LBL-34797
UC-414

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Physics Division

To be presented at the 1993 Kazimierz Meeting on Elementary Particle Physics, Kazimierz, Poland, May 24–25, 1993, and to be published in the Proceedings

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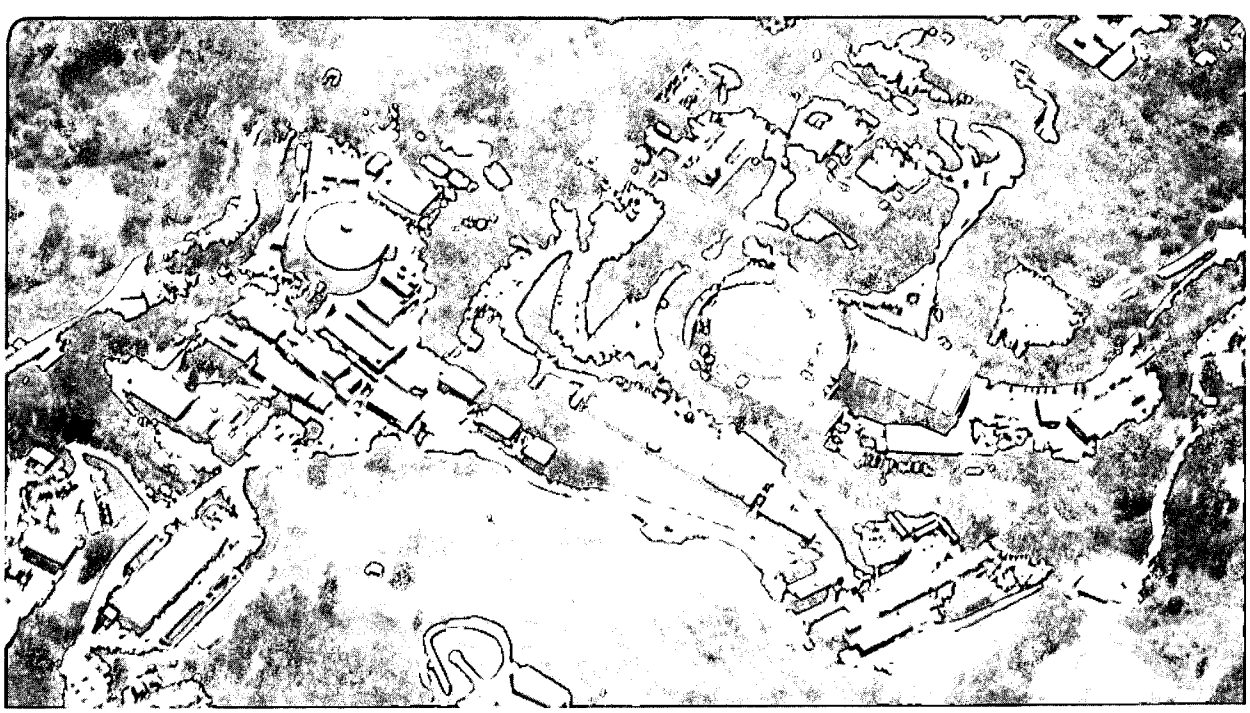
November 1992



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November 1, 1993 LBL-34797

OPERATOR ANALYSIS FOR QUARK AND LEPTON MASSES IN SO(10). * †

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Abstract

A sequence of simple assumptions leads to a supersymmetric SO(10) theory with 8 predictions in addition to $\sin^2\theta$: $\tan\beta, m_t, V_{cb}$, $m_s, m_s/m_d$, $m_u/m_d, V_{ub}$ and the amount of CP violation J. These predictions are presented, together with experiments which will test them.

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY90-21139.

†Contribution to the 1993 Kazimierz meeting on Elementary Particle Physics, Kazimierz, Poland, 24-25th May 1993. , in collaboration with G. Anderson, S. Dimopoulos, S. Raby and G. Starkman.

The standard model, while extremely successful, has 18 free parameters, 13 of which are in the flavor sector. In seeking a more fundamental theory we can be guided by the requirement that at least some of these parameters should be predicted. Symmetries provide essentially the only known tool which is sufficiently developed to yield such predictions: imposing extra symmetries on a theory leads to a reduction in the number of free parameters. Such symmetries have been studied in the flavor sector for over 20 years [1–7]. We believe that experiment has provided a hint as to which symmetries should be imposed. Of the 18 parameters of the standard model, only one has been successfully predicted to a high level of significance: the weak mixing angle [8]. This suggests that we should pursue theories which have both supersymmetry and grand unified symmetry.

Unfortunately this is not sufficient information to constrain the flavor sector of the theory. Further parameter reduction in this sector may be achieved with family symmetries, which we envisage imposing as follows. Suppose that the theory just beneath the Planck scale has a set of renormalizable interactions coupling quarks and leptons to additional fields in a way which obeys certain family symmetries. If these additional fields acquire large masses and are integrated out of the theory, they give rise to non-renormalizable operators involving quark and lepton fields. Furthermore, these operators, which we will use to generate the light quark and lepton masses, will have a very restricted form, reflecting the effects of the flavor symmetry in the high energy theory.

One of the difficulties associated with understanding the flavor sector is that we do not know most of the parameters very accurately. Only the electron, muon and tau masses and the Cabibbo angle are known to 1% or better, and grand unified theories do not provide relations amongst these. Mass relations from grand unified theories can typically be checked only at the 10% to 30% level. Thus one successful prediction is not very significant. A clear advantage of studying theories of maximal predictivity is that they will be greatly constrained by present experimental measurements. Successful candidate models will already have achieved considerable significance, and will be subject to future experimental tests.

We are therefore motivated to carry out *a general operator analysis for the most predictive flavor sector of supersymmetric grand unified SO(10) theories.*

In this talk I give a summary of this work[9]. The framework for the operator analysis is as follows:

- The gauge group is SO(10)[10]. This is the smallest gauge group that allows an entire family to be described by a single irreducible representation. Thus the three families are written as 16_i with i=1,2,3 and 16_3 being the heaviest family. Perhaps the most elegant feature of SO(10) is the way in which all the measured gauge charges of the fermions can be simply understood in terms of this 16 dimensional spinor.

- The grand unified theory is supersymmetric. Below the grand unification scale we take the theory to be the minimal supersymmetric standard model, as this is the unique minimal possibility for obtaining the successful $\sin^2\theta$ prediction.

- The two low energy Higgs doublets of this theory lie in a single 10 dimensional representation of SO(10). This is the unique minimal possibility.

- The masses of the heavy generation (m_t, m_b and m_τ) come from a single renormalizable operator

$$A\ 16_3\ 10\ 16_3 \tag{1}$$

  where 10 is the multiplet containing the light doublets. This elegant picture of the unification of the Yukawa couplings λ_t, λ_b and λ_τ is reminiscent of the unification of the three gauge couplings g_1, g_2 and g_3 and is due to Ananthanarayan, Lazarides and Shafi [11,12].

- All the masses of the quarks and leptons of the lightest two generations, and the mixing angles of the KM matrix, are entirely due to non-renormalizable operators in the effective theory at the GUT scale, which give masses suppressed compared to those from (1) by powers of M_G/M where M is a mass scale close to the Planck scale. Thus the mass hierarchy between generations and the smallness of the KM angles is to be understood in terms of powers of M_G/M. We study those models with the fewest such operators required for consistency with the known masses and mixings.

- These non-renormalizable operators have the form

$$O_{ij} \equiv 16_i \frac{45_1}{M_1} ... \frac{45_k}{M_k} 10 \frac{45_{k+1}}{M_{k+1}} ... \frac{45_\ell}{M_\ell} 16_j \tag{2}$$

  The mass terms result when the various 45 dimensional adjoint representations acquire vacuum expectation values (vevs) of order M_G.

- Simple relations amongst the masses in the up, down and electron sectors follow because each 45 vev lies in a definite direction in the SO(10) group space: in the hypercharge, $B-L$, T_{3R} or X direction, where X preserves an SU(5) subgroup. To see this, recall that when a 45 vev acts on a fermion in a 16, it gives a numerical "Clebsch", which is the charge of the fermion under the particular group generator corresponding to the direction of this vev. It is the SO(10) group theory "Clebschs" which allow an understanding of the regularities of the fermion mass matrix [13].

- SO(10) is broken to SU(5) by a vev of 45_X at a scale v_{10} larger than the scale v_5 at which SU(5) is broken. In constructing the effective theory at scale v_5, particles of mass of order v_{10} must also be integrated out of the theory. This means that the objects appearing in the denominators in (2) can be $< 45_X >$ as well masses of order M. Thus the hierarchy of fermion masses is due to v_5/v_{10} as well as v_{10}/M.

At least two operators of the form (2) are needed in order to give all quark and leptons a mass. Two such operators, together with (1), allow the 3×3 Yukawa matrices to have non-zero determinants. However the coefficients of these three operators can all be made real by rotating the phases of the three 16_i fields. Hence this case is excluded because the CP violation in the KM matrix, J, vanishes.

The most predictive theories of this sort therefore have three operators of type (2) in addition to the operator (1). Now only three of the four operator coefficients can be made real, so that there are five independent GUT flavor parameters. In addition, the quark and lepton masses depend on $\tan\beta$, the ratio of the two Higgs doublet vevs, so there are a total of six independent flavor parameters. We choose to determine these from the six best measured flavor parameters: $m_e, m_\mu, m_\tau, \theta_c, m_c$ and m_b. Hence the theory predicts $\tan\beta$ and the

seven standard model flavor parameters $m_t, V_{cb}, m_s, m_d, m_u, V_{ub}$ and J. We have performed a search for all successful theories of this type, and find that they fall into two classes each with a certain texture. In this talk I will discuss only the favored class, which is selected by the additional requirement that

- there must be a natural understanding of why $m_c/m_t \approx V_{cb}^2 \ll m_s/m_b \approx m_\mu/m_\tau$ for quantities renormalized at the GUT scale.

A lengthy but straightforward argument shows that the set of assumptions marked above by bullets leads to Yukawa coupling matrices renormalized at the GUT scale of the form:

$$\mathbf{U} = \begin{pmatrix} 0 & -\frac{1}{27}C & 0 \\ -\frac{1}{27}C & 0 & x_u B \\ 0 & x'_u B & A \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} 0 & C & 0 \\ C & Ee^{i\phi'} & x_d B \\ 0 & x'_d B & A \end{pmatrix}$$

$$\mathbf{E} = \begin{pmatrix} 0 & C & 0 \\ C & 3Ee^{i\phi'} & x_e B \\ 0 & x'_e B & A \end{pmatrix} \tag{3}$$

where A occurs in (1), and B, C and $Ee^{i\phi'}$ are proportional to the coefficients of the three non-renormalizable operators of type (2), which must be chosen to contribute to the 23, 12, and 22 entries of the matrices respectively. Notice that while operator (1) yields $U_{33} = D_{33} = E_{33}$, a similar equality is not found for the non-renormalizable contributions. We can prove quite generally that O_{12} is unique and the 45 vevs lead to the Clebsch factors $U_{12} : D_{12} : E_{12} = -1 : 27 : 27$. While four operators O_{22} are possible, they lead to the same Clebsch factors: $U_{22} : D_{22} : E_{22} = 0 : 1 : 3$ and therefore to the same predictions. The 22 entry is infact the one similarity of this scheme with the Georgi-Jarlskog pattern [4]. We have found nine possible operators O_{23} and hence we have nine SO(10) models with this maximal level of predictivity. The corresponding Clebschs are denoted in (3) by x_i and x'_i. In fact, the low energy predictions depend essentially on only two combinations of these Clebsch parameters.

We have explicitly shown the Yukawa matrices in eqtn. (3) because it leads quite directly to our predictions. Nevertheless, we stress that underlying such

Yukawa matrices are a set of four SO(10) invariant operators. If one wishes to ponder the flavor problem in our scheme, then it is these operators which one should study. The operators for one of our models are shown below in eqtn. (23), and following this equation we discuss a possible generation mechanism.

To demonstrate the power of these theories I will write down the analytic formulas for the eight predictions. A technical problem is that the dynamical renormalization group factors, η_a, depend not only on α_s, but also on the third generation Yukawa parameter A. Hence the determination of A, and of the η_a, is a non-linear problem, which has no analytic solution. Of course A and η_a can be numerically computed with good accuracy. Hence I will give the predictions in terms of the 6 input parameters, A and η_a and one should simply remember that A and η_a are understood to be computed numerically from the inputs.

The predictions take the form

$$\begin{pmatrix}\text{predicted}\\ \text{quantity}\end{pmatrix} = \begin{pmatrix}\text{group theory}\\ \text{GUT Clebsch}\end{pmatrix}\begin{pmatrix}\text{input}\\ \text{parameter}\end{pmatrix}\begin{pmatrix}\text{dynamical}\\ \text{RG factor}\end{pmatrix} \tag{4}$$

The eight predictions are as follows. The ratio of electroweak vevs $\tan\beta$ is obtained from

$$\cos\beta = \frac{\sqrt{2}m_\tau}{v}\,\frac{\eta_1}{A} \tag{P1}$$

where $v = 247$ GeV. The top quark mass parameter is

$$m_t = \tan\beta\; m_\tau\, \eta_2 \tag{P2}$$

with β determined from eq. (P1). These two predictions follow from just operator (1) for the heaviest generation and in these cases the GUT Clebsch factor is unity. The mixing between the two heaviest generations is given by

$$V_{cb} = \chi\sqrt{\frac{m_c}{m_t}}\,\eta_3 \tag{P3}$$

Thus the GUT relation[5] $V_{cb} = \sqrt{\frac{m_c}{m_t}}$ is modified by a Clebsch factor χ, which we discuss below. Whenever m_t appears on the right-hand side of a prediction, it is understood that the value given by (P2) is to be used. The strange mass is given by

$$m_s = \frac{1}{3}(1+\delta)\frac{m_b}{m_\tau}m_\mu\,\eta_4 \tag{P4}$$

which is one of the Georgi-Jarlskog relations [4], except for a small correction $1+\delta$ which we discuss below. From the determinant of **D** and **E** one finds

$$\frac{m_s}{m_d} = \frac{1}{9}(1+2\delta)\frac{m_\mu}{m_e} \tag{P5}$$

which is a small modification of the second Georgi-Jarlskog relation. In this prediction the renormalization factors cancel. The prediction for m_u/m_d follows from the determinants of the Yukawa matrices, with m_s substituted from (P4):

$$\frac{m_u}{m_d} = \frac{1}{3^7}(1+\delta)\frac{m_\mu}{m_\tau}\frac{m_t^2}{m_c m_b}\eta_6 \tag{P6}$$

The last two predictions are for parameters of the KM matrix. Diagonalization of **U** and **D** yield a KM matrix of the form

$$V = \begin{pmatrix} c_1c_2 - s_1s_2e^{-i\phi} & s_1 + c_1s_2e^{-i\phi} & s_2s_3 \\ -c_1s_2 - s_1e^{-i\phi} & c_1c_2c_2e^{-i\phi} - s_1s_2 & c_2s_3 \\ s_1s_3 & -c_1s_3 & c_3e^{i\phi} \end{pmatrix} \tag{5}$$

where s_1, s_2 and ϕ are renormalization group invariants, and $s_3 = V_{cb}$ has a simple scaling behaviour. The CP violating phase ϕ is derived from, but not identical to, the phase ϕ' of eq. (3). The angles s_1 and s_2 are given by

$$s_1 = \sqrt{\frac{m_d}{m_s}} \tag{6}$$

$$s_2 = \sqrt{\frac{m_u}{m_c}} \tag{7}$$

and ϕ is determined from the Cabibbo angle via

$$\sin\theta_c = |V_{us}| = |s_1 + c_1s_2e^{-i\phi}| \tag{8}$$

The two quantities of **V** which are predicted are

$$|\frac{V_{ub}}{V_{cb}}| = s_2 \tag{9}$$

and the amount of CP violation

$$J = s_1s_2s_3^2s_\phi \tag{10}$$

It is very interesting to note that equations (5) - (10) also hold in the Georgi-Jarlskog scheme [6]. Indeed it has recently been shown that the successful predictions (9) and (10) follow from very simple assumptions about the form of U and D [14]. However, the class of theories under study here is much more predictive and makes specific predictions for $\sqrt{\frac{m_d}{m_s}}$ and $\sqrt{\frac{m_u}{m_c}}$ which can then be substituted in eq. (6) and (7). The prediction for $\sqrt{\frac{m_d}{m_s}}$ is obtained from (P5) while the prediction for $\sqrt{\frac{m_u}{m_c}}$ gives

$$|\frac{V_{ub}}{V_{cb}}| = s_2 = \frac{1}{27}\frac{m_e^{1/2}m_\mu^{1/2}}{m_\tau}\frac{m_t}{m_c}\eta_7 \qquad (P7)$$

The final prediction is for the amount of CP violation in the KM matrix obtained by using the above expressions for s_1, s_2 and $s_3 = V_{cb}$ in eq. (10)

$$J = \frac{\chi^2}{9}(1-\delta)\frac{m_e}{m_\tau}s_\phi\,\eta_8 \qquad (P8)$$

where ϕ is obtained from (8).

The class of models under discussion does not have a unique operator contributing to the 23 and 32 entries of the Yukawa matrices. This is reflected in (3) by the appearance of the Clebschs x_i and x_i' which can assume a set of discrete values. Nevertheless all models of this class lead to the above 8 predictions and the only dependence on these Clebschs is through the two parameters

$$\chi = \frac{|x_u - x_d|}{\sqrt{|x_u x_u'|}} \qquad (11)$$

which only enters the relation for V_{cb} and

$$\delta = \frac{3x_d x_d' - x_e x_e'}{x_u x_u'}\frac{m_\tau m_c}{m_\mu m_t}\cos\phi\,\eta_9 \qquad (12)$$

These Clebsch factors are listed for the nine models (1 - 9) in the table. The prediction (P3) implies that the only theoretically allowed values of χ which are experimentally acceptable are: $\chi = 2/3, 5/6$ and $8/9$. The case $\chi = 1$ [5,6,7] is disfavored in the present theories which contain operator (1) because the resulting values for V_{cb} are uncomfortably large. For all models of interest $\delta \ll 1$ and hence the δ dependence of m_s in (P4) and of J in (P8) is much less than the experimental uncertainties on these quantities, and can be dropped. The

more interesting effects of δ are to be found in (P5) and in (P6), where they give small modifications to the ratios m_u/m_d and m_s/m_d. The values of m_s/m_d are shown in the table.

Table

	1	2	3	4	5	6	7	8	9
$\frac{x_e x'_e}{x_u x'_u}$	-1.5	-13.5	$1/2$	4.5	-6.75	2.25	$1/9$	1	9
$\frac{x_d x'_d}{x_u x'_u}$	$1/6$	$1/6$	$-1/18$	$-1/18$	$-1/12$	$1/36$	$1/9$	$1/9$	$1/9$
$c = \frac{3x_d x'_d - x_e x'_e}{x_u x'_u}$	2	14	$-2/3$	-4.7	6.5	-2.2	$2/9$	$-2/3$	-8.7
$\frac{m_s}{m_d}$	26.7	D	24.8	23.6	D	24.2	25.3	24.8	
χ	$2/3$	$2/3$	$2/3$	$2/3$	$5/6$	$5/6$	$8/9$	$8/9$	$8/9$
$\cos\phi$	0.35		0.24	0.16		0.20	0.27	0.24	
$\sin\phi$	0.94		0.97	0.99		0.98	0.96	0.97	

For a detailed study of the predictions of these nine models the reader should consult the figures of ref. 9. Here we give approximate results which follow from the above analytic analysis, using inputs of $m_b = 4.25$ GeV, $m_c = 1.27$ GeV and $\alpha_s(M_Z) = 0.115 \pm .005$.

The top quark mass prediction is essentially the same for all nine models. For the above inputs the pole mass is

$$M_t = 179 \pm 9\ GeV$$

where the superpartners have been taken degenerate at 200 GeV. Supersymmetric threshold corrections of order 10 GeV are to be expected, however it is not easy to push the top quark mass beneath 165 GeV[16].

In the remaining predictions we show the dependence on the top quark mass explicitly:

$$V_{cb} \simeq .059\chi \sqrt{\frac{180GeV}{m_t}}\ (1 \mp .03) \tag{13}$$

$$\frac{V_{ub}}{V_{cb}} \simeq .063\ \frac{m_t}{180GeV}\ (1 \pm 0.1) \tag{14}$$

where the range corresponds to $\alpha_s(M_Z) = 0.115 \pm 0.005$. The m_s/m_d prediction is listed in the Table . From (P4), (P6) and (P8)

$$m_s \simeq (168 \pm 19) \sqrt{\frac{m_s/m_d}{23}}\, MeV \tag{15}$$

$$\frac{m_u}{m_d} \simeq 0.71 \left(\frac{m_t}{180GeV}\right)^2 \sqrt{\frac{m_s/m_d}{23}}\, (1 \pm 0.13) \tag{16}$$

$$J \simeq 4.6 \times 10^{-5} \chi^2 \sqrt{\frac{23}{m_s/m_d}} \sin\phi (1 \pm 0.03) \tag{17}$$

for $\alpha_s(M_Z) = 0.115 \pm 0.005$. Values of $\cos\phi$ and $\sin\phi$ are listed in the table. For all models $\sin\phi \simeq 1$.

Finally we give approximate formulas for CP violation in K and B meson processes, and for $B^0\overline{B}^0$ mixing. Using the same central values as above, we find the kaon CP impurity parameter ϵ is given by

$$|\epsilon| \simeq 2.26\ 10^{-3} \left(\frac{\hat{B}_K}{0.51}\right) \left(\frac{\chi}{5/6}\right)^4 \left(\frac{23}{m_s/m_d}\right)^{3/2} \sqrt{\frac{m_t}{180GeV}}\, (1 \mp 0.03) \tag{18}$$

for $\alpha_s(M_Z) = 0.115 \pm 0.005$. A lattice calculation[17] gives $\hat{B}_K = 0.72 \pm 0.06$, where the error includes only the uncertainty due to the continuum extrapolation.

For $B_d^0\overline{B}_d^0$ mixing we find

$$x_d \equiv \frac{\Delta m}{\Gamma} \simeq 0.67 \left(\frac{\tau_B}{1.28ps}\right) \left(\frac{\sqrt{B} f_B}{175MeV}\right)^2 \frac{23}{m_s/m_d} \left(\frac{\chi}{5/6}\right)^2 \sqrt{\frac{m_t}{180GeV}}\, (1 \mp 0.06) \tag{19}$$

showing that large χ prefers smaller f_B (which is normalized such that $f_\pi = 135$ MeV).

The CP violating parameters $\sin 2\alpha$ and $\sin 2\beta$ measured in $B^0 \to \pi^+\pi^-$ and $B^0 \to \psi K_s$ are given in our models by

$$\sin 2\alpha = -2\cos\phi \sin\phi \tag{20}$$

$$\sin 2\beta = \frac{2c_1 s_1 s_2}{s_c^2} \sin\phi \left(1 + \frac{c_1 s_2}{s_1} \cos\phi\right). \tag{21}$$

From the Table it can be seen that six models have $|\cos\phi| \leq 0.35$. In this case the factor $\frac{c_1 s_2}{s_1} \cos\phi$ is less than 0.1 and can be neglected in (21). Since these

models all have $\sin\phi \simeq 1$ we find $\sin 2\beta \simeq 2s_1 s_2/s_c^2$. Since $s_1 \simeq 0.2\ (1 \pm 0.05)$ and $s_2 \simeq 0.063\ (1 \pm 0.2)$ we find that these models all lead to essentially the same β:

$$\sin 2\beta \simeq 0.54\sqrt{\frac{23}{m_s/m_d}}\,(1 \pm 0.15) \tag{22}$$

On the other hand the models do have variation in $\cos\phi$, so they can be distinguished by the values of $\sin 2\alpha$, which are listed in the Table.

In summary our eight predictions for $\beta, m_t, V_{cb}, m_s, m_s/m_d, m_u/m_d, V_{ub}/V_{cb}$ and J are given in (P1) - (P8). They all agree with present experimental values, and the predictions are sufficiently accurate that future experiments will provide critical tests of these theories. The most important advances which can test our scheme via predictions (P1) - (P8) are

- A measurement of m_t.
- A high statistics study of semi-leptonic B meson decay to measure V_{cb}. In addition, better theoretical understanding of this matrix element is required, which looks likely in view of recent developments in heavy quark effective field theory.
- A measurement of the CP violating decays of neutral B mesons, which will test our predictions for the KM matrix. In particular, all five of our models require that $\sin 2\beta$ be in the range 0.52 ± 0.13, while $\sin 2\alpha$ differentiates between the models.
- A better theoretical understanding of the values for m_u/m_d and m_s/m_d implied by experiment.

There are obvious objections to the above scheme:

1) While there are only six independent continuous flavor parameters, there are millions of operators of the form of (2), and therefore there are extra discrete variables: the Clebschs. If Clebschs can be found to fit any values of the standard model parameters, then there is no significance to our results.

In fact we find the set of possible Clebschs to be very coarse-grained. As Clebschs are varied from one set to the next set, the value of a predicted quantity is found to jump by amounts typically much larger than its experimental error

bar, hence successful predictions are significant. A case where this is not true is the prediction (P3) for V_{cb}. In this case the experimental error bar is of order the interval generated by successive possible values of the Clebsch χ. A modest decrease in the experimental error bar will simply serve to choose one of the three presently allowed values of χ. Furthermore, it is not true that models could be found for any values of the masses and mixing parameters. For example, if the muon mass were doubled (or trebled) there would be no successful model.

2) Our scheme is based on a large number (9) of assumptions, suggesting that it is unlikely to be the one chosen by nature.

My response to this is mixed. It may well be that the "zeroth order" assumption is wrong and that this whole approach to fermion masses is incorrect. However, if perturbative supersymmetric GUTs are correct, I would argue that the set of 9 assumptions which we have made is the simplest that leads to models of such high predictivity. There are undoubtedly more complicated sets of assumptions, and obviously there are less predictive theories, but *without major additions to the basic tools it is unlikely that there is a simpler, more predictive model.* The success of the predictions gives me optimism that nature may have chosen the very simplest direction. Even if additional operators contribute to fermion masses, it may be that they only affect some of the predictions, or it may be that they only give perturbations to the predictions given here.

3) Our operator search was phenomenological in nature and yielded a set of operators which appears ad hoc, and which is far from providing a fundamental theory of flavor.

We have no apology for using experiment to guide our search. Nevertheless the resulting operators which describe the effective theory of flavor at the Planck scale have no particular form or symmetries which suggest they arise from a special and appealing theory of flavor. For example, our model 9 is specified by the following four operators of the effective theory at the GUT scale:

$$O_{33} = 16_3 \; 10 \; 16_3$$

$$O_{23} = 16_2 \; 10 \; \frac{45^2_{B-L}}{45^2_{\chi}} \; 16_3$$

$$O_{22} = 16_2 \; 10 \; \frac{45_{B-L} \; S}{45_X^2} \; 16_2$$

$$O_{12} = 16_1 \left(\frac{45_X}{M}\right)^3 \; 10 \left(\frac{45_X}{M}\right)^3 \; 16_2 \tag{23}$$

where S is an $SO(10)$ singlet fields with VEV of order M_G. The 45 dimensional adjoint representations are labeled by the direction of their vevs. We think it remarkable that all of flavor physics can be described by such a simple and predictive structure. However, it is not at all clear why nature chose this particular effective theory of flavor rather than one of a host of other similar possibilities. There is no flavor symmetry of the effective theory that can guarantee the form of (23). For example, any symmetry which allows 0_{23} will also allow an extra operator $0'_{23}$ in which the 10 is next to 16_3 rather than 16_2. It is possible that the answer to this puzzle lies one step deeper: perhaps the symmetries responsible for this structure become apparent only in the renormalizable theory above the scale M. If these flavor symmetries are spontaneously broken at M then they will not be manifest in the effective theory beneath M. Nevertheless, the effective theory will be greatly restricted in a way which reflects the original flavor symmetry. In the rest of this talk I will make a few general observations about the construction of the theory at scale M.

In weak interaction physics the structure of the non-renormalizable four-fermion operators in the effective theory beneath M_W appears somewhat ad hoc, and only becomes transparent and compelling when the renormalizable $SU(2) \times U(1)$ gauge interactions of the standard model are written down. However the analogy with the flavor interactions at the GUT scale is clearly not perfect: in the weak interaction case there are certain regularities amongst the four fermion operators, for example the universal size of the coefficient G_F. These regularities are due to the underlying gauge interaction. Such regularities are absent from (23), suggesting that the relevant interaction at scale M is a Yukawa interaction rather than a gauge interaction.

There is no guarantee that these Yukawa interactions at scale M will be simple and compelling. If they arise as a low energy remnant of a string theory, then they are still not fundamental and might reflect some complicated compactification scheme. Nevertheless, we believe that it is of interest to take this

next stage in the "bottom up" approach for several reasons

(a) It is important to study whether it is possible to obtain flavor structures, such as those of (23), from renormalizable theories.

(b) Such renormalizable theories may result in a simpler picture of flavor.

(c) In our scheme, it is only at the level of the renormalizable theory that the flavor symmetry can be studied.

(d) It is the theory at scale M which is of interest from the viewpoint of comparing with string inspired models. It is possible that the representation content and flavor symmetry of this theory will suggest a certain string compactification scheme.

(e) By explicitly integrating out particles of mass M, one might discover a natural mechanism for generating operators other than those which we have studied. This might suggest modifications to our predictions, or it might suggest an entirely new phenomenological analysis.

Rather than describing an ongoing attempt to construct a complete theory at scale M [15], I will illustrate a simple mechanism for generating the non renormalizable operators.

We envisage that, just beneath the Planck scale, the physics of flavor resides in renormalizable F type interactions between SO(10) 16 and $\overline{16}$ spinor multiplets and 1, 10 and 45 dimensional tensor multiplets. Only one of these interactions, $16_3\ 10\ 16_3$, has components containing only light fields and survives as a Yukawa coupling of the low energy effective theory. It is responsible for the masses of the heaviest generation 16_3. All the remaining interactions involve at least some fields which acquire masses at the GUT scale.

Suppose that when SO(10) is broken by the vev of 45_X, a 16 (ψ) and $\overline{16}$ ($\bar{\psi}$) multiplet become superheavy, with a mass of order V_{10}, via the interaction

$$\bar{\psi}\ 45_X\ \psi \tag{24}$$

Suppose that 16_2 has a coupling to ψ and 16_3 with $\bar{\psi}$:

$$16_2\ 10\ \psi\ +\ 16_3\ 45_{B-L}\ \bar{\psi} \tag{25}$$

Although these interactions do not survive as renormalizable couplings of the effective theory beneath V_{10}, when the ψ and $\bar{\psi}$ are integrated out they lead to

the non-renormalizable interaction

$$16_2 \; 10 \; \frac{45_{B-L}}{45_X} \; 16_3 \tag{26}$$

This is the basic mechanism we have in mind for generating the operators O_{ij}. While this operator is not precisely that of O_{23} in eqtn. (23), it is simple to alter the theory so that it is in fact O_{23} which is generated. This example shows how operators can be generated which have a particular ordering of fields. The operator of (26) has the 10 next to the 16_2, no similar operator was generated with the 10 next to the 16_3. This is because of the choice made in (25): the 10 appears in an interaction with 16_2 but not 16_3. The couplings of (24) and (25) can easily be guaranteed by a symmetry. However this symmetry is spontaneously broken at V_{10}, and so is not apparent in the effective theory beneath V_{10}. The supersymmetric non-renormalization theorems guarantee that $16_2 \frac{45_{B-L}}{45_X} 1016_3$ is not generated by radiative corrections.

Extraordinary effort is involved in measuring the 18 parameters of the standard model. Why bother? Two answers are frequently given:

- because they are there and they are fundamental.
- By measuring them more accurately, via a variety of methods, one could uncover inconsistencies in the standard model which would indicate new physics.

While both of these arguments have considerable merit, a third reason is also important:

- the accurate determination of the 18 parameters of the standard model may lead us to a deeper understanding of particle physics. The accurate determination of $\sin^2\theta$ has led to the possibility that we have already started down this path. Significant comparisons between theoretical and experimental values of quark and lepton masses and mixing angles may lead us towards a fundamental theory of fermion masses.

Acknowledgments

This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY90-21139.

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