The important role of intrinsic spin in providing experimental investigations of parity conservation, charge symmetry, and time-reversal invariance is discussed.

1. Introduction

Spin polarization effects in nuclear reactions and scattering have been studied during the past forty-odd years in order to provide quantitative descriptions of the various spin dependences of the nuclear interactions, i. e., spin-orbit, spin-spin, and spin-tensor. During this period, polarization studies were fairly consistently pursued in nuclear physics experiments. The available polarized beams of (mainly) protons and deuterons, over the broad spectrum of nuclear targets and the regularly expanding accessible energy range, have provided ample reason and opportunity for a continuous program of polarization experiments in nuclear reactions and scattering. The basic intent of this research, that of yielding specific details of the spin-dependences of the various interactions involved or of the theoretical models invoked, has been, and continues to be, richly achieved.

As particle physics diverged and then essentially separated from nuclear physics, polarization experiments in nucleon-nucleon *(NN)* scattering, which had been among the first such nuclear studies, continued to be pursued at the ever higher energy accelerators where beams of polarized protons could be achieved. *NN* scattering, however, became a smaller subfield of particle physics with the ever expanding number of new mesons and baryons discovered. As a result, few were interested at that time in the spin dependence of a relevant strong interaction in the absence of any viable theory to test; and in this context the semi-serious remark was made that spin was an inessential complication in particle physics, and even in *NN* scattering the theoretical expectation was that the spin effects would diminish with increasing energy, eventually to vanish. This point of view changed completely during the mid to late 1970s, when, for example, it was found that the Λ hyperon produced in the inclusive process $pA \rightarrow \Lambda X$ was polarized (Bunce et al 1976), and after the deep inelastic electron scattering experiments showed the parton substructure of the nucleon (Miller *et al* 1972, Bodek *et al* 1979). This validated the concept of the quark substructure of hadrons, and the development of the strong-interaction theory of quantum chromodynamics (QCD), with its spin-*1/2* quarks and spin-*1* gluons, clearly has moved the questions of spin-dependence to the forefront. More recently, determinations of the quark polarizations relative to that of the proton have been inferred from the measurements of the appropriate spin observable in the deep inelastic scattering of polarized muons from polarized protrons (Ashman et al.1988, 1989). The result, contrary to intuition, that essentially none of the proton's spin was carried by the spins of the valence quarks, came as a complete surprise and stimulated an intense interest and activity in the subject. Thus, during the past 15 years or so, the realization and development of the essential role of spin in particle physics has been characterized by remarks

such as: "Among the most critical tests of any dynamical theory of hadronic phenomena is the correct description of spin effects." (Brodsky and Lepage 1981); and, concerning a proposed program to measure various spin observables in deep inelastic lepton scattering, "There is no other program which rivals it in precision or clarity of interpretation within the framework of QCD. The experiments to date only scratch the surface of this rich and challenging subject" (Jaffe 1992).

Electron scattering, which has components in both nuclear and particle physics, has really expanded to generate a broad investigation of spin effects only within the past decade. There was, of course, a fundamental reason working against an earlier such development. Due to the relativistic nature of the electron at the wavelengths, thus momenta, of interest even in nuclear physics, the electron transverse-spin observable, the analyzing power A_y, vanishes as $1/\gamma$, with γ the Lorentz factor (Scofield 1959). Since the longitudinal (helicity) analyzing power, A_z, is a parity nonconserving *(PNC)* observable, there was little incentive to develop beams of polarized electrons for the sole purpose of looking for a nonzero value of A_z as a test of parity conservation in the electromagnetic interaction. This view changed quickly with the development of the unified electroweak theory, because then a measurement of A_z (in inclusive inelastic electron-deuteron scattering) could, and did, provide a quantitative determination of the interference between the *PC* electromagnetic one-photon-exchange amplitude and the *PNC* weak Z^o-exchange amplitude (Prescott et al 1978, 1979). In nuclear physics, by contrast, where no such condition inhibited nonzero values of A_y, polarized beams of protons and deuterons were early developed for the express purpose of providing measurements of A_y in a very wide variety of nuclear scattering and reactions. The development of polarized targets and efficient polarimeters then made it possible to measure some of the other polarization observables that are required for any quantitative determination of the various components of spin-dependent interactions. It is this development that has opened the way for measurements of spin observables in electron scattering, utilizing both polarized electrons and polarized targets or measuring the polarization of the recoil nuclear target (Donnelly and Raskin 1986).

Thus, during the past decade, there has been developing a clear unifying element among the otherwise seemingly disparate fields of nuclear, particle, and electron-scattering physics. That element is "spin physics", a generic term that includes all spin-polarization investigations. It has not been at all obvious that the same spin physics applies universally here, but this difficulty derives mainly from the different "languages" , i. e., treatments, terminology, and notations, that have been used. Also, the relativistic nature of electron scattering and particle reactions simplifies considerably the spin aspects as compared to those of nuclear physics. That is, the number of scattering or transition amplitudes is reduced substantially, leading to a correspondingly large reduction in the number of spin-polarization observables. Thus,

electron scattering and particle reactions can be viewed as special cases of the more general description of the spin observables and the spin formalism of nuclear reactions and scattering.

My design, then, in this review is to emphasize this commonality of spin physics and to present the spin formalism in a manner that demonstrates the required correspondence between the theoretical and experimental descriptions of the observables. Also, this treatment is relatively self-contained, because it is addressed more to the broader community than to that of the spin-physics specialists.

2. Spin-$\frac{1}{2}$ and spin-1 polarization observables

It is possible, and useful, to describe the basic content of spin physics in terms of scattering or reactions with the simplest spin structures, i. e.,

$$\frac{1}{2} + 0 \rightarrow \frac{1}{2} + 0 \tag{2.1}$$

and

$$1 + 0 \rightarrow 1 + 0. \tag{2.2}$$

The increased complexity associated with the more complicated spin structures can then be recognized as the natural extensions of the basic content. This point of view, in fact, is the only rationale for this nontraditional review, since comprehensive reviews and reports that develop the entire formal structure of spin physics are available (Ohlsen 1972, Simonius 1973, Bystricky *et al* 1978, Bourrely *et al* 1980). This development of the basic content of spin physics is made with an emphasis on the experimentally defined observables and on the requirement that their formal theoretical counterparts show an exact equivalence.

2.1 Spin dependent cross-sections and spin observables, $\frac{1}{2} + 0 \rightarrow \frac{1}{2} + 0$

Experimentally, one measures particle yields; that is, decay rates or cross-sections, and this review will be concerned with the latter. With the spin structure (2.1), if there is no selection made among the possible spin states (orientations) of the initial state particle, the unpolarized differential cross-section is simply the sum of the spin-dependent differential cross-sections, averaged over the two possible initial states,

$$I(\theta) = \frac{1}{2}\,[I_{jk}(++) + I_{jk}(+-) + I_{jk}(-+) + I_{jk}(--)]. \tag{2.3}$$

Here, $I_{jk}(+-)$, for example, is the cross-section for the transition from the positive spin state along the j-axis to the negative state along the k-axis, where $j,k = x, y,$ or z in a reference coordinate frame. Clearly, the spin-state cross-sections I_{jk} are the fundamental experimental quantities, but $I(\theta)$ measures only the sum of these basic cross-sections, which provides minimal information on their individual values, so the entire program of experimental spin physics is engaged in their determination. Although the initial-state spin polarization can be selected, e.g., with a polarized-ion source, the final state polarization is determined by the dynamics of the interactions involved. Thus, the individual I_{jk} are not directly measureable, but the various spin-polarization observables are, like $I(\theta)$, simply different linear combinations of them. So, if there should be, as in (2.3), four spin-state cross-sections†, the determination of their values requires measurements of four different observables, only one of which is the unpolarized cross-section. Thus, even in this example with the simplest spin structure, one sees that the complete experimental determination of the basic cross-sections requires the measurement of three more spin observables. Since $I(\theta)$ is always just one linear combination of the basic cross-sections, the required number of spin observables increases rapidly as one goes to more complicated spin structures, with the correspondingly increased number of basic cross-sections. There, then, the role of spin physics becomes increasingly important.

Consider a polarized beam, with its polarization defined as

$$p_j \equiv n_j(+) - n_j(-), \qquad \text{with} \qquad n_j(+) + n_j(-) = 1, \qquad -1 \le p_j \le 1, \tag{2.4}$$

where the n_j are the fractional numbers of particles in the indicated spin states of quantization direction j. Then the polarized cross-section is

$$I_j(\theta) = I(\theta)[1 + p_j A_j(\theta)], \tag{2.5}$$

which defines the analyzing power A_j as the relative change in the cross-section in going from $p_j = 0$ to 1 and, also, satisfies the requirements that $I_j(\theta) = I(\theta)$ for either p_j or $A_j = 0$, and that $I(\theta)$ is the average of $I_j(p_j)$ and $I_j(-p_j)$. The observable A_j is thus determined from the experimental asymmetry

† As will be shown, parity conservation reduces (2.3) to two independent terms. However, the inclusion of other polarization orientations results in the same number, four, of independent observables.

$$\varepsilon \equiv \frac{I_j(p_j) - I_j(-p_j)}{I_j(p_j) + I_j(-p_j)} = p_j A_j, \tag{2.6}$$

and, in terms of the pure spin states $p_j = \pm 1$,

$$A_j = \frac{I_j(+) - I_j(-)}{I_j(+) + I_j(-)}, \tag{2.7}$$

where $I_j(+)$ is the spin-state cross-section for the designated initial spin state but with the final-state spin undetermined, thus summed over. Then with $I_j(+) = I_{jk}(++) + I_{jk}(+-)$, etc.,

$$A_j = [I_{jk}(++) + I_{jk}(+-) - I_{jk}(-+) - I_{jk}(--)]/2I \tag{2.8}$$

is the corresponding linear combination of the spin-state cross-sections.

With an unpolarized beam, the final-state particle can be (and usually is) polarized, and this polarization, the polarizing power of the scattering, again from (2.4), is

$$P_k = [I_k(+) - I_k(-)]/I\,. \tag{2.9}$$

So,

$$P_k = [I_{jk}(++) + I_{jk}(-+) - I_{jk}(+-) - I_{jk}(--)]/2I\,. \tag{2.10}$$

Using (2.6), the polarization of this (scattered) beam of particles is determined in a second scattering, for which the analyzing power is known.

There is one other type of spin observable available in this simple system, that corresponding to the determination of the final-state polarization when the beam itself is polarized. With $p_j\,(p_k)$ designating the initial (final) polarization, the final polarization is

$$p_k = \frac{n_k(+) - n_k(-)}{n_k(+) + n_k(-)}, \tag{2.11}$$

so, suppressing for the moment the I_{jk} subscripts,

$$p_k = \frac{n_j(+)I(++) + n_j(-)I(-+)] - n_j(+)I(+-) - n_j(-)I(--)}{n_j(+)I(++) + n_j(-)I(-+)] + n_j(+)I(+-) + n_j(-)I(--)}, \tag{2.12}$$

and noting from (2.4) that in the initial state

$$n_j(\pm) = \frac{1}{2}(1 \pm p_j) , \tag{2.13}$$

$$p_k = \frac{I(++)+I(-+)-I(+-)-I(--) + p_j[I(++)-I(-+)-I(+-)+I(--)]}{I(++)+I(-+)+I(+-)+I(--) + p_j[I(++)-I(-+)+I(+-)-I(--)]} \tag{2.14}$$

And, with (2.3), (2.8), and (2.10),

$$p_k = \frac{P_k + p_j K_{jk}}{1 + p_j A_j} = \frac{I}{I_j}(P_k + p_j K_{jk}), \tag{2.15}$$

so the observable K_{jk}, the polarization-transfer coefficient, is

$$K_{jk} = [I_{jk}(++) + I_{jk}(--) - I_{jk}(-+) - I_{jk}(+-)]/2I . \tag{2.16}$$

Also,

$$K_{jk} = 1-2S_{jk} , \tag{2.17}$$

with the spin-flip probability defined as

$$S_{jk} = [I_{jk}(-+) + I_{jk}(+-)]/I . \tag{2.18}$$

Then, in terms of these four observables, the basic spin-state cross-sections are

$$I_{jk}(++) = \frac{1}{4} I(1 + A_j + P_k + K_{jk}), \tag{2.19a}$$

$$I_{jk}(+-) = \frac{1}{4} I(1 + A_j - P_k - K_{jk}), \tag{2.19b}$$

$$I_{jk}(-+) = \frac{1}{4} I(1 - A_j + P_k - K_{jk}), \tag{2.19c}$$

$$I_{jk}(--) = \frac{1}{4} I(1 - A_j - P_k + K_{jk}), \tag{2.19d}$$

and the straightforward connection to any dynamical theory is made, in principle, through the corresponding (calculated) spin-state amplitudes, for example

$$I_{jk}(++) = |M_{jk}(++)|^2, \tag{2.20}$$

where the M_{jk} are the amplitudes (matrix elements) for transitions between the indicated spin states. However, as is discussed in the next section , the formal theoretical structure of spin physics obscures this simplicity.

In arriving at eqns. (2.19), the restrictions imposed by parity conservation have not been invoked, but this treatment provides an illustrative example that is easily extended to more complicated spin structures.

2.2 Spin formalism and spin obsrvables, $\frac{1}{2} + 0 \rightarrow \frac{1}{2} + 0$

Since a single spin-$\frac{1}{2}$ particle is always completely polarized in some (arbitrary) direction, the polarization of an ensemble of beam (or target) particles is defined, as in (2.4), as the ensemble average of the difference between the two spin-state populations. Correspondingly, the quantum mechanical spin-function, the Pauli spinor

$$X = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = c_1 \; Z(+) + c_2 \; Z(-) \; , \tag{2.21}$$

is understood to be averaged over the ensemble, so that c_1 and c_2 are the *amplitudes* for the fractional populations of the $Z(+)$ and $Z(-)$ states, respectively, so

$$|c_1|^2 = n_z(+), \qquad |c_2|^2 = n_z(-), \qquad X^\dagger X = |c|^2 + |c_2|^2 = 1. \tag{2.22}$$

The Pauli spin matrices,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \tag{2.23}$$

which operate on the base states $Z(\pm)$, have the multiplication properties

$$\sigma_j \sigma_k = i\sigma_l \quad j,k,l \text{ cyclic in } x, y, z; \qquad \sigma_j^2 = 1 = \sigma_o; \qquad Tr\; \sigma_j \sigma_k = 2\delta_{jk} \, . \tag{2.24}$$

In terms of the base states, the eigenstates of these operators are defined by

$$\sigma_x \, X(\pm) = \pm X(\pm), \quad \text{so} \quad X(\pm) = [Z(+) \pm Z(-)]/\sqrt{2} \tag{2.25a}$$

$$\sigma_y \, Y(\pm) = \pm Y(\pm), \quad \text{so} \quad Y(\pm) = [Z(+) \pm i\, Z(-)]/\sqrt{2} \tag{2.25b}$$

$$\sigma_z \, Z(\pm) = \pm Z(\pm), \tag{2.25c}$$

and, just as $Z(\pm)$ represent states of spin quantized along the z direction, $X(\pm)$ and $Y(\pm)$ represent states quantized along the x and y directions. Then, the polarization components

p_j, which are the expectation values of the σ_j, all have the same form when the spin function (2.21) is expressed in terms of the corresponding eigenstates $J(\pm)$,

$$p_j = \langle\sigma_j\rangle \equiv X_j^\dagger\, \sigma_j\, X_j = X_j^\dagger\, [c_{1j}\, J(+) - c_{2j}\, J(-)]$$
$$= |c_{1j}|^2 - |c_{2j}|^2 = n_j(+) - n_j(-), \qquad (2.26)$$

in complete agreement with the experimental definition (2.4). However, formally, one generally uses only the base eigenstates $Z(\pm)$, resulting in

$$p_x = X_z^\dagger\, \sigma_x\, X_z = 2\, Re\, c_1^* c_2 \qquad (2.27a)$$
$$p_y = X_z^\dagger\, \sigma_y\, X_z = 2\, Im\, c_1^* c_2 \qquad (2.27b)$$
$$p_z = X_z^\dagger\, \sigma_z\, X_z = |c_1|^2 - |c_2|^2, \qquad (2.27c)$$

where the subscript z has been dropped from the amplitudes c_1, c_2. Thus, the unphysical expressions for p_x and p_y simply result from this choice.

The polarization (pseudo)vector, with the components (2.27), has the absolute value of unity since $p_x^2 + p_y^2 + p_z^2 = 1$, so there must be some restriction on the amplitudes in order for (2.27) to represent an arbitrary polarization. For example, using

$$\frac{1}{2}(p_x \pm i p_y) = c_1^* c_2 / c_1 c_2^*, \qquad (2.28)$$

the polarization in the transverse (x,y) plane, $p_t = p_x \pm i p_y$, can be set to zero without c_1 or $c_2 = 0$, but with $c_1^* c_2$ or $c_1 c_2^* = 0$; that is $Re\, c_1^* c_2 = \pm\, i\, Im\, c_1 c_2^*$. Thus, arbitrary values of of the polarization components correspond to the conditions

$$p_x = 2\, Re\, c_1^* c_2\,; \quad p_y = p_z = 0, \quad so \quad Im\, c_1^* c_2 = 0, \ |c_1| = |c_2| \qquad (2.29a)$$
$$p_y = 2\, Im\, c_1^* c_2\,; \quad p_x = p_z = 0, \quad so \quad Re\, c_1^* c_2 = 0, \ |c_1| = |c_2| \qquad (2.29b)$$
$$p_z = |c_1|^2 - |c_2|^2; \quad p_x \pm i p_y = 0, \quad so \quad Re\, c_1^* c_2 = \pm\, i\, Im\, c_1 c_2^*. \qquad (2.29c)$$

It is convenient to replace the matrix operations of (2.27) and the conditions (2.29) on the spinor amplitudes c_1 and c_2 by the density matrix ρ. Since an observable is defined as the expectation value of the corresponding hermitian operator Ω, as in (2.27)

$$<\Omega> = X_z^{\dagger}\,\Omega\,X_z = \begin{pmatrix} c_1^* & c_2^* \end{pmatrix} \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{12}^* & \Omega_{22} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \tag{2.30}$$

$$<\Omega> = |c_1|^2\,\Omega_{11} + c_1^* c_2\,\Omega_{12} + c_1 c_2^*\,\Omega_{12}^* + |c_2|^2\,\Omega_{22}\,, \tag{2.31}$$

$$<\Omega> = Tr\,\rho\Omega, \quad \text{with} \quad \rho \equiv \begin{pmatrix} |c_1|^2 & c_1 c_2^* \\ c_1^* c_2 & |c_2|^2 \end{pmatrix} = X_z X_z^{\dagger} \tag{2.32}$$

Then, since ρ is a 2×2 matrix it can be expanded in terms of the set of σ_j, $j = o,x,y,z$, as

$$\rho = \sum_j b_j\,\sigma_j \tag{2.33}$$

And, using (2.24),

$$p_j \equiv <\sigma_j> = Tr\,\rho\,\sigma_j = 2\,b_j\,, \tag{2.34}$$

so,

$$\rho = \frac{1}{2}\sum_j p_j\,\sigma_j = \frac{1}{2}\begin{pmatrix} 1+p_z & p_x - ip_y \\ p_x + ip_y & 1-p_z \end{pmatrix} \tag{2.35}$$

Thus, comparison with with (2.32) shows that the density matrix, expressed simply in terms of the polarization components of the ensemble, automatically includes the conditions (2.29) on the spinor amplitudes c_1 and c_2, which can then disappear from the further development of the formalism.

So far, this discussion has described only the initial-state spinor and density matrix, which can be prepared for the experiment. All experimental observables are described in terms of the amplitudes for transitions between individual initial and final spin states. That is, the final spinor is given in terms of the initial spinor by $X_f = M(\theta)\,X_i$, so $M(\theta)$ is the 2×2 matrix of transition (or scattering) amplitudes $M(if)$:

$$M(\theta) = \begin{pmatrix} M(++) & M(-+) \\ M(+-) & M(--) \end{pmatrix} \tag{2.36}$$

This 2×2 matrix can, also, be expressed in terms of the set of σ_j,

$$M(\theta) = \sum_j a_j(\theta)\,\sigma_j = \begin{pmatrix} a_o + a_z & a_x - ia_y \\ a_x + ia_y & a_o - a_z \end{pmatrix} \tag{2.37}$$

and this is a convenient form in which to apply the conditions that are imposed by the fundamental symmetries of parity conservatiion *(PC)* and time-reversal invariance *(TRI)*. Choosing now a coordinate frame for the generic reaction $a + b \rightarrow c + d$, the center of mass helicity frame, Fig. 1, is used, since it is the one in which the conditions imposed by *TRI* on the scattering/reaction amplitudes are most naturally expressed (Ohlsen et al 1972, Simonius 1974). Then, unit vectors along the coordinate axes are

$$\mathbf{z}_i\,(\mathbf{z}_f) = \mathbf{k}_i\,(\mathbf{k}_f) \qquad \mathbf{y} = \mathbf{k}_i \times \mathbf{k}_f \qquad \mathbf{x}_i\,(\mathbf{x}_f) = \mathbf{y} \times \mathbf{z}_i\,(\mathbf{z}_f)\,, \tag{2.38}$$

where $\mathbf{k}_i\,(\mathbf{k}_f)$ is the c.m. momentum of particle $a\,(c)$, and the base states $Z(\pm)$ are states of spin quantized along the direction of the particle's momentum; that is, helicity states, so the corresponding M-matrix amplitudes (2.36) are helicity amplitudes. The P and T transformations are $\mathbf{k}_{i,f} \rightarrow -\mathbf{k}_{i,f}$, $\sigma \rightarrow \sigma$ and $\mathbf{k}_i \leftrightarrow -\mathbf{k}_f$, $\sigma \rightarrow -\sigma$, respectively. Here, $\sigma = (\sigma_x, \sigma_y, \sigma_z)$, $\sigma_j \equiv \sigma \cdot j$, so the transformations of σ_j under these P, T symmetry operations are:

$$\begin{aligned} P:&\quad \sigma_x, \sigma_y, \sigma_z \rightarrow -\sigma_x, \sigma_y, -\sigma_z \\ T:&\quad \sigma_x, \sigma_y, \sigma_z \rightarrow -\sigma_x, \sigma_y, \sigma_z \end{aligned} \tag{2.39}$$

Thus, in order that the M-matrix (2.37) be invariant under these operations, the amplitude a_j changes sign wherever σ_j changes sign, and it can be classified according to its P and/or T symmetry. That is, an amplitude a_j is

$$P\text{-odd } (T\text{-odd}) \text{ if } n_x + n_z\ (n_x) \text{ is odd,} \tag{2.40}$$

where $n_x\,(n_z)$ is the number of $x\,(z)$ subscripts†. *PC* requires the P-odd (P-even) amplitudes to vanish when the product of the particles' intrinsic parities is even (odd), but *TRI* imposes no such condition on the T-odd amplitudes. Consider the M^t-matrix for the time-reversed reaction, with amplitudes a^t_j, as in Eq. (2.37). Then *TRI* requires that $k_f M^t = k_i M$

† For this simple spin structure the amplitudes carry only one subscript. With more complex spin structures, for which the amplitudes carry two or more subscripts, (2.42) is valid in general.

($k_i = k_f$ in elastic scattering), so the T-odd amplitudes satisfy the condition $k_f a^t_x = - k_i a_x$. Only in the case of elastic scattering, which is its own inverse reaction, does this condition force the amplitude to vanish.

For a reaction, then, in which there is no net change of intrinsic parities between the initial and final states, the PC condition (2.40) requires a_x and a_z to vanish, so (2.37) becomes

$$M = a_o \sigma_o + a_y \sigma_y, \tag{2.41}$$

and the helicity amplitudes satisfy

$$M(++) = M(--) \quad , \quad M(+-) = - M(-+). \tag{2.42}$$

Correspondingly, the basic helicity cross-sections (2.20) are reduced in number to two.

Since the experimental observables are formally defined as the final-state expectation values of the corresponding hemitian operators, the final density matrix is required in (2.32); and it, defined in the same way as the initial density matrix in (2.32), is

$$\rho_f = X_f X_f^\dagger = M X_i X_i^\dagger M^\dagger = M \rho_i M^\dagger. \tag{2.43}$$

Thus, the final density matrix, which describes the polarization components of the final ensemble, is naturally given in terms of the initial polarization components followed by their transition probabilities to the final states. So, with (2.35) and

$$<\Omega_f> = Tr\, M \rho_i M^\dagger \Omega \ , \tag{2.44}$$

the observables are the unpolarized differential cross-section, $p_j = 0$,

$$I(\theta) = <\sigma_o \equiv 1> = \frac{1}{2} Tr\ MM^\dagger, \tag{2.45}$$

and, with a beam polarization p_j,

$$I_j(\theta) = <\sigma_o> = \frac{1}{2} Tr\ M(1 + p_j \sigma_j)M^\dagger = I(\theta)\left[1 + p_j \frac{Tr\ M\sigma_j M^\dagger}{Tr\ MM^\dagger}\right]. \tag{2.46}$$

Thus, the analyzing power in (2.5) is

$$A_j = Tr\ M\sigma_j M^\dagger \ / \ Tr\ MM^\dagger = Tr\ M\sigma_j M^\dagger \ / \ 2I\ . \tag{2.47}$$

In order to calculate the final-state polarization, which has the limits ± 1, the final density matrix is normalized to unit trace by $\rho_f \rightarrow \rho_f / Tr\, \rho_f$. Noting that $Tr\, \rho_f = I\, (I_j)$ when the beam is unpolarized (polarized), the polarizing power (2.10) is

$$P_k = \langle \sigma_k \rangle = Tr\, MM^\dagger \sigma_k / 2I, \tag{2.48}$$

and, with a polarized beam, the final polarization is

$$\begin{aligned} p_k &= \langle \sigma_k \rangle = \frac{1}{2} Tr\, M(1 + p_j \sigma_j) M^\dagger \sigma_k / I_j \\ p_k &= \frac{I}{I_j} [(Tr\, MM^\dagger \sigma_k + p_j\, Tr\, M\sigma_j M^\dagger \sigma_k) / 2I] \\ p_k &= \frac{I}{I_j} (P_k + p_j K_{jk}), \end{aligned} \tag{2.49}$$

as in (2.16). Thus,

$$K_{jk} = Tr\, M\sigma_j M^\dagger \sigma_k / 2I \tag{2.50}$$

Consider, now, the experimental observables (2.45), (2.47), (2.48), and (2.50), expressed in the general form

$$X(j,k) = Tr\, M\sigma_j M^\dagger \sigma_k / Tr\, MM^\dagger\,, \qquad j,k = o,x,y,z\,, \tag{2.51}$$

where j labels the polarization component of the initial-state particle, k labels the observed final-state polarization component, and $j\,(k) = o$ for unpolarized incident particles (unobserved final polarization). For example, $X(y,o) \equiv A_y$ and $X(x,z) \equiv K_{x,z}$. Again, just as the spinor amplitudes c_1 and c_2 were replaced by the density matrix, now ρ_i does not appear explicitly in (2.51), and an observable is calculated simply with the specification of the initial and final polarizations, σ_j and σ_k, and the matrix of transition amplitudes.

Since, by definition, the P transformation of the M-matrix is $M \rightarrow M$, the combination $M, M^\dagger$ contributes no change of sign in the P transformation of an observable, so its P-symmetry is determined by the explicit spin-operators, σ_j and σ_k, in (2.51). Its T-symmetry is determined in the same manner. Thus, with (2.39), it follows from (2.51) that these observables can be classified according to their P and T symmetries, in exactly the same way as was found for the amplitudes in (2.40):

$$P: \quad X(j,k) = (-1)^{(n_x+n_z)}\, X(j,k) \tag{2.52a}$$

$$T: \quad X(j,k) = (-1)^{n_x}\, X^t(k,j) \tag{2.52b}$$

So, *PC* requires a *P*-odd *observable* to be zero, but the condition imposed by *T*-symmetry is that an observable is equal to *(+/-)* the corresponding observable in the inverse process *(k,j)*; for example, $A_y = P^t_y$. This holds, also, for elastic scattering, which is its own inverse process but with the initial and final (spin) states interchanged.

In order to see the specific equivalence between the experimental observables, defined in terms of the basic cross-sections of section 2.1, and their formal counterparts, one must use in (2.51) the *M*-matrix form (2.36) with the direct connection between its amplitudes and the basic cross-sections (2.20). Since the base spin states have been chosen to be the helicity states $Z(\pm)$, the matrix of helicity amplitudes is designated $M_Z(\theta)$. Then, for example, the analyzing power components are

$$IA_x = \frac{1}{2} Tr\ M_Z \sigma_x M_Z^\dagger = Re[M(++)M(-+)^* + M(+-)M(--)^*] \tag{2.53a}$$

$$IA_y = \frac{1}{2} Tr\ M_Z \sigma_y M_Z^\dagger = Im[M(++)M(-+)^* + M(+-)M(--)^*] \tag{2.53b}$$

$$IA_z = \frac{1}{2} Tr\ M_Z \sigma_z M_Z^\dagger = \frac{1}{2}[|M(++)|^2 + |M(+-)|^2 - |M(-+)|^2 - |M(--)|^2]. \tag{2.53c}$$

Thus, A_z agrees, term by term, with its experimental definition (2.8) in terms of the basic cross-sections, whereas the expressions for A_x and A_y are again unphysical for the same reason as in (2.27), that of describing all components A_j in terms of the helicity amplitudes. From *PC*, (2.52a) and (2.42), A_x and A_z vanish, so one does not generally see the expression (2.53c) for an analyzing power component, and

$$IA_y = 2\ Im[M(++)M(-+)^*] \tag{2.54}$$

is the standard expression for A_y. This, and similar expressions for other observables in terms of such bilinear combinations of different amplitudes, as opposed to sums of the absolute squares of amplitudes, (2.53c), have led to assertions that polarization effects are interference phenomena. It is clear, however, that these "interference terms" result simply from the disparity between the experimental quantization axes and the helicity quantization frame that is generally chosen for the formalism. Experimentally, the quantization axis for each initial-state particle is that of its prepared polarization direction, and the axis for each final-state particle is that of its measured polarization direction. If the same choice of a separate

quantization direction for each particle is made in the formal description of the observables, the spin operator in (2.51) for each particle is σ_z. Then, since σ_z is a diagonal matrix, it does not change the positions of any of the M-matrix amplitudes in the multiplications indicated in (2.51); it simply introduces the appropriate minus signs. As an example,

$$IK_{zz} = \frac{1}{2} Tr\, M\sigma_z M^\dagger \sigma_z = \frac{1}{2}[|M(++)|^2 + |M(--)|^2 - |M(-+)|^2 - |M(+-)|^2], \qquad (2.55)$$

in agreement with (2.16).

Since , for good reason (Jacob and Wick 1959), the helicity frame was first chosen to be the standard reference frame for the description of polarization observables (Madison convention, Barschall and Haeberli 1971), most of the polarization observables allowed by PC are expressed in terms of the sums of bilinear combinations of amplitudes. It is now clear, however, that one should not try to interpret the "physics" of such expressions, knowing that the physics is contained in the the experimental definitions of the observables in terms of the basic cross-sections. That is, all of the observables of spin physics are simply different linear combinations of the basic cross-sections. This has been demonstrated here for a reaction with the simplest spin structure, but it should be clear that this holds true for any more complicated spin structure. The only difference is that the number of basic cross-sections and corresponding amplitudes increases, so the number of observables increases in accord with the number of independent linear combinations of the basic cross-sections.

One sees that the choice of the M-matrix expanded in terms of the σ_j, (2.37), is convenient for more reasons than that of the application of the P and T symmetries. That is, the calculation of any observable (2.51) is reduced to the trace of sums of products $\sigma_j\sigma_k$. Then, using the properties (2.24), there is no actual multiplication of matrices required. Also, the symmetry of M in terms of the σ_j results in a symmetry in the expressions for the different components of an observable. For example, to calculate IA_j, with j,k,l cyclic in x,y,z,

$$M = a_o\sigma_o + a_j\sigma_j + a_k\sigma_k + a_l\sigma_l \qquad (2.56)$$
$$\sigma_j M^\dagger = a_o^*\sigma_j + a_j^*\sigma_o + ia_k^*\sigma_l - ia_l^*\sigma_k;$$

but twelve of the sixteen terms of the product $M\sigma_j M^\dagger$ are traceless, so

$$I A_j = \frac{1}{2} Tr\, M\sigma_j M^\dagger = a_o a_j^* + a_o^* a_j - ia_k a_l^* + ia_k^* a_l = 2(Re\ a_o a_j^* + Im\ a_k a_l^*), \qquad (2.57)$$

showing the equivalent expression for each component. Then with the P-odd amplitudes $a_x = a_z = 0$, the surviving component is

$$IA_y = 2Re\ a_o a_y^*, \tag{2.58}$$

in agreement with (2.54) from (2.36) and (2.37).

2.3 Spin dependent cross-sections and spin observables, $1 + 0 \rightarrow 1 + 0$

Although the differences between the spin-$\frac{1}{2}$ and spin-1 (and higher spin) polarization descriptions and formalism are sometimes emphasized (e.g. Bourrely et al 1980), I choose to emphasize the similarities. One has, now, the fractional populations, $n_j(+)$, $n_j(o)$, $n_j(-)$ of the three indicated spin states $(\pm 1,0)$ of quantization direction j. As before, the vector polarization is

$$p_j \equiv n_j(+) - n_j(-), \qquad with \qquad n_j(+) + n_j(o) + n_j(-) = 1, \qquad -1 \leq p_j \leq 1. \tag{2.59}$$

Since an unpolarized ensemble has $n_j(+) = n_j(o) = n_j(-) = \frac{1}{3}$, consider the case with $n_j(+) = n_j(-)$ but $n_j(o) \neq \frac{1}{3}$. Although $p_j = 0$, this is not an unpolarized, but an aligned or tensor polarized, ensemble; and this tensor polarization is defined quantitatively as the difference of the population $n_j(o)$ from its unpolarized value,

$$p_{jj} \equiv 1 - 3n_j(o) \tag{2.60}$$
$$p_{jj} = 1,\ 0,\ -2 \quad for\ n_j(o) = 0,\ \frac{1}{3},\ 1.$$

It immediately follows from (2.59) that a purely vector-polarized ensemble, i.e. with the tensor polarization $p_{jj} = 0$, is limited to the range $-\frac{2}{3} \leq p_j \leq \frac{2}{3}$. Now, the polarized cross-section is

$$I_j(\theta) = I(\theta)[1 + \frac{3}{2} p_j A_j(\theta) + \frac{1}{2} p_{jj} A_{jj}(\theta)], \tag{2.61}$$

which defines the tensor analyzing power A_{jj} as the relative change in cross-section due to the tensor polarization. The numerical coefficients in (2.61) correspond to the noted limits on p_j and p_{jj} and the requirement that $I(\theta)$ be the average over the (pure) spin-state cross-sections

$$
\begin{aligned}
I_j(+) &= I\,[1+ \tfrac{3}{2} A_j + \tfrac{1}{2} A_{jj}], \\
I_j(o) &= I\,[1- A_{jj}], \\
I_j(-) &= I\,[1- \tfrac{3}{2} A_j + \tfrac{1}{2} A_{jj}].
\end{aligned}
\tag{2.62}
$$

There are now nine basic cross-sections

$$
I_{jk}(\alpha,\beta), \qquad \alpha,\, \beta = +,o,- \,. \tag{2.63}
$$

Then, as for spin-$\frac{1}{2}$, the vector analyzing power is one linear combination of them,

$$
A_j = I_j(+) - I_j(-)]/3I = \sum_{\beta} [I_{jk}(+,\beta) - I_{jk}(-,\beta)]/3I \,, \tag{2.64}
$$

and the tensor analyzing power is another,

$$
A_{jj} = 1- I_j(o)/I = 1- \sum_{\beta} I_{jk}(o,\beta)/I. \tag{2.65}
$$

The nine basic cross-sections can be expressed, as in (2.19), as different linear combinations of the required nine observables. In a straightforward extension of the treatment of section 2.1, these include the four observables I, A_j, P_j, K_{jk} and the five additional ones associated with the tensor polarization, A_{jj}, P_{kk}, $K_{j,kk}$, $K_{jj,k}$, *and* $K_{jj,kk}$. P_{kk} is the tensor polarizing power and the polarization transfer coefficients are vector-to-tensor, tensor-to-vector, and tensor-to-tensor, respectively.

2.4 Spin formalism and spin observables, $1 + 0 \rightarrow 1 + 0$

Extending the procedure of section 2.2, the spin-*1* spinor,

$$
X = c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = c_1\, Z(+) + c_2\, Z(o) + c_3\, Z(-), \tag{2.66}
$$

is averaged over the ensemble, and

$$|c_1|^2 = n_z(+), \quad |c_2|^2 = n_z(o), \quad |c_3|^2 = n_z(-), \quad X^\dagger X = |c_1|^2 + |c_2|^2 + |c_3|^2 = 1. \qquad (2.67)$$

The basic spin-*1* matrix operators are $P_o \equiv 1$ and

$$P_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad P_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad P_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \qquad (2.68)$$

As before, the base states $Z(\alpha)$ are eigenstates of P_z: $P_z Z(\alpha) = \alpha Z(\alpha)$, and these operators satisfy the commutation relations

$$P_j P_k - P_k P_j = iP_l, \quad j,k,l \text{ cyclic}; \quad \text{with } Tr\, P_j P_k = 2\delta_{jk} \text{ and } Tr\, P_j P_k P_l = i\varepsilon_{jkl}, \qquad (2.69)$$

with $\varepsilon_{jkl} = 1,-1,0$ for jkl cyclic, anticyclic, or neither, respectively.

Since the expectation values of these hermitian P_j provide only the vector polarization components in the helicity frame of Fig. 1, additional matrix operators are required to represent the tensor polarization components. These operators are chosen (Ohlsen 1972) to be the symmetric tensors formed from the P_j,

$$P_{jk} = \frac{3}{2}(P_j P_k + P_k P_j) - 2\delta_{jk}, \qquad j,k = x,y,z, \qquad (2.70)$$

and, for example, P_{xx} (P_{xz}) represents an alignment, thus quantization, along the x $(x = z)$ direction. Only five of the six possible tensors, $P_{xx}, P_{yy}, P_{zz}, P_{xy}, P_{yz}, P_{zx}$, are independent, since

$$P_{xx} + P_{yy} + P_{zz} = 3(P_x^2 + P_y^2 + P_z^2) - 6 = 3\mathbf{P}^2 - 6 = 3P(P+1) - 6 = 0. \qquad (2.71)$$

Then, the $P_{o,x,y,z}$ and the five independent P_{jk} form a convenient set of nine independent matrix operators in terms of which any 3×3 matrix can be expanded. This set of operators, Ω_j, normalized so that

$$Tr\, \Omega_j = 3\delta_{oj}, \qquad Tr\, \Omega_j \Omega_k = 3\delta_{jk}, \qquad (2.72)$$

is

$$\Omega_j\colon P_o, \sqrt{\frac{3}{2}}(P_x, P_y, P_z), \frac{1}{\sqrt{6}}(P_{xx} - P_{yy}), \frac{1}{\sqrt{2}}P_{zz}, \sqrt{\frac{2}{3}}(P_{xy}, P_{yz}, P_{zx}). \qquad (2.73)$$

Just as before, after expanding the spin-*1* density matrix (2.35) in terms of this set of Ω_j,

$$\rho_j = \frac{1}{3}\sum_j p_j \Omega_j , \tag{2.74}$$

and expanding the M matrix (2.37) as

$$M(\theta) = \sum_j a_j(\theta)\, P_j , \tag{2.75}$$

the procedure from (2.43) to (2.51) yields the same expression for the general spin-1 observable, with σ_j, σ_k replaced by P_j, P_k,

$$X(j,k) = Tr\ MP_jM^{\dagger}P_k / Tr\ MM^{\dagger}, \tag{2.76}$$

but now with $j,k = o, x, y, z, xx, yy, zz, xy, yz, zx$.

The initial (final) polarizations $p_j\,(p_k)$ are now the expectation values of the operators $P_j\,(P_k)$.

Since the P and T transformations of $\boldsymbol{P}$ are $\boldsymbol{P} \to \boldsymbol{P}$ and $\boldsymbol{P} \to -\boldsymbol{P}$, respectively, just as for σ, the transformations of the P_j are the same as those of the σ_j, (2.39). Then, noting (2.70), it follows that the symmetry conditions (2.40) and (2.52) apply, as well, to the spin-*1* amplitudes (2.75) and the observables (2.76). Thus, the P-symmetry condition (2.52a), with (2.71), reduces the number of nonvanishing spin-*1* amplitudes to five. Thus, when there is no net change of intrinsic parities, the parity conserving M-matrix can be expressed as

$$M = a_o + a_y P_y + a_{xx}(P_{xx} - P_{yy}) + a_{zz} P_{zz} + a_{zx} P_{zx} . \tag{2.77}$$

Explicitly,

$$M = \begin{array}{c} \\ + \\ 0 \\ - \end{array} \begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} a_o+a_{zz} & -(ia_y-(\frac{3}{2})a_{zx})/\sqrt{2} & 3a_{xx} \\ (ia_y+(\frac{3}{2})a_{zx})/\sqrt{2} & a_o-2a_{zz} & -(ia_y+(\frac{3}{2})a_{zx})/\sqrt{2} \\ 3a_{xx} & (ia_y-(\frac{3}{2})a_{zx})/\sqrt{2} & a_o+a_{zz} \end{pmatrix} \end{array} \tag{2.78}$$

which shows the helicity amplitudes $M(\alpha,\beta)$ in terms of the invariant (with respect to the choice of coordinate frame) amplitudes a_j. In this form it is easy to see the helicity change that

is provided by a particular a_j. For example, only the far off-diagonal tensor term a_{xx} provides the transitions in which the helicity changes by two units.

Following the same procedure as for spin-$\frac{1}{2}$ in (2.56) and (2.57), any spin-*1* observable (2.76) can be expressed in terms of the amplitudes (2.77) by using the properties (2.69) and (2.70).

Finally, in order to see again the specific equivalence between a spin-*1* experimental observable, defined in terms of the basic cross-sections, and its formal counterpart, consider

$$IA_{zz} = \frac{1}{3} Tr\, M_z P_{zz} M_z^{\dagger}, \tag{2.79}$$

with

$$M \equiv [M(\alpha, \beta)] \tag{2.80}$$

as the matrix of spin-*1* helicity amplitudes analogous to (2.36). With

$$P_{zz} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2.81}$$

$$IA_{zz} = \frac{1}{3} \sum_{\beta} [|M(+,\beta)|^2 + |M(-,\beta)|^2 - 2|M(o,\beta)|^2] \tag{2.82}$$

$$IA_{zz} = \frac{1}{3} \sum_{\alpha,\beta} [|M(\alpha,\beta)|^2 - \sum_{\beta} |M(o,\beta)|^2 \tag{2.83}$$

$$IA_{zz} = I - \sum_{\beta} I_{jk}(o,\beta), \tag{2.84}$$

in agreement with (2.65).

Thus, it is clear that the description of the spin-*1* observables, in both the experimental and spin-formalism definitions, can indeed be the natural extension of the description of the spin-$\frac{1}{2}$ observables that follows in going from the two to three base spin states.

2.5 Photons

Although the photon is a (massless) spin-1 particle, the real photon with momentum in the z direction has only two states of linear polarization, corresponding to its transverse electric field being along either the x-axis or the y-axis of figure 1. Thus, the description of its polarization can follow exactly that of the spin-$\frac{1}{2}$ particles, section 2.2. So with

$$X_\gamma = c_x X(+) + c_y Y(-), \quad |c_x|^2 + |c_y|^2 = 1, \tag{2.85}$$

for pure "spin" states the linear polarizations are

$$\varepsilon_x(+) = |c_x|^2 = n_x(+), \quad \varepsilon_y(-) = |c_y|^2 = n_y(-), \tag{2.86}$$

and an arbitrary linear polarization is given by

$$\varepsilon = \varepsilon_x - \varepsilon_y = n_x(+) - n_y(-). \tag{2.87}$$

Then, the circular polarizations (helicities) correspond to the states

$$\varepsilon_z(\pm) = \frac{1}{2}(\varepsilon_x \pm i\varepsilon_y), \tag{2.88}$$

and, in the spin-1 framework $n_z(o) = 0$, so a beam of real photons is tensor polarized, with $p_{zz} = 1$.

Virtual photons have the additional longitudinal component of linear polarization, so the treatment of their polarization requires the full spin-1 three-state description, and the corresponding 3 X 3 density matrix has been given by Dombey (1969).

3. More complex spin structures

3.1 $\frac{1}{2} + \frac{1}{2} \to \frac{1}{2} + \frac{1}{2}$

The entire foregoing discussion is easily extended to reactions/scattering of more complex spin-structures. Consider, for example, the case of particular interest in particle physics, $a + b \to c + d$, with four spin-$\frac{1}{2}$ particles. Now, with j,k,l,m referring to the polarization components of particles a,b,c,d, respectively, the polarized cross-section (2.5) becomes

$$I_{jk}(\theta) = I(\theta)[1 + p_j A_{jo} + p_k A_{ok} + p_j p_k A_{jk}], \tag{3.1}$$

which defines, as before, the additional observables A_{ok}, the target analyzing power, and A_{jk}, the initial-state spin-correlation coefficients. Similarly, additional observables are

experimentally defined in the extension of (2.15) to include determinations of the polarization components of particles c and d, both separately and in correlation. Experimentally, these observables are all expressed in terms of the basic spin-state cross-sections, $I_{jk,lm}(\alpha\beta,\gamma\delta)$, with $\alpha, \beta, \gamma, \delta = \pm$

Now that it is clear that the formal expression for any observable, (2.51), will, with the appropriate choices M_z and σ_z, yield the proper linear combination of the basic cross-sections, this extension to more complex spin-structures will use the formally more convenient helicity-frame M-matrix (with its base helicity states $Z(\pm)$), expanded in terms of the (hermitian) spin operators σ_j. The initial-state (final-state) spinor is now the two-particle four component spinor formed as the direct product of the individual particle spinors $X_{ab} = X_a \otimes X_b$ $(X_{cd} = X_c \otimes X_d)$. The required 4 X 4 M-matrix can now be expanded in terms of direct products of the 2 X 2 (a,c) and (b,d) matrices σ_j and σ_k, respectively (MacGregor et al 1960),

$$M(\theta) = \sum_{j,k} a_{jk}(\theta)\ \sigma_j \otimes \sigma_k\ , \qquad j,k = o,x,y,z. \tag{3.2}$$

In a more compact form, with the 4 X 4 matrix $\sigma_{jk} \equiv \sigma_j \otimes \sigma_k$,

$$M = \sum_{j,k} a_{jk}\ \sigma_{jk}\ , \tag{3.3}$$

and the 16 M-matrix amplitudes,

$$\begin{aligned} &a_{oo},\ \underline{a}_{ox},\ a_{oy},\ \underline{a}_{oz},\ \underline{a}_{xo},\ a_{xx},\ \underline{a}_{xy},\ a_{xz} \\ &a_{yo},\ \underline{a}_{yx},\ a_{yy},\ \underline{a}_{yz},\ \underline{a}_{zo},\ a_{zx},\ \underline{a}_{zy},\ a_{zz}, \end{aligned} \tag{3.4}$$

can then be classified, from (2.40), according to their P and/or T symmetries. For example, the eight underlined amplitudes are P-odd (with $a_{ox}, a_{xo}, a_{xy}, a_{yx}$ also T-odd) and a_{xz} and a_{zx} are T-odd. Also, the experimental observables, in the now familiar form,

$$X(jk,lm) = Tr\ M\ \sigma_{jk}\ M^{\dagger}\sigma_{lm}\ /\ Tr\ MM^{\dagger}, \qquad j,k,l,m = o,x,y,z\ , \tag{3.5}$$

have, in an obvious extension of (2.52), the symmetries

$$P: \qquad X(jk,lm) = (-1)^{(n_x+n_z)}\, X(jk,lm) \tag{3.6a}$$

$$T: \qquad X(jk,lm) = (-1)^{n_x}\, X^t(lm,jk). \tag{3.6b}$$

For the case in which π_{tot}, the product of the four particles' intrinsic parities, is even, so that the eight P-odd amplitudes of (3.4) vanish, the parity conserving M-matrix is

$$M = a_{oo} + a_{yo}\,\sigma_{yo} + a_{oy}\,\sigma_{oy} + a_{xx}\,\sigma_{xx} + a_{yy}\,\sigma_{yy} + a_{zz}\,\sigma_{zz} + a_{xz}\,\sigma_{xz} + a_{zx}\,\sigma_{zx}. \tag{3.7}$$

It is useful to display M in its matrix form, so with

$$\sigma_{xz} = \begin{pmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad \text{etc.,} \tag{3.8}$$

$$M = \begin{array}{c} \\ ++ \\ +- \\ -+ \\ -- \end{array} \begin{array}{c} \begin{array}{cccc} ++ & +- & -+ & -- \end{array} \\ \begin{pmatrix} a_{oo}+a_{zz} & a_{zx}-ia_{oy} & a_{xz}-ia_{yo} & a_{xx}-a_{yy} \\ a_{zx}+ia_{oy} & a_{oo}-a_{zz} & a_{xx}+a_{yy} & -a_{xz}-ia_{yo} \\ a_{xz}+ia_{yo} & a_{xx}+a_{yy} & a_{oo}-a_{zz} & -a_{zx}-ia_{oy} \\ a_{xx}-a_{yy} & -a_{xz}+ia_{yo} & -a_{zx}+ia_{oy} & a_{oo}+a_{zz} \end{pmatrix} \end{array}, \tag{3.9}$$

Where the column (row) labels are now the helicities $\alpha\beta$ $(\gamma\delta)$ of particles a,b (c,d). Here, then, the helicity amplitudes $M_{if} = M(\alpha\beta,\gamma\delta)$ are explicitly displayed in terms of the invariant amplitudes a_{jk}. Simonius (1974) has shown directly that the conditions imposed by P-symmetry on these helicity amplitudes are

$$M(\alpha\beta,\gamma\delta) = \pi_{tot}\,(-1)^{\alpha+\beta+\gamma+\delta}\, M(-\alpha\ -\beta,-\gamma\ -\delta), \tag{3.10}$$

and these conditions are automatically satisfied in (3.9), as well as in (2.78) and (2.41).

There is much more information available from (3.7) that is seen in (3.9). Since the off-diagonal operators σ_x and σ_y change the single-particle helicity states, i.e., $\sigma_x Z(+) = Z(-)$, $\sigma_y Z(+) = iZ(-)$, etc., the corresponding a_x and a_y are helicity-flip amplitudes while a_o and a_z are non-flip amplitudes. Thus, a_{xx} is a double helicity-flip amplitude; that is, both particles a, b flip helicities in the transition to c, d; and, e.g., a_{yo} is an amplitude for the process in which helicity-flip occurs only in the a to c transition. These properties are conveniently displayed in (3.9). It is , also, very useful to make the connection between the invariant amplitudes a_{jk} and the spin-dependent interactions, spin-orbit, spin-spin, and

spin-tensor. Since orbital angular momentum is along $\boldsymbol{k_i} \times \boldsymbol{k_f}$ (Fig. 1), a spin-orbit term $\sigma \cdot \boldsymbol{l} = \sigma_y$, so a_{yo} (a_{oy}) is the projectile (target) spin-orbit amplitude. A spin-spin term, $a_{ab}\, \sigma_{\boldsymbol{a}} \cdot \sigma_{\boldsymbol{b}} = a_{ab}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$, by itself would require that $a_{xx} = a_{yy} = a_{zz} = a_{ab}$ in (3.7), so it is the spin-tensor term which removes these equalities and, also, provides the amplitudes a_{jk} with $j \neq k$. Then one sees again in (3.9) that a change in total helicity of two units, provided by the far off-diagonal amplitudes, is possible only through the tensor interaction. Thus, the amplitude subscripts describe their P and T symmetries, their explicit helicity transitions, and their corresponding spin-dependent interactions.

The calculation of an observable (3.5) in terms of the M-matrix amplitudes (3.7) proceeds just as for the simpler example of (2.56) and (2.57) since the matrix operations factor, as they must, into the operations of σ_j and σ_k in the separate (a,c) and (b,d) spin-spaces, respectively. For example, consider the spin correlation coefficient

$$IA_{jk} \equiv IX(jk,oo) = \frac{1}{4}\, Tr\, M\, \sigma_{jk}\, M^{\dagger}\, \sigma_{oo}. \tag{3.11}$$

With (3.3), a representative term of (3.11) has the matrix structure

$$\begin{aligned} Tr\, [(\sigma_{j'} \otimes \sigma_{k'})(\sigma_j \otimes \sigma_k)(\sigma_{l'} \otimes \sigma_{m'})(\sigma_o \otimes \sigma_o)] &= Tr\, [(\sigma_{j'}\sigma_j\sigma_{l'}) \otimes (\sigma_{k'}\sigma_k\sigma_{m'})] \\ &= [Tr\, \sigma_{j'}\sigma_j\sigma_{l'}][Tr\, \sigma_{k'}\sigma_k\sigma_{m'}], \end{aligned} \tag{3.12}$$

showing the reduction to the two separate trace factors. Then, writing the general expression

$$IX(jk,lm) = \frac{1}{4}\, Tr\, M\, \sigma_{jk}\, M^{\dagger}\, \sigma_{lm} \tag{3.13}$$

as the product of the two eight-term sums $M\, \sigma_{jk}$ and $M^{\dagger}\sigma_{lm}$, only eight of the sixty-four product terms have nonvanishing traces and contribute to (3.13). These are selected by the trace properties

$$\begin{aligned} Tr[\sigma_{jk}\, \sigma_{j'k'}] \equiv\ & Tr[(\sigma_j \otimes \sigma_k)(\sigma_{j'} \otimes \sigma_{k'})] = Tr[(\sigma_j\, \sigma_{j'}) \otimes (\sigma_k\, \sigma_{k'})] \\ &= (Tr\, \sigma_j\, \sigma_{j'})(Tr\, \sigma_k\, \sigma_{k'}) = 4\, \delta_{jj'}\, \delta_{kk'}\,. \end{aligned} \tag{3.14}$$

3.2 $\frac{1}{2} + 1 \rightarrow \frac{1}{2} + 1$

Extending now to the spin-structure $\frac{1}{2}+1 \rightarrow \frac{1}{2}+1$, one can proceed in the same manner as in subsection 3.1. The polarized cross-section (2.61) now becomes

$$I_{jk}(\theta) = I(\theta)[1 + p_j A_{jo} + \frac{3}{2} p_k A_{ok} + \frac{1}{2} p_{kk} A_{kk} + \frac{3}{2} p_j p_k A_{j,k} + p_j p_{kk} A_{j,kk}], \tag{3.15}$$

which defines the target vector (tensor) analyzing power A_{ok} (A_{kk}) and the vector-vector (vector-tensor) spin correlation coefficient $A_{j,k}$ $(A_{j,kk})$. Again, additional observables are defined experimentally in an extension of (2.15) that includes determinations of the polarization components of particles c and d, both vector and tensor for d. These observables are all expressed in terms of the basic spin-state cross-sections $I_{jk,lm}(\alpha\beta,\gamma\delta)$, now with $\alpha,\gamma = \pm$ and $\beta,\delta = +,o,-$.

Eq. (3.2) now becomes the 6 x 6 M-matrix expanded in terms of direct products of the 2 X 2 (a,c) matrices σ_j and the 3 X 3 (b,d) matrices P_k.

$$M(\theta) = \sum_{j,k} a_{jk}\ \sigma_j P_k, \tag{3.16}$$

$$j = o,\ x,\ y,\ z; \quad k = o,\ x,\ y,\ z,\ xx/yy,\ zz,\ xy,\ xz,\ yz\ ,$$

and the observables are

$$X(jk,lm) = Tr\, M\, \sigma_j P_k\, M^{\dagger} \sigma_l P_m\ /\ Tr\, MM^{\dagger}, \tag{3.17}$$

with l (m) having the same range of components as j (k). Just as before, the symmetry conditions (3.6) apply to these observables.

The calculation of an observable (3.17) in terms of the amplitudes (3.16) proceeds just as for the case of the spin structure $\frac{1}{2}+\frac{1}{2} \rightarrow \frac{1}{2}+\frac{1}{2}$, (3.11) to (3.14). In addition to the properties (2.24) and (2.69), useful relations of the spin-1 matrices are

$$P_j^3 = P_j, \quad P_j^4 = P_j^2\ . \tag{3.18}$$

3.3 Arbitrary spin structure

So far, we have considered only those cases where the final-state spins are the same as the initial-state spins, and this is not generally the case in inelastic scattering and reactions. As a representative example, we consider the spin structure $1+\frac{1}{2} \rightarrow 0+\frac{1}{2}$, which has been

treated in exhaustive detail by Keaton *et al* (1974). The six component initial-state spinor and two component final-state spinor require a 6 X 2 M-matrix with the helicity amplitudes $M(\alpha\beta,\delta)$, where $\alpha = \pm 1,0;\ \beta, \delta = \pm\frac{1}{2}$. This matrix can be expanded in terms of direct products of the 3 X 1 (particle a) row matrices $X_j^\dagger$ and the 2 X 2 (b,d) matrices σ_k,

$$M(\theta) = \sum_{j,k} a_{jk} X_j^\dagger \sigma_k\ ;\ j = x,y,z\ ;\ \ k = o,\ x,y,z\ . \tag{3.19}$$

The cartesian matrices $X_j^\dagger$ are derived from the spherical forms

$$X(+)^\dagger = (1\ \ 0\ \ 0),\ \ X(o)^\dagger = (0\ \ 1\ \ 0),\ \ X(-)^\dagger = (0\ \ 0\ \ 1) \tag{3.20}$$

with

$$\begin{aligned} X_x^\dagger &= [-X(+)^\dagger + X(-)^\dagger]/\sqrt{2} = (-1\ \ 0\ \ 1)/\sqrt{2} \\ X_y^\dagger &= -i[X(+)^\dagger + X(-)^\dagger]/\sqrt{2} = -i(1\ \ 0\ \ 1)/\sqrt{2} \\ X_z^\dagger &= X(o)^\dagger = (0\ \ 1\ \ 0)\ . \end{aligned} \tag{3.21}$$

For example, then, the $a_{x,z}$ term in (3.19) is

$$a_{xz} X_x^\dagger \otimes \sigma_z = a_{xz}\,(-\sigma_z\ \ 0\ \ \sigma_z)/\sqrt{2} = \frac{1}{\sqrt{2}}\, a_{xz}\begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}. \tag{3.22}$$

Again, the P and T transformations of X are $X \to X$ and $X \to -X$, respectively, as for σ, so the symmetry conditions (2.40) and (2.52) apply to the amplitudes in (3.19) and to the observables

$$X(jk,l) = Tr\, M\, P_j \sigma_k M^\dagger\, \sigma_l / Tr\, MM^\dagger. \tag{3.23}$$

Thus, with the six P-odd amplitudes vanishing, the parity conserving M-matrix is

$$M = a_{yo} X_y^\dagger \sigma_o + a_{xx} X_x^\dagger \sigma_x + a_{yy} X_y^\dagger \sigma_y + a_{zz} X_z^\dagger \sigma_z + a_{xz} X_x^\dagger \sigma_z + a_{zx} X_z^\dagger \sigma_x\,, \tag{3.24}$$

so

$$M = \frac{1}{\sqrt{2}} \begin{array}{c} + \\ - \end{array} \begin{array}{c} \begin{array}{cccccc} ++ & +- & 0+ & 0- & -+ & -- \end{array} \\ \begin{pmatrix} -ia_{yo}-a_{xz} & -a_{xx}-a_{yy} & \sqrt{2}a_{zz} & \sqrt{2}a_{zx} & -ia_{yo}+a_{xz} & a_{xx}-a_{yy} \\ -a_{xx}+a_{yy} & -ia_{yo}+a_{xz} & \sqrt{2}a_{zx} & -\sqrt{2}a_{zz} & a_{xx}+a_{yy} & -ia_{yo}-a_{xz} \end{pmatrix} \end{array}, \tag{3.25}$$

where the column (row) labels indicate the helicities $\alpha\beta(\delta)$ of the particles $ab(d)$. Again, (3.25) satisfies the conditions (3.10) and explicitly shows the helicity amplitudes $M(\alpha\beta,\delta)$ in terms of the invariant amplitudes. The tensor term $(a_{xx} - a_{yy})$ again provides a change of two in the total helicity, and there is no a_{oo} amplitude because all of the transitions are helicity changing.

Finally, the calculation of the observables (3.23) in terms of the amplitudes (3.24) proceeds in the same way as for all the previous examples, now with the additional relations

$$P_j X_k = i\varepsilon_{jkl} X_l \ , \quad X_j^\dagger X_k = \delta_{jk} \ . \tag{3.26}$$

3.4 Inclusive reactions, $a + b \rightarrow c + X$

In view of the fact that many inclusive experiments are pursued, especially in particle physics, it is of interest to know whether or not there are P and/or T imposed symmetries on the available experimental observables in such reactions, $a + b \rightarrow c + X$, where only particle c is detected in the final state of three or more particles. Goldstein et al (1976) have provided a formalism to describe the inclusive reaction spin observables. The constraints imposed by P-symmetry are included but those imposed by T-symmetry are not. From energy and momentum conservation, X can be treated as a composite "particle" of known mass and momentum, with, however, unobservable spin. This latter fact has no effect on the observables involving particles $a, b,$ and $c,$ and it will be seen that these observables retain the same symmetries as in the $2 \rightarrow 2$ exclusive reactions, namely (3.6a) and (3.6b).

Consider a reaction in which $a, b,$ and c are spin-$\frac{1}{2}$ particles, i. e., fermions. Then from baryon and lepton conservation, "particle" X is also a "fermion" and , for the purpose of illustration, is taken to be spin-$\frac{1}{2}$. Then the available observables are given as in (3.5) with $m = o,$ corresponding to the fact that the "polarization" of X is not observed ,

$$X(jk,lo) = Tr\, M\, \sigma_{jk}\, M^\dagger \sigma_{lo} \,/\, Tr\, MM^\dagger . \tag{3.27}$$

Then, just as before, these observables have the symmetries given in (3.6a) and (3.6b). In order to better understand the specific details of these results, we consider, for example, the expressions for the analyzing power A_{yo} and the inverse-reaction polarizing power $P^t{}_{yo}$, even though the latter cannot be determined experimentally. These are

$$IA_{yo} \equiv IX(yo,oo) = \frac{1}{4} Tr\, M\, \sigma_{yo}\, M^{\dagger}, \tag{3.28a}$$

$$I^tP^t_{yo} \equiv I^tX^t(oo,yo) = \frac{1}{4} Tr\, M^tM^{t\dagger}\, \sigma_{yo}\ , \tag{3.28b}$$

and with (3.3), (3.28a) becomes

$$IA_{yo} = \frac{1}{4} Tr\ \ [(\sum_{j,k} a_{jk}\, \sigma_{jk})\, \sigma_{yo}\, (\sum_{j'k'} a_{j'k'}^{*}\, \sigma_{j'k'})]\ . \tag{3.29}$$

Then, using

$$Tr\ \sigma_{jk}\, \sigma_{lm} = Tr\ [(\sigma_j\sigma_l) \otimes (\sigma_k\sigma_m)] = Tr\ \sigma_j\sigma_l\ Tr\ \sigma_k\sigma_m\ , \tag{3.30}$$

we have

$$IA_{yo} = \frac{1}{4} \sum_{j,k} \sum_{j',k'} a_{jk}\ \ a_{j'k'}^{*}\ Tr\ \sigma_j\sigma_y\sigma_{j'}\ Tr\ \sigma_k\sigma_{k'}\ , \tag{3.31}$$

showing the matrix operations factored into operations in the separate *(a,c)* and *(b,X)* spin-spaces. Then using the properties (2.24), one finds

$$\begin{aligned} Tr\ \sigma_j\sigma_y\sigma_{j'} &= 2i\ (-2i) \quad \text{for} \quad (j,j') = (x,z)\ ((z,x)), \\ &= 2 \quad \text{for} \quad (j,j') = (o,y) \ \text{ or } \ (y,o)\ , \\ &= 0 \quad \text{otherwise}\ , \end{aligned} \tag{3.32}$$

and (3.31) becomes

$$IA_{yo} = \sum_k\ 2(Re\ a_{ok}\ a_{yk}^{*} + Im\ a_{zk}\ a_{xk}^{*}), \tag{3.33}$$

and, similarly,

$$I^tP^t_{yo} = \sum_k\ 2(Re\ a^t_{ok}\ a^t_{yk}{}^{*} - Im\ a^t_{zk}\ a^t_{xk}{}^{*}) \tag{3.34}$$

for the inverse reaction. Then, for all values of k, the condition (2.42) imposed by T-symmetry on the amplitudes in (3.33) and (3.34) provides the result $A_{yo} = P^t_{yo}$, in agreement with (3.6b). Comparing (3.33) with the same observable (2.57) of Section 2.2,

$$IA_y = 2(Re\ a_o a_y^{*} + Im\ a_z a_x^{*}), \tag{3.35}$$

one sees that they have identical forms, with the additional summation over k coming from taking the *trace* over the (b,X) part of the spin-space, which performs the sums over the spin projections of particles b and X. One then recovers the symmetries (3.6), with the restriction $m = o$, among these inclusive observables, and these are independent of the "spin" of "particle" X.

4 Representative examples of spin physics results

Expressions for the polarized cross-sections like (3.1) and (3.15) show that the spin observables represent the specific "response" of the system to the corresponding selected polarization component. Just as the basic spin-state cross-sections can be determined from linear combinations of the observables, as in (2.19), so can specific components of the spin-dependent interactions, as expressed via the amplitudes a_{jk}, be isolated in the same manner. Then, such a combination of observables represents the response function corresponding to the selected component of the spin-dependent interaction that is represented by the a_{jk}. This represents, perhaps, the most powerful and useful technique of spin physics, in that combinations of observables can be chosen in order to examine specific components of the interaction, i.e., spin-orbit, spin-spin, or spin-tensor. Thus, where available, I have selected examples illustrative of this procedure in the following discussion of examples of spin physics results in nuclear, electron-scattering, and particle physics.

4.1 Nuclear scattering/reactions

Polarization experiments in nucleon-nucleon (NN) scattering were among the first such nuclear studies, and they have continued to the present as new ranges of energy have become accessible. The basic motivation remains the same, that of determining the M-matrix amplitudes $a_{jk}(E,\theta)$ of (3.7) from the measured observables via the expression (3.5) Then the comparison can be made with the calculated a_{jk}, where the detailed results of the dynamical content of the theoretical model appear. In practice, usually, linear combinations of the a_{jk}, such as the helicity amplitudes of (3.9), are determined in terms of the phase shifts in their partial wave expansions. Since NN elastic scattering is its own time-reversed process, the T-odd amplitudes a_{xz} and a_{zx} vanish in the M-matrix (3.7). Also, under identical particle exchange, including np from isospin symmetry, $a_{yo} = a_{oy}$, so (3.7) reduces to the five terms

$$M = a_{oo} + a_{yo}(\sigma_{yo} + \sigma_{oy}) + a_{xx}\,\sigma_{xx} + a_{yy}\,\sigma_{yy} + a_{zz}\,\sigma_{zz}. \qquad (4.1)$$

In proton-nucleus inelastic scattering, *(p,p')*, and charge-exchange *(p,n)* reactions the intent has been to interpret, where possible, the experimental results in terms of the underlying *NN* interactions. In the medium energy range of 200 to 500 MeV, and at forward angles where the *NN* single-scattering approximation in the nucleus is most valid, there has been developed a very interesting and useful application of spin physics (Moss 1982, Bleszynski et al 1982) that is designed to determine the nuclear response functions from an appropriate set of measured *(p,p')* and *(p,n)* polarization-transfer coefficients. Here I describe their development from the point of view just noted, that of determining, in this case the individual $|a_{jk}|^2$, from linear combinations of the unpolarized cross-section and polarization-transfer coefficients.

The generally adopted notation designates the polarization-transfer coefficients as

$$X(jo,om) \equiv K_{jm}\,, \qquad X(jo,lo) \equiv D_{jl}\,; \tag{4.2}$$

that is, the D_{jl} (K_{jm}) correspond to polarization transfer from particle *a* to *c(d)*. Then from (3.5) and (4.1) the *NN* observables of interest are

$$I = |a_{oo}|^2 + |a_{yo}|^2 + |a_{oy}|^2 + |a_{xx}|^2 + |a_{yy}|^2 + |a_{zz}|^2, \tag{4.3a}$$

$$I\,D_{xx} = |a_{oo}|^2 - |a_{yo}|^2 + |a_{oy}|^2 + |a_{xx}|^2 - |a_{yy}|^2 - |a_{zz}|^2, \tag{4.3b}$$

$$I\,D_{yy} = |a_{oo}|^2 + |a_{yo}|^2 + |a_{oy}|^2 - |a_{xx}|^2 + |a_{yy}|^2 - |a_{zz}|^2, \tag{4.3c}$$

$$I\,D_{zz} = |a_{oo}|^2 - |a_{yo}|^2 + |a_{oy}|^2 - |a_{xx}|^2 - |a_{yy}|^2 + |a_{zz}|^2. \tag{4.3d}$$

Also, $a_{yo} = a_{oy}$, but it is useful here to keep both terms. Then, combinations of of these observables that isolate the $|a_{jk}|^2$ are

$$I\,D_o \equiv \frac{1}{4} I\,(1 + D_{xx} + D_{yy} + D_{zz}) = |a_{oo}|^2 + |a_{oy}|^2, \tag{4.4a}$$

$$I\,D_x \equiv \frac{1}{4} I\,(1 + D_{xx} - D_{yy} - D_{zz}) = |a_{xx}|^2, \tag{4.4b}$$

$$I\,D_y \equiv \frac{1}{4} I\,(1 - D_{xx} + D_{yy} - D_{zz}) = |a_{yo}|^2 + |a_{yy}|^2, \tag{4.4c}$$

$$I\,D_z \equiv \frac{1}{4} I\,(1 - D_{xx} - D_{yy} + D_{zz}) = |a_{zz}|^2, \tag{4.4d}$$

and the similarity to equations (2.19) is clear. Since the other D_{jl} available is

$$I\,D_{zx} = -I\,D_{xz} = 2\,Im(a_{oo}a_{yo}^{*} + a_{yo}a_{yy}^{*}), \tag{4.5}$$

it does not provide the means for any further isolation of $|a_{oo}|^2$, $|a_{yo}|^2 = |a_{oy}|^2$, or $|a_{yy}|^2$. Note in (4.4) that the combination $I\,D_j$ has selected those amplitudes a_{jk} that correspond to the projectile spin operator σ_j.

Expressing the same combinations of observables (4.4) from nucleon-nucleus (p,p') and (n,p) reactions as $(I\,D)_J$, $J = O, X, Y, Z$, the plane-wave single NN scattering approximation gives

$$(I\,D)_J = I\,D_j\,R_k\,N_{eff}\,, \tag{4.6}$$

where N_{eff} is the effective number of participating target nucleons. The spin response functions for the nuclear transitions $|i)$ to $|f)$ are

$$R_k(q,E) = |(f\,|\,\sigma_k\,e^{i\,\mathbf{q}\cdot\mathbf{r}}\,|\,i)|^2, \tag{4.7}$$

where **q** (E) is the momentum (energy) transfer, **r** is the target nucleon coordinate, and the σ_k are the target nucleon spin operators. Thus, measurements of $(I\,D)_J$ and knowledge of the $I\,D_j$ or the $|a_{jk}|^2$ in (4.4) yields $N_{eff}\,R_k(q,E)$. Typically, then ratios of response functions are compared with those calculated, eliminating N_{eff}. Experimental determinations of these spin response functions have now provided important checks on the details of the spin-dependent effective *NN* interactions that are employed in the calculations (McClelland *et al* 1984, Carey *et al* 1984, McClelland et al 1992, Green *et al* 1993).

4.2 Electron scattering

Electron scattering has been used for a long time to probe the electromagnetic structure of nucleons and nuclei. However, except for experiments designed to search for parity nonconserving effects, serious consideration of the use of polarized electrons in nuclear and particle physics (Arnold et al 1981, Cheung and Woloshyn 1983 and Donnelly and Raskin 1986, for example) is a relatively recent development when compared with the use of polarized nucleons and deuterons. This delayed application is easily understood, of course, when it is noted that (in the one-photon exchange approximation) the terms in the cross-section that depend on the transverse polarizations, p_x and p_y, vanish as $1/\gamma = m_e/E$ in the scattering of polarized electrons (Scofield 1959, Donnelly and Raskin 1986). This relativistic behavior may be understood, heuristically, from the Lorentz transformation of the electron spin four-

vector defined in the rest frame, $S^o = (0;\mathbf{s}) = (0;s_x,s_y,s_z)$. Under a Lorentz boost of β in the z direction,

$$S = L_z(\beta)\, S^o = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} 0 \\ s_x \\ s_y \\ s_z \end{pmatrix} = \gamma \begin{pmatrix} \beta s_z \\ s_x/\gamma \\ s_y/\gamma \\ s_z \end{pmatrix}, \tag{4.8}$$

and, relative to the helicity s_z, the transverse spin components vanish as $1/\gamma$. Thus, at the electron energies of interest in nuclear and particle physics, except for a measurement of the PNC longitudinal (helicity) analyzing power A_z, nothing more is learned with polarized electrons (alone) than is available from the scattering of unpolarized electrons.

As has become clear, however, during the past two decades of hadronic scattering, and as I have emphasized here, there are other (two-spin) observables such as polarization-transfer coefficients, $K_{jm} \equiv X(jo,om)$, and spin-correlation coefficients, $A_{jk} \equiv X(jk,oo)$, which provide information concerning the spin dependence of the interactions that can never be gleaned from unpolarized cross-sections alone. These observables became experimentally accessible with useful precision during that period only through the development of efficient polarimeters and of polarized targets. It is that development, then, that has really made it possible for electron scattering to join, experimentally, the field of spin physics. However, as has been indicated in section 1, the standard spin-physics notation and terminology of nuclear physics, which I summarize in the appendix, has been little used in electron-scattering papers. This, I feel, is detrimental to the important goal of fruitful communication and interaction with the broader spectrum of nuclear and particle spin-physics practitioners, in that it constitutes a "language" barrier. It may be both instructive and useful to "translate" an example from the most complete formulation of electron-scattering spin physics (Donnelly and Raskin 1986) to the notation and terminology of nuclear physics as follows.

The cross-section for polarized electrons is

$$\left(\frac{d\sigma}{d\Omega}\right)^h = \Sigma_o + h\Delta = \Sigma_o\left(1 + h\frac{\Delta}{\Sigma_o}\right) \quad \text{--->} \quad I_{zo} = I(1 + p_z A_z), \tag{4.9}$$

so there is no difficulty at this point. However, with a polarized target of arbitrary spin and arbitrary orientation with respect to the chosen z-axis, the final result is

$$\left(\frac{d\sigma}{d\Omega}\right)^{h,p} = \Sigma_p + h\Delta_p, \tag{4.10a}$$

$$\Sigma_p = \Sigma_o[1 + \sum_{even} P_{J0} R_{J0} + P_{J1} R_{J1} \cos \phi^* + P_{J2} R_{J2} \cos 2\phi^*] , \tag{4.10b}$$

$$h\Delta_p = h\Sigma_o[\sum_{odd} P_{J0} R_{J0} + P_{J1} R_{J1} \cos \phi^*] , \tag{4.10c}$$

where the sums are over $J \neq 0$, J is the rank of the polarization tensor, the P_{JM} are Legendre functions, and I have suppressed the functional dependences of all terms. Comparison with (3.15), for example, shows that Σ_p, which does not depend on the electron polarization, must contain terms which are products of the target polarizations and target analyzing powers, while $h\Delta_p$, which depends on both polarizations, must have the spin-correlation terms. The R_{JM} are, indeed, products of the target polarization components and the corresponding nuclear form factors that make up the observables.

For eN elastic scattering, equations (4.10) simplify to

$$\Sigma_p = \Sigma_o , \tag{4.11a}$$

$$h\Delta_p = -h\Sigma_o f_1(\sqrt{2}\, v'_T F_T^2 \cos \theta^* + 2\sqrt{2}\, v'_{TL} F_T F_L \sin \theta^* \cos \phi^*) / F^2, \tag{4.11b}$$

with

$$F^2 = v_L F_L^2 + v_T F_T^2 \tag{4.11c}$$

where f_1 is the target (spherical tensor) polarization, θ^* and ϕ^* are the polar and azimuthal angles of the polarization with respect to the virtual photon momentum, i.e., the momentum transfer direction; v'_T, v'_{TL}, v_L, and v_T are electron kinematical factors, and F_T (F_L) is the nucleon transverse (longitudinal) form factor. Equation (4.11a) reveals the interesting fact that the target analyzing power vanishes. Then, identifying the target polarization components as

$$p_{x'} = \sqrt{2}\, f_1 \sin \theta^* \cos \phi^* , \qquad p_{z'} = \sqrt{2}\, f_1 \cos \theta^* , \tag{4.12}$$

where the primes signify that the (x', y', z') frame is not the helicity frame, (4.10a) becomes

$$\left(\frac{d\sigma}{d\Omega}\right)^{h,p} = \Sigma_o(1 + p_z p_{z'} A_{zz'} + p_z p_{x'} A_{zx'}), \tag{4.13}$$

with

$$A_{zz'} = v'_T F_T^2/F^2, \qquad A_{zx'} = v'_{TL} F_L F_T/F^2, \tag{4.14}$$

for comparison with (3.1). Thus, in elastic eN scattering, there are only the three nonvanishing observables of equation (4.13).

It is, also, most interesting to view elastic electron scattering from spin-$\frac{1}{2}$ nuclei within the framework of the general spin structure, notation, and terminology of section 3.1. Then, since elastic scattering is its own time-reversed process, the T-odd amplitudes a_{xz} and a_{zx} vanish in the M-matrix (3.7), reducing it to the six term

$$M = a_{oo} + a_{yo}\,\sigma_{yo} + a_{oy}\,\sigma_{oy} + a_{xx}\,\sigma_{xx} + a_{yy}\,\sigma_{yy} + a_{zz}\,\sigma_{zz}. \tag{4.15}$$

With six amplitudes there are $6^2 = 36$ independent observables, of which the following, from (3.13) and (4.15) are listed for consideration in electron scattering:

$$\begin{aligned}
I \equiv X(oo,oo) &= |a_{oo}|^2 + |a_{yo}|^2 + |a_{oy}|^2 + |a_{xx}|^2 + |a_{yy}|^2 + |a_{zz}|^2,\\
IA_{yo} \equiv IX(yo,oo) &= 2Re(a_{oo}a_{yo}^* + a_{oy}a_{yy}^*),\\
IA_{oy} \equiv IX(oy,oo) &= 2Re(a_{oo}a_{oy}^* + a_{yo}a_{yy}^*),\\
IA_{zx} \equiv IX(zx,oo) &= 2Im(a_{oy}a_{zz}^* - a_{yo}a_{xx}^*),\\
IK_{zx} \equiv IX(zo,ox) &= -2Im(a_{oy}a_{zz}^* + a_{yo}a_{xx}^*),\\
IA_{zz} \equiv IX(zz,oo) &= 2Re(a_{oo}a_{zz}^* - a_{xx}a_{yy}^*),\\
IK_{zz} \equiv IX(zo,oz) &= 2Re(a_{oo}a_{zz}^* + a_{xx}a_{yy}^*).
\end{aligned} \tag{4.16}$$

Imposing, now, the relativistic condition (for $1/\gamma << 1$) that the amplitudes corresponding to the transverse components of the *electron* polarization vanish: $a_{yo} = a_{xx} = a_{yy} = 0$, so (4.15) reduces to

$$M = a_{oo} + a_{oy}\,\sigma_{oy} + a_{zz}\,\sigma_{zz}\,. \tag{4.17}$$

Thus, the relativistic nature of the electron results in a remarkable simplification of the scattering process. With the number of amplitudes reduced to three, the independent observables are reduced to nine. In the process, all of the amplitudes that flip the *electron* helicity have vanished, so the electron helicity is conserved. This is conveniently seen in (3.9) where the amplitudes in the off-diagonal quadrants have vanished, leaving the M-matrix diagonal with respect to the electron helicity. The correlated result from (4.16) is that

$$A_{yo} = 0, \tag{4.18a}$$

$$A_{zx} = -K_{zx}, \quad A_{zz} = K_{zz} . \tag{4.18b}$$

These results are all known (Donnelly and Raskin 1986), but the relativistic origin of the "turn around" relations (4.18b) had not been noted explicitly. These relations have important experimental consequences, since, with them, a double-scattering experiment to determine a polarization-transfer coefficient K_{jk} can, in principle, always be replaced by a single-scattering experiment to determine the equivalent spin-correlation coefficient A_{jk}.

Further simplification results from the dynamical description of the electron scattering process in the one-photon exchange plane-wave Born approximation. One can associate the surviving amplitudes of (4.17) with components of the hadronic electromagnetic current. For the very interesting case of electron-nucleon scattering (Dombey 1969), the nucleon electromagnetic current, in terms of the two-component nucleon spinors for particles b and d is

$$J_\mu = X_d^\dagger M^\mu X_b , \tag{4.19}$$

with

$$\begin{aligned} M^0 &= 2imG_E \sigma_o , \\ M^1 &= iqG_M \sigma_y , \\ M^2 &= -iqG_M \sigma_x , \\ M^3 &= 0 . \end{aligned} \tag{4.20}$$

$G_E(q^2) = \sqrt{4\pi}\, F_L/(1+\tau)$ and $G_M(q^2) = -\sqrt{2\pi}\, F_T/\sqrt{\tau(1+\tau)}$ are the charge and magnetic form factors of the nucleon, m is the nucleon mass, q is the four-momentum transfer, and $\tau = q^2/4m^2$. The longitudinal, z, coordinate direction is taken along the momentum transfer $\boldsymbol{q}$, which is effectively the $\boldsymbol{x}$ coordinate (2.38) with respect to the transformation properties (2.39). Thus, the transformation from the helicity frame to this in-plane transversity frame (figure 1) is given by $\sigma_z \to \sigma_{x'}$ and $\sigma_x \to -\sigma_{z'}$, where the primed coordinates refer to this final-state transversity frame, in which (4.17) becomes

$$M = a_{oo} + a_{oy}\, \sigma_{oy} + a_{zx'}\, \sigma_{zx'}. \tag{4.21}$$

By inspection, noting that the σ components in (4.20) are the σ_k of (3.2), the hadronic-current contributions to these amplitudes are shown explicitly in

$$a_{oo} = 2imG_E\, C_{oo}(\theta), \quad a_{oy} = qG_M\, C_{oy}(\theta), \quad a_{zx'} = -iqG_M\, C_{zx'}(\theta) , \tag{4.22}$$

where the $C_{jk}(\theta)$ are the electron kinematical factors. Thus, the independent amplitudes are reduced to the two that correspond to G_E and G_M, which has long been established. However, it is interesting to see these results emerge, as they must, from the general formalism of this spin structure, and to see that the one-photon exchange electron-scattering process is a particularly simple example. From (4.16) and (4.22), then,

$$
\begin{aligned}
I = |a_{oo}|^2 + |a_{oy}|^2 + |a_{zx'}|^2 &= C_{oo}^2\, 4m^2 G_E^2 + (C_{oy}^2 + C_{zx}^2) q^2 G_M^2, \\
IA_{oy} &= 2Re\ a_{oo} a_{oy}^* = 0\ , \\
IA_{zx'} &= 2Re\ a_{oo} a_{zx'}^* = -\, C_{oo} C_{zx'}\, 4mqG_M\, G_E\ , \\
IA_{zz'} &= 2Im\ a_{oy} a_{zx'}^* = C_{oy} C_{zx'}\, 2q^2 G_M^2\ . \qquad (4.23)
\end{aligned}
$$

In this example of elastic electron scattering as a special case of the general formalism, some results emerge more transparently than from the detailed calculations themselves. Here, one readily sees that the projectile analyzing power $A_{yo} = 0$ from the relativistic electron helicity conservation, while the target analyzing power $A_{oy} = 0$ from the dynamics of the one-photon exchange process. One final consequence of the results shown in (4.22) is that only two of the three surviving observables are independent. In view of the recognized importance of providing a more accurate determination of the charge form factor of the neutron, (4.23) shows that $A_{zx'}$ is the observable most sensitive to G_E^n, depending linearly on it (Arnold et al 1981, Cheung and Woloshyn 1983).

4.3 Particle reactions

In addition to the spin observables that can be determined when polarized beams and/or polarized targets are available, there are some unique opportunities in selected particle reactions to determine final-state spin observables without any initial-state polarization. This is achieved in the production of unstable particles that decay via the PNC weak interaction, which results in an asymmetry in the yield of the decay particles with respect to the polarization of the parent particle. The first observed (Bunce *et al* 1976) and most prominent example is the Λ hyperon (strangeness $S = -1$ baryon), whose $\Delta S = 1$ weak decay to $p\pi^-$ provides an angular distribution of the form $(1 + \alpha P \cos\theta)$, where θ is the angle between the proton (or pion) momentum and the Λ polarization P. The "analyzing power" α having been determined, a measurement of the proton asymmetry yields P, just as in (2.6).

This self-analyzing feature of the Λ and its antiparticle $\bar{\Lambda}$ has been used at the CERN low energy antiproton ring (LEAR) in a remarkable and important investigation of the reaction $\bar{p}p \rightarrow \bar{\Lambda}\Lambda$ near the reaction threshold (Barnes et al 1991). This reaction has the spin

structure of section 3.1. Among the measured final-state observables were the three spin-correlation coefficients

$$X(oo,lm) \equiv C_{lm}, \quad with \quad lm = xx, yy, zz\ , \tag{4.24}$$

from which it is possible to determine the the fractions of $\bar{\Lambda}\Lambda$ pairs produced in the singlet and the triplet spin configurations. The singlet and triplet fractions are the expectation values of the corresponding projection operators,

$$F_s = \frac{1}{4}[1 - <\sigma_1 \cdot \sigma_2>], \qquad F_t = \frac{1}{4}[3 + <\sigma_1 \cdot \sigma_2>], \tag{4.25}$$

and with

$$\sigma_1 \cdot \sigma_2 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}\ , \tag{4.26}$$

as in (2.48)

$$I\, C_{lm} = I <\sigma_{lm}> = \frac{1}{4} Tr\, MM^{\dagger} \sigma_{lm}\ . \tag{4.27}$$

Thus,

$$I\, F_s = \frac{1}{4} I[1 - (C_{xx} + C_{yy} + C_{zz})], \qquad I\, F_t = \frac{1}{4}\ I\ [3 + (C_{xx} + C_{yy} + C_{zz})], \tag{4.28}$$

so these combinations of observables again select specific features of the spin dependent interactions. The remarkable experimental result is that F_s is consistent with zero, so the $\bar{\Lambda}\Lambda$ pairs are produced only in the triplet state. Tabakin, Eisenstein, and Lu (1991) have provided a complete theoretical description and analysis of this reaction and its spin observables in terms of the helicity amplitudes, and they show that

$$I\, F_s = |E|^2, \tag{4.29}$$

where $E(\theta)$ is the singlet-to-singlet transition amplitude in the (coupled) singlet/triplet representation of the M-matrix. They noted suggestions that the vanishing of F_s in this isospin $T = 0$ channel is due to the presence of a strong tensor force, in accord with the expectation that it be enhanced in the pp $T = 0$ channel (Buck, Dover and Richard 1979). Since the association of a particular helicity amplitude with a particular component, e.g. tensor, of the interaction is not transparent, one can express this $F_s \approx 0$ result in terms of the a_{jk}, whose connections with the components of the spin-dependent interactions have been noted in section 3.1.

For this reaction, the M-matrix is that of (3.7). Under particle-antiparticle interchange, charge-conjugation (C) symmetry requires that $a_{yo} = a_{oy}$ and $a_{zx} = a_{xz}$, and (3.7) reduces to the six terms

$$M = a_{oo} + a_{yo}(\sigma_{yo} + \sigma_{oy}) + a_{xx}\,\sigma_{xx} + a_{yy}\,\sigma_{yy} + a_{zz}\,\sigma_{zz} + a_{zx}(\sigma_{zx} + \sigma_{xz}) \tag{4.30}$$

With (4.27),

$$\begin{aligned} I &= |a_{oo}|^2 + 2|a_{yo}|^2 + |a_{xx}|^2 + |a_{yy}|^2 + |a_{zz}|^2 + 2|a_{zx}|^2, \\ I\,C_{xx} &= 2Re(a_{oo}a_{xx}^* - a_{yy}a_{zz}^*) - 4Im(a_{yo}a_{zx}^*), \\ I\,C_{yy} &= 2Re(a_{oo}a_{yy}^* - a_{xx}a_{zz}^*) + 2|a_{yo}|^2 + 2|a_{zx}|^2, \\ I\,C_{zz} &= 2Re(a_{oo}a_{zz}^* - a_{xx}a_{yy}^*) + 4Im(a_{yo}a_{zx}^*), \end{aligned} \tag{4.31}$$

and from (4.28)

$$\begin{aligned} I\,F_s = \frac{1}{4}[&|a_{oo}|^2 + |a_{xx}|^2 + |a_{yy}|^2 + |a_{zz}|^2 \\ &- 2Re(a_{oo}a_{xx}^* + a_{oo}a_{yy}^* + a_{oo}a_{zz}^*) \\ &+ 2Re(a_{xx}a_{yy}^* + a_{xx}a_{zz}^* + a_{yy}a_{zz}^*)], \end{aligned} \tag{4.32}$$

so

$$I\,F_s = \frac{1}{4}|a_{oo} - a_{xx} - a_{yy} - a_{zz}|^2, \tag{4.33}$$

and $(a_{oo} - a_{xx} - a_{yy} - a_{zz})$ is the singlet-to-singlet helicity amplitude M_{ss}. Similarly,

$$I\,F_t = \frac{1}{4}[4\,I - |M_{ss}|^2], \tag{4.34}$$

Thus, quite generally, for $F_s = 0$, $F_t = 1$ the spin-independent and spin-spin amplitudes, a_{oo} and a_{jj}, must vanish, and only the spin-orbit, a_{yo}, and tensor, a_{zx}, terms contribute in the reaction.

Another experiment carried out at the LEP collider at CERN made use of the self analyzing property of the τ lepton in the reaction $e^+e^- \rightarrow \tau^+\tau^-$ (Decamp *et al* 1991). This experiment provided a determination of the ratio of the neutral current vector (v) and axial-vector (a) coupling constants of the τ lepton from a measurement of the τ polarization P_z, thus providing an important specific test of the theoretical model of electroweak interactions . The measurement was made at the Z resonance energy where the weak-interaction Z^o-exchange amplitude is dominant. This reaction, again with the spin structure of section 3.1, provides another example of the large reduction in the number of independent amplitudes that follows, as in electron scattering, from the relativistic nature of both the e and τ leptons. Further

simplicity results from the requirement that the e^+e^- annihilation (and $\tau^+\tau^-$ production) state has $J^P = 1^-$, that of the Z^o (and the photon). (Note that fermions and antifermions have opposite parities). Since parity is not conserved here, one starts with all sixteen amplitudes (3.4). Then with relativistic helicity conservation for both leptons and the $J = 1$ requirement that the total helicity be ± 1 in the initial and final states, only four helicity amplitudes (3.9) survive:

$$M(++,++) = 2(a_{oo} + a_{zo}), \qquad M(--,++) = 2(a_{xx} - ia_{xy}),$$
$$M(++,--) = 2(a_{xx} + ia_{xy}), \qquad M(--,--) = 2(a_{oo} - a_{zo}), \tag{4.35}$$

and each has its PC and PNC component. From helicity conservation $P_x = P_y = 0$, and only the PNC component P_z is nonzero. With $P_z(\tau^+) = P_z(\tau^-)$ from C-symmetry, the P_z averaged over all τ production angles, $<P_z>$, has the very direct connection to the ratio v/a of the coupling constants,

$$<P_z> = -2\frac{v/a}{1 + (v/a)^2}, \tag{4.36}$$

and, thus, to the weak mixing angle θ_W from $v/a = 1 - 4\sin^2\theta_W$. The experimental result, at the Z mass, of $<P_z> = -0.150 \pm 0.045$ then gave $v/a = 0.076 \pm 0.023$ and $\sin^2\theta_W = 0.2302 \pm 0.0058$. Also, with

$$I\,P_z = 4(\mathrm{Re}\; a_{oo}a_{zo}^* - \mathrm{Im}\; a_{xx}a_{xy}^*),$$
$$I\,A_z = 4(\mathrm{Re}\; a_{oo}a_{zo}^* + \mathrm{Im}\; a_{xx}a_{xy}^*), \tag{4.37}$$

it is clear that a measurement of A_z with polarized electrons would provide the same information, and this observable has been measured in a very recent experiment at SLAC with an electron beam polarization $p_z = 0.224 \pm 0.006$ (Abe K et al SLD Collaboration 1993). The measurement of A_z has clear advantages over that of P_z, in that all of the events identified as Z decays, both hadronic and leptonic, can be counted, and there is no need for an analysis of the decay asymmetries. The SLD results, $A_z = -0.110 \pm 0.044 \pm 0.004$, $\sin^2\theta_W = 0.2378 \pm 0.0056 \pm 0.0005$ show this advantage in the very small systematic error. This indicates the considerable improvement in statistical precision that can be attained both with more events and with a higher beam polarization, since the statistical figure of merit is Lp_z^2, with L the luminosity. These P_z and A_z results provide prime examples of the importance of spin observables.

Clearly, in terms of the parton model of the nucleon, with its quark and gluon constituents, the question of how the nucleon's spin is made up from those of its constituents is really a most fundamental question to be addressed by QCD. The simplest view would be that the three (valence) spin-$\frac{1}{2}$ quarks couple to $J=\frac{1}{2}$, just as (to a good approximation) the three nucleons couple to give $J=\frac{1}{2}$ for ^3He. However, it has long been known that about half of the nucleon's momentum is carried by gluons which implies that they could also contribute to its spin. A more sophisticated quark model estimate was that 60 ± 12 % of the nucleon's spin is carried by the quarks (Jaffe and Manohar 1990), so the European Muon Collaboration result, $(12 \pm 9 \pm 14)$%, from deep-inelastic muon scattering (Ashman et al 1988, 1989), stimulated an intense interest and activity in the subject.

First, though, how does one "measure" the spin fraction carried by the quarks? The answer was to measure the spin correlation coefficient A_{zz} in the deep inelastic scattering of polarized muons from polarized protons, selecting the region of momentum and energy transfer that is associated with scattering from the individual quarks. This is an inclusive scattering process, $\mu + p \rightarrow \mu + X$, as is discussed in section 3.4, and $A_{zz}(v,q)$ is now a continous function of the energy and four-momentum transfer, v *and* q $(Q^2 \equiv -q^2)$. Then, with $A_{zo} = A_{oz} = 0$ from PC, (3.1) gives

$$A_{zz}(v,q) = \frac{I_{zz}(++) - I_{zz}(+-)}{I_{zz}(++) + I_{zz}(+-)}. \tag{4.38}$$

In the quark-parton model, with one-photon exchange lepton-quark scattering,

$$A_{zz}(x) = D\frac{2x\, g_1(x)}{F_2(x)} = 2x\, D\, \frac{\frac{1}{2}\Sigma\, e_i^2 [q_i(x,+) - q_i(x,-)]}{\frac{1}{2}\Sigma\, e_i^2 [q_i(x,+) + q_i(x,-)]}, \tag{4.39}$$

where D is the (known) virtual photon depolarization factor, e_i is the charge of the quark of flavor i, $x = \frac{Q^2}{2m_p v}$ is the momentum fraction of the quark within an infinite-momentum proton, $q_i(x,\pm)$ is the momentum-fraction distribution of quarks (plus antiquarks) with $\pm$ helicities within a proton of $+$ helicity, and $g_1(x)$ is the spin dependent nucleon structure function. The similarity between (4.39) and (4.38) is clear since the lepton-quark cross-sections are proportional to e_i^2 and the previously determined unpolarized (i.e. spin averaged) nucleon structure function $F_2(x)$ corresponds to the unpolarized cross-section.

Then, for example, including only $u(e = \frac{2}{3})$ and $d(e = -\frac{1}{3})$ quarks, with $\Delta q_i(x) \equiv q_i(x,+) - q_i(x,-)$,

$$\frac{F_2(x)}{2xD} A_{ZZ}(x) = g_1(x) = \frac{1}{2}[\frac{4}{9} \Delta u(x) + \frac{1}{9} \Delta d(x)]. \tag{4.40}$$

The experimental result, extrapolated and integrated over all x, is

$$\Gamma_1^p = \int_0^1 dx\, g_1^p(x) = 0.126 \pm 0.010 \pm 0.015 = \frac{1}{2}[\frac{3.82}{9} \Delta u + \frac{1.08}{9} \Delta d], \tag{4.41}$$

where the integrated model result on the right includes first order QCD corrections. Then, with the Bjorken sum rule, which connects the proton and neutron spin structure functions by the ratio of the axial to vector coupling constants from neutron β-decay (Bjorken 1966, 1970),

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6}|G_A/G_V|(1- \alpha_S) = 0.191 \pm 0.002, \tag{4.42}$$

the neutron value (with the interchange $\Delta_u \leftrightarrow \Delta_d$) is

$$\Gamma_1^n = -0.065 \pm 0.010 \pm 0.015 = \frac{1}{2}[\frac{1.08}{9} \Delta u + \frac{3.82}{9} \Delta d]. \tag{4.43}$$

Thus, the fractions of the nucleon's spin carried by the u and d quarks in this example would be

$$\Delta u = 0.74 \pm 0.03 \pm 0.05, \qquad \Delta d = -0.52 \pm 0.03 \pm 0.05. \tag{4.44}$$

The complete analysis included s quarks and gave the final result that only $(12 \pm 9 \pm 14)\%$ of the proton's spin is carried by the quarks, with the again surprising contribution from the strange quark sea, $\Delta s = -0.19 \pm 0.03 \pm 0.05$.

This quite unexpected result stimulated a variety of theoretical explanations, but experimental confirmation is clearly needed. Recent results from two subsequent experiments have been reported, one from the Spin Muon Collaboration (SMC) at CERN (Adeva *et al* 1993) the other from SLAC (Souder et al 1993). The SMC group measured $A_{ZZ}^d(x)$ in deep inelastic muon-deuteron scattering and determined Γ_1^d. Then from $2\Gamma_1^d \cong (\Gamma_1^p + \Gamma_1^n)(1 - 1.5\, P_D)$, with P_D the deuteron D-state probability, Γ_1^n was inferred. Their result, in agreement

with the EMC proton result, is that $(6 \pm 20 \pm 15)\%$ of the nucleon's spin is carried by the quarks. The preliminary SLAC result does not agree. That group measured $A_{zz}^{3He}(x)$ in deep inelastic electron-^3He scattering. This provided a "direct" measurement of Γ_1^n, in that the neutron provides the ^3He spin, again corrected for the ^3He D-state probability. Their result is that approximately 63% of the spin is carried by the quarks, so a definite answer concerning the composition of the nucleon's spin from the spins of its parton constituents is yet to be revealed.

5. Spin and symmetries

Certainly, in scattering and reactions one of the most important aspects of the intrinsic spin of particles is the almost indispensible feature of providing spin observables that constitute the most sensitive tests of parity conservation, charge symmetry, and time-reversal invariance. This derives simply from the fact that the (axial vector) spin $\boldsymbol{S}$ and the available momentum vectors $\boldsymbol{k_i}$ and $\boldsymbol{k_f}$ have the different transformations that are shown in arriving at (2.39), so they can then provide the P-odd and T-odd observables.

5.1 Parity

Parity is conserved in the theoretical descriptions of the strong and electromagnetic interactions, and there is no experimental evidence to suggest any *PNC* component. However, the *PNC* weak interaction provides a weak-current contribution, for example, to both nucleon-nucleon *(NN)* and electron-nucleon *(eN)* scattering. During the past several years, there have been very significant advances in the level of sensitivity achieved in experiments that were designed to determine the *PNC* contribution quantitatively.

In principle, from (3.6a) either the transverse or longitudinal analyzing power, A_x or A_z, provides a null test, in which a measured nonzero value is a direct determination of the *PNC* contribution. In practice, A_x corresponds to an up-down asymmetry of the detected particles in the yz plane (fig. 1), whereas A_z is invariant with respect to rotation around the z-axis. This latter fact permits the use of a large solid-angle cylindrical detector which reduces considerably both the statistical and systematic errors, and determinations of A_z have reached levels of precision three orders of magnitude better than that achieved with respect to the transverse analyzing power A_y, for example.

Determinations of A_z in proton-proton scattering have reached the remarkable precision of $\pm 2 \times 10^{-8}$. The actual experimental values are

$$A_Z(13.6\ MeV) = (-0.93 \pm 0.20 \pm 0.05)\ X\ 10^{-7} \text{ (Eversheim et al 1991, 1993)},$$
$$A_Z(45\ MeV) = (-1.50 \pm 0.22)\ X\ 10^{-7} \text{ (Kistryn et al 1987)}, \qquad (5.1)$$

and from these quantitative determinations of the *PNC* hadronic weak-interaction contribution, and other *PNC* experimental results, it is possible to derive specific information about the interaction. That is, in the meson-exchange description of the *NN* interaction the *PNC* contribution is provided by a diagram with one strong and one weak-interaction vertex. There are six "parameters" to be determined, the weak meson-nucleon coupling constants for π, ρ, and ω exchanges with isospin $\Delta T = 0,1,$ *or* 2, as allowed. At present, there are not six linearly independent experimental results from which to determine the coupling constants, so further experiments are planned (van Oers 1992)

Experiments on the transmission of slow neutrons through various nuclear targets have shown very large enhancements above the $\approx 10^{-7}$ values anticipated for *PNC* observables. Since an *s-p* wave interference is required for *PNC*, these enhancements occur near a *p*-wave resonance with an *s*-wave admixture. At the $J = \frac{1}{2}^-$ *p*-wave resonance at $0.73\ eV$ in neutron-^{139}La scattering, an amazingly large value of $A_Z \approx 0.10$ has been found (Alfimenkov et al 1983, Masuda et al 1989, Bowman et al 1989, Yuan et al 1991), both for the neutron capture (n,γ) reaction and for the total cross-section measured in a neutron transmission experiment. This result is explained in terms of parity mixing with nearby $J = \frac{1}{2}^+$ *s*-wave nuclear levels and the *p*-wave barrier hindrance of the *PC* transitions. Following this discovery, neutron transmission experiments at the Los Alamos Neutron Scattering Center (LANSCE) have found many *p*-wave resonances in n-^{238}U *and* n-^{232}Th with values of A_Z from 0.01 *to* 0.10 (Zhu et al 1992, Frankle et al 1992). The intent, of course, is to convert these results, via a plausible and tractable model calculation, from the compound nucleus system to a *PNC* component in the underlying nucleon-nucleon effective interaction (Johnson et al 1991), but it is not at all clear that such a circuitous route to a better determination of the weak meson-nucleon coupling constants is feasible.

The measurement of A_Z in inclusive electron-deuteron scattering, $e + d \rightarrow e' + X$, at 16 to $22\ GeV$ (Prescott et al 1978, 1979) was, at that time, a most important test and verification of the present standard (Weinberg-Salam) model of the electroweak interaction. The result for $A_Z(Q^2) = A_Z\ Q^2$, removing the known Q^2 dependence of the electromagnetic amplitude, was $A_Z = (-9.5 \pm 1.6)\ X\ 10^{-5}$. This nonzero value eliminated some models which could explain the difference between neutrino and antineutrino scattering but which were parity conserving in electron scattering. The derived value of the Weinberg (weak mixing) angle was $sin^2\ \theta_W =$

0.224 ± 0.020, consistent with the *W-S* model and with the value obtained from neutrino experiments to that time. It is interesting to compare this value of A_Z with that of $\langle P_Z \rangle = -0.15$ in the reaction $e^+e^- \rightarrow \tau^+\tau^-$ at the Z resonance, as is discussed in section 4.3, showing the dominance of the weak Z^o-exchange amplitude there.

5.2 Charge symmetry

Conceptually, with respect to scattering/reactions charge symmetry *(CS)* is similar to identical-particle *(IP)* symmetry and to charge-conjugation *(C)* symmetry in that each symmetry imposes the condition that an observable be invariant under the interchange of particles. *IP*-symmetry is exact, and *C*-symmetry (particle-antiparticle interchange) seems to be valid in the strong interaction, but is broken, along with *P*, in the weak interaction. *CS* is a symmetry under interchange of the "mirror" members of an isospin multiplet. In nuclear reactions this entails the interchange $p \leftrightarrow n$ for all the reaction participants. Clearly, this interchange alters the electromagnetic energy, but with the system corrected for this electromagnetic change there had been, for a long time, no convincing evidence of *CS* breaking. The problem has been that the corrections are large and/or model dependent. For example, in the long-researched comparison between the *nn* and *pp* scattering lengths, $a_{pp} = -7.81$ *fm* is corrected to $a_{pp}^c = -17.3 \pm 0.4$ *fm* for comparison with a_{nn}. Finally, a very clever way was devised to remove the necessity for such a large correction by using a comparison of spin observables in *np* elastic scattering itself. *CS* then requires that the neutron analyzing power $A_n = A_p$, the proton analyzing power, and determinations of $\Delta A \equiv A_n - A_p \neq 0$ have been made in two technically challenging and demanding experiments. The results are

$$\Delta A = (4.7 \pm 2.2 \pm 0.8) \times 10^{-3} \quad \text{at} \quad E_n = 477\ MeV \quad \text{(Abegg } et\ al \text{ 1986, 1989)},$$
$$\Delta A = (3.31 \pm 0.59 \pm 0.43) \times 10^{-3} \quad \text{at} \quad E_n = 183\ MeV \quad \text{(Knutson } et\ al \text{ 1991)}, \qquad (5.2)$$

which have provided direct evidence of *CS* breaking at the level of about 4×10^{-3}.

At the quark level the $p(uud) \leftrightarrow n(udd)$ interchange is just the $u \leftrightarrow d$ interchange, and the origin of the (non-electromagnetic) strong-interaction *CS* breaking is the *u-d* quark mass difference. But *CS* is badly broken at the quark level, with $\frac{1}{2}(m_u + m_d) \approx 5$ *MeV*, $m_d - m_u \approx 3$ *MeV*, $\frac{m_d}{m_u} \approx 1.8$. However, because *QCD* has a scale $\Lambda \approx 1$ *GeV* it is the smallness of these quark masses with respect to Λ (an accidental symmetry) that results in *CS* breaking of the order $(m_d - m_u)/\Lambda$ (de Teramond and Gabioud 1987), which is consistent with the

experimental values (5.2). A comprehensive review of the charge symmetry of strong interactions has been provided by Miller *et al* (1990)

5.3 Time reversal

It is important to note that the measurements of the *PNC* A_z in *the NN* interaction have reached the precision of $\pm 2 \times 10^{-8}$ (4.45), while those of *CS* breaking $A_n - A_p$, have attained $\pm 7 \times 10^{-4}$ (5.2). The principal reason for this very substantial difference in the attained precision is that the A_z measurement is a null test of *PC;* that is, $A_z = 0$ from *PC,* so any nonzero value is an immediate signal of *PNC*. By contrast, *CS* requires neither A_n nor A_p to vanish, only that they be equal, so both must be measured and compared.

Tests of time-reversal invariance, *T*-symmetry, in scattering/reactions have all been of the latter category, and there is a proof of the nonexistance of a null test of *T*-symmetry (Arash et al 1985). This result can be seen in the condition (3.6b) that *T*-symmetry equates a reaction observable to an observable in the inverse reaction, so the difference (or sum) is zero. Even in elastic scattering, which is its own inverse reaction, two different observables are related by *T*-symmetry; for example, the analyzing and polarizing powers, so that $A_y - P_y = 0$. Since,here, the final state P_y is determined via a second scattering, it is easy to understand why such *T*-tests have rarely attained the 10^{-2} level of experimental error.

There is, however one feature of elastic scattering, not included in the nonexistence proof, that makes possible a null test of *T*-symmetry . As is discussed in connection with the conditions (2.40), a T-odd amplitude vanishes in elastic scattering, so an observable proportional to it would also vanish from T-symmetry. The spin-dependent total cross-section, with both beam and target polarized, provides just such an observable through its connection to the forward elastic amplitude by the spin-dependent optical theorem (Phillips 1963),

$$I_T(p_j,p_k) = \frac{4\pi}{k} \, Im \; Tr \; [\rho_{j,k} \, M(0)], \tag{5.3}$$

where $\rho_{j,k}$ is the density matrix representing the inital polarizations, $M(0)$ is the forward scattering *M*-matrix, and $I_T(p_j, p_k)$ is the corresponding total cross-section. Then with (3.7), for example, and

$$I_T(p_j,p_k) = I \, [1 + p_j p_k \, A_{jk}], \tag{5.4}$$

$$A_{jk} = Im \; a_{jk}(0) / \; Im \; a_{oo}(0). \tag{5.5}$$

Unfortunately, the T-odd amplitudes a_{zx} and a_{xz} are odd functions of θ and vanish at $\theta = 0$, so the total cross-section spin-correlation coefficient A_{zx} cannot provide a null test of T-symmetry. It is necessary to have a spin structure, like that of section 3.2, where tensor polarization is available. Then, the T-odd amplitude $a_{x,yz}$ is an even function of θ so the corresponding observable $A_{x,yz} = 0$ from T-symmetry alone, thus providing a null test. The required total-cross section ratio $I_T(+,+)/I_T(-,+)$ can be measured in a transmission experiment with the beam polarization p_x and the target tensor polarization (alignment) along the direction $y = z$, as the notation indicates. It seems clear that such a null test will permit an improvement of several orders of magnitude in the experimental precision achievable in tests of T-symmetry (Conzett 1993).

6. Summary

One objective of this review has been to present the experimental description and definitions of spin-polarization observables along with the corresponding spin formalism in a manner that reveals the required equivalence of the two. Then, the principal focus has been to emphasize the important common role that spin physics plays in the seemingly dissimilar disciplines of nuclear, particle, and electron- scattering physics. That of nuclear physics is the more complicated in the sense that all of the reaction/scattering amplitudes that are allowed by parity conservation, T-symmetry, and other specific symmetries (e. g., identical particles), are nonvanishing in general. Helicity conservation, a relativistic condition, reduces substantially the number of nonvanishing amplitudes in electron scattering and in particle reactions. Thus these disciplines are spin physics subsets of that of nuclear reactions. In order to emphasize this common ground, it is important to implement the use of a common terminology and notation. Finally, examples are given to illustrate the important use of spin observables in tests of the fundamental symmetries.

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Appendix. Spin physics terminology and notation

The most commonly encountered polarization observables, as introduced at the appropriate points in this review, are listed as follows:

A1. General observable for the generic reaction $a + b \rightarrow c + d$.

$X(jk,lm)$; j,k (l,m) designate the prepared (measured) polarizations of a,b (c,d).

That is, $j = y$ means that particle a has polarization p_y, etc., with $j = o$ unpolarized.

Spin-$\frac{1}{2}$: $j,k,l,m = o,x,y,z$. Spin-1: $j,k,l,m = o,\ x,y,z,\ xx,yy,zz,\ xy,yz,zx$.

$X(jk,lm) = 0$ for $n_x + n_z =$ odd , from PC.

A2. Specific observables

A2.1. Analyzing powers

Particle a : $A_y = A_{yo} \equiv X(yo,oo)$

Particle b : $A_y = A_{oy} \equiv X(oy,oo)$

A2.2. Polarizing powers

Particle c : $P_y = P_{yo} \equiv X(oo,yo)$

Particle d : $P_y = P_{yo} \equiv X(oo,oy)$

A2.3. Polarization-transfer coefficients

a to c ; b to d : $D_{jl} \equiv X(jo,lo)$; $D_{km} \equiv X(ok,om)$

a to d ; b to c : $K_{jm} \equiv X(jo,om)$; $K_{kl} \equiv X(ok,lo)$

A2.4. Spin-correlation coefficients

Initial state : $A_{jk} \equiv X(jk,oo)$

Final state : $C_{lm} \equiv X(oo,lm)$

The initial and final (x, y, z) triads are defined by equations (2.38) and are shown in figure 1. At intermediate energies and in high energy NN scattering (S, N, L) usually replaces (x, y, z). Some different coordinate frames used are also shown in figure 1. Common to all is that $\boldsymbol{y}, \boldsymbol{N}, \boldsymbol{n} = \boldsymbol{k_i} \times \boldsymbol{k_f}$, so they are all related by a simple rotation around this axis.

In the above listing, the left-hand-side simpler notation for the observable is the common usage. Wherever some confusion might result one can describe the observable more clearly or specify it early by its $X(jk,lm)$ form.

When spin-1 tensor components are involved a simple change in the notation is made. For example, a tensor to tensor polarization-transfer coefficient is $D_{xx,zz} \equiv X(xx,o;zz,o)$.

For the spin structure $\frac{1}{2} + \frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2}$ a complete listing of the observables has been given by Hoshizaki (1986).

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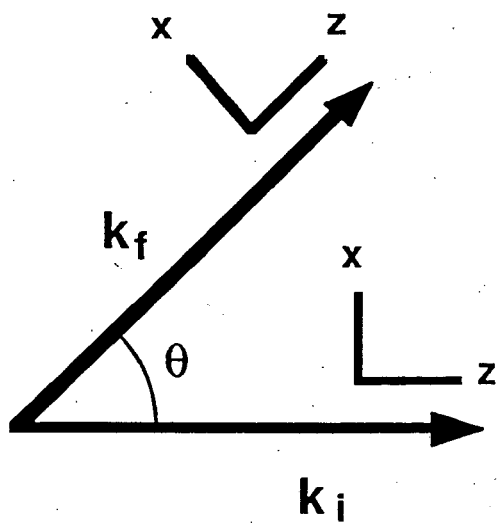
Figure caption

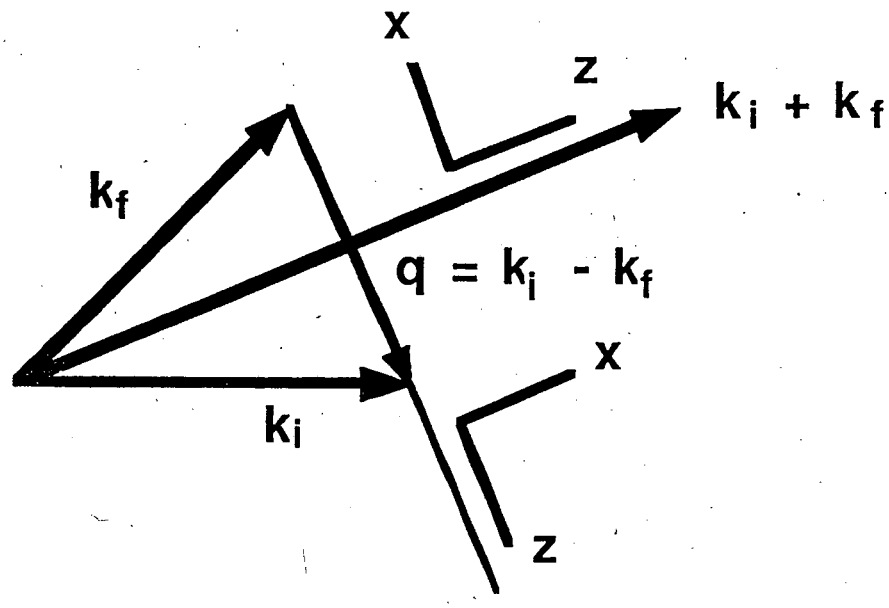
Figure 1. a) The initial and final center of mass helicity frames. b) The usual NN elastic scattering frame with $\boldsymbol{z}$ along $\boldsymbol{k_i} + \boldsymbol{k_f}$, with $(x, y, z) \leftrightarrow (q, n, p)$. In electron scattering the target polarization reference frame is chosen with $\boldsymbol{z}$ along the momentum transfer $\boldsymbol{q}$. This is the (in plane) transversity frame. All of these frames, including their $\theta_L \leftrightarrow \theta_C$ transformations, are connected by rotations around the y-axis.

a)



$$y = k_i \times k_f$$

b)



LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
TECHNICAL INFORMATION DEPARTMENT
BERKELEY, CALIFORNIA 94720