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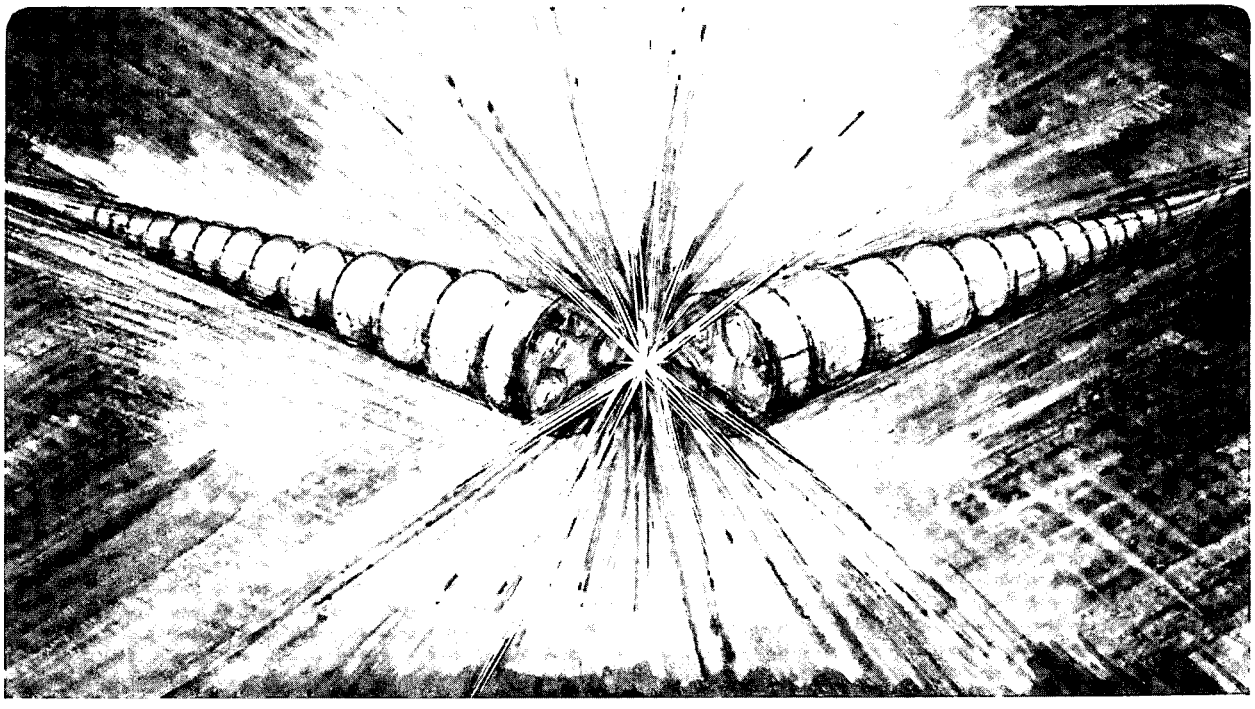

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COULOMB FIELD EFFECT ON PLASMA FOCUSING AND WAKE FIELD ACCELERATION*†

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Abstract

It is shown that the fields generated by relativistic electron (positron) bunches moving in overdense cold plasma have two components - wake and Coulomb. The existence of the Coulomb component is caused by the absence of the Debay screening of the charge moving in plasma with the velocity greater than the thermal velocity of the plasma electrons [1]. It is shown that at some conditions the contribution of the Coulomb component to focusing and self-focusing of the electron (positron) bunches, and wake field generation could be essential. This conclusion is valid for different descriptions of cold plasma - relativistic electron bunch system.

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1. Introduction

It is well known (see e.g. [1]) that the screening of the Coulomb field of the point charge moving in plasma with the velocity greater than the thermal velocity of plasma electrons $V_o > (KT_e/m)^{1/2}$ is absent. Therefore the Coulomb field of the relativistic electron (positron) bunches moving in cold neutral plasma must be taken into account in calculation of wake field generation and the focusing properties of the plasma lens.When the transverse dimensions of the bunch are infinite, the total charge (charge of bunch plus net charge of the plasma inside and behind the bunch) is zero, hence the effect of bunch Coulomb field is absent [2-4]. But for finite transverse dimensions of the bunch this compensation does not take place and Coulomb field of the bunch must be considered. Coulomb field component will emerge automatically from any properly written basic equation, describing the plasma-bunch system. The main goal of the present work is to consider such eqs. and to estimate the role of the Coulomb field component for plasma focusing and wake field generation. The focusing of charged particle beams moving in plasma is under discussion since the thirties [5,6]. The modern approaches are stimulated by the problems of the luminosity enhancement and beamstrahlung suppression in future linear colliders [7-17]. The idea to use longitudinal wake fields, excited by electrons or electron bunches moving in plasma to accelerate charged particles was set up in the early fifties [18,19]. Recently this idea has been further developed in numerous theoretical works [2-4,20-24] and experimentally tested at ANL [11,25,26] and KEK [15,27].

2. The basic equation and its solution

Let us consider for simplicity a flat electron bunch with horizontal dimensions σ_x which are much larger than the vertical dimensions σ_y, $\sigma_y \ll \sigma_x$ and an arbitrary longitudinal dimension σ_z. The electron bunch has a density $n_b \ll n_o$ (n_o is the equilibrium density of the plasma electrons) and is moving with a constant velocity $\vec{V}(0,0,V_z=V_o)$, $\beta \equiv V_o/c$ through the cold neutral plasma which will be described in the hydrodynamic approximation. In the considered case the components of the electric field $E_z \neq 0$, $|E_x| \ll |E_y|$, the magnetic field $B_z=0$, $|B_y| \ll |B_x|$ and it is possible also to neglect the x -dependence of all the components of the fields. All quantities will be considered as functions of y and $\tilde{z} = z - V_o t$ only, which corresponds to the stationary (steady state) case. We will also neglect plasma ion motion.

In order to make the mathematical formulation of the problem more transparent we assume that plasma electrons have a velocity $\vec{V}_e(0,0,V_z=V_e)$. It is possible to realize by applying a strong constant magnetic field $\vec{B}(0,0,B_o)$ along the direction of the bunch velocity. The strength of the constant magnetic field should suffice the condition that Larmour radius of the plasma electrons must be smaller than plasma wave length and/or bunch transverse dimensions. In this case the flow of plasma electrons out of the beam region is impeded and charge neutralization will diminish. Nevertheless, adopted model, as we shall show below, preserves qualitative mathematical features of more realistic three-dimensional equations [24,28] describing cold plasma - relativistic elec-

tron (positron) bunch system. Note also, that in experimental installations for wake field generation at ANL [11,25,26] and KEK [15,27] plasma chambers have had a constant axial solenoidal magnetic fields to confine the plasma column. Denoting the density of the plasma electrons by n_e one can obtain from the continuity eq. that for $\beta_e = 0$ $(n_e = n_0)$, at $\tilde{z} \to +\infty$

$$n_e = n_0 \beta / (\beta - \beta_e) \quad . \tag{1}$$

In order to obtain the fields generated by the electron bunch, let us introduce the vector potential $\vec{A}(0,0,A_z)$ and scalar potential φ, obeing the Lorentz condition, and adopt the boundary condition $B_x = 0$, $E_y = 0$, $\varphi = 0$, $A_z = 0$ at $\tilde{z} \to +\infty$

Then from the Maxwell eqs. it follows that

$$B_x = -\beta E_y \quad . \tag{2}$$

The solution of the relativistic eq. of motion for the plasma electrons, neglecting transverse motion, for $\rho_e = 0$, $\varphi = 0$ at $\tilde{z} \to +\infty$ is:

$$(1+\rho_e^2)^{1/2} - \beta \rho_e = 1 + \frac{e\varphi}{mc^2\gamma^2} \equiv \chi \, , \quad \rho_e = \frac{\beta_e}{(1-\beta_e^2)^{1/2}} \quad . \tag{3}$$

Using the Maxwell eqs., relations (2), (3) and introducing the dimensionless coordinates $y' = y/\sqrt{2}\sigma_y$ and $z' = \tilde{z}/\sqrt{2}\sigma_z$, we will have the following nonlinear eq. for the potential χ :

$$\frac{\gamma^2}{2K_p^2\sigma_y^2}\frac{\partial^2\chi}{\partial y'^2}+\frac{1}{2K_p^2\sigma_z^2}\frac{\partial^2\chi}{\partial z'^2}+\gamma^2\left(1-\frac{\beta\chi}{\sqrt{\chi^2-1/\gamma^2}}\right)=\frac{n_b}{n_0}, \quad (4)$$

where $K_p^2=\frac{\omega_p^2}{c^2}=\left(\frac{2\pi}{\lambda_p}\right)^2$, $\omega_p^2=\frac{4\pi n_0 e^2}{m}$ is the plasma frequency, $\gamma^2=(1-\beta^2)^{-1}$ is the Lorentz factor of the bunch electrons. Eq.(4) corresponds to the analogous one obtained earlier [29] for the cylindrical bunch. Note the difference between the first and the second terms in eq. (4): The Lorentz factor $\gamma=(1-\beta)^{-1/2}$ at the second derivative over z' is canceled out as a consequence of the stationarity condition.

For $e\varphi/mc^2\gamma^2\equiv\overset{*}{\varphi}/\gamma^2\ll 1$, $\overset{*}{\varphi}=e\varphi/mc^2<2$ it is possible to linearize eq. (4) and obtain the following resulting eq. for $\overset{*}{\varphi}$:

$$\frac{(\beta\gamma)^2}{2(K_p\sigma_y)^2}\frac{\partial^2\overset{*}{\varphi}}{\partial y'^2}+\frac{\beta^2}{2(K_p\sigma_z)^2}\frac{\partial^2\overset{*}{\varphi}}{\partial z'^2}+\overset{*}{\varphi}=(\beta\gamma^2)\frac{n_b}{n_0}, \quad (5)$$

which is the basic eq. in considered case.

Now consider the case when $n_b(y',z')=n_b=const\neq 0$ for $-1<y'<1$, $-1<z'<1$ and $n_b=0$ outside these intervals (the rigid fixed bunch approximation).

Performing the Fourier transformation over y' and denoting by

$$g(\lambda_y,z')=\frac{1}{2}\int_{-\infty}^{\infty}e^{i\lambda_y y'}\overset{*}{\varphi}(y',z')\,dy', \quad (6)$$

$$\overset{*}{\varphi}(y',z')=\frac{2}{\pi}\int_0^{\infty}\cos(\lambda_y y')\,g(\lambda_y,z')\,d\lambda_y \quad (6')$$

and taking into account that

$$\overset{*}{\varphi}(y' \to \pm\infty, z') = 0 \quad , \quad \frac{\partial \overset{*}{\varphi}(y' \to \pm\infty, z')}{\partial z'} = 0 \tag{7}$$

we arrive at the following eq. for $g(\lambda_y, z)$ (in what follows we drop out the superscripts on y and z)

$$\frac{\partial^2 g}{\partial z^2} + \frac{2(k_p \sigma_z)^2}{\beta^2}\left(1 - \frac{\beta^2\gamma^2\lambda_y^2}{2(k_p\sigma_y)^2}\right) g = 2(k_p\sigma_z)^2 \int_0^\infty \frac{n_b}{n_0} \cos\lambda_y y \, dy . \tag{8}$$

The fundamental solution of the homogeneous part of eq. (8) are periodic for $0 \leq \lambda_y < \lambda_c = \frac{\sqrt{2} k_p \sigma_y}{\beta\gamma}$ and nonperiodic for $\lambda_y \geq \lambda_c$. Note that nonperiodic solutions disappear for very wide bunches, when $\sqrt{2}\sigma_y \to \infty$ and the periodic solutions disappear for very narrow bunches, when $\sqrt{2}\sigma_y \to 0$. Using the fundamental solutions of the homogeneous part of eq. (8) and conventional technique it is possible to construct the solution of eq. (8) which will be the sum of both Fourier components - periodic and nonperiodic. It is impossible to drop out, for example, the nonperiodic solution, as it was done e.g. in [30], for the completeness property of the basic functions in (6), (6') eq. (8) will not be fulfilled, if in (6') the limits of integration are taken $(0, \lambda_c)$ which corresponds only to the periodic solutions. As it will be shown in the sequal, the nonperiodic components contribute to the Coulomb field of the bunch. It is known [1] that the screening of Coulomb field for the point charge is absent, when it moves in plasma with the velocity larger than the plasma electron thermal velocity $V_0 > (kT_e/m)$; hence the screening of the Coulomb field in the consi-

dered case is absent too.

The periodic components of the field contribute to the wake, built up inside and behind the bunch. The wake field does not exist in front of the driving bunch: this is due to the fact that in considered case the plasma wave has zero group velocity and does not propagate in space, therefore can not overtake the driving bunch [2].

Our next step is the choice of the boundary conditions for $g(\lambda_y, z)$ in eq. (8). We take $g=0, \partial g/\partial z = 0$ for $z \to +\infty$ and arbitrary $\lambda_y \geq 0$ for both of the solutions. For the periodic solution of eq. (8) which we shall denot by $g^w(\lambda_y, z)$ and will call the wake field component, it means that $g^w(\lambda_y, z) = 0$ for $1 \leq z < +\infty$, i.e. in front of the bunch. The nonperiodic solution (strictly speaking, the part of the solution of eq. (8) which is constructed from the nonperiodic solution of the homogeneous eq. (8)) which we will denote by $g^c(\lambda_y, z)$ and call the Coulomb component, is symmetric in z and y and disappears for $\sqrt{2}\,\sigma_y \to \infty$ $(\lambda_c \to \infty)$.

It is worth to note that only the continuous and finite solutions of eq. (8) are under consideration. It is also possible to construct a noncontinuous solution of eq. (8) which will be, for example, zero in front of the bunch for the nonperiodic case too, but in this case a surface charge of an unknown and not understandable origin should exist on the front of the bunch.

The existence of two types of solutions outlined above is the main difference between our approach and approaches presented previously. As we shall see in the following sections, under certain conditions this difference may be essential.

3. The potentials and fields

The components of the electric fields may be calculated from the scalar potential using the following relations:

$$E_z = -(1-\beta^2)\frac{mc^2}{\sqrt{2}\,\sigma_z e}\frac{\partial \overset{*}{\varphi}}{\partial z} = -\frac{1}{\sqrt{2}\,\sigma_z \gamma^2}\frac{mc^2}{e}\frac{\partial \overset{*}{\varphi}}{\partial z}, \tag{9}$$

$$E_y = \frac{1}{\sqrt{2}\,\sigma_y}\frac{mc^2}{e}\frac{\partial \overset{*}{\varphi}}{\partial y}. \tag{10}$$

The scalar potential $\overset{*}{\varphi}_1$ in front (subscript 1) of the bunch $1 \leq z \leq +\infty$, $(n_b = 0)$, is completely Coulombic (superscript c)

$$\overset{*}{\varphi}_1(y,z) = \overset{*}{\varphi}{}^c_1(y,z) =$$
$$= \frac{2}{\pi}(k_p\sigma_z)^2\gamma^2\int_{\lambda_c}^{\infty}\frac{\sin\lambda_y y\cos\lambda_y y}{\lambda_y\lambda^2}\left[e^{-\lambda(z-1)} - e^{-\lambda(z+1)}\right]d\lambda_y, \tag{11}$$

where

$$\lambda^2 \equiv \frac{2}{\beta^2}(k_p\sigma_z)^2\left(\frac{\lambda_y^2}{\lambda_c^2} - 1\right) \equiv -\lambda_z^2,$$
$$\lambda_c^2 \equiv \frac{2}{\beta^2\gamma^2}(k_p\sigma_y)^2,\quad \lambda_k^2 \equiv \frac{2}{\beta^2}(k_p\sigma_z)^2; \tag{12}$$

$$\frac{\lambda_z^2}{\lambda_k^2} + \frac{\lambda_y^2}{\lambda_c^2} = 1. \tag{13}$$

The potential inside $(-1 \leq z \leq 1, -1 \leq y \leq 1)$ and at the side $(-1 \leq z \leq 1, |y| > 1$ of the bunch is (subscript 0) the sum of the wake (superscrip w) and Coulomb components:

$$\overset{*}{\varphi}_0(y,z) = \overset{*}{\varphi}{}_0^{w}(y,z) + \overset{*}{\varphi}{}_0^{c}(y,z) =$$

$$= \frac{4}{\pi}(k_p\sigma_z)^2\frac{n_b}{n_0}\gamma^2\int_0^{\lambda_c}\frac{\sin\lambda_y\cos\lambda_y y[1-\cos\lambda_z(1-z)]d\lambda_y}{\lambda_y\lambda_z^2} - \tag{14}$$

$$-\frac{2}{\pi}(k_p\sigma_z)^2\frac{n_b}{n_0}\gamma^2\int_{\lambda_c}^{\infty}\frac{\sin\lambda_y\cos\lambda_y y}{\lambda_y\lambda^2}\left[2-e^{-\lambda(1+z)}-e^{-\lambda(1-z)}\right]d\lambda_y .$$

The potential behind (subscript 2) the bunch $(-\infty < z \leq -1)$ is given by

$$\overset{*}{\varphi}_2(y,z) = \overset{*}{\varphi}{}_2^{w}(y,z) + \overset{*}{\varphi}{}_2^{c}(y,z) =$$

$$= -\frac{8}{\pi}(k_p\sigma_z)^2\frac{n_b}{n_0}\gamma^2\int_0^{\lambda_c}\frac{\sin\lambda_y\cos\lambda_y y\,\sin\lambda_z z\,\sin\lambda_z\,d\lambda_y}{\lambda_y\lambda_z^2} - \tag{15}$$

$$-\frac{2}{\pi}(k_p\sigma_z)^2\frac{n_b}{n_0}\gamma^2\int_{\lambda_c}^{\infty}\frac{\sin\lambda_y\cos\lambda_y y}{\lambda_y\lambda^2}\left[e^{\lambda(z+1)}-e^{\lambda(z-1)}\right]d\lambda_y .$$

It is interesting to note that for the periodic solutions λ_y lies inside the ellipse (13) and for the nonperiodic solutions outside it. If we first perform the Fourier transformation in z, then the periodic solutions in y will have λ_z inside the ellipse (13) and the nonperiodic ones - outside it.

In general, it is impossible to calculate analytically the integrals in (11), (14), (15). But if we shall not persue on quantitative precision of the calculations, it is possible to perform an approximate evaluation of these integrals. One of the possible approaches is the following. Let us introduce a new variable $x = \lambda_y/\lambda_c$; the limits of integration in the periodic component will be from 0 to 1; let us divide this interval in two $(0, x_1)$ and $(x_1, 1)$, $x_1 \simeq 1/3$. In the first interval we approximate $1-x^2$ by 1, in the second interval we shall introduce the new variable $\tau^2 = 1-x^2$, $0 \leq \tau \leq \tau_1 = (1-x_1^2)^{1/2}$ and also approximate $1-\tau^2$ by 1.

The resulting integrals can be expressed through known functions.

The integrals corresponding to the nonperiodic components, after introducing the variable $\tau^2 = x^2 - 1$, $(x = \lambda y / \lambda_c)$, are divided into an integral from 0 to 1 and from 1 to ∞. In the first interval we approximate $\tau^2 + 1$ by 1, in the second interval by τ^2. Then the results can also be expressed through known functions.

For example, let us write down the expression for the potential inside the bunch, evaluated in the outlined way. For the wake field component we have

$$\overset{*}{\varphi}{}_0^{w} = \frac{1}{\pi}\frac{n_b}{n_0}(\beta\gamma)^2\left[1-\cos\frac{\sqrt{2}(k_p\sigma_z)}{\beta}(1-z)\right]\left[Si(\lambda_c(1-y)x_1)-Si(\lambda_c(1+y)x_1)\right]+$$

$$+\frac{1}{\pi}\frac{n_b}{n_0}(\beta\gamma)^2 Cin\left(\frac{\sqrt{2}(k_p\sigma_z)}{\beta}(1-z)\tau_1\right)\left[\sin\lambda_c(1-y)+\sin\lambda_c(1+y)\right]. \tag{16}$$

For the Coulomb component

$$\overset{*}{\varphi}{}_0^{c} = -\frac{1}{2\pi}\frac{n_b}{n_0}(\beta\gamma)^2\left[\sin\lambda_c(1-y)+\sin\lambda_c(1+y)\right]\left[E_1\left(\frac{\sqrt{2}(k_p\sigma_z)}{\beta}(1+z)+\right.\right.$$

$$\left.+E_1\left(\frac{\sqrt{2}(k_p\sigma_z)}{\beta}(1-z)\right)+\ln\frac{2}{\beta^2}(k_p\sigma_z)^2(1-z^2)+2\gamma\right]- \tag{17}$$

$$-\frac{1}{2\pi}\frac{n_b}{n_0}(\beta\gamma)^2\left[\sin\lambda_c(1-y)+\lambda_c(1-y)\cos\lambda_c(1-y)+\lambda_c^2(1-y)^2 Si\lambda_c(1-y)+\right.$$

$$\left.+\sin\lambda_c(1+y)+\lambda_c(1+y)\cos\lambda_c(1+y)+\lambda_c^2(1+y)^2 Si\lambda_c(1+y)\right]\times$$

$$\times\left[1-e^{-\sqrt{2}(k_p\sigma_z)/\beta}\cdot ch\frac{\sqrt{2}(k_p\sigma_z)}{\beta}z\right],$$

where $Cin\,z = -Ci\,z + \ln z + \gamma$, $Si\,z = si\,z + \pi/2$, $E_1(z)$ are the integral cosine, sine and the exponent functions respectively (see for e.g. [31]; the γ is Euler constant). The potential behind the bunch (15) is also expressed through the same functions. Note, that the condition for linearization of eq. (5) $\overset{*}{\varphi}/\gamma^2 \ll 1$ leads

to the fulfillment of the condition $n_b/n_0 \ll 1$, i.e. the plasma should be overdense.

In order to obtain the electric fields we use relations (9-10) and differentiate the potentials (11, 14, 15) under the integrals, then evaluate the obtained expressions in the same way outlined above we shall give the expression for the longitudinal field behind the bunch, $-\infty < z \leq -1$ which will be useful for the estimation of the accelerating wake fields generated by the charged particle bunches.

Denoting the longitudinal component of the electric field behind the bunch by E_{2z} and introducing $x = \lambda_y/\lambda_c$ we have

$$E_{2z} = E_{2z}^{w} + E_{2z}^{c},$$

$$\sqrt{2}\sigma_z E_{2z}^{w} = \frac{8}{\pi}(k_p\sigma_z)^2 \frac{n_b}{n_0}\frac{mc^2}{e}\int_0^{\lambda_c} \frac{\sin\lambda_y \cos\lambda_y y \cos\lambda_z z \sin\lambda_z \, d\lambda_y}{\lambda_y \lambda_z} =$$

$$= \frac{4\sqrt{2}\beta}{\pi}(k_p\sigma_z)^2 \frac{n_b}{n_0}\frac{mc^2}{e}\int_0^1 \frac{\sin\lambda_c x \cos\lambda_c x y \cos\frac{\sqrt{2}k_p\sigma_z(1-x^2)^{1/2}}{\beta} z \cos\frac{\sqrt{2}k_p\sigma_z}{\beta}(1-x^2)^{1/2} dx}{x(1-x^2)^{1/2}}, \tag{18}$$

$$\sqrt{2}\sigma_z E_{2z}^{c} = -\frac{2}{\pi}(k_p\sigma_z)^2 \frac{n_b}{n_0}\frac{mc^2}{e}\int_{\lambda_c}^{\infty} \frac{\sin\lambda_y \cos\lambda_y y}{\lambda_y \lambda}\left[e^{\lambda(z+1)} - e^{-\lambda(z-1)}\right] d\lambda_y =$$

$$= -\frac{\sqrt{2}\beta}{\pi}(k_p\sigma_z)\frac{n_b}{n_0}\frac{mc^2}{e}\int_1^{\infty} \frac{\sin\lambda_c x \cos\lambda_c x y}{x(x^2-1)^{1/2}}\left[e^{\frac{\sqrt{2}k_p\sigma_z}{\beta}(z-1)(x^2-1)^{1/2}} - e^{\frac{\sqrt{2}k_p\sigma_z}{\beta}(z+1)(x^2-1)^{1/2}}\right] dx. \tag{19}$$

For $\lambda_c \to 0$, $(\sigma_y \to 0)$ $E_{2z}^{w} \sim \lambda_c \to 0$, for $\lambda_c \to \infty$, $(\sigma_y \to \infty)$ $E_{2z}^{c} \to 0$; using the relation

$$\lim_{\lambda_c \to \infty} \frac{\sin\lambda_c x}{x} = \pi\delta(x)$$

and performing the integration in (18) over x we shall have

$$E_{2z}^{w} = 2\sqrt{2}\,\beta(k_p\sigma_z)\frac{n_e}{n_0}\frac{mc^2}{e}\cos\left(\frac{\sqrt{2}\,k_p\sigma_z}{\beta}\right)z\sin\left(\frac{\sqrt{2}\,k_p\sigma_z}{\beta}\right),$$

which coincides in the linear approximation with the result obtained previously [21] for the wake field from the bunch with infinite transverse dimensions. The estimation of the integrals in (18)-(19) by the above described method gives:

$$\sqrt{2}\,\sigma_z E_{2z}^{w} = \frac{2\sqrt{2}\,\beta}{\pi}(k_p\sigma_z)\frac{n_e}{n_0}\frac{mc^2}{e}\cos\frac{\sqrt{2}\,k_p\sigma_z}{\beta}z\sin\frac{\sqrt{2}\,k_p\sigma_z}{\beta}\Big[Si\,\lambda_c(1-y)x_1 + Si\,\lambda_c(1+y)x_1\Big] + \frac{4}{\pi}\beta^2\frac{n_e}{n_0}\frac{mc^2}{e}\left[\frac{\sin^2\frac{\sqrt{2}}{2\beta}(k_p\sigma_z)(1-z)\tau_1}{1-z} + \frac{\sin\frac{\sqrt{2}}{2\beta}(k_p\sigma_z)(1+z)\tau_1}{1+z}\right]\Big[\sin\lambda_c(1-y)+\sin\lambda_c(1+y)\Big] \tag{18'}$$

and

$$\sqrt{2}\,\sigma_z E_{2z}^{e} = -\frac{\beta^2}{\pi}\frac{n_e}{n_0}\frac{mc^2}{e}\Big[\sin\lambda_c(1-y)+\sin\lambda_c(1+y)\Big]\times \left[\frac{2}{z^2-1} + \frac{\exp\{-\frac{\sqrt{2}}{\beta}(k_p\sigma_z)(|z|+1)\}}{|z|+1} - \frac{\exp\{-\frac{\sqrt{2}}{\beta}(k_p\sigma_z)(|z|-1)}{|z|-1}\right] - \frac{\beta}{\pi}\frac{n_e}{n_0}(k_p\sigma_z)\Big[\sin\lambda_c(1-y)+\sin\lambda_c(1+y)-\lambda_c(1-y)Ci(\lambda_c(1-y)) - \lambda_c(1+y)Ci(\lambda_c(1+y))\Big]\left[e^{-\frac{\sqrt{2}k_p\sigma_z}{\beta}(|z|+1)} - e^{-\frac{\sqrt{2}k_p\sigma_z}{\beta}(|z|-1)}\right]. \tag{19'}$$

When $\frac{\sqrt{2}(k_p\sigma_z)}{\beta}(|z|-1) \gg 1$ which means that the considered point is far from the rear end of the bunch, the expressions (18') and (19') are simplified

$$\sqrt{2}\,\sigma_z E_{2z}^{w} \simeq \frac{2\sqrt{2}\,\beta}{\pi}(k_p\sigma_z)\frac{n_b}{n_0}\frac{mc^2}{e}\cos\frac{\sqrt{2}\,k_p\sigma_z}{\beta}z\sin\frac{\sqrt{2}\,k_p\sigma_z}{\beta}\times$$

$$\times\left[Si\,\lambda_c(1-y)x_1 + Si\,\lambda_c(1+y)x_1\right], \qquad (18'')$$

$$\sqrt{2}\,\sigma_z E_{2z}^{c} \simeq -\frac{2\beta^2}{\pi}\frac{n_b}{n_0}\frac{mc^2}{e}\left[\sin\lambda_c(1-y)+\sin\lambda_c(1+y)\right]\frac{1}{|z|^2}. \qquad (19'')$$

For $\lambda_c < \pi/2$ and $y = 0$ from (18''), (19''), we have for the ratio

$$\frac{|E_{2z}^{w}|}{|E_{2z}^{c}|} \gtrsim \sqrt{2}\,(k_p\sigma_z)|z|^2, \quad \beta = 1. \qquad (20)$$

and when $|z|^2 \gg \left[\sqrt{2}(k_p\sigma_z)\right]^{-1}$ the influence of the Coulomb component of the field is negligible. In the KEK experiment [14, 15] $\lambda_c \simeq 10^{-2} - 10^{-3}$, $\lambda_k \simeq 10^{-1} - 10^{-2}$ so, in order to escape the influence of the Coulomb component, the witness bunch should be placed behind the driving one on distances $z_d \gg$ 1.5 - 15 cm. The bunch separation in the KEK experiment [15, 27] was 10.5 cm, so the influence of the Coulomb component may be present in some cases. In the ANL experiment [26] $\lambda_c \simeq (3\text{-}7)\cdot 10^{-2}$ and $\lambda_k = 4.2$ so $z_d \gg$ $\gg$ 0.2 cm; the delay time between the driving and witness bunches was 0.2 - 0.5 nsec in this experiment and at corresponding distances (6-10 cm) the influence of the Coulomb component is negligible. In both of the experiments [14, 26] the bunches were cylindrical with a nonuniform charge distribution inside them, hence the presented estimates may have only a qualitative meaning.

Let us note in conclusion of this section that the longitudinal component of the field in front of the bunch (denoted by sub-

script 1) is

$$E_{1z} = E_{1z}^{c} = -E_{2z}^{c}, \quad (E_{1z}^{w} = 0),$$

4. Self-focusing and focusing in the presence of the Coulomb field

Using (1) it is possible to find the total charge and the current densities of the plasma-electron bunch system in the model considered:

$$\rho = e(n_0 - n_e - n_b) = -\frac{e}{\beta - \beta_e}\left[(n_0 - n_b)\beta_e + n_b\beta\right], \tag{21}$$

$$j = -e(n_e v_e + n_b v_0) = -\frac{e v_0}{\beta - \beta_e}\left[(n_0 - n_b)\beta_e + n_b\beta\right] = v_0\rho. \tag{22}$$

Hence, in the considered case from local charge neutralization ($\rho = 0$) follows local neutralization of the current and vice versa.

In the linear approximation from (21) and (3) it follows that

$$\rho \simeq -\frac{e}{\beta}(n_0\beta_e + n_b\beta), \tag{21'}$$

$$\beta_e \simeq \beta_e \simeq -\overset{*}{\varphi}/\beta\gamma^2. \tag{3'}$$

So the condition for charge neutralization or positive charge excess is $\rho \geq 0$, i.e.,

$$\beta_e \leq -\frac{n_b}{n_0}\beta, \quad \overset{*}{\varphi} \geq \frac{n_b}{n_0}\beta^2\gamma^2, \quad \frac{\overset{*}{\varphi}}{\frac{n_b}{n_0}\beta^2\gamma^2} \geq 1. \tag{23}$$

(Note that $\overset{*}{\varphi}/\gamma^2 \ll 1$ in adopted linear approximation).

From (11), (14), (15) it is possible to find out the domain of the values of parameters involved when the condition (23) will be fulfilled. For the values of parameters similar to that of the future linear colliders like NLC or CLIC $\lambda_c = \sqrt{2} k_p \sigma_y / \beta\gamma$ is very small. For CLIC $\lambda_c = 1.0\ 10^{-6}$, for NLC $\lambda \simeq 2\ 10^{-6}$, for PB-TLC discussed in [13] $\lambda_c = 3\ 10^{-6}$. (We also take the plasma densities used in [13] which are high enough). For the interesting case of self-focusing of electron bunches the expressions (16), (17) for the potentials inside the bunches should be used. They can be expanded in series leaving the first terms in $O(\lambda_c)$ when $|y| < 1$. In the case of short bunches $\sqrt{2}(k_p \sigma_z) \ll 1$, $(\beta = 1)$, using the series expansion for $E_1(z)$ and $Ci_n(z)$ [31] it is possible to show that condition (23) is not fulfilled. The same is true also for long $\sqrt{2}(k_p \sigma_z) \gg 1$ bunches (we use in this case the asymptotic expansions of $E_1(z)$ and $Ci_n(z)$ [31] practically for all points inside the bunches excluding only the rear end of the bunch where $\sqrt{2}(k_p \sigma_z)(1+z) \ll 1$ $(z \to -1)$ and if $\frac{1}{\pi} \lambda_c \ln 2\sqrt{2}(k_p \sigma_z) \gtrsim 1$. Generally speaking, the condition (23) may be fulfilled for the interior points of the bunch, when $\lambda_c = \sqrt{2} k_p \sigma_y / \beta\gamma \gtrsim 0.5$ and $\sqrt{2}(k_p \sigma_z)(1+z) \simeq$ 5-10. From this analysis it follows that for the values of the parameters which are similar to that of NLC or CLIC, even for large values of the plasma density, λ_c, which is inversely proportional to the Lorentz factor, is so small that the compensation condition (23) is never fulfilled. This is due to the presence of the defocusing Coulomb field, which is not screened.

When condition (23) is fulfilled, it is necessary to estimate the quantity and the space behaviour of the focusing field $E_{oy} = E_{oy}^{w} + E_{oy}^{c}$. The focusing force for the bunch electrons is

$$F_y = -e\left(E_{oy} + \frac{v_o}{c} B_{ox}\right) = -\frac{eE_{oy}}{\gamma^2} . \qquad (24)$$

The Coulomb component of the field E^c_{oy} is always negative inside the bunch $|y| \leq 1$, so it has a defocusing character. The wake field component inside the bunch has focusing properties. For $\lambda_c < \pi/2$, E_{oy} is practically (within 10% accuracy) linear in , so linear focusing takes place, but the z -dependence of the focusing field is complicated, E^w_{oy} =0 at the front of the bunches (z =1) and the defocusing field is around its maximum. For short bunches E^w_{oy} quadratically depends on z , for long bunches the z -dependence is more complicated and has periodic component.

In the intermediate case when, for example, λ_c=0.5 and λ_k= 30 the attainuable focusing field gradient, when $\sqrt{2}\,\sigma_y \equiv b/2$ = 0.005 cm, $\sqrt{2}\,\sigma_z \equiv d/2$ =0.01 cm, γ =30, n_b/n_o =10^{-1} and n_o= 2.7 10^{18} cm , is

$$G = \frac{F_y}{ey} \simeq \frac{8}{\pi}\beta^2 \frac{n_b}{n_o}\frac{mc^2}{eb^2} = 2\cdot 10^6 \frac{GS}{cm} = 2\,\frac{MGS}{cm}$$

When n_o =10^{15} cm^{-3}, n_b =10^{14} cm^{-3}, γ =6, d =1 cm, b =0.1 cm, G =2 $10^4 \frac{GS}{cm}$ =20 $\frac{KGS}{cm}$.

For the case of a positron driving bunch it is necessary to substitute $n_b \to -n_b$ in all the formulae, in particular in (21') and (23). Then the condition of negative charge excess,i.e., the condition of self-focusing of the positron bunch, will be

$$\varphi^* / \frac{n_b}{n_o}(\beta\gamma)^2 < -1 . \qquad (23')$$

Taking into account that n_b enters in the expressions for the potential $\overset{*}{\varphi}$ as a simple factor and we need to change $n_b \to -n_b$, it is easier to show that the conditions for the fulfillment of the unequality (23') are the same as for (23).

Behind the bunch $n_b = 0$, so from (21') and (3') follows the condition for the existence of the positive charge excess in the plasma

$$\beta_e < 0 \ , \quad \overset{*}{\varphi}_2 > 0 \ , \tag{25}$$

The eq. (25) is the condition for the focusing of electrons, following the driving electron bunches with a velocity $V_z = V'$. The condition for focusing of positrons, following the driving electron bunches, will be the existence of an excess of negative charge in the plasma which is

$$\beta_e > 0 \ , \quad \overset{*}{\varphi}_2 < 0 \tag{26}$$

so the conditions (25) and (26) are opposite to each other and in this sense complimentary. The focusing force is

$$F_y = \mp e(1 - \beta'\beta)E_y \ , \quad \beta' = V'/c \ , \tag{27}$$

where the upper sign is for electrons and the lower one for positrons. If $\beta' \ll \beta$, the focusing force, being uncompensated by the magnetic field, could be large. The force (27) is acting also (with the upper sign) for the plasma electrons. The potential $\overset{*}{\varphi}_2$ is given by (15) and the field E_{2z} by (10). For the estimate of these quantities we shall use the same technique as described above.

Let us consider the case of a high energy driving bunch ($\gamma \gg 1$) and a not extremely dense plasma when $\lambda_c = \sqrt{2} k_p \sigma_y / \beta\gamma \lesssim \frac{\pi}{2}$. Then for the case when the distances from the rear end of the driving bunch are large

$$|z| \gg \left[\sqrt{2} k_p \sigma_y\right]^{-1} \tag{28}$$

the effect of the Coulomb component of the potential is smaller than the effect from the wake field component and for the long bunches $\sqrt{2}(k_p \sigma_z) \gg 1$

$$\overset{*}{\varphi}_2 = \overset{*}{\varphi}{}_2^{w} + \overset{*}{\varphi}{}_2^{c} \approx$$

$$\approx \frac{1}{\pi} \beta^2 \gamma^2 \frac{n_b}{n_0} \left\{ 4\left(\lambda_c - \frac{1}{6}\lambda_c^3 y^2\right) \sin\frac{\sqrt{2}(k_p\sigma_z)}{\beta} |z| \sin\frac{\sqrt{2}(k_p\sigma_z)}{\beta} + \right.$$
$$\left. + \left(\lambda_c - \frac{1}{2}\lambda_c^3 y^2\right) \ln\frac{|z|+1}{|z|-1} \right\}. \tag{29}$$

Hence, the condition (25) will be fulfilled in the regions where

$$\sin\frac{\sqrt{2}(k_p\sigma_z)}{\beta} |z| \sin\frac{\sqrt{2}(k_p\sigma_z)}{\beta} > 0 \tag{30}$$

or, for the focusing of the positrons (condition (26)), when

$$\sin\frac{\sqrt{2}(k_p\sigma_z)}{\beta} |z| \sin\frac{\sqrt{2}(k_p\sigma_z)}{\beta} < 0 . \tag{30'}$$

The focusing field gradient in both cases is

$$G = \frac{E_y}{y} \simeq \frac{16}{3} \frac{n_b}{n_0} \frac{mc^2}{eb^2} \gamma^2 \lambda_c^3 = \frac{2}{3} \frac{n_b}{n_0} \frac{mc^2 b}{e\gamma} . \tag{31}$$

Let us take for estimates

$n_0 = 6.0\ 10^{15}$, $n_b = 6.0\ 10^{13}$, $b = 0.28$ cm, $\gamma = 42$, $\lambda_c \approx 0.5$, and

$$G = 2{,}56 \cdot 10^5 \frac{Gs}{cm} \approx 0.26 \frac{MGs}{cm}$$

For short electron bunches when $\sqrt{2}(k_p \delta_z) \ll 1$ for far distanes $|z| \gg [\sqrt{2}(k_p \delta_z)]^{-1}$ the result coincides with (29)-(31). For distances near the rear end of the driving bunch the difference between short and long bunches is essential. For short bunches $\sqrt{2}(k_p \delta_z) \ll$ $\ll 1$ and $\sqrt{2}(k_p \delta_z)(|z|-1) \ll 1$ the force (27) is small and defocusing for electrons (and focusing for positrons) and for long bunches, when $\sqrt{2}(k_p \delta_z) \gg 1$ and $\sqrt{2}(k_p \delta_z)(|z|-1) \gg 1$

$$\overset{*}{\varphi}_2 \approx \frac{2}{\pi} \frac{n_b}{n_0} (\beta\gamma)^2 \left(\lambda_c - \frac{1}{2}\lambda_c^3 y^2\right) \ln\left(\frac{\sqrt{2}(k_p \delta_z)}{\beta^2}(|z|+1)\tau_1\right) > 0 \qquad (32)$$

i.e., the focusing of electrons takes place. It is natural, if we remember that for long bunches the field at the rear end of the bunch has self-focusing property, as it was shown in the preceeding section and potentials (and fields) are continuous. The focusing field gradient in the case is ($\beta = 1$):

$$G = \frac{E_y}{y} = \frac{8}{\pi} \frac{n_b}{n_0} \lambda_c^3 \ln\left(2\sqrt{2}(k_p \delta_z)\right) \frac{mc^2}{eb^2} ; \qquad (33)$$

When $n_0 = 3\ 10^{15} cm^{-3}$, $n_b = 3\ 10^{14}\ cm^{-3}$, $\gamma = 50$, $b = 0.2$ cm, $\lambda_c = 0.2$, $d = 0.7$ cm, $\lambda_z = 500$, $G = 10^4 \frac{Gs}{cm} = 10 \frac{kGs}{cm}$.

5. Coloumb component existence in other descriptions of the cold plasma - relativistic electron bunch system

As it is seen from the preceeding sections the Coulomb component of the field exists as a solution of the basic eqs.(5)(which is eq. of Helmholtz type) and at certain conditions gives an essential contribution to wake fields and focusing forces. The simplification inherent to adopted model of the plasma-bunch system,i.e. $V_y=0$ is not crucialy essential for the very existence of the Coulomb componentof the field. As it was mentioned above the physical reason for its existence is the absence of the screening of the charge moving in plasma with the velocity greater than the thermal velocity of plasma electrons [1].The description in linear approximation of the problem in question by means of a Helmholtz type equation for the relevant quantity (e.g. potential $\overset{*}{\varphi}$ in our case, (5)) is the formal indication of Coulomb component existence at certain, innate for considered problem boundary (initial) conditions.. Performing Fourier transformation over one variable (y in our case(see section 2), we came to second order non homogeneous ordinary differential equation for Fourier transformants as functions of other variable. This eq. automatically has both non-periodic (Coulomb) and periodic (wake wave) solutions.

Having this in mind, let us consider the alternative approaches to the description of cold plasma-bunch system, presented in [24, 28].

Following the ideas of the work [24] we again, as in the previous section, consider the flat electron bunch moving in cold plasma with the constant velocity $\vec{V}_0(0, 0, V_0)$ with the horizontal dimensions σ_x much larger than vertical dimensions σ_y .

Plasma electrons will have the velocity $\vec{V}_e(0, V_{ey}, V_{ez})$, electric field is $\vec{E}(0, E_y, E_z)$ and magnetic field is $\vec{B}(B, 0, 0)$. All quantities are functions of y and $\tilde{z} = z - V_0 t$. (Note that in contrast to [24] we take $V_0 \neq c$, $V_0 < c$). Using relativistic eqs. of motion of plasma electrons, continuity eq. and Maxwell eqs., introducing dimensionless variables

$$t = \sqrt{\frac{m}{4\pi n_0 e^2}}\, t', \quad y = \sqrt{\frac{mc^2}{4\pi n_0 e^2}}\, y', \quad \tilde{z} = \sqrt{\frac{mc^2}{4\pi n_0 e^2}}\, z',$$

$$\vec{p} = mc\vec{p}\,', \quad \vec{V} = c\vec{V}\,', \tag{34}$$

$$n = n_0 n', \quad \vec{j} = e n_0 c \vec{j}\,', \quad \vec{E} = \sqrt{4\pi n_0 m c^2}\,\vec{E}\,', \quad \vec{B} = \sqrt{4\pi n_0 m c^2}\,\vec{B}\,'$$

and introducing new variables by modified Breizman-Tajima-Fisher-Chebotarev transformation

$$V_z = \frac{V_z'}{\beta - V_z'}, \quad V_y = \frac{V_y'}{\beta - V_z'}, \quad N\beta = n'(\beta - V_z'), \quad \beta = \frac{V_0}{c} \tag{35}$$

it is possible to obtain the focusing system of nonlinear equations, describing our problem:

$$-\frac{\partial V_z}{\partial z} + V_y \frac{\partial V_z}{\partial y} = \frac{W^{1/2}}{\beta^2}\left[-E_z\left(1 + 2V_z + \frac{V_z^2}{\gamma^2}\right) + \beta^2 V_z V_y \left(E_y + \frac{1}{\beta}B\right) + \beta V_y B\right], \tag{36}$$

$$-\frac{\partial V_y}{\partial z} + V_y \frac{\partial V_y}{\partial y} = \frac{W^{1/2}}{\beta^2}\left[-E_y\left(1 + V_z - \beta^2 V_y^2\right) - E_z V_y\left(1 + \frac{V_z}{\gamma}\right) - \beta\left(V_z^2 - V_y^2\right)B\right], \tag{37}$$

$$-\frac{\partial}{\partial z} N + \frac{\partial}{\partial y}(N V_y) = 0, \tag{38}$$

$$\frac{\partial}{\partial z}(B+\beta E_y) = -\beta N V_y \,, \tag{39}$$

$$\frac{\partial}{\partial z}(\beta B+E_y) = \frac{\partial E_z}{\partial y} \,, \tag{40}$$

$$\frac{\partial B}{\partial y} = \beta N V_z + \beta \frac{n_b}{n_0} + \beta \frac{\partial E_z}{\partial z} \,, \tag{41}$$

where $W \equiv (1+2V_z + V_z^2/\gamma^2 - \beta^2 V_y^2)$ and subscripts "prime" are omitted. When $\beta \to 1$, $\gamma^2 = (1-\beta^2)^{-1} \to \infty$ eqs. (36)-(41) coincide with the subsequent eqs. in [24], with the obvious difference in signs, which comes from the different definitions (34).

In linear approximation, when $n_b/n_0 \ll 1$ (overdense plasma), N is put equal to 1 in [24], which means that n_e is determined by eq. (1) of the present work. It also means (38) that the change of the V_y with y is small

$$\left|\frac{\partial V_y}{\partial y}\right| \ll 1 \qquad \text{or} \qquad V_y \ll \frac{V_0}{c} b\omega_p = V_0 (k_p b). \tag{42}$$

The system (36)-(41) in linear approximation defined in such a way takes the form:

$$\frac{\partial V_z}{\partial z} = \frac{1}{\beta^2} E_z \,, \tag{36'}$$

$$\frac{\partial V_y}{\partial z} = \frac{1}{\beta^2} E_y \,, \tag{37'}$$

$$\frac{\partial}{\partial z}(B+\beta E_y) = -\beta V_y \,, \tag{39'}$$

$$\frac{\partial}{\partial z}(\beta B+E_y) = \frac{\partial E_z}{\partial y} \,, \tag{40'}$$

$$\frac{\partial B}{\partial y} = \beta V_z + \beta \frac{n_b}{n_0} + \beta \frac{\partial E_z}{\partial z} \,. \tag{41'}$$

From (39')-(40') it follows

$$\frac{1}{\gamma^2}\frac{\partial B}{\partial z} = -\beta V_y - \beta \frac{\partial E_z}{\partial y} \,. \tag{43}$$

Taking the partial derivative on from (43) and using condition (42) we have

$$\frac{1}{\gamma^2}\frac{\partial^2 B}{\partial y \partial z} \approx -\beta \frac{\partial^2 E_z}{\partial y^2} \,. \tag{44}$$

Taking the partial derivative on z from (41'), using (36') we get

$$(\beta\gamma)^2 \frac{\partial^2 E}{\partial y^2} + \beta^2 \frac{\partial^2 E_z}{\partial z^2} + E_z = -\beta \frac{\partial}{\partial z}\left(\frac{n_b}{n_0}\right). \tag{45}$$

Introducing the "potential" $\Phi(y, z)$ (Φ is defined up to additional arbitrary function of y) by

$$E_z = -\frac{\partial \Phi}{\partial z} \tag{46}$$

we come to the eq. (5) for $\Phi(y,z)$.

Thus all consequencies of eq. (5) are valid for the model considered in linear approximation in [24], in particular the main result of the previous section, concerning to the existence and sometimes essential role of the Coulomb component. The expressions for E_y and B components of the electromagnetic field will be in general different from those obtained in our model, because they must be found from eqs. (37'),(39'),(40').

Let us consider now another formulation of the problem arising from the work [28].

Assuming the validity of relations

$$\mathrm{rot}\left(\vec{p} - \frac{e}{c}\vec{A}\right) = 0 \quad , \quad \mathrm{rot}\,\mathrm{rot}\left(\vec{p} - \frac{e}{c}\vec{A}\right) = 0 \tag{48}$$

adopted in [28] for laser wake field acceleration for the description of cold plasma - relativistic electron bunch system, it is possible to obtain the following eq. for momenta $\vec{p}$ of the plasma electrons (compare with [28]):

$$\left(\vec{\nabla}\vec{\nabla} - \nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{p} + \frac{\vec{p}}{\gamma}\left(\frac{\omega_p^2}{c^2} + \frac{1}{mc^2}\frac{\partial}{\partial t}\vec{\nabla}\vec{p} + \nabla^2(\gamma)\right) + m\frac{\partial}{\partial t}\vec{\nabla}(\gamma) = -\frac{\omega_p^2}{c^2}\frac{n_e}{n_0}\frac{\vec{p}_0}{\gamma_0}, \tag{49}$$

where $\gamma = \sqrt{1 + \frac{p^2}{m^2c^2}}$ is Lorentz factor for plasma electrons, γ_0 - bunch Lorentz factor, p_0 - electron bunch momenta. Considering, as in previous sections, the flat bunch, introducing dimensionless variables as in (4), assuming that $\vec{p} = \vec{P}/mc$ are the functions of y' and $z' - v_0 t'$ only (in what follows we omit the super-

script "prime") we come in linear approximation to the following set of differential eqs. for quantities in question ρ_y, ρ_z:

$$\frac{\partial^2 \rho_z}{\partial y^2} - \beta_0^2 \frac{\partial^2 \rho_z}{\partial z^2} - \frac{\partial^2 \rho_y}{\partial y \partial z} - \beta_0^2 k_p^2 \rho_z = -\beta_0^3 \frac{n_b}{n_0},$$
$$\frac{1}{\gamma_0^2} \frac{\partial^2 \rho_y}{\partial y^2} - \beta_0^2 k_p^2 \rho_y - \frac{\partial^2 \rho_z}{\partial y \partial z} = 0. \qquad (50)$$

In (50) n_b is constant inside the bunch and equals to zero outside it (rigid bunch approximation), $\beta_0 = V_0/c$, $\gamma_0 = (1-\beta_0^2)^{-1/2}$, where V_0 is the bunch velocity. Perfoming Fourier transformation on y and z in eqs. (50) we come to the set of the linear algebraic eqs. for transformants, which have the solution. Performing inverse Fourier transformation we can use the Caushy theorem and take the residues at the zeros of the determinant of the algebraic system of eqs. for transformants. It is very easy to show that the determinant has two real and two imaginary roots.One of the imaginary roots will be inside the integration contour and the residue will give the nonperiodic solution for ρ_z and ρ_y as functions of z.

It is necessary to mention, however, that the validity of the basic assumptions (48) for the problem of the rigid electron bunch moving in plasma needs some additional consideration (see, e.g. [32]).

The results of this section show, as it was expected, that the very existence of the nonperiodic (Coulomb) component of the field, is independent from the way of plasma-electron bunch system description. As it was shown in previous section at some condit-

ions this component must be taken into account in the consideration of wake field generation and plasma focusing. At the nonlinear treatment, when linear approach sometimes is taken as a zero approximation, the role of the Coulomb field component may be even more essential.

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